829. Consecutive Numbers Sum (/problems/consecutive-numbers-sum/)

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Given a positive integer `N`, how many ways can we write it as a sum of consecutive positive integers?

Example 1:

```
Input: 5
Output: 2
Explanation: 5 = 5 = 2 + 3
```

Example 2:

```
Input: 9
Output: 3
Explanation: 9 = 9 = 4 + 5 = 2 + 3 + 4
```

Example 3:

```
Input: 15
Output: 4
Explanation: 15 = 15 = 8 + 7 = 4 + 5 + 6 = 1 + 2 + 3 + 4 + 5
```

Note: `1 <= N <= 10 ^ 9`.

Approach #1: Brute Force [Time Limit Exceeded]

Intuition and Algorithm

For each starting number, we scan forward until we meet or exceed the target `N`. If we meet it, then it represents one way to write `N` as a sum of consecutive numbers.

For example, if `N = 6`, and we scan forward from `1`, we'll get `1 + 2 + 3 = 6` which contributes to the answer. If we scan forward from `2`, we'll get `2 + 3 + 4` (the first time that the sum is `>= N`) which is too big.

暴力枚举(超时)

Java Python Copy

```
class Solution(object):
      def consecutiveNumbersSum(self, N):
            ans = 0
            for start in xrange(1, N+1):
                  target = N
                  while target > 0:
                        target -= start
                        start += 1
                  if target == 0: ans += 1
            return ans
```

Complexity Analysis

- Time Complexity: $O(N^2)$.
- Space Complexity: $O(1)$.

Approach #2: Mathematical (Naive) [Time Limit Exceeded]

Intuition and Algorithm

We can model the situation by the equation $N = (x+1) + (x+2) + \cdots + (x+k)$. Here, $x \geq 0, k \geq 1$. Using the identity $1 + 2 + \cdots + k = \frac{k(k+1)}{2}$, we can simplify this equation to $2 * N = k(2 * x + k + 1)$.

From here, clearly $1 \geq k \geq 2 * N$. We can try every such k. We need $x = \frac{\frac{2*N}{k} - k - 1}{2}$ to be a non-negative integer for a solution to exist for the k we are trying.

采用等差数列求和公式

Java Python Copy

```python
class Solution(object):
    def consecutiveNumbersSum(self, N):
        # 2N = k(2x + k + 1)
        ans = 0
        for k in xrange(1, 2*N + 1):
            if 2*N % k == 0:
                y = 2 * N / k - k - 1
                if y % 2 == 0 and y >= 0:
                    ans += 1
        return ans
```

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Complexity Analysis

- Time Complexity: $O(N)$.
- Space Complexity: $O(1)$.

Approach #3: Mathematical (Fast) [Accepted]

Intuition and Algorithm

As in *Approach #2*, $2 * N = k(2 * x + k + 1)$ with $x \geq 0, k \geq 1$. Call k the first factor, and $2 * x + k + 1$ the second factor. We are looking for ways to solve this equation without trying all $2 * N$ possibilities.

Now notice that the parity of k and $(2 * x + k + 1)$ are different. That is, if k is even then the other quantity is odd, and vice versa. Also, $2 * x + k + 1 \geq k + 1 > k$, so the second factor must be bigger.

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Now write $2N = 2^{\alpha} * M$ where M is odd. If we factor $M = a * b$, then two candidate solutions are $k = a, 2x + k + 1 = b * 2^{\alpha}$, or $k = a * 2^{\alpha}, 2x + k + 1 = b$. However, only one of these solutions will have the second factor larger than the first. (Because $\alpha \geq 1$, we are guaranteed that one factor is strictly larger.)

Thus, the answer is the number of ways to factor the odd part of N.

使用根号来加速

Java Python Copy

```python
class Solution(object):
    def consecutiveNumbersSum(self, N):
        while N & 1 == 0:
            N >>= 1

        ans = 1
        d = 3
        while d * d <= N:
            e = 0
            while N % d == 0:
                N /= d
                e += 1
            ans *= e + 1
            d += 2

        if N > 1: ans *= 2
        return ans
```

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Complexity Analysis

- Time Complexity: $O(\sqrt{(N)})$.
- Space Complexity: $O(1)$.

Analysis written by: @awice (https://leetcode.com/awice).

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yao2001626 (/yao2001626) ★ 3 May 6, 2018, 12:37 PM

It should be 1 <= k <= 2∗N for Approach 2.

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