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829. Consecutive Numbers Sum [↗](#)

(/problems/consecutive-numbers-sum/)

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Given a positive integer N , how many ways can we write it as a sum of consecutive positive integers?

Example 1:

Input: 5
Output: 2
Explanation: $5 = 5 = 2 + 3$

Example 2:

Input: 9
Output: 3
Explanation: $9 = 9 = 4 + 5 = 2 + 3 + 4$

Example 3:

Input: 15
Output: 4
Explanation: $15 = 15 = 8 + 7 = 4 + 5 + 6 = 1 + 2 + 3 + 4 + 5$

Note: $1 \leq N \leq 10^9$.

Approach #1: Brute Force [Time Limit Exceeded]

Intuition and Algorithm

For each starting number, we scan forward until we meet or exceed the target N . If we meet it, then it represents one way to write N as a sum of consecutive numbers.

For example, if $N = 6$, and we scan forward from 1, we'll get $1 + 2 + 3 = 6$ which contributes to the answer. If we scan forward from 2, we'll get $2 + 3 + 4$ (the first time that the sum is $\geq N$) which is too big.

暴力枚举(超时)

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```

1 class Solution(object):
2     def consecutiveNumbersSum(self, N):
3         ans = 0
4         for start in xrange(1, N+1):
5             target = N
6             while target > 0:
7                 target -= start
8                 start += 1
9             if target == 0: ans += 1
10        return ans

```

Complexity Analysis

- Time Complexity: $O(N^2)$.
- Space Complexity: $O(1)$.

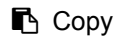
Approach #2: Mathematical (Naive) [Time Limit Exceeded]

Intuition and Algorithm

We can model the situation by the equation $N = (x + 1) + (x + 2) + \dots + (x + k)$. Here, $x \geq 0, k \geq 1$. Using the identity $1 + 2 + \dots + k = \frac{k(k+1)}{2}$, we can simplify this equation to $2 * N = k(2 * x + k + 1)$.

From here, clearly $1 \leq k \leq 2 * N$. We can try every such k . We need $x = \frac{2 * N - k - 1}{2}$ to be a non-negative integer for a solution to exist for the k we are trying.

采用等差数列求和公式



Java

Python

```

1 class Solution(object):
2     def consecutiveNumbersSum(self, N):
3         # 2N = k(2x + k + 1)
4         ans = 0
5         for k in xrange(1, 2*N + 1):
6             if 2*N % k == 0:
7                 y = 2 * N / k - k - 1
8                 if y % 2 == 0 and y >= 0:
9                     ans += 1
10        return ans

```

Complexity Analysis

- Time Complexity: $O(N)$.
- Space Complexity: $O(1)$.

Approach #3: Mathematical (Fast) [Accepted]

Intuition and Algorithm

As in *Approach #2*, $2 * N = k(2 * x + k + 1)$ with $x \geq 0, k \geq 1$. Call k the first factor, and $2 * x + k + 1$ the second factor. We are looking for ways to solve this equation without trying all $2 * N$ possibilities.

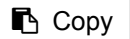
Now notice that the parity of k and $(2 * x + k + 1)$ are different. That is, if k is even then the other quantity is odd, and vice versa. Also, $2 * x + k + 1 \geq k + 1 > k$, so the second factor must be bigger.

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Now write $2N = 2^\alpha * M$ where M is odd. If we factor $M = a * b$, then two candidate solutions are $k = a, 2x + k + 1 = b * 2^\alpha$, or $k = a * 2^\alpha, 2x + k + 1 = b$. However, only one of these solutions will have the second factor larger than the first. (Because $\alpha \geq 1$, we are guaranteed that one factor is strictly larger.)

Thus, the answer is the number of ways to factor the odd part of N .

使用根号来加速



Java

Python

```

1 class Solution(object):
2     def consecutiveNumbersSum(self, N):
3         while N & 1 == 0:
4             N >>= 1
5
6         ans = 1
7         d = 3
8         while d * d <= N:
9             e = 0
10            while N % d == 0:
11                N /= d
12                e += 1
13            ans *= e + 1
14            d += 2
15
16        if N > 1: ans *= 2
17        return ans

```

Complexity Analysis

- Time Complexity: $O(\sqrt{N})$.
- Space Complexity: $O(1)$.

Analysis written by: @awice (<https://leetcode.com/awice>).

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yao2001626 (/yao2001626) ★ 3 🕒 May 6, 2018, 12:37 PM

It should be $1 \leq k \leq 2 \cdot N$ for Approach 2.

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