Machine Learning and Financial Applications

Lecture 3
Logistic regression

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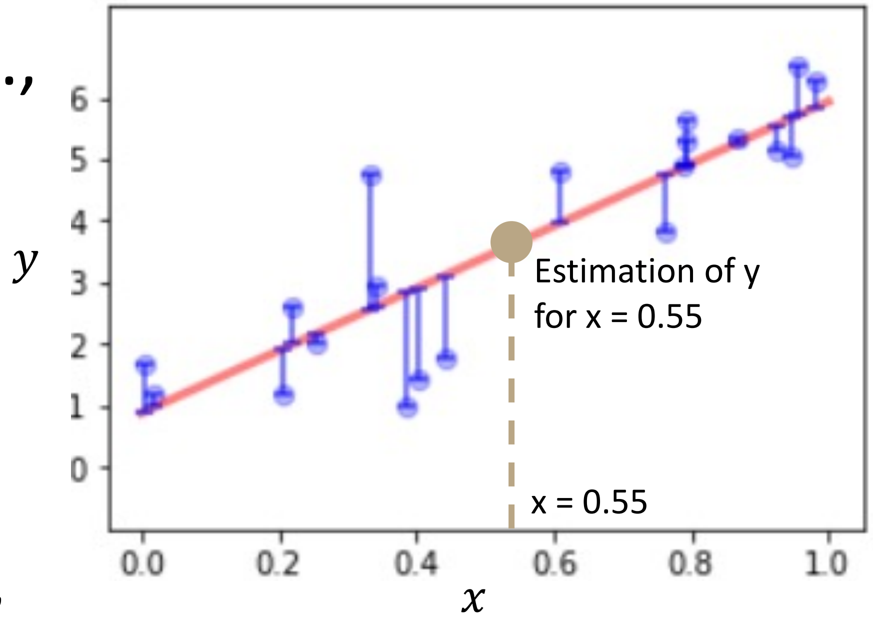
Video tutorial

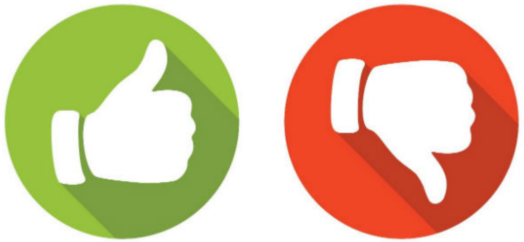
- https://youtu.be/I51DDBeZ-VU

Two categories of predictive models

- An **regressor** predicts a numerical target, e.g.,
  - monthly revenue
  - GDP of a country
  - crime rate of a region
  - duration of a phone call



- A **classifier** predicts a categorical target, e.g.,
  - fraud, non-fraud
  - default, non-default
  - high risk, low-risk
  - good, satisfactory, poor



Classifier modelling in financial applications

	Description	Solution	End goal
Internet fraud detection	▪ Stealing of credit card, login, personal details over internet ▪ A growing concern in banking and online payment, as it is hard to verify identity online	▪ Use classifier modelling in real time to **identify fraudulent transactions**, based on input variables, e.g., transaction amount, location, type of goods/services	▪ To flag/reject the suspicious transactions
Insurance fraud detection	• Concealing, deceiving, and misrepresenting information to make a claim	▪ Use classifier modelling to **predict the likelihood of insurance fraud** based on input variables, e.g., size of claim, premium, previously reported fraud, insurer employment status, health conditions	▪ To flag/reject the suspicious claims

Is Linear Regression suitable as a classifier?

Simple linear regression

$$y = \beta_0 + \beta_1 x_1$$

Multiple linear regression

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_k x_k$$

Application

- Sales figure forecast
- No. of times being streamed for a new song on Spotify within the first month
- …

Advantages

- One simple equation
- Easy, fast and transparent to users

Limitations

- Target variable must be a continuous value
- Target variable can be $(-\infty, \infty)$; in practice usually within a specified range

Conclusion

- Is it the best model to predict categorical target variable? $\boxed{}$

Applying MLR to a classification problem

Social_Network_Ads.csv

- Purchasing behaviour of people who have been exposed to a social media marketing advertisement campaign
- Input variables: Gender, Age, EstimatedSalary (UserID is not useful)
- Target variable: Purchased (0 or 1)

Exercise time to extract columns for x_orig and y

- Convert 'Gender' column to 0 and 1
- Scale EstimatedSalary

```
x_encoded = x_orig.copy()
x_encoded.loc[:,"Gender"] =
(x_encoded.loc[:,"Gender"] ==
"Female").astype(int)


x_encoded.loc[:,"EstimatedSalary"] =
x_encoded.loc[:,"EstimatedSalary"] / 1000
```

Train and Test - model evaluation method

- Training data
  - For training the predictive model – supervised or unsupervised learning?
  - Contains input variables and **known** target variable
  - Training data is **seen by the model**
- Testing data
  - For checking how well the model predicts target variable in the future
  - Contains input variables; but target variable is **hidden**
  - **NOTE:** Testing data is **unseen by the model during the training phase**
- How to select training and testing data sets
  - Usually by random sampling, e.g., randomly select 70% of the records as training data, and remaining 30% as testing data
  - Ratio of Training vs. Testing: usually more data in training set

Train and Test - implementation

- Separate the data set into train vs. test subsets

```
x_train, x_test, y_train, y_test =
train_test_split(x_encoded, y,
test_size=0.3, random_state=12345)
```

Exercise time to Train the model

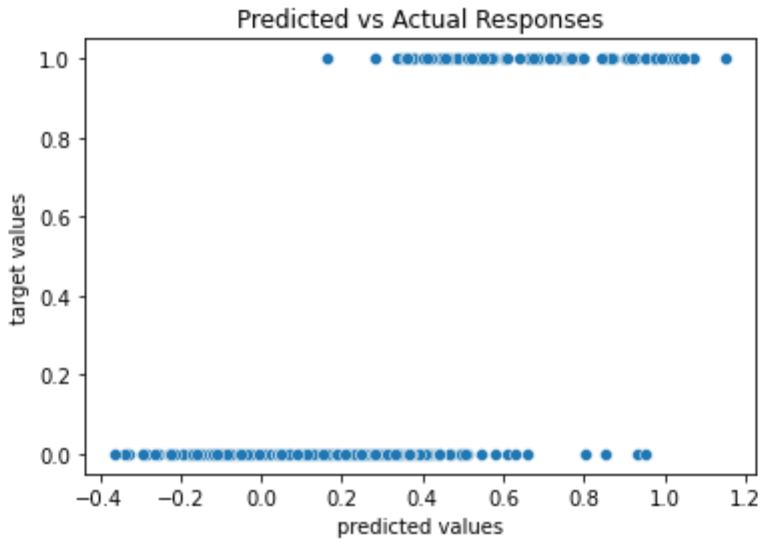
- Feed training data set into the model to get the model's prediction
- Feed testing data set into the model to get the model's prediction

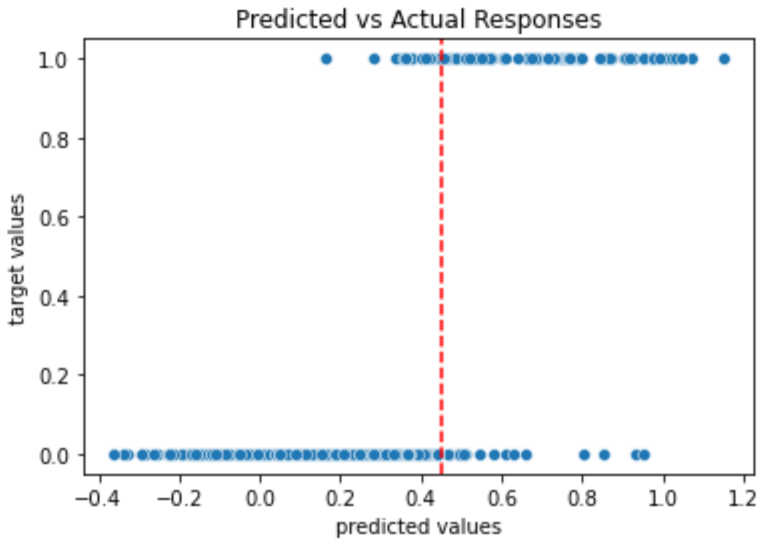
```
linreg_ols_pred_train =
linreg_ols.predict(x_train)
linreg_ols_pred_test =
linreg_ols.predict(x_test)
```

Transforming the model output to categorical target variable (1/2)

- How can we equate these predictions to the categorical target variable (Purchased should 0/1)?

- Find a threshold 0.45, based on the graph of actual values vs. predicted value of target variable 'Purchased'





Transforming the model output to categorical target variable (2/2)

- If a regression output is larger than 0.45, it would be transformed to True, then to a y value of 1
- Compare the predicted y to the actual y value, to check accuracy
- Do the same for test data set

```
y_pred_train = (linreg_ols_pred_train >
0.45).astype(int))

acc_train = y_pred_train == y_train
print("Accuracy:", acc_train.mean())

y_pred_test = (linreg_ols_pred_test >
0.45).astype(int)
acc_test = y_pred_test == y_test
print("Accuracy:", acc_test.mean())
```

Issues

- The cut-off value for regression output to be considered 1 or 0 was chosen by observation, without a guiding principle
- The Linear Regression model's output can be $(-\infty, \infty)$

Logistic Regression to the rescue

Logistic Regression

$$p = f(z) = \frac{1}{1 + e^{-z}}$$

where $z = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_k x_k$

- z is the linear combination of k input variables x_i, and it measures the overall effect of all the input variables
- z is linear regression → place this z through an activation function to get a probability (a continuous value between 0 and 1)

Key facts

- Binary classification model to predict the probability of occurrence of an event, e.g.
  - probability of default on a loan
  - probability of buying insurance in response to an advertisement
- The target variable y is binary (1 or 0, Yes or No), depending on whether p is above or below a threshold
- y follows a Bernoulli distribution

$$y = \begin{cases} 1 & \text{with probability } p \\ 0 & \text{with probability } 1 - p \end{cases}$$

The loss function

- Quantifies the difference between the predicted probabilities and the actual class labels in the dataset.
- Minimize this cost function during the training process to obtain the best-fitting model.
- Binary cross-entropy loss:
  - **$Loss(y, \hat{y}) = -[y * \log(\hat{y}) + (1 - y) * \log(1 - \hat{y})]$**
- To obtain the overall cost function for logistic regression, average the binary cross-entropy loss over all data points in the training dataset:
  - **$Cost = (1/N) * \Sigma\ Loss(y_i, \hat{y}_i)$**

More on Logistic Regression

How Logistic Regression model is trained

- Purpose of training is to obtain the coefficients $\beta_i, i = 0, 1, 2, \ldots, k$
- Once β_i is calculated, the logistic regression model $f(z)$ is trained
- Usually, if $f(z) \geq 0.5$, then y = 1; else, y = 0

Exercise time to train logistic regression using logit() from statsmodels.formula.api

Assumptions

- The observations (rows) are independent of each other, and their target outcome follow the same Bernoulli distribution
- Little or no collinearity (i.e., low correlation) among the input variables
- No linear relationship between the target y and input variables
- Log odds of the probability of a target value y being 1 is linearly related to input variables

$$\text{log odds} = \log\left(\frac{p}{1-p}\right) = z$$
$$= \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_k x_k$$

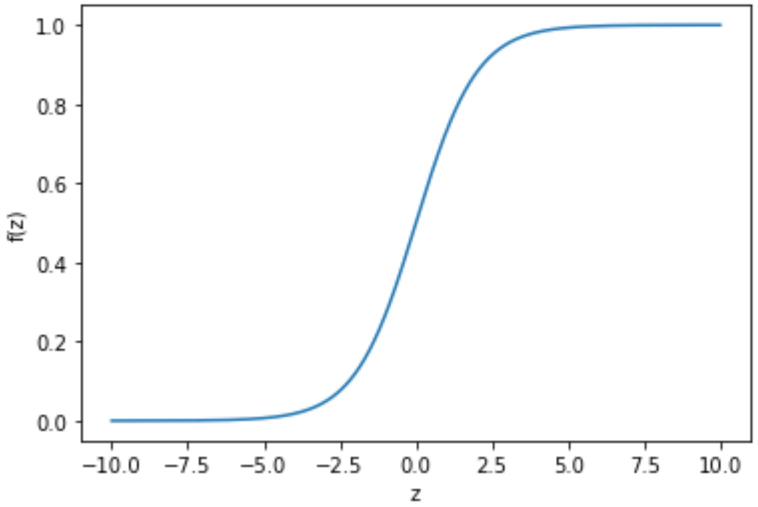
(details later)

Sigmoid function and odds

Definition of Sigmoid function

$$p = f(z) = \frac{1}{1 + e^{-z}}$$

- $z \rightarrow \infty, f(z) \rightarrow 1; z \rightarrow -\infty, f(z) \rightarrow 0$
- Hence $f(z) \in [0,1]$
- $f(z)$ represents the probability of an event



Definition of odds

$$\text{odds} = \frac{prob\ of\ event\ \text{happening}}{prob\ of\ event\ not\ \text{happening}} = \frac{p}{1-p}$$

$$p = \frac{1}{1 + e^{-z}}$$

$$1 - p = 1 - \frac{1}{1+e^{-z}} = \frac{e^{-z}}{1+e^{-z}}$$

$$\text{odds} = \frac{p}{1-p} = e^{z}$$

$$\text{i.e.} \log(\text{odds}) = \text{z}$$

Odds Ratio

- Suppose x_i is a binary input variable, x_i = 1 or 0
- Odds of x_i = 1: $\frac{p_1}{1-p_1} = e^z | x_i$ = 1, measures the chance of an event for x_i = 1, over the chance of a non-event
- Odds of x_i = 0: $\frac{p_0}{1-p_0} = e^z | x_i$ = 0, measures the chance of an event for x_i = 0 **over** the chance of a non-event
- **Odds Ratio of $\boldsymbol{x_i}$:** the ratio of the odds of x_i = 1 to the odds of x_i = 0

$$\frac{p_1}{1-p_1} / \frac{p_0}{1-p_0} = \frac{e^{\beta_0+\beta_1 x_1+\ldots+\ \beta_i * 1+\cdots+\beta_k x_k}}{e^{\beta_0+\beta_1 x_1+\ldots+\ \beta_i * 0+\cdots+\beta_k x_k}} = \ e^{\beta_i}$$

- e^{β_i} measures the quantified impact of the **binary** input variable x_i on the odds of the outcome y being 1, while all other input variables remain unchanged
- Recall for Multiple Linear Regression:

$$y + \Delta y = \beta_0 + \beta_1 x_1 + \cdots + \beta_j (x_j + \Delta x_j) + \cdots$$
$$\Delta y = \beta_j \ \Delta x_j$$

How to interpret Odds Ratio

- For categorical input variable
  - E.g., gender (0: male, 1: female) may be related to whether the insurance is purchased (1: yes; 0: no)
  - Base category: male set as 0
  - If the estimated β of gender is 0.2, then OR = $e^{0.2} \approx 1.22$
  - Odds of female customers to purchase the insurance is 1.22 times the odds of their male counterparts to purchase the insurance, assuming Ceteris Paribus
- For numerical input variable
  - E.g., age may be related to whether the insurance is purchased
  - No need to set base category
  - If the estimated β of age is 0.3, then OR = $e^{0.3} \approx 1.35$
  - Odds of a client to purchase is 1.35 times the odds of similar people who are 1 year younger, assuming Ceteris Paribus
  - What if β is -0.3?

How to evaluate Logistic Regression model

Metrics

- Accuracy rate
- Error rate
- Precision
- Recall
- Sensitivity (true positive rate)
- Specificity (true negative rate)
- AUC

Principle of parsimony

- If two competing models provide the similar level of fit to the data, the one with fewer input variables should be picked
- The most accurate model is not necessarily the best model

Model metrics – various rates

Confusion matrix

	Predicted $Y=0$	Predicted $Y=1$	Total
Non-Event $Y=0$	a	c	$a+c$
Event $Y=1$	b	d	$b+d$
Total	$a+b$	$c+d$	n

- **Accuracy (ACC) rate**
  - ACC = $(a+d)/n$
- **Error rate**
  - Error rate = 1 - ACC = $(b+c)/n$
- Positive predictive value (PPV), or **Precision**
  - **Precision** = $d/(c+d)$
  - Out of all the positive predictions, what percentage is actually positive?

- True positive rate (TPR), or sensitivity, **Recall**
  - **Recall** = $d/(b+d)$
  - Out of all the events, what percentage are predicted correctly as positive?
- True negative rate (TNR), or **Specificity**, selectivity
  - **Specificity** = $a/(a+c)$
  - Out of all the non-events, what percentage are predicted correctly as negative?

Exercise time to manually calculate ACC, Precision, Recall

How changing the classification threshold might impact the metrics of a classifier

Exercise time to calculate for threshold of 0.3

Low threshold 0.3
- Accuracy: 0.79
- Precision: 0.79
- Recall: 0.68

Default threshold 0.5
- Accuracy: 0.74
- Precision: 0.85
- Recall: 0.46

High threshold 0.8

Accuracy: 0.69

Precision: 0.88

Recall: 0.3

In this particular example, increasing the threshold leads to

- Accuracy decreasing
- Recall decreasing
- Precision increasing

Implication for classifiers in general

- Precision and Recall usually have an inverse relationship with respect to the adjustment of the classification threshold
- Reviewing both precision and recall is useful for cases where there is a huge imbalance in the target variable’s values

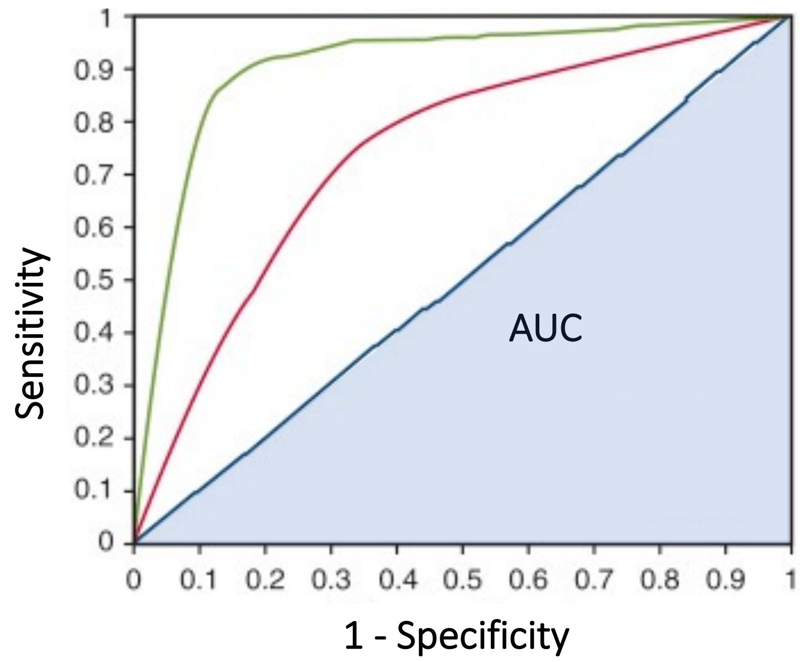
Optimize for Precision or Recall?

Recall (True positive rate TPR) = 1 – False Negative Rate (FNR)

	Positive class	Interpreting False Negative	How bad is FN?	Optimize for?
Spam filter	Spam	Spam goes to the inbox	Acceptable	Precision
Fraudulent transaction detector	Fraud	Fraudulent transactions that are not detected	Very bad	Recall
Cancer Diagnose	Cancer	Test for cancer shows up as negative even though the patient has cancer	Very bad	Recall

Model metrics – Area under ROC curve (AUC)

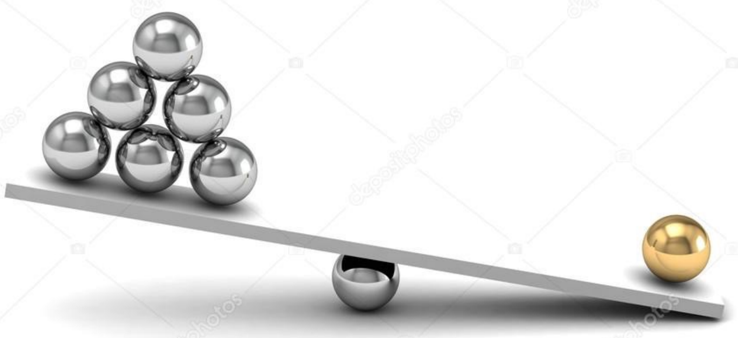
- ROC curve (receiver operating characteristic curve)
  - Plot Sensitivity vs. (1-Specificity), i.e., TPR vs. (1-TNR), or TPR vs. FPR
- As the classification threshold goes up
  - FPR goes down
  - Leftward movement on the curve
- A perfect classifier (0,1) has AUC score of 1
  - 1 – specificity: 0, i.e., FPR is 0 (no false positive, i.e., all negative cases are not predicted as positive)
  - Sensitivity: 1, i.e., TPR is 1 (all positive cases are predicted as positive correctly)
- A randomly guessing classifier has AUC score of 0.5 (area under the **blue** ROC)



Dealing with imbalanced data

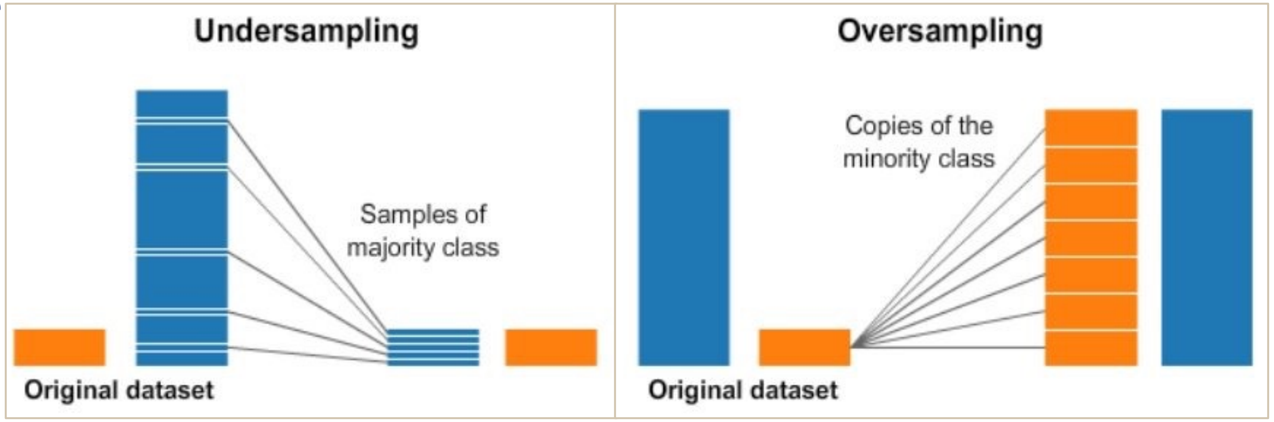
Issues

- Target variable's values are not distributed equally
- Very common in real-world applications
  - Fraudulent transactions in banks
  - Spam/phishing emails for employees in banks
  - Identification of rare diseases, e.g., cancer
  - Natural disasters, e.g., earthquakes
- Classification performance may be **dominated by the majority class**, i.e., metric fool



Popular solutions

- Re-design the data collection or collect more data
- Change the performance evaluation method
- **Data resampling**: oversampling and/or under-sampling to make the distribution less imbalanced



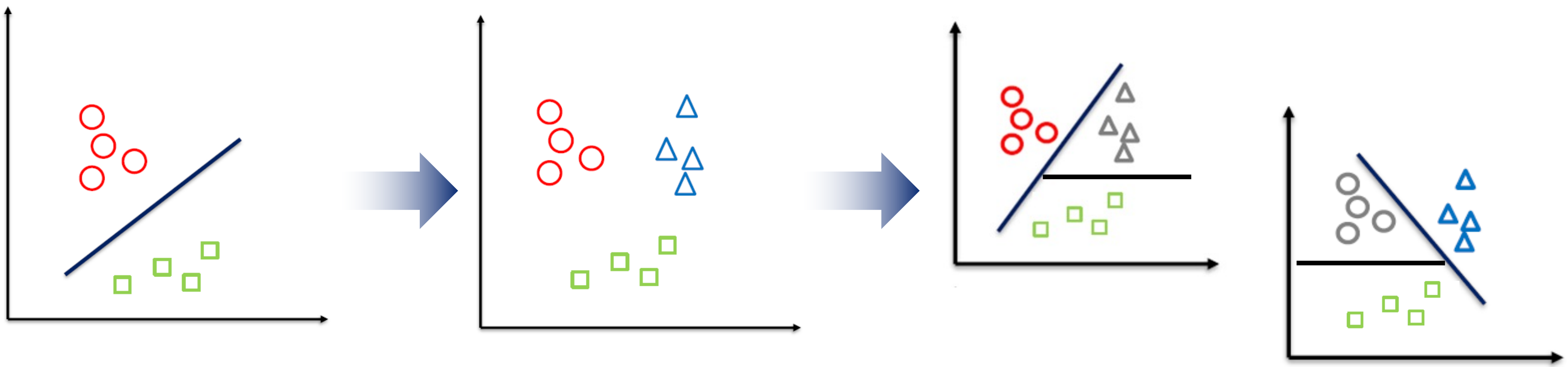
Extending binary classification to multiclass classification

Multiclass classification

- S&P bond rating: AAA, AA, A…
- Corporate client accounts in a bank: good credit, past due, overdue and doubtful

Multinomial logistic regression

- Generalizes logistic regression to multiclass problems
- Decomposed as a set of independent binary logistic regressions



Mathematical derivation to obtain the multinomial logistic regression

For binary target variable y

- $\log(\text{odd}) = \log\left(\frac{p}{1-p}\right) = \log\left(\frac{P(y=1)}{P(y=0)}\right) = \text{z}$
- $\frac{P(y=1)}{P(y=0)} = e^{z}$

- Since $P(y = 1) + P(y = 0) = 1$
- Therefore:

$$P(y = 1) = \frac{1}{1 + e^{-z}}$$

$$P(y = 0) = \frac{e^{-z}}{1 + e^{-z}}$$

Assume target variable y have 3 values, 0, 1, 2

- Choose 0 as the pivot value
- $\frac{P(y=1)}{P(y=0)} = e^{z_1}$ (z_1 is a linear combination of all input x_i)
- $\frac{P(y=2)}{P(y=0)} = e^{z_2}$ (z_2 is another linear combination of all input x_i)

- $P(y = 2): P(y = 1): P(y = 0) = e^{z_2}: e^{z_1}: 1$
- Since $P(y = 2) + P(y = 1) + P(y = 0) = 1$
- Therefore:

$$P(y = 2) = e^{z_2}/(e^{z_2} + e^{z_1} + 1)$$
$$P(y = 1) = e^{z_1}/(e^{z_2} + e^{z_1} + 1)$$
$$P(y = 0) = 1/(e^{z_2} + e^{z_1} + 1)$$

Recap and next lecture

Recap

- Linear Regression
- Train and test model evaluation
- Limitation for Linear Regression on classification problem
- Logistic Regression
- Probability and odds
- Odds Ratio
- Model metrics: Precision vs. Recall, AUC
- Multinomial logistic regression

Next lecture

- Regularization

Lab session on logistic regression

Homework

- Revisit video tutorial and class recording week 3 lecture
- Start thinking about final project topic (see project guideline in Elearn)
- Post learning reflections and questions if any