Objective: The goal of this exercise is to demonstrate the effect of regularization on model complexity and generalization. Students will work with a simple linear regression problem and apply different regularization techniques, such as Lasso (L1) and Ridge (L2) regularization.

1. Generate synthetic data: Create a synthetic dataset with a known linear relationship between a single input feature (X) and the target variable (y). Add some Gaussian noise to the target variable to simulate real-world noise. Split the dataset into training and validation sets.

2. Create a high-dimensional feature space: To demonstrate the effect of regularization, it's helpful to create a high-dimensional feature space. Transform the single input feature (X) into a high-dimensional feature space by creating polynomial features (e.g., X^2, X^3, ..., X^n) for some n > 1.

3. Fit an unregularized linear regression model: Train a linear regression model on the training dataset using the high-dimensional feature space. Evaluate the model's performance on the validation set, and plot the learned coefficients.

4. Apply L1 (Lasso) regularization: Train a Lasso regression model on the training dataset, experimenting with different regularization strengths (alpha). Evaluate the model's performance on the validation set, and plot the learned coefficients. Discuss how L1 regularization affects the model's complexity and sparsity of the coefficients.

5. Apply L2 (Ridge) regularization: Train a Ridge regression model on the training dataset, experimenting with different regularization strengths (alpha). Evaluate the model's performance on the validation set, and plot the learned coefficients. Discuss how L2 regularization affects the model's complexity and the magnitude of the coefficients.

6. Model selection: Compare the performance of the unregularized, Lasso, and Ridge regression models on the validation set, and choose the best model based on validation performance. Discuss the trade-offs between model complexity and generalization, and how regularization can help balance these trade-offs.

7. Optional extension: Introduce Elastic Net regularization, which combines L1 and L2 regularization, and have students experiment with different combinations of L1 and L2 penalties.