GRADE 7 MATH TEACHING GUIDE

Lesson 8: Multiplication and Division of Rational Numbers **Time: 2 hours**

Prerequisite Concepts: addition and subtraction of rational numbers, expressing rational numbers in different forms

Objectives:

In this lesson, you are expected to:

1. Multiply rational numbers;
2. Divide rational numbers;
3. Solve problems involving multiplication and division of rational numbers.

NOTE TO THE TEACHER:

This lesson reinforces what they learned in elementary mathematics. It starts with the visualization of the multiplication and division of rational numbers using the area model. Use different, yet appropriate shapes when illustrating using the area model. The opening activity encourages the students to use a model or drawing to help them solve the problem. Although, some students will insist they know the answer, it is a whole different skill to teach them to visualize using the area model.

Lesson Proper

A. Models for the Multiplication and Division

I. Activity:

Make a model or a drawing to show the following:

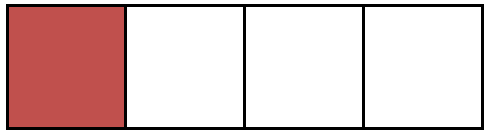
1. A pizza is divided into 10 equal slices. Kim ate $\frac{3}{5}$ of $\frac{1}{2}$ of the pizza. What part of the whole pizza did Kim eat?
2. Miriam made 8 chicken sandwiches for some street children. She cut up each sandwich into 4 triangular pieces. If a child can only take a piece, how many children can she feed?

Can you make a model or a drawing to help you solve these problems?

A model that we can use to illustrate multiplication and division of rational numbers is the area model.

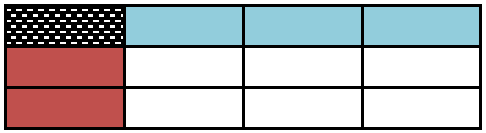
What is $\frac{1}{4} \times \frac{1}{3}$? Suppose we have one bar of chocolate represent 1 unit.

Divide the bar first into 4 equal parts vertically. One part of it is $\frac{1}{4}$



Then, divide each fourth into 3 equal parts, this time horizontally to make the divisions easy to see. One part of the horizontal division is $\frac{1}{3}$.

$\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$



There will be 12 equal-sized pieces and one piece is $\frac{1}{12}$. But, that one piece is $\frac{1}{3}$ of $\frac{1}{4}$, which we know from elementary mathematics to mean $\frac{1}{3} \times \frac{1}{4}$.

NOTE TO THE TEACHER

The area model is also used in visualizing division of rational numbers in fraction form. This can be helpful for some students. For others, the model may not be easily understandable. But, do not give up. It is a matter of getting used to. In fact, this is a good way to help them use a non-algorithmic approach to dividing rational numbers in fraction form: by using the idea that division is the reverse of multiplication.

What about a model for division of rational numbers?

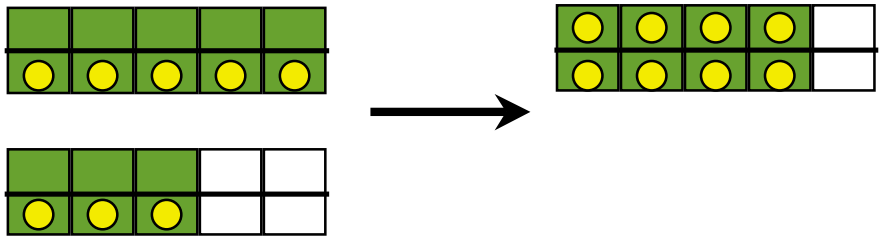
Take the division problem: $\frac{4}{5} \div \frac{1}{2}$. One unit is divided into 5 equal parts and 4 of them are shaded.

Each of the 4 parts now will be cut up in halves

Since there are 2 divisions per part (i.e. $\frac{1}{5}$) and there are 4 of them (i.e. $\frac{4}{5}$), then there will be 8 pieces out of 5 original pieces or $\frac{4}{5} \div \frac{1}{2} = \frac{8}{5}$.

NOTE TO THE TEACHER

The solution to the problem $\frac{4}{5} \div \frac{1}{2}$ can be easily checked using the area model as well. Ask the students, what is $\frac{1}{2} \times \frac{8}{5}$. The answer can be obtained using the area model



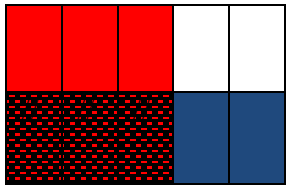
$\frac{1}{2} \times \frac{8}{5} = \frac{4}{5}$

NOTE TO THE TEACHER:
It is important for you to go over the answers of your students to the questions posed in the opening activity in order to process what they have learned for themselves. Encourage discussions and exchanges in the class. Do not leave questions unanswered.

II. Questions to Ponder (Post-Activity Discussion)

Let us answer the questions posed in the opening activity.

1. A pizza is divided into 10 equal slices. Kim ate $\frac{3}{5}$ of $\frac{1}{2}$ of the pizza. What part of the whole pizza did Kim eat?

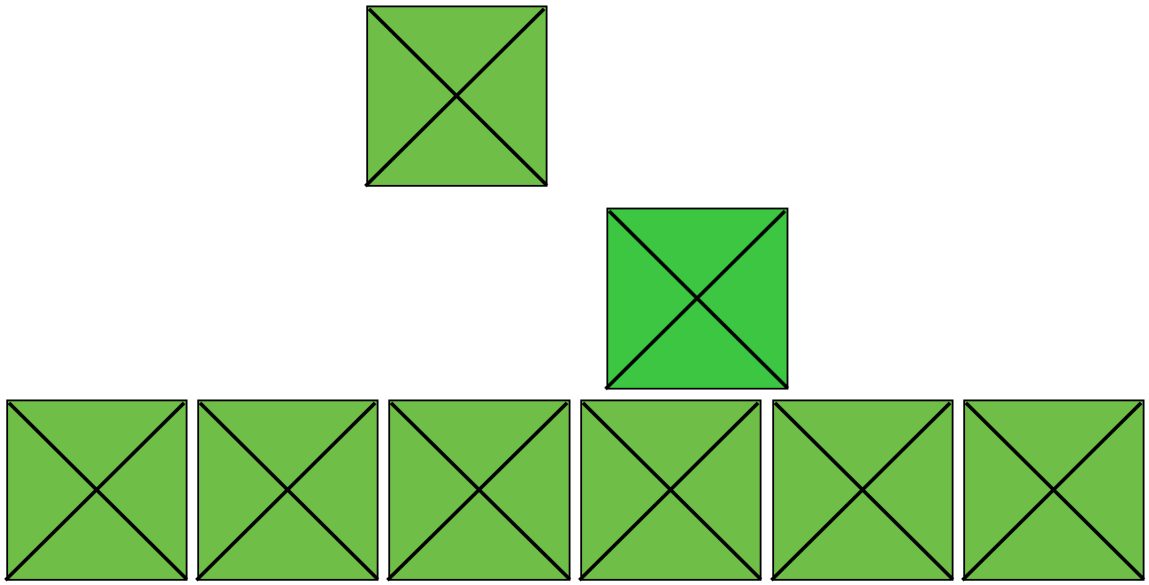


$$\frac{3}{5} \times \frac{1}{2} = \frac{3}{10}$$

Kim ate $\frac{3}{10}$ of the whole pizza.

NOTE TO THE TEACHER
The area model works for multiplication of rational numbers because the operation is binary, meaning it is an operation done on two elements. The area model allows for at most "shading" or "slicing" in two directions.

2. Miriam made 8 chicken sandwiches for some street children. She cut up each sandwich into 4 triangular pieces. If a child can only take a piece, how many children can she feed?



The equation is $8 \div \frac{1}{4} = 32$. Since there are 4 fourths in one sandwich, there will be 4 x 8 = 32 triangular pieces and hence, 32 children will be fed.

How then can you multiply or divide rational numbers without using models or drawings?

NOTE TO THE TEACHER:
Below are important rules or procedures that the students must remember. From here on, be consistent in your rules so that your students will not be confused. Give plenty of examples.

Important Rules to Remember
The following are rules that you must remember. From here on, the symbols to be used for multiplication are any of the following: •, x, ×, or *x*.

1. To multiply rational numbers in fraction form simply multiply the numerators and multiply the denominators.

In symbol, $\frac{a}{b} \bullet \frac{c}{d} = \frac{ac}{bd}$ where: b and d are NOT equal to zero, ($b \neq 0; d \neq 0$)

2. To divide rational numbers in fraction form, you take the reciprocal of the second fraction (called the divisor) and multiply it by the first fraction.

In symbol, $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \bullet \frac{d}{c} = \frac{ad}{bc}$ where: b, c, and d are NOT equal to zero.

Example:
Multiply the following and write your answer in simplest form

a. $\frac{3}{7} \bullet \frac{2}{5}$

$\frac{3}{7} \bullet \frac{2}{5} = \frac{3 \times 2}{7 \times 5} = \frac{6}{35}$

b. $4\frac{1}{3} \bullet 2\frac{1}{4}$

$\frac{13}{3} \bullet \frac{9}{4} = \frac{13 \bullet 3 \bullet 3}{3 \bullet 4} = \frac{13 \bullet 3}{4}$

$= \frac{39}{4}$ *or* $9\frac{3}{4}$

The easiest way to solve for this number is to change mixed numbers to an improper fraction and then multiply it. Or use prime factors or the greatest common factor, as part of the multiplication process.

Divide: $\frac{8}{11} \div \frac{2}{3}$

$\frac{8}{11} \div \frac{2}{3} = \frac{8}{11} \bullet \frac{3}{2}$

$= \frac{2 \bullet 4}{11} \bullet \frac{3}{2}$

$\frac{4 \bullet 3}{11} = \frac{12}{11}$ *or* $1\frac{1}{11}$

Take the reciprocal of $\frac{2}{3}$*, which is* $\frac{3}{2}$ *then multiply it with the first fraction. Using prime factors, it is easy to see that 2 can be factored out of the numerator then cancelled out with the denominator, leaving 4 and 3 as the remaining factors in the numerator and 11 as the remaining factors in the denominator.*

III. Exercises.

Do the following exercises. Write your answer on the spaces provided:

1. Find the products. Express in lowest terms (i.e. the numerator and denominators do not have a common factor except 1). Mixed numbers are acceptable as well:

a. $\frac{5}{6} \bullet \frac{2}{3} = \frac{5}{9}$

b. $7 \bullet \frac{2}{3} = \frac{14}{3} = 4\frac{2}{3}$

c. $\frac{4}{20} \bullet \frac{2}{5} = \frac{2}{25}$

d. $10\frac{5}{6} \bullet 3\frac{1}{3} = \frac{325}{9} = 36\frac{1}{9}$

e. $-\frac{9}{20} \bullet \frac{25}{27} = -\frac{5}{12}$

f. $4\frac{1}{2} \bullet 5\frac{2}{3} = \frac{51}{2} = 25\frac{1}{2}$

g. $\frac{2}{15} \bullet \frac{3}{4} = \frac{1}{10}$

h. $\frac{1}{6} \bullet \frac{2}{3} \bullet \frac{1}{4} = \frac{1}{36}$

i. $-\frac{5}{6} \bullet \frac{2}{3} \bullet \left(-\frac{12}{15}\right) = \frac{4}{9}$

j. $\frac{9}{16} \bullet \frac{4}{15} \bullet (-2) = -\frac{3}{10}$

B. Divide:

1. $20 \div \frac{2}{3} = 30$

2. $\frac{5}{12} \div \left(-\frac{3}{4}\right) = -\frac{5}{9}$

3. $\frac{5}{50} \div \frac{20}{35} = \frac{7}{40}$

4. $5\frac{3}{4} \div 6\frac{2}{3} = \frac{69}{80}$

6. $\frac{8}{15} \div \frac{12}{25} = \frac{10}{9} = 1\frac{1}{9}$

7. $13\frac{1}{6} \div (-2) = -\frac{79}{12} = -6\frac{7}{12}$

8. $-\frac{5}{6} \div \left(-\frac{10}{14}\right) = \frac{7}{6} = 1\frac{1}{6}$

9. $-\frac{2}{9} \div \frac{11}{15} = -\frac{10}{33}$

5. $\frac{9}{16} \div \frac{3}{4} \div \frac{1}{6} = \frac{9}{2} = 4\frac{1}{2}$

10. $\frac{15}{6} \div \frac{2}{3} \div \frac{5}{8} = 6$

C. Solve the following:

1. Julie spent $3\frac{1}{2}$ hours doing her assignment. Ken did his assignment for $1\frac{2}{3}$ times as many hours as Julie did. How many hours did Ken spend doing his assignment? $\frac{35}{6} = 5\frac{5}{6}$ hours
2. How many thirds are there in six-fifths? $\frac{18}{5} = 3\frac{3}{5}$
3. Hanna donated $\frac{2}{5}$ of her monthly allowance to the Iligan survivors. If her monthly allowance is P3500, how much did she donate? P1,400.00
4. The enrolment for this school year is 2340. If $\frac{1}{6}$ are sophomores and $\frac{1}{4}$ are seniors, how many are freshmen or juniors? 1,365 students are freshmen or juniors
5. At the end of the day, a store had 2/5 of a cake leftover. The four employees each took home the same amount of leftover cake. How much of the cake did each employee take home? $\frac{1}{10}$ of the cake.

B. Multiplication and Division of Rational Numbers in Decimal Form

NOTE TO THE TEACHER

The emphasis here is on what to do with the decimal point when multiplying or dividing rational numbers in decimal form. Do not get stuck on the rules. Give a deeper explanation. Consider:

$$6.1 \times 0.08 = 6\frac{1}{10} \times \frac{8}{100} = \frac{488}{1000} = 0.488$$

The decimal places indicate the powers of 10 used in the denominators hence, the rule for determining where to place the decimal point in the product.

This unit will draw upon your previous knowledge of multiplication and division of whole numbers. Recall the strategies that you learned and developed when working with whole numbers.

Activity:

1. Give students several examples of multiplication sentences with the answers given. Place the decimal point in an incorrect spot and ask students to explain why the decimal place does not go there and explain where it should go and why.

   Example:

   215.2 x 3.2 = 68.864

2. Five students ordered buko pie and the total cost was P135.75. How much did each student have to pay if they shared the cost equally?

Questions and Points to Ponder:

1. In multiplying rational numbers in decimal form, note the importance of knowing where to place the decimal point in a product of two decimal numbers. Do you notice a pattern? *Take the sum of the decimal places in each of the multiplicand and the multiplier and that is the number of places in the product.*
2. In dividing rational numbers in decimal form, how do you determine where to place the decimal point in the quotient? *The number of decimal places in the quotient depends on the number of decimal places in the divisor and the dividend.*

NOTE TO THE TEACHER
Answer to the Questions and Points to Ponder is to be elaborated when you discuss the rules below.

Rules in Multiplying Rational Numbers in Decimal Form

1. Arrange the numbers in a vertical column.
2. Multiply the numbers, as if you are multiplying whole numbers.
3. Starting from the rightmost end of the product, move the decimal point to the left the same number of places as the sum of the decimal places in the multiplicand and the multiplier.

Rules in Dividing Rational Numbers in Decimal Form

1. If the divisor is a whole number, divide the dividend by the divisor applying the rules of a whole number. The position of the decimal point is the same as that in the dividend.
2. If the divisor is not a whole number, make the divisor a whole number by moving the decimal point in the divisor to the rightmost end, making the number seem like a whole number.
3. Move the decimal point in the dividend to the right the same number of places as the decimal point was moved to make the divisor a whole number.
4. Lastly divide the new dividend by the new divisor.

Exercises:

A. Perform the indicated operation

1. 3.5 ÷ 2 = 1.75
2. 78 x 0.4 = 31.2
3. 9.6 x 13 = 124.8
4. 3.24 ÷ 0.5 = 6.48
5. 1.248 ÷ 0.024 = 52
6. 27.3 x 2.5 = 68.25
7. 9.7 x 4.1 = 39.77
8. 3.415 ÷ 2.5 = 1.366
9. 53.61 x 1.02 = 54.6822
10. 1948.324 ÷ 5.96 = 326.9

B. Finds the numbers that when multiplied give the products shown.

1. □.□ x □ = 10.6

3. □.□ x □ = 21.6

5. .□□□ x □ = 21.98

2. □.□ x □ = 16.8

4. □.□ x □ = 9.5

Answers: (1) 5.3 x 2 ; (2) 8.4 x 2 or 5.6 x 3; (3) 5.4 x 4; (4) 3.5 x 3; (5) 3.14 x 7

NOTE TO THE TEACHER: These are only some of the possible pairs. Be open to other pairs of numbers.

NOTE TO THE TEACHER

Give a good summary to this lesson emphasizing how this lesson was meant to deepen their understanding of rational numbers and develop better skills in multiplying and dividing rational numbers.

Summary

In this lesson, you learned to use the area model to illustrate multiplication and division of rational numbers. You also learned the rules for multiplying and dividing rational numbers in both the fraction and decimal forms. You solved problems involving multiplication and division of rational numbers.