On the Persistence of Higher-Order Interactions in Real-World Hypergraphs - Supplementary Document

A Randomized Hypergraphs (Null Models)

For comparisons with real-world hypergraphs, we consider the hypergraphs randomized from them in the following two different ways:

- **CL**: We adapt the Chung-Lu (CL) model [1], where the degree sequence of the nodes is expected to be preserved. Specifically, for each hyperedge, the nodes in it are replaced with nodes drawn independently with probability proportional to their degrees. The size and timestamp of the hyperedge remain unchanged.
- **Time-Shuffled:** We randomly shuffle the timestamps of hyperedges without changing the nodes in the hyperedges.

B Observations

B.1 Global Analysis: Persistence vs. Frequency The distributions of the persistence of HOIs in all 13 real-world hypergraphs are shown in Table 1 and Fig. 1. While the distributions from most datasets clearly obey power-laws, there exist anomalies that deviate from the fitted lines in the distributions from the Eu and Classes datasets. The anomalies from the Eu dataset indicate the surprising abundance of highly persistent HOIs.

The distributions of the persistence of HOIs in the randomized hypergraphs are given in Fig. 2 and Fig. 3. Additionally, in Table 2 and Table 3, we report (a) the goodness-of-fit R^2 of straight lines fitted on a log-log scale, (b) the exponents (i.e., k in $f(x) = ax^{-k}$) of the fitted power-law distributions, and (c) the average persistence of HOIs of size 2, 3, or 4.

B.2 Local Analysis (1): Group Features vs. Group Persistence.

Observations. The mutual information (MI) and Pearson correlation coefficients (CC) between each structural group feature and the persistence in each dataset are shown in Table 4. Most features are positively correlated with persistence, and on average, the CC is strongest for #, (i.e., the number of hyperedges containing each HOI S), followed by $\mathcal{H}$ (i.e., the entropy in the sizes of hyperedges containing each HOI S), and then $\Sigma/\cap$. Notably, $\Sigma/\#$ (i.e., the average size of the hyperedges containing each HOI S) is the only feature that is negatively correlated with persistence. We show in Fig. 2 the distributions of # and $\Sigma/\#$ of HOIs with each level of persistence in all 13 real-world hypergraphs.

Table 1: **Distributions of HOIs of each Size and HOIs of each Persistence.**

Dataset	Percentage of HOIs (in %)												
	Size of HOIs			Persistence of HOIs									
	2	3	4	1	2	3	4	5	6	7	8	9	10
DBLP	25.5	29.9	44.6	97.2	1.9	0.6	0.2	0.1	0.0	0.0	0.0	0.0	0.0
Geology	17.0	28.3	54.7	99.1	0.7	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0
History	9.1	21.8	69.1	100.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
High	69.7	27.4	2.9	79.9	13.2	4.6	1.6	0.7	0.1	0.1	0.0	0.0	0.0
Primary	61.5	35.8	2.6	68.2	15.7	7.0	4.0	2.0	1.3	0.8	0.5	0.4	0.1
Enron	23.4	34.3	42.3	68.3	17.2	5.5	2.8	2.3	1.7	0.9	0.6	0.4	0.4
Eu	2.7	21.5	75.8	95.7	3.1	0.7	0.3	0.1	0.1	0.0	0.0	0.0	0.0
Classes	15.4	33.8	50.8	84.3	10.1	3.4	1.4	0.6	0.2	0.1	0.0	0.0	0.0
Substances	4.6	21.0	74.5	94.0	4.9	0.7	0.2	0.1	0.0	0.0	0.0	0.0	0.0
Ubuntu (Tag)	27.1	51.5	21.4	91.0	6.0	1.6	0.6	0.3	0.2	0.1	0.1	0.1	0.1
Math.sx (Tag)	18.5	54.6	26.9	87.9	8.5	2.2	0.8	0.3	0.2	0.1	0.0	0.0	0.0
Ubuntu (Thr)	69.9	22.4	7.7	99.7	0.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Math.sx (Thr)	50.8	31.2	17.9	98.9	1.0	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Average	**30.4**	**31.8**	**37.8**	**89.6**	**6.4**	**2.0**	**0.9**	**0.5**	**0.3**	**0.2**	**0.1**	**0.1**	**0.0**

Observation 1. (Group Features vs. Group Persistence) *In real-world hypergraphs, the persistence of each HOI S is positively correlated with (a) the number of hyperedges containing S and (b) the entropy in the sizes of hyperedges containing S.*

B.3 Local Analysis (2): Node Features vs. Group Persistence

Node Features. As described in Section 4.1, for each HOI appearing for the first time at time t, we consider the hypergraph H consisting of all hyperedges appearing between time $t+1$ and $t+T_s$. The structural node features are obtained from the projected graph H' (see Section 3.1) of H, as described below. Recall that, H' is a pairwise graph, and for each node v, we use $N'(v)$ to denote the set of its neighbors in H'. We define the features as follows:

Table 2: **The goodness-of-fit R^2 of fitted lines, the exponents of the fitted power-law distributions, the average persistence of HOIs of size 2, 3, or 4 in Randomized Hypergraphs (CL).**

Dataset	R^2 of Fitted Line			Power-Law Exponent (Relative)			Average Persistence (Relative)		
Size of HOIs	2	3	4	2	3	4	2	3	4
DBLP	1.00	-	-	-	-	-	1.00	1.00	1.00
Geology	1.00	-	-	-	-	-	1.00	1.00	1.00
History	0.98	-	-	-	-	-	1.00	1.00	1.00
High	0.78	-	-	1.00	0.39	-	1.00	0.46	0.46
Primary	0.78	-	-	1.00	0.36	-	1.00	0.40	0.40
Enron	0.70	0.97	-	1.00	0.58	0.32	1.00	0.32	0.30
Eu	0.76	0.94	0.98	1.00	0.39	0.27	1.00	0.49	0.49
Classes	0.82	0.98	-	1.00	0.56	0.38	1.00	0.51	0.50
Substances	0.91	0.99	-	1.00	0.54	0.33	1.00	0.77	0.77
Ubuntu (Tag)	0.92	0.99	-	1.00	0.49	0.29	1.00	0.70	0.70
Math.sx (Tag)	0.86	0.97	-	1.00	0.41	0.24	1.00	0.44	0.43
Ubuntu (Thr)	0.97	-	-	-	-	-	1.00	0.99	0.99
Math.sx (Thr)	0.99	-	-	1.00	0.37	0.28	1.00	0.94	0.94
Average	**0.88**	**0.97**	**0.98**	**1.00**	**0.46**	**0.30**	**1.00**	**0.69**	**0.69**

-: not enough HOIs.

Table 3: **The goodness-of-fit R^2 of fitted lines, the exponents of the fitted power-law distributions, the average persistence of HOIs for each size in Randomized Hypergraphs (Time-Shuffled).**

Dataset	R^2 of Fitted Line			Power-Law Exponent (Relative)			Average Persistence (Relative)		
Size of HOIs	2	3	4	2	3	4	2	3	4
DBLP	0.93	0.99	0.97	1.00	0.76	0.51	1.00	0.73	0.71
Geology	0.97	0.99	0.96	1.00	0.80	0.66	1.00	0.91	0.90
History	0.97	0.88	0.96	1.00	0.83	0.40	1.00	0.99	0.99
High	0.89	0.97	0.93	1.00	0.44	0.52	1.00	0.49	0.39
Primary	0.99	0.98	0.99	1.00	0.47	0.41	1.00	0.29	0.22
Enron	0.66	0.87	0.89	1.00	0.74	0.62	1.00	0.61	0.46
Eu	0.97	0.97	0.97	1.00	0.52	0.45	1.00	0.48	0.43
Classes	0.81	0.90	0.84	1.00	0.97	0.78	1.00	0.74	0.67
Substances	0.93	0.91	0.94	1.00	0.82	0.69	1.00	0.73	0.65
Ubuntu (Tag)	0.99	0.99	0.97	1.00	0.67	0.53	1.00	0.56	0.48
Math.sx (Tag)	0.98	0.97	0.98	1.00	0.61	0.46	1.00	0.48	0.34
Ubuntu (Thr)	0.95	-	-	1.00	0.33	-	1.00	0.98	0.98
Math.sx (Thr)	0.98	0.92	-	1.00	0.54	0.26	1.00	0.90	0.90
Average	**0.92**	**0.95**	**0.95**	**1.00**	**0.65**	**0.52**	**1.00**	**0.68**	**0.62**

-: not enough HOIs.

- **Degree** (d): The *degree* $d(v)$ of a node v is the number of the edges adjacent to v in H'.

$$d(v) := |N'(v)|.$$

- **Weighted degree** (w): The *weighted degree* $w(v)$ of a node v is the sum of the weights of the edges incident to v in H'.

$$w(v) := \sum\nolimits_{u \in N'(v)} \Omega(u, v).$$

- **Number of occurrences** (o): The *number of occurrences* $o(v)$ of a node v is the number of hyperedges including v in H.

$$o(v) := |\{e \in E : v \in e\}|.$$

- **Core number** (c): The *k-core* H'_k of H' is its maximal subgraph where every node is adjacent to at least k nodes in it. The *core number* of a node v is the largest k such that H'_k contains v.

- **PageRank** (r): The PageRank $r(v)$ of a node v in H' is the stationary probability of a random walker on H' being at v. At each time, the random walker either follows (with probability $\beta = 0.85$) an incident edge chosen uniformly at random or jumps (with probability $1 - \beta$) to a node chosen uniformly at random.

- **Average degree of neighbors** ($\bar{d}$): We denote the *average degree of the neighbors* of a node v by $\bar{d}(v)$.

$$\bar{d}(v) := \frac{1}{d(v)} \sum\nolimits_{u \in N'(v)} d(u).$$

- **Average weighted degree of neighbors** ($\bar{w}$) : We denote the *average weighted degree of the neighbors* of a node v by $\bar{w}(v)$.

$$\bar{w}(v) := \frac{1}{d(v)} \sum\nolimits_{u \in N'(v)} w(u).$$

- **Local clustering coefficient** (l): The *local clustering coefficient* $l(v)$ of a node v is defined as

$$l(v) := |\{\{u, w\} \in E' : u \in N'(v) \text{ and } w \in N'(v)\}| \Big/ \binom{d(v)}{2},$$

where the denominator is the number of pairs of the neighbors, and the numerator is the number of such pairs that are directly joined by an edge. That is, $l(v)$ quantifies how close the neighbors of v are, and equivalently, their tendency to form a clique together.

Observations. The mutual information (MI) and Pearson correlation coefficients (CC) between each structural node feature, which is averaged over the nodes involved in each HOI, and the persistence in each dataset are shown in Table 4. On average, the MI is largest for $\bar{w}$ (i.e., the average weighted degree of neighbors), $\bar{d}$ (i.e., the average degree of neighbors), and r (i.e., PageRank). Notably, $\bar{w}$ and $\bar{d}$ are negatively correlated with persistence. In addition to r, w (i.e., weighted degree), and o (i.e., the number of occurrences) are positively correlated with persistence. The distributions of averaged w and $\bar{w}$ of HOIs with each level of persistence in all 13 real-world hypergraphs are shown in Fig. 3.

Observation 2. (Node Features vs. Group Persistence) *In real-world hypergraphs, the persistence of each HOI is negatively correlated with the average (weighted) degree of neighbors of each node involved in the HOI.*

The MI and CC between each structural node feature and the persistence in the randomized hypergraphs with shuffled timestamps (see Appendix A) are summarized in Table 5. For the hypergraphs randomized using the Chung-Lu (CL) model, as shown in Fig. 2, the persistence values are not diverse enough (i.e., the distribution of the persistence is highly concentrated in small values particularly for the HOIs of size 3 or 4) to measure the MI and the CC between each structural feature and the persistence.

B.4 Local Analysis (3): Node Features vs. Node Persistence

Observations. We report in Table 4 the mutual information (MI) and Pearson correlation coefficients (CC) between each structural node feature and the k-node persistence in each dataset. Overall, the MIs are larger than those obtained in the previous subsections. On average, the MI is largest for r (i.e., PageRank), followed by $\bar{w}$ (i.e., the average weighted degree of neighbors), and then $\bar{d}$ (i.e., the average degree of neighbors). The correlation is strongest for o (i.e., the number of occurrences) and w (weighted node degree), which are positively correlated with k-node persistence. Among the features, only $\bar{w}$, $\bar{d}$, and l (i.e., the local clustering coefficient) are negatively correlated with k-node persistence. The distributions of w and $\bar{w}$ of nodes with each level of k-node persistence in all 13 real-world hypergraphs are shown in Fig. 4.

Observation 3. (Node Features vs. Node Persistence) *In real-world hypergraphs, the weighted degree and number of occurrences of each node are positively correlated with the persistence of HOIs that the node is involved in.*

The MI and CC between each structural node feature and the k-node persistence in the randomized hypergraphs with shuffled timestamps (see Appendix A) are summarized in Table 5. Due to the aforementioned reason, we could not measure the MI and CC in the hypergraphs randomized using the Chung-Lu (CL) model.

C Linear Regression Analysis

In Table 6, we report the average coefficient, standard error, and p-value of each structural feature obtained by linear regression analysis of each dataset. The results are summarized in Table 7.

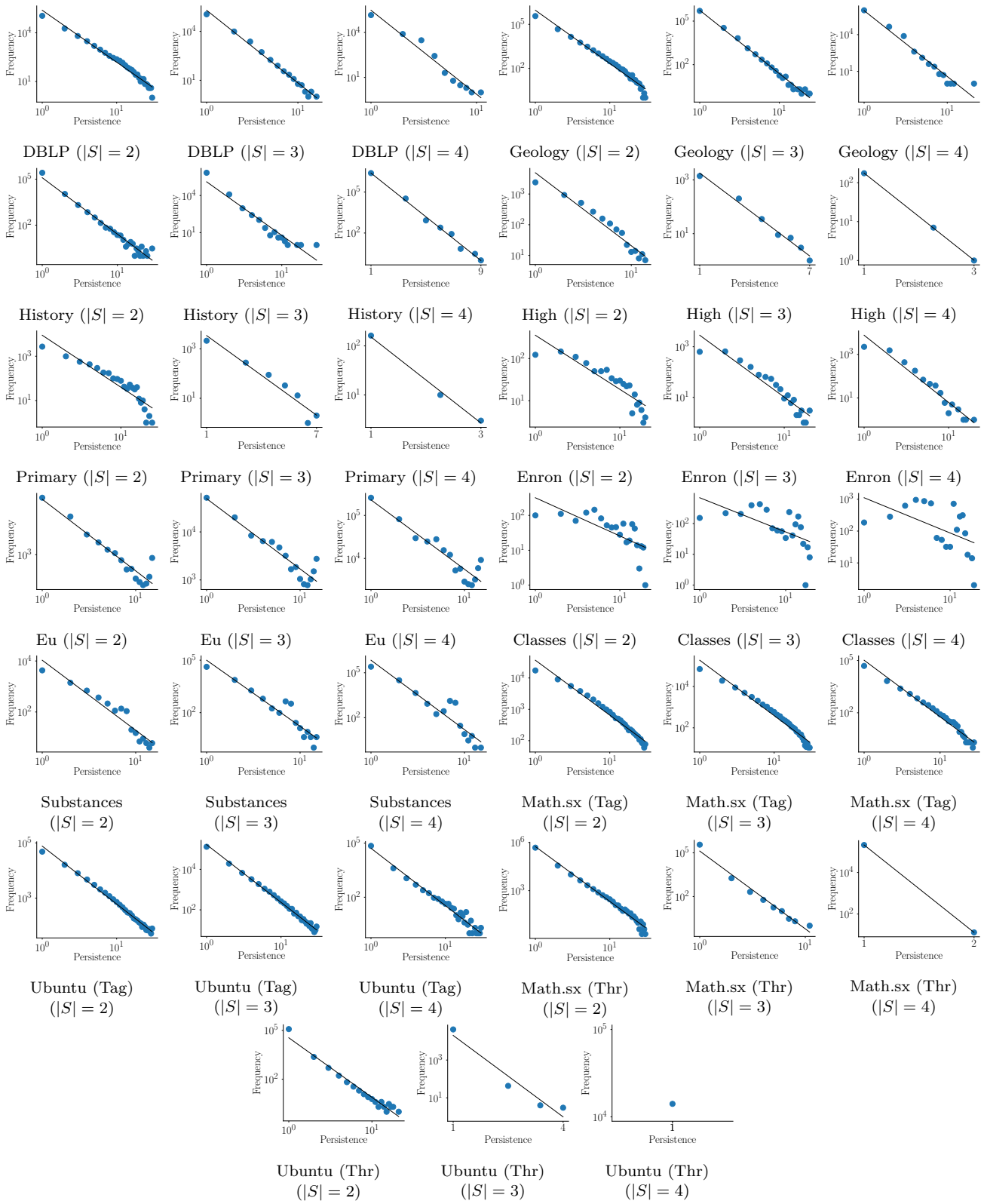


Figure 1: **Distributions of the Persistence of HOIs in Real-world Hypergraphs.**

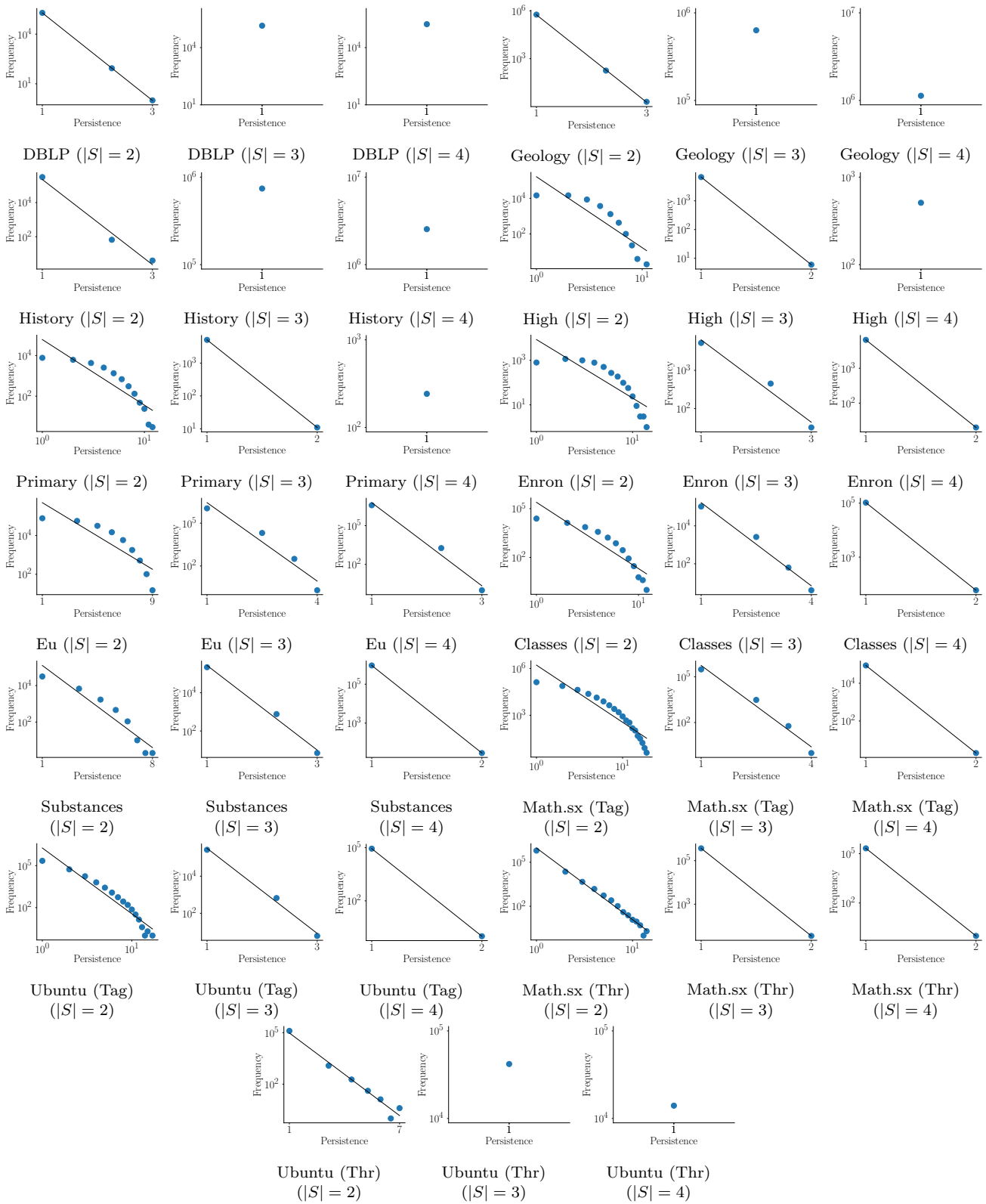


Figure 2: **Distributions of the Persistence of HOIs in Randomized Hypergraphs (CL).**

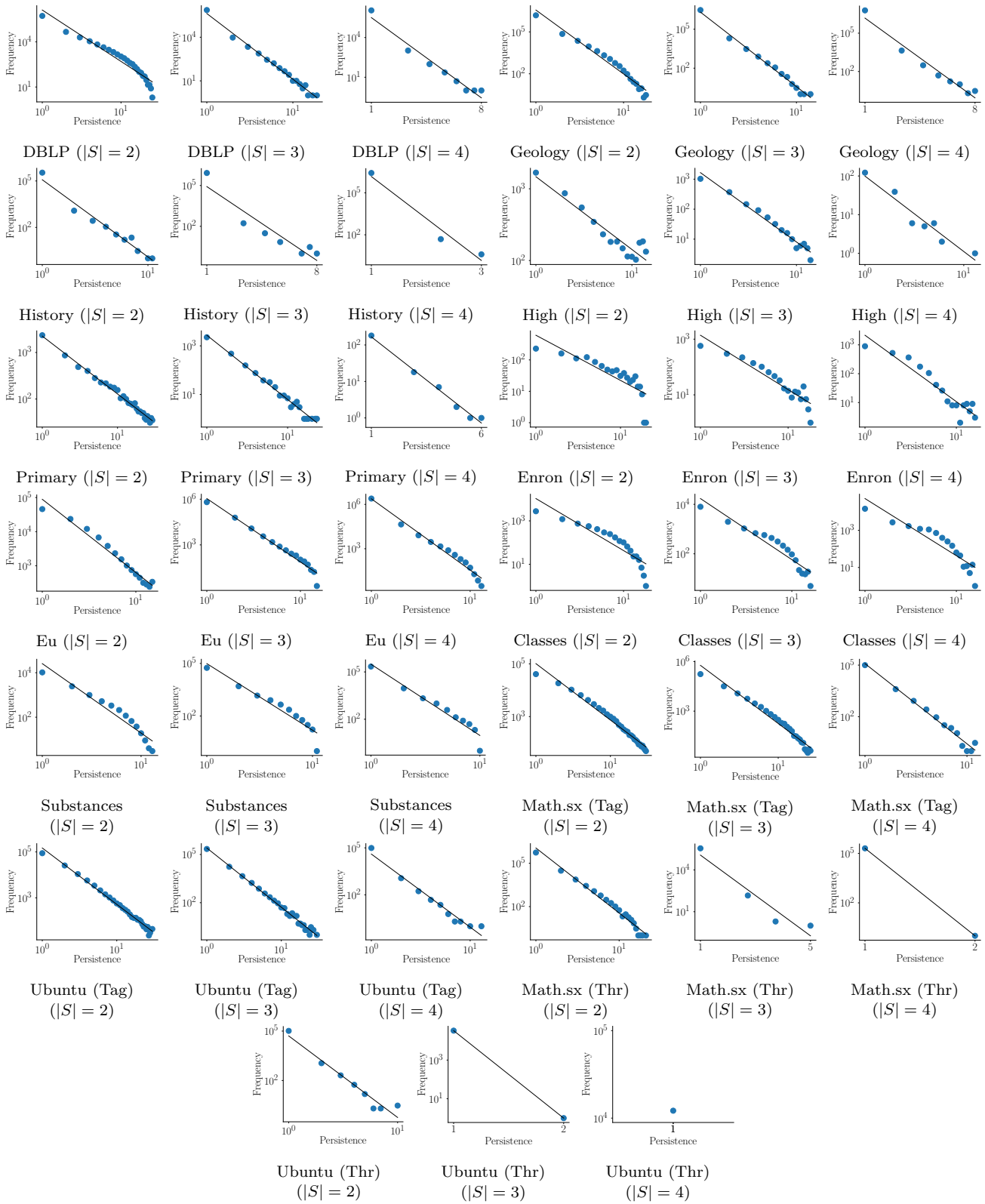


Figure 3: **Distributions of the Persistence of HOIs in Randomized Hypergraphs (Time-Shuffled).**

Table 4: **Features vs. Persistence.** Mutual information (MI) and correlation coefficients (CC) in all 13 real-world hypergraphs. In each case, the first and second most strongly correlated features are in **bold** and underlined, respectively.

DBLP

		Group Features vs. Group Persistence								Node Features vs. Group Persistence								Node Features vs. Node Persistence							
	Size of HOIs	$\#$	$\frac{\#}{\cup}$	$\frac{\Sigma}{\Sigma\cup}$	$\cap$	$\frac{\#}{\cap}$	$\frac{\Sigma}{\cap}$	$\frac{\Sigma}{\#}$	$\mathcal{H}$	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l
MI	2	**0.12**	0.03	0.02	0.01	0.04	0.04	0.04	0.11	0.01	0.01	0.02	0.00	**0.07**	0.04	0.06	0.05	0.07	0.08	0.11	0.04	**0.21**	0.10	0.13	0.12
	3	0.07	0.01	0.01	0.00	0.01	0.02	0.01	**0.07**	0.00	0.00	0.00	0.00	**0.02**	0.02	0.02	0.02	0.05	0.06	0.07	0.03	**0.12**	0.07	0.09	0.09
	4	0.04	0.00	0.00	0.01	0.01	0.01	0.01	**0.05**	0.00	0.00	0.00	0.00	0.01	0.01	0.01	**0.01**	0.02	0.03	0.03	0.02	**0.04**	0.03	0.04	0.04
	Avg.	0.08	0.02	0.01	0.00	0.02	0.02	0.02	**0.08**	0.00	0.00	0.01	0.00	**0.03**	0.02	0.03	0.03	0.05	0.06	0.07	0.03	**0.12**	0.07	0.09	0.09
CC	2	**0.53**	-0.03	-0.04	0.12	0.26	0.34	-0.09	0.38	0.11	0.16	**0.19**	0.02	0.00	0.02	-0.06	-0.16	0.15	0.25	**0.33**	0.07	0.00	0.06	-0.02	-0.09
	3	**0.39**	-0.02	-0.02	0.00	0.20	0.25	-0.06	0.25	0.05	**0.10**	0.10	-0.02	0.00	0.00	-0.05	-0.10	0.06	0.14	**0.15**	0.02	-0.01	0.03	-0.02	-0.09
	4	**0.28**	0.00	0.01	0.03	0.11	0.17	-0.02	0.21	0.02	**0.07**	0.04	0.00	0.00	0.01	-0.03	-0.04	0.03	**0.08**	0.07	0.01	0.00	0.02	-0.01	-0.05
	Avg.	**0.40**	-0.01	-0.02	0.05	0.19	0.26	-0.06	0.28	0.06	**0.11**	0.11	0.00	0.00	0.01	-0.05	-0.10	0.08	0.16	**0.19**	0.03	0.00	0.04	-0.02	-0.07

Geology

		Group Features vs. Group Persistence								Node Features vs. Group Persistence								Node Features vs. Node Persistence							
	Size of HOIs	$\#$	$\frac{\#}{\cup}$	$\frac{\Sigma}{\Sigma\cup}$	$\cap$	$\frac{\#}{\cap}$	$\frac{\Sigma}{\cap}$	$\frac{\Sigma}{\#}$	$\mathcal{H}$	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l
MI	2	0.13	0.03	0.02	0.01	0.03	0.03	0.03	**0.13**	0.01	0.01	0.02	0.01	**0.06**	0.05	0.05	0.05	0.06	0.07	0.11	0.04	**0.16**	0.09	0.11	0.12
	3	0.08	0.01	0.01	0.00	0.01	0.02	0.01	**0.11**	0.00	0.00	0.01	0.00	0.02	0.02	0.02	**0.02**	0.05	0.06	0.08	0.03	0.09	0.07	0.08	**0.11**
	4	0.06	0.00	0.00	0.00	0.01	0.01	0.01	**0.09**	0.00	0.00	0.00	0.00	0.01	0.01	0.01	**0.01**	0.02	0.03	0.03	0.02	0.03	0.03	0.03	**0.04**
	Avg.	0.09	0.01	0.01	0.00	0.02	0.02	0.02	**0.11**	0.00	0.01	0.01	0.00	**0.03**	0.02	0.03	0.03	0.05	0.05	0.07	0.03	**0.10**	0.06	0.08	0.09
CC	2	**0.50**	-0.09	-0.10	0.17	0.12	0.24	-0.04	0.44	0.19	0.21	**0.24**	0.10	-0.01	0.09	0.03	-0.19	0.21	0.27	**0.34**	0.13	0.00	0.13	0.06	-0.08
	3	**0.37**	-0.05	-0.06	0.04	0.10	0.17	-0.05	0.33	0.11	0.15	**0.15**	0.02	-0.01	0.05	0.00	-0.13	0.12	0.17	**0.19**	0.07	0.00	0.08	0.03	-0.12
	4	0.26	-0.03	-0.04	0.01	0.08	0.13	-0.04	**0.27**	0.08	**0.12**	0.11	0.00	-0.01	0.03	0.00	-0.09	0.06	**0.11**	0.10	0.04	0.00	0.05	0.02	-0.06
	Avg.	**0.38**	-0.06	-0.06	0.07	0.10	0.18	-0.04	0.35	0.13	0.16	**0.17**	0.04	-0.01	0.06	0.01	-0.14	0.13	0.19	**0.21**	0.08	0.00	0.09	0.04	-0.09

History

		Group Features vs. Group Persistence								Node Features vs. Group Persistence								Node Features vs. Node Persistence							
	Size of HOIs	$\#$	$\frac{\#}{\cup}$	$\frac{\Sigma}{\Sigma\cup}$	$\cap$	$\frac{\#}{\cap}$	$\frac{\Sigma}{\cap}$	$\frac{\Sigma}{\#}$	$\mathcal{H}$	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l
MI	2	**0.07**	0.01	0.01	0.00	0.01	0.01	0.01	0.07	0.00	0.00	0.01	0.00	0.02	0.01	0.02	**0.02**	0.02	0.02	**0.06**	0.01	0.04	0.03	0.03	0.04
	3	0.05	0.00	0.01	0.00	0.01	0.01	0.00	**0.07**	0.00	0.00	0.00	0.00	**0.01**	0.01	0.01	0.01	0.01	0.02	0.03	0.01	0.02	0.02	0.02	**0.03**
	4	0.03	0.00	0.00	0.00	0.00	0.01	0.00	**0.06**	0.00	0.00	0.00	0.00	**0.01**	0.01	0.01	0.01	0.01	0.02	**0.02**	0.01	0.01	0.01	0.01	0.01
	Avg.	0.05	0.01	0.01	0.00	0.01	0.01	0.01	**0.07**	0.00	0.00	0.01	0.00	**0.01**	0.01	0.01	0.01	0.02	0.02	**0.04**	0.01	0.03	0.02	0.02	0.03
CC	2	0.12	-0.05	-0.05	0.02	0.07	0.07	-0.02	**0.22**	0.08	**0.13**	0.10	-0.01	0.01	0.02	0.00	-0.08	-0.01	0.05	**0.08**	-0.02	-0.01	0.01	0.00	-0.06
	3	0.14	-0.01	-0.02	0.04	0.04	0.07	0.01	**0.21**	0.08	**0.12**	0.07	0.02	-0.01	0.04	0.02	-0.05	0.02	0.08	**0.08**	0.01	0.00	0.03	0.02	-0.01
	4	0.10	0.02	0.01	0.03	0.02	0.04	0.01	**0.17**	0.05	**0.06**	0.03	0.02	-0.02	0.03	0.01	-0.01	0.01	**0.07**	0.06	0.01	0.00	0.03	0.03	0.00
	Avg.	0.12	-0.01	-0.02	0.03	0.04	0.06	0.00	**0.20**	0.07	**0.10**	0.07	0.01	0.00	0.03	0.01	-0.05	0.01	0.06	**0.07**	0.00	0.00	0.02	0.02	-0.02

High

		Group Features vs. Group Persistence								Node Features vs. Group Persistence								Node Features vs. Node Persistence							
	Size of HOIs	$\#$	$\frac{\#}{\cup}$	$\frac{\Sigma}{\Sigma\cup}$	$\cap$	$\frac{\#}{\cap}$	$\frac{\Sigma}{\cap}$	$\frac{\Sigma}{\#}$	$\mathcal{H}$	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l
MI	2	0.02	0.03	**0.03**	0.01	0.02	0.03	0.03	0.03	0.01	0.02	0.02	0.01	**0.03**	0.03	0.03	0.03	0.43	0.61	0.60	0.30	**0.65**	0.63	**0.65**	0.61
	3	0.01	0.01	0.01	0.00	0.01	0.01	0.01	**0.01**	0.00	0.01	0.01	0.00	0.01	0.01	0.01	**0.01**	0.14	0.16	0.15	0.11	0.16	0.16	0.16	**0.16**
	Avg.	0.01	0.02	0.02	0.00	0.01	0.02	0.02	**0.02**	0.01	0.02	0.01	0.01	**0.02**	0.02	0.02	0.02	0.28	0.38	0.38	0.20	**0.41**	0.40	**0.41**	0.38
CC	2	0.08	0.06	0.06	0.05	0.08	**0.08**	-0.01	0.05	0.07	0.09	**0.10**	0.07	0.00	0.07	-0.01	0.05	0.03	-0.01	-0.01	0.03	0.02	0.06	**0.07**	0.02
	3	**0.06**	0.02	0.02	0.01	0.05	0.05	-0.02	0.02	-0.03	0.03	0.03	-0.01	**-0.04**	-0.01	-0.04	0.04	0.02	**0.24**	0.24	0.04	-0.03	0.08	-0.09	0.11
	Avg.	**0.07**	0.04	0.04	0.03	0.06	0.06	-0.02	0.04	0.02	0.06	**0.07**	0.03	-0.02	0.03	-0.02	0.05	0.02	**0.11**	0.11	0.04	0.00	0.07	-0.01	0.06

Primary

		Group Features vs. Group Persistence								Node Features vs. Group Persistence								Node Features vs. Node Persistence							
	Size of HOIs	$\#$	$\frac{\#}{\cup}$	$\frac{\Sigma}{\Sigma\cup}$	$\cap$	$\frac{\#}{\cap}$	$\frac{\Sigma}{\cap}$	$\frac{\Sigma}{\#}$	$\mathcal{H}$	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l
MI	2	0.09	0.15	**0.18**	0.04	0.10	0.11	0.10	0.12	0.03	0.07	0.06	0.04	**0.20**	0.20	0.20	0.20	0.76	0.93	0.93	0.58	**0.99**	0.98	0.99	0.95
	3	0.03	0.04	**0.04**	0.01	0.03	0.03	0.03	0.03	0.02	0.03	0.03	0.01	0.05	0.05	**0.05**	0.05	0.62	0.77	0.76	0.38	**0.83**	0.82	**0.83**	0.82
	Avg.	0.06	0.09	**0.11**	0.02	0.04	0.07	0.04	0.08	0.02	0.05	0.05	0.03	0.13	0.12	**0.13**	0.12	0.69	0.85	0.85	0.48	**0.91**	0.90	0.91	0.88
CC	2	0.34	**0.40**	0.39	0.28	0.34	0.34	0.01	0.21	-0.11	-0.02	-0.01	-0.13	-0.14	-0.17	-0.16	**0.18**	0.31	**0.44**	0.43	0.31	0.17	0.27	-0.11	0.13
	3	**0.14**	0.13	0.13	0.04	0.12	0.12	-0.03	0.04	0.03	0.06	**0.06**	0.05	0.00	0.03	-0.02	0.00	0.05	0.08	0.07	-0.01	0.06	0.00	**-0.13**	0.01
	Avg.	0.24	**0.26**	0.26	0.16	0.23	0.23	-0.01	0.12	-0.04	0.02	0.02	-0.04	-0.07	-0.07	-0.09	**0.09**	0.18	**0.26**	0.25	0.15	0.11	0.13	-0.12	0.07

Enron

		Group Features vs. Group Persistence								Node Features vs. Group Persistence								Node Features vs. Node Persistence							
	Size of HOIs	$\#$	$\frac{\#}{\cup}$	$\frac{\Sigma}{\Sigma\cup}$	$\cap$	$\frac{\#}{\cap}$	$\frac{\Sigma}{\cap}$	$\frac{\Sigma}{\#}$	$\mathcal{H}$	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l
MI	2	0.15	0.31	**0.38**	0.09	0.23	0.28	0.26	0.25	0.10	0.29	0.23	0.08	0.40	0.39	**0.40**	0.38	0.65	0.81	0.75	0.58	0.92	0.88	**0.92**	0.74
	3	0.10	0.14	0.18	0.10	0.16	**0.20**	0.16	0.15	0.06	0.12	0.09	0.09	0.20	0.20	**0.20**	0.20	0.64	0.79	0.75	0.55	0.86	0.84	**0.86**	0.79
	4	0.08	0.07	0.09	0.11	0.13	**0.15**	0.13	0.11	0.04	0.06	0.04	**0.11**	0.10	0.10	0.10	0.10	0.55	0.70	0.69	0.46	0.73	0.72	**0.74**	0.70
	Avg.	0.11	0.17	**0.22**	0.10	0.17	0.21	0.18	0.17	0.07	0.16	0.12	0.09	0.23	0.23	**0.24**	0.23	0.61	0.76	0.73	0.53	0.84	0.81	**0.84**	0.74
CC	2	0.36	0.32	0.24	-0.07	0.29	0.30	-0.33	**0.39**	-0.20	-0.06	0.00	-0.28	0.21	-0.33	**-0.34**	0.01	-0.12	0.24	**0.34**	-0.12	0.19	-0.25	-0.26	0.01
	3	0.34	0.24	0.20	-0.14	0.37	0.32	-0.35	**0.38**	-0.19	0.00	0.04	-0.28	0.24	-0.27	**-0.31**	0.07	-0.21	0.33	**0.37**	-0.18	0.19	-0.20	-0.28	0.12
	4	0.21	0.26	0.23	-0.20	0.28	0.21	-0.33	**0.34**	-0.23	-0.05	-0.05	-0.31	**0.34**	-0.31	-0.30	0.16	-0.22	0.11	0.13	-0.21	0.16	**-0.23**	-0.19	0.12
	Avg.	0.30	0.27	0.22	-0.14	0.31	0.28	-0.34	**0.37**	-0.21	-0.04	0.00	-0.29	0.27	-0.30	**-0.32**	0.08	-0.18	0.22	**0.28**	-0.17	0.18	-0.23	-0.24	0.08

Eu

		Group Features vs. Group Persistence								Node Features vs. Group Persistence								Node Features vs. Node Persistence							
	Size of HOIs	$\#$	$\frac{\#}{\cup}$	$\frac{\Sigma}{\Sigma\cup}$	$\cap$	$\frac{\#}{\cap}$	$\frac{\Sigma}{\cap}$	$\frac{\Sigma}{\#}$	$\mathcal{H}$	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l
MI	2	0.18	0.19	**0.29**	0.05	0.15	0.21	0.18	0.23	0.02	0.12	0.07	0.02	0.31	0.31	**0.31**	0.31	0.68	0.83	0.78	0.53	0.89	0.88	**0.89**	0.85
	3	0.20	0.11	0.20	0.07	0.18	**0.24**	0.17	0.23	0.01	0.05	0.02	0.05	0.23	0.23	**0.23**	0.23	0.67	0.82	0.78	0.51	0.86	0.86	**0.86**	0.85
	4	0.22	0.08	0.15	0.08	0.22	**0.27**	0.19	0.24	0.01	0.02	0.01	0.07	0.18	0.18	**0.18**	0.18	0.61	0.74	0.70	0.44	**0.77**	0.77	**0.77**	0.76
	Avg.	0.20	0.13	0.21	0.07	0.18	**0.24**	0.18	0.23	0.02	0.06	0.03	0.05	0.24	0.24	**0.24**	0.24	0.65	0.80	0.75	0.49	0.84	0.83	**0.84**	0.82
CC	2	0.58	0.43	0.49	0.34	0.18	0.49	0.04	**0.59**	-0.02	0.19	0.07	0.02	-0.01	-0.15	**-0.32**	0.09	0.32	**0.52**	0.33	0.45	0.30	-0.10	-0.38	0.18
	3	**0.66**	0.49	0.58	0.15	0.49	0.64	-0.02	0.46	-0.16	0.16	-0.06	-0.14	-0.15	-0.20	**-0.32**	0.14	0.11	**0.40**	0.11	0.24	0.09	-0.15	-0.31	0.05
	4	**0.68**	0.55	0.61	0.07	0.58	0.66	-0.07	0.41	-0.18	0.16	-0.09	-0.19	-0.18	-0.21	**-0.32**	0.14	0.05	**0.36**	0.06	0.16	0.02	-0.11	-0.27	0.09
	Avg.	**0.64**	0.49	0.56	0.19	0.42	0.59	-0.02	0.49	-0.12	0.17	-0.02	-0.11	-0.11	-0.18	**-0.32**	0.12	0.16	**0.43**	0.16	0.28	0.14	-0.12	-0.32	0.11

Classes

		Group Features vs. Group Persistence								Node Features vs. Group Persistence								Node Features vs. Node Persistence							
	Size of HOIs	$\#$	$\frac{\#}{\cup}$	$\frac{\Sigma}{\Sigma\cup}$	$\cap$	$\frac{\#}{\cap}$	$\frac{\Sigma}{\cap}$	$\frac{\Sigma}{\#}$	$\mathcal{H}$	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l
MI	2	0.31	0.36	0.41	0.21	0.38	**0.43**	0.34	0.31	0.25	0.41	0.35	0.24	0.45	0.41	**0.46**	0.32	0.31	0.51	0.46	0.26	**0.62**	0.47	0.55	0.26
	3	0.32	0.35	0.38	0.27	0.40	**0.42**	0.38	0.34	0.26	0.38	0.32	0.31	0.37	0.36	**0.41**	0.28	0.38	0.59	0.52	0.30	**0.70**	0.60	0.66	0.39
	4	0.32	0.31	0.33	0.33	0.40	**0.43**	0.41	0.33	0.23	0.31	0.25	**0.36**	0.30	0.30	0.33	0.24	0.43	0.62	0.56	0.36	**0.69**	0.63	0.66	0.47
	Avg.	0.32	0.34	0.37	0.27	0.39	**0.43**	0.37	0.32	0.25	0.37	0.31	0.30	0.37	0.36	**0.40**	0.28	0.37	0.57	0.51	0.31	**0.67**	0.56	0.62	0.37
CC	2	0.08	0.15	0.19	-0.08	**0.23**	0.12	-0.15	0.12	-0.19	-0.17	-0.15	-0.10	-0.18	**-0.27**	-0.20	-0.03	0.00	0.05	**0.26**	-0.04	-0.13	-0.16	-0.13	-0.06
	3	0.07	0.18	0.26	-0.24	0.13	0.09	**-0.31**	0.06	-0.36	-0.22	-0.17	-0.27	-0.09	**-0.44**	-0.29	0.10	-0.04	0.03	0.21	-0.11	-0.16	-0.23	-0.15	**-0.29**
	4	0.17	0.28	0.36	-0.40	0.19	0.19	**-0.49**	0.13	-0.47	-0.18	-0.15	-0.45	-0.01	**-0.58**	-0.37	0.17	0.06	0.05	0.21	-0.07	-0.14	-0.19	-0.14	**-0.41**
	Avg.	0.10	0.20	0.27	-0.24	0.18	0.13	**-0.32**	0.10	-0.34	-0.19	-0.15	-0.28	-0.09	**-0.43**	-0.29	0.08	0.01	0.05	0.23	-0.07	-0.14	-0.20	-0.14	**-0.25**

Substances

		Group Features vs. Group Persistence								Node Features vs. Group Persistence								Node Features vs. Node Persistence							
	Size of HOIs	#	$\frac{\#}{\cup}$	$\frac{\Sigma}{\Sigma\cup}$	$\cap$	$\frac{\#}{\cap}$	$\frac{\Sigma}{\cap}$	$\frac{\Sigma}{\#}$	$\mathcal{H}$	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l
MI	2	0.08	0.13	**0.20**	0.09	0.15	0.19	0.15	0.13	0.08	0.14	0.09	0.10	**0.26**	0.25	0.25	0.24	0.35	0.42	0.36	0.31	**0.53**	0.46	0.51	0.40
	3	0.08	0.08	0.11	0.07	0.10	**0.13**	0.09	0.10	0.06	0.08	0.05	0.09	**0.13**	0.13	0.13	0.12	0.30	0.37	0.31	0.28	**0.45**	0.41	0.44	0.34
	4	0.08	0.06	0.07	0.05	0.09	**0.10**	0.07	0.08	0.06	0.05	0.04	**0.08**	0.08	0.08	0.08	0.07	0.24	0.28	0.27	0.23	**0.34**	0.31	0.33	0.23
	Avg.	0.08	0.09	0.13	0.07	0.11	**0.14**	0.10	0.10	0.06	0.09	0.06	0.09	**0.16**	0.15	0.15	0.15	0.30	0.36	0.31	0.27	**0.44**	0.40	0.43	0.32
CC	2	0.08	0.05	0.03	-0.16	0.26	0.18	**-0.32**	0.01	-0.12	-0.17	-0.11	**-0.26**	-0.03	-0.24	-0.19	-0.25	-0.03	-0.01	**0.25**	-0.13	-0.05	-0.07	-0.07	-0.15
	3	0.17	**0.24**	0.23	-0.05	0.21	0.20	-0.22	0.11	-0.21	-0.15	-0.14	-0.24	-0.05	**-0.28**	-0.19	0.07	-0.11	-0.01	**0.22**	-0.16	-0.09	-0.17	-0.10	-0.10
	4	0.21	0.29	**0.30**	0.01	0.21	0.22	-0.15	0.16	-0.26	-0.17	-0.17	-0.25	-0.06	**-0.30**	-0.18	0.23	-0.13	0.02	**0.28**	-0.15	-0.11	-0.18	-0.09	-0.01
	Avg.	0.15	0.19	0.19	-0.06	0.23	0.20	**-0.23**	0.10	-0.20	-0.16	-0.14	-0.25	-0.05	**-0.27**	-0.19	0.02	-0.09	0.00	**0.25**	-0.15	-0.09	-0.14	-0.09	-0.09

Math.sx (Tags)

		Group Features vs. Group Persistence								Node Features vs. Group Persistence								Node Features vs. Node Persistence							
	Size of HOIs	#	$\frac{\#}{\cup}$	$\frac{\Sigma}{\Sigma\cup}$	$\cap$	$\frac{\#}{\cap}$	$\frac{\Sigma}{\cap}$	$\frac{\Sigma}{\#}$	$\mathcal{H}$	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l
MI	2	**0.15**	0.08	0.12	0.05	0.08	0.09	0.07	0.14	0.01	0.06	0.04	0.01	0.20	0.20	**0.20**	0.20	0.50	0.56	0.48	0.46	**0.93**	0.90	0.93	0.61
	3	**0.12**	0.04	0.07	0.03	0.05	0.06	0.05	0.11	0.01	0.03	0.02	0.00	0.11	0.11	**0.11**	0.11	0.51	0.58	0.51	0.47	**0.93**	0.90	0.92	0.64
	4	**0.10**	0.03	0.05	0.03	0.04	0.06	0.04	0.08	0.01	0.04	0.03	0.00	0.06	0.06	**0.06**	0.06	0.51	0.58	0.53	0.45	**0.81**	0.79	0.81	0.65
	Avg.	**0.12**	0.05	0.08	0.04	0.06	0.07	0.05	0.11	0.01	0.04	0.03	0.00	0.12	0.12	**0.12**	0.12	0.51	0.58	0.50	0.46	**0.89**	0.86	0.89	0.63
CC	2	**0.57**	0.06	0.06	0.42	0.36	0.51	-0.11	0.52	0.08	0.07	0.08	-0.06	**0.34**	-0.26	-0.23	-0.18	0.32	0.33	0.31	**0.34**	0.20	0.13	0.03	-0.05
	3	**0.51**	0.03	0.03	0.21	0.26	0.34	-0.07	0.40	0.06	0.08	0.09	-0.02	0.16	-0.15	**-0.17**	-0.10	0.14	**0.19**	0.17	0.18	0.05	0.10	0.01	0.00
	4	**0.36**	0.02	0.02	0.12	0.22	0.25	-0.04	0.29	0.06	0.10	0.11	0.01	0.10	-0.10	**-0.14**	-0.08	0.12	**0.20**	0.18	0.13	0.07	0.02	-0.04	-0.02
	Avg.	**0.48**	0.04	0.03	0.25	0.28	0.37	-0.08	0.40	0.07	0.09	0.09	-0.02	**0.20**	-0.17	-0.18	-0.12	0.19	**0.24**	0.22	0.21	0.11	0.08	0.00	-0.03

Ubuntu (Tags)

		Group Features vs. Group Persistence								Node Features vs. Group Persistence								Node Features vs. Node Persistence							
	Size of HOIs	#	$\frac{\#}{\cup}$	$\frac{\Sigma}{\Sigma\cup}$	$\cap$	$\frac{\#}{\cap}$	$\frac{\Sigma}{\cap}$	$\frac{\Sigma}{\#}$	$\mathcal{H}$	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l
MI	2	**0.17**	0.07	0.09	0.08	0.09	0.10	0.08	0.17	0.03	0.05	0.05	0.01	**0.15**	0.15	0.15	0.15	0.43	0.47	0.41	0.38	**0.83**	0.80	0.82	0.59
	3	**0.16**	0.03	0.05	0.06	0.06	0.07	0.05	0.14	0.01	0.03	0.02	0.01	**0.07**	0.07	0.07	0.07	0.46	0.50	0.45	0.41	**0.74**	0.72	0.73	0.62
	4	**0.18**	0.02	0.03	0.05	0.05	0.08	0.04	0.11	0.01	0.02	0.02	0.01	0.04	0.04	0.04	**0.04**	0.42	0.45	0.42	0.35	0.51	0.50	0.51	**0.52**
	Avg.	**0.17**	0.04	0.06	0.07	0.07	0.08	0.06	0.14	0.02	0.04	0.03	0.01	**0.09**	0.09	0.09	0.09	0.44	0.47	0.43	0.38	**0.69**	0.67	0.69	0.58
CC	2	0.52	-0.03	-0.03	0.55	0.27	0.52	-0.04	**0.56**	0.35	0.36	**0.36**	0.19	0.32	-0.24	-0.21	-0.26	0.35	0.31	0.31	**0.42**	0.05	-0.05	-0.07	-0.11
	3	**0.57**	-0.02	-0.02	0.44	0.18	0.32	-0.01	0.42	0.21	**0.25**	0.25	0.15	0.19	-0.15	-0.17	-0.16	0.20	0.19	0.19	**0.26**	0.07	-0.02	-0.06	-0.12
	4	**0.58**	0.00	0.00	0.26	0.20	0.28	0.00	0.29	0.11	**0.16**	0.15	0.12	0.09	-0.09	-0.13	-0.11	0.14	0.15	0.14	**0.15**	0.12	0.00	-0.02	-0.08
	Avg.	**0.56**	-0.02	-0.02	0.42	0.22	0.37	-0.02	0.42	0.23	**0.25**	0.25	0.15	0.20	-0.16	-0.17	-0.18	0.23	0.22	0.21	**0.28**	0.08	-0.02	-0.05	-0.10

Math.sx (Threads)

		Group Features vs. Group Persistence								Node Features vs. Group Persistence								Node Features vs. Node Persistence							
	Size of HOIs	#	$\frac{\#}{\cup}$	$\frac{\Sigma}{\Sigma\cup}$	$\cap$	$\frac{\#}{\cap}$	$\frac{\Sigma}{\cap}$	$\frac{\Sigma}{\#}$	$\mathcal{H}$	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l
MI	2	**0.11**	0.02	0.02	0.02	0.02	0.03	0.02	0.11	0.01	0.01	0.01	0.01	0.03	0.03	0.03	**0.03**	0.15	0.16	0.19	0.13	0.09	0.10	0.10	**0.21**
	3	**0.09**	0.00	0.00	0.00	0.00	0.00	0.00	0.08	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**0.00**	0.03	0.03	**0.04**	0.02	0.01	0.01	0.01	0.03
	4	0.05	0.00	0.00	0.00	0.00	0.00	0.00	**0.07**	0.00	**0.00**	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**0.00**	0.00	0.00	0.00	0.00	0.00
	Avg.	0.08	0.01	0.01	0.01	0.01	0.01	0.01	**0.09**	0.00	0.00	0.00	0.00	0.01	0.01	0.01	**0.01**	0.06	0.06	0.08	0.05	0.03	0.04	0.04	**0.08**
CC	2	**0.46**	-0.04	-0.04	0.33	0.00	0.08	0.04	0.35	0.23	0.22	0.21	0.19	**0.25**	-0.12	-0.12	-0.12	0.25	0.25	0.25	**0.26**	0.22	-0.01	-0.02	-0.03
	3	**0.24**	-0.01	-0.01	0.02	-0.01	0.00	0.00	0.19	0.08	0.09	0.08	0.04	**0.10**	-0.06	-0.06	-0.04	0.07	0.08	0.07	0.05	**0.09**	-0.01	-0.01	-0.03
	4	**0.18**	0.00	0.00	0.00	0.00	0.00	0.00	0.17	0.01	**0.01**	0.01	0.00	0.01	0.00	-0.01	0.00	0.01	**0.01**	0.01	0.01	0.01	0.00	0.00	0.00
	Avg.	**0.29**	-0.02	-0.02	0.11	0.00	0.03	0.01	0.24	0.11	0.11	0.10	0.08	**0.12**	-0.06	-0.06	-0.06	0.11	**0.11**	0.11	0.11	0.10	-0.01	-0.01	-0.02

Ubuntu (Threads)

		Group Features vs. Group Persistence								Node Features vs. Group Persistence								Node Features vs. Node Persistence							
	Size of HOIs	$\#$	$\frac{\#}{\cup}$	$\frac{\Sigma}{\Sigma\cup}$	$\cap$	$\frac{\#}{\cap}$	$\frac{\Sigma}{\cap}$	$\frac{\Sigma}{\#}$	$\mathcal{H}$	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l
MI	2	0.11	0.01	0.01	0.01	0.01	0.02	0.01	**0.11**	0.01	0.01	0.01	0.01	0.01	0.01	0.01	**0.01**	0.06	0.07	**0.09**	0.05	0.02	0.03	0.03	0.08
	3	0.15	0.00	0.00	0.00	0.00	0.00	0.00	**0.17**	0.00	**0.00**	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	**0.01**	0.01	0.00	0.00	0.00	0.01
	4	**0.12**	0.00	0.00	0.00	0.00	0.00	0.00	0.11	0.00	**0.00**	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**0.00**	0.00	0.00	0.00	0.00	0.00
	Avg.	0.12	0.00	0.00	0.00	0.01	0.01	0.01	**0.13**	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**0.01**	0.03	0.03	**0.03**	0.02	0.01	0.01	0.01	0.03
CC	2	**0.43**	-0.04	-0.04	0.25	-0.01	0.05	0.04	0.29	0.15	**0.16**	0.14	0.15	0.14	-0.02	-0.02	-0.05	0.13	**0.13**	0.13	0.13	0.03	0.02	0.02	0.00
	3	**0.40**	-0.01	-0.01	0.09	-0.02	-0.01	0.03	0.35	0.09	**0.10**	0.09	0.06	0.08	-0.01	-0.01	-0.05	0.11	**0.12**	0.12	0.08	0.09	0.01	0.01	-0.03
	4	0.25	0.00	-0.01	0.02	-0.01	0.00	0.00	**0.25**	0.06	**0.07**	0.06	0.03	0.05	-0.01	-0.01	-0.02	0.18	**0.21**	0.19	0.07	0.13	-0.01	-0.01	-0.03
	Avg.	**0.36**	-0.02	-0.02	0.12	-0.01	0.01	0.02	0.30	0.10	**0.11**	0.10	0.08	0.09	-0.01	-0.02	-0.04	0.14	**0.15**	0.14	0.09	0.08	0.01	0.01	-0.02

Table 5: **Features vs. Persistence in Randomized Hypergraphs (Time-Shuffled).** Mutual information (MI) and correlation coefficients (CC) in all 13 randomized hypergraphs. In each case, the first and second most strongly correlated features are in **bold** and underlined, respectively.

DBLP

		Group Features vs. Group Persistence								Node Features vs. Group Persistence								Node Features vs. Node Persistence							
	Size of HOIs	$\#$	$\frac{\#}{\cup}$	$\frac{\Sigma}{\Sigma\cup}$	$\cap$	$\frac{\#}{\cap}$	$\frac{\Sigma}{\cap}$	$\frac{\Sigma}{\#}$	$\mathcal{H}$	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l
MI	2	**0.17**	0.02	0.01	0.00	0.02	0.03	0.02	0.15	0.00	0.00	0.01	0.00	**0.05**	0.03	0.03	0.04	0.05	0.05	0.05	0.03	**0.22**	0.08	0.07	0.08
	3	**0.14**	0.00	0.00	0.00	0.00	0.01	0.00	0.12	0.00	0.00	0.00	0.00	**0.01**	0.01	0.01	0.01	0.03	0.04	0.03	0.03	**0.12**	0.06	0.06	0.08
	4	**0.09**	0.00	0.00	0.00	0.00	0.00	0.00	0.08	0.00	0.00	0.00	0.00	**0.00**	0.00	0.00	0.00	0.02	0.02	0.02	0.02	0.03	0.03	0.03	**0.04**
	Avg.	**0.13**	0.01	0.00	0.00	0.01	0.01	0.01	0.11	0.00	0.00	0.00	0.00	**0.02**	0.01	0.01	0.01	0.04	0.03	0.03	0.03	**0.13**	0.05	0.05	0.07
CC	2	**0.50**	-0.10	-0.11	0.00	0.11	0.21	-0.08	0.45	0.07	0.08	0.09	0.05	0.00	0.06	0.05	**-0.14**	0.05	0.08	**0.20**	0.00	0.00	0.06	0.05	-0.09
	3	**0.34**	-0.03	-0.05	-0.05	0.08	0.10	-0.06	0.30	0.03	0.04	0.05	-0.02	0.00	0.02	0.01	**-0.07**	0.00	0.02	**0.07**	-0.02	0.00	0.03	0.02	-0.05
	4	**0.19**	-0.01	-0.01	-0.03	0.04	0.04	-0.03	0.18	0.00	0.00	0.01	**-0.02**	0.00	0.00	0.00	-0.02	-0.01	0.00	**0.02**	-0.01	0.00	0.01	0.01	-0.01
	Avg.	**0.34**	-0.05	-0.06	-0.02	0.08	0.11	-0.06	0.31	0.04	0.04	**0.05**	0.00	0.00	0.03	0.02	**-0.08**	0.02	0.03	**0.10**	-0.01	0.00	0.03	0.03	-0.05

Geology

		Group Features vs. Group Persistence								Node Features vs. Group Persistence								Node Features vs. Node Persistence							
	Size of HOIs	$\#$	$\frac{\#}{\cup}$	$\frac{\Sigma}{\Sigma\cup}$	$\cap$	$\frac{\#}{\cap}$	$\frac{\Sigma}{\cap}$	$\frac{\Sigma}{\#}$	$\mathcal{H}$	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l
MI	2	**0.12**	0.01	0.00	0.00	0.01	0.01	0.01	0.11	0.00	0.00	0.00	0.00	**0.02**	0.01	0.01	0.02	0.02	0.02	0.03	0.02	**0.12**	0.05	0.05	0.06
	3	**0.11**	0.00	0.00	0.00	0.00	0.00	0.00	0.11	0.00	0.00	0.00	0.00	**0.00**	0.00	0.00	0.00	0.03	0.03	0.03	0.02	0.07	0.04	0.04	**0.07**
	4	**0.13**	0.00	0.00	0.00	0.00	0.00	0.00	0.12	0.00	0.00	0.00	0.00	**0.00**	0.00	0.00	0.00	0.02	0.02	0.03	0.02	0.03	0.02	0.02	**0.05**
	Avg.	**0.12**	0.00	0.00	0.00	0.00	0.00	0.00	0.11	0.00	0.00	0.00	0.00	**0.01**	0.01	0.01	0.01	0.02	0.02	0.03	0.02	**0.07**	0.04	0.04	0.06
CC	2	**0.33**	-0.08	-0.09	-0.02	0.06	0.10	-0.06	0.31	0.06	0.07	0.09	0.00	-0.01	0.03	0.03	**-0.12**	0.00	0.01	**0.11**	-0.02	0.01	0.02	0.02	-0.05
	3	**0.25**	-0.03	-0.04	-0.03	0.05	0.06	-0.04	0.23	0.03	0.03	0.05	-0.02	0.00	0.01	0.00	**-0.06**	-0.01	-0.01	**0.04**	-0.02	0.00	0.01	0.01	-0.03
	4	**0.21**	-0.01	-0.01	-0.02	0.03	0.03	-0.02	0.21	0.01	0.01	0.02	-0.01	0.00	0.00	0.00	**-0.02**	-0.01	-0.01	**0.01**	-0.01	0.00	0.01	0.01	-0.01
	Avg.	**0.27**	-0.04	-0.05	-0.02	0.05	0.06	-0.04	0.25	0.03	0.04	**0.05**	-0.01	0.00	0.01	0.01	**-0.06**	-0.01	0.00	**0.06**	-0.02	0.00	0.01	0.01	-0.03

History

		Group Features vs. Group Persistence								Node Features vs. Group Persistence								Node Features vs. Node Persistence							
	Size of HOIs	$\#$	$\frac{\#}{\cup}$	$\frac{\Sigma}{\Sigma\cup}$	$\cap$	$\frac{\#}{\cap}$	$\frac{\Sigma}{\cap}$	$\frac{\Sigma}{\#}$	$\mathcal{H}$	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l
MI	2	**0.15**	0.00	0.00	0.00	0.00	0.00	0.00	0.14	0.00	0.00	0.00	0.00	**0.00**	0.00	0.00	0.00	0.01	0.01	**0.02**	0.01	0.01	0.01	0.01	0.01
	3	**0.19**	0.00	0.00	0.00	0.00	0.00	0.00	0.17	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**0.00**	0.00	0.00	0.01	0.00	0.00	0.00	0.00	**0.01**
	4	**0.38**	0.00	0.00	0.00	0.00	0.00	0.00	0.36	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**0.00**	0.00	0.00	**0.01**	0.00	0.00	0.00	0.00	0.01
	Avg.	**0.24**	0.00	0.00	0.00	0.00	0.00	0.00	0.22	0.00	0.00	0.00	0.00	**0.00**	0.00	0.00	0.00	0.00	0.00	**0.01**	0.00	0.01	0.01	0.01	0.01
CC	2	**0.32**	-0.02	-0.02	-0.02	0.03	0.04	-0.03	0.29	0.00	0.00	0.01	-0.02	-0.01	-0.01	-0.01	**-0.02**	-0.01	-0.01	**0.05**	-0.01	-0.01	0.00	0.00	-0.01
	3	**0.35**	-0.01	-0.01	-0.01	0.02	0.02	-0.01	0.31	0.00	0.00	0.00	-0.01	0.00	-0.01	-0.01	**-0.01**	-0.01	-0.01	**0.03**	-0.01	0.00	-0.01	-0.01	-0.02
	4	**0.49**	0.00	0.00	0.00	0.01	0.01	-0.01	0.46	0.00	0.00	0.00	**-0.01**	0.00	0.00	0.00	-0.01	-0.01	-0.01	**0.02**	-0.01	0.00	-0.01	-0.01	-0.01
	Avg.	**0.39**	-0.01	-0.01	-0.01	0.02	0.02	-0.02	0.36	0.00	0.00	**0.01**	-0.01	0.00	-0.01	-0.01	**-0.01**	-0.01	-0.01	**0.03**	-0.01	-0.01	0.00	-0.01	-0.01

High

		Group Features vs. Group Persistence								Node Features vs. Group Persistence								Node Features vs. Node Persistence							
	Size of HOIs	$\#$	$\frac{\#}{\cup}$	$\frac{\Sigma}{\Sigma\cup}$	$\cap$	$\frac{\#}{\cap}$	$\frac{\Sigma}{\cap}$	$\frac{\Sigma}{\#}$	$\mathcal{H}$	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l
MI	2	0.01	0.03	**0.04**	0.01	0.01	0.01	0.01	0.01	0.01	0.03	0.03	0.02	**0.06**	0.05	0.06	0.05	0.55	0.79	0.77	0.30	**0.84**	0.82	**0.84**	0.81
	3	0.03	0.04	0.04	0.01	0.02	0.03	0.03	**0.04**	0.02	0.03	0.04	0.02	0.05	**0.05**	0.05	0.05	0.41	0.55	0.55	0.24	**0.59**	0.58	**0.59**	0.57
	4	**0.18**	0.05	0.04	0.01	0.12	0.11	0.01	0.00	0.05	0.04	**0.05**	0.04	0.05	0.05	0.05	0.05	0.15	0.17	0.17	0.12	0.18	**0.18**	0.18	0.17
	Avg.	**0.07**	0.04	0.04	0.01	0.05	0.05	0.02	0.02	0.02	0.04	0.04	0.03	**0.05**	0.05	0.05	0.05	0.37	0.50	0.50	0.22	**0.54**	0.53	**0.54**	0.52
CC	2	-0.04	-0.01	-0.01	0.00	-0.04	-0.04	**0.06**	0.02	-0.08	-0.05	-0.05	-0.11	0.03	**-0.12**	-0.05	-0.06	-0.01	-0.03	-0.03	-0.07	-0.01	-0.04	**0.08**	-0.03
	3	0.06	0.01	0.02	0.08	0.03	0.04	0.05	**0.12**	**-0.03**	-0.01	-0.01	-0.02	-0.03	-0.02	0.00	0.00	-0.04	-0.04	-0.05	**-0.07**	-0.03	-0.06	-0.02	-0.06
	4	0.31	0.02	0.02	-0.05	0.31	**0.31**	-0.05	-0.01	0.06	0.03	0.03	0.01	0.05	0.02	0.09	**-0.10**	0.07	0.01	0.01	0.03	0.05	0.05	0.07	**-0.10**
	Avg.	**0.11**	0.01	0.01	0.01	0.10	0.10	0.02	0.04	-0.02	-0.01	-0.01	-0.04	**0.02**	-0.04	0.01	**-0.05**	0.01	-0.02	-0.02	-0.04	0.00	-0.02	0.04	**-0.06**

Primary

		Group Features vs. Group Persistence								Node Features vs. Group Persistence								Node Features vs. Node Persistence							
	Size of HOIs	$\#$	$\frac{\#}{\cup}$	$\frac{\Sigma}{\Sigma\cup}$	$\cap$	$\frac{\#}{\cap}$	$\frac{\Sigma}{\cap}$	$\frac{\Sigma}{\#}$	$\mathcal{H}$	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l
MI	2	**0.29**	0.21	0.26	0.10	0.22	0.24	0.18	0.21	0.02	0.08	0.07	0.05	**0.32**	0.31	0.32	0.32	0.78	0.92	0.91	0.34	**0.99**	0.98	0.99	0.98
	3	**0.15**	0.08	0.09	0.02	0.10	0.12	0.06	0.09	0.03	0.06	0.05	0.02	0.13	0.13	0.13	**0.13**	0.74	0.89	0.87	0.41	**0.95**	0.95	**0.95**	0.95
	4	**0.13**	0.10	0.10	0.01	0.12	0.11	0.01	0.00	0.07	0.10	0.09	0.04	**0.10**	**0.10**	**0.10**	**0.10**	0.33	0.42	0.41	0.16	**0.45**	0.45	**0.45**	0.45
	Avg.	**0.19**	0.13	0.15	0.04	0.14	0.16	0.08	0.10	0.04	0.08	0.07	0.04	**0.18**	0.18	0.18	0.18	0.62	0.74	0.73	0.30	**0.80**	0.79	0.80	0.79
CC	2	**0.75**	0.63	0.64	0.53	0.60	0.64	0.05	0.51	0.01	0.10	0.10	0.07	-0.02	0.08	-0.08	**0.13**	0.15	0.40	**0.42**	0.32	0.11	0.24	-0.39	0.33
	3	**0.58**	0.10	0.10	0.16	0.44	0.48	0.04	0.28	0.12	**0.19**	0.18	0.12	0.07	0.13	-0.02	-0.03	0.39	**0.55**	0.51	0.19	0.41	0.19	-0.33	-0.30
	4	0.32	0.00	-0.01	-0.08	0.37	**0.38**	-0.08	-0.02	0.05	0.12	0.12	0.12	-0.03	**0.14**	0.03	0.06	0.06	0.12	0.11	0.13	0.03	**0.14**	-0.04	-0.01
	Avg.	**0.55**	0.24	0.24	0.20	0.47	0.50	0.01	0.25	0.06	**0.14**	0.13	0.10	0.00	0.11	-0.02	0.05	0.20	**0.35**	0.34	0.21	0.19	0.19	-0.25	0.01

Enron

		Group Features vs. Group Persistence								Node Features vs. Group Persistence								Node Features vs. Node Persistence							
	Size of HOIs	$\#$	$\frac{\#}{\cup}$	$\frac{\Sigma}{\Sigma\cup}$	$\cap$	$\frac{\#}{\cap}$	$\frac{\Sigma}{\cap}$	$\frac{\Sigma}{\#}$	$\mathcal{H}$	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l
MI	2	0.16	0.26	**0.35**	0.07	0.15	0.21	0.16	0.16	0.09	0.22	0.17	0.06	**0.41**	0.40	0.41	0.38	0.57	0.65	0.60	0.48	**0.97**	0.87	0.91	0.63
	3	0.18	0.20	**0.28**	0.12	0.16	0.23	0.19	0.20	0.08	0.17	0.14	0.11	0.31	0.31	**0.31**	0.31	0.66	0.75	0.70	0.55	**0.91**	0.88	0.90	0.82
	4	0.15	0.16	0.24	0.17	0.19	**0.27**	0.21	0.21	0.07	0.14	0.11	0.13	0.26	0.26	**0.26**	0.26	0.66	0.75	0.73	0.51	**0.87**	0.85	0.87	0.82
	Avg.	0.17	0.20	**0.29**	0.12	0.16	0.24	0.18	0.19	0.08	0.18	0.14	0.10	0.32	0.32	**0.33**	0.32	0.63	0.72	0.68	0.52	**0.92**	0.87	0.89	0.76
CC	2	**0.64**	0.33	0.39	0.21	0.26	0.42	-0.02	0.52	-0.17	-0.18	-0.19	-0.25	0.28	-0.39	**-0.41**	0.14	0.08	0.24	0.27	0.03	**0.43**	-0.28	-0.39	0.19
	3	**0.57**	0.50	0.55	0.24	0.18	0.39	0.05	0.53	-0.22	-0.17	-0.21	-0.25	0.06	-0.35	**-0.41**	0.35	0.05	0.27	0.09	0.01	0.23	-0.34	**-0.46**	0.27
	4	0.45	0.59	**0.60**	0.15	0.20	0.35	0.00	0.47	-0.28	-0.23	-0.25	-0.27	-0.03	-0.38	**-0.40**	0.38	-0.07	0.05	-0.10	-0.10	0.11	-0.38	**-0.42**	0.38
	Avg.	**0.55**	0.48	0.52	0.20	0.21	0.38	0.01	0.51	-0.23	-0.19	-0.22	-0.26	0.10	-0.37	**-0.41**	**0.29**	0.02	0.19	0.08	-0.02	0.26	-0.33	**-0.43**	0.28

Eu

		Group Features vs. Group Persistence								Node Features vs. Group Persistence								Node Features vs. Node Persistence							
	Size of HOIs	$\#$	$\frac{\#}{\cup}$	$\frac{\Sigma}{\Sigma\cup}$	$\cap$	$\frac{\#}{\cap}$	$\frac{\Sigma}{\cap}$	$\frac{\Sigma}{\#}$	$\mathcal{H}$	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l
MI	2	**0.17**	0.07	0.14	0.03	0.06	0.07	0.06	0.16	0.06	0.06	0.05	0.05	**0.22**	0.22	0.22	0.22	0.84	0.86	0.82	0.69	**0.94**	0.94	0.94	0.93
	3	**0.13**	0.01	0.02	0.01	0.01	0.02	0.01	0.12	0.01	0.01	0.01	0.01	**0.05**	0.05	0.05	0.05	0.85	0.88	0.84	0.68	0.92	0.92	0.92	**0.94**
	4	**0.19**	0.00	0.01	0.01	0.01	0.01	0.01	0.19	0.00	0.00	0.00	0.00	0.01	0.01	0.01	**0.01**	0.82	0.85	0.82	0.64	0.88	0.88	0.88	**0.90**
	Avg.	**0.16**	0.03	0.06	0.01	0.03	0.04	0.03	0.16	0.02	0.02	0.02	0.02	**0.09**	0.09	0.09	0.09	0.84	0.86	0.83	0.67	0.91	0.91	0.91	**0.92**
CC	2	0.51	0.21	0.11	0.38	0.07	0.23	-0.12	**0.59**	0.46	0.43	0.44	0.22	**0.51**	-0.38	-0.46	-0.42	0.84	0.82	0.81	0.70	**0.84**	-0.17	-0.51	-0.25
	3	**0.50**	0.10	0.06	0.12	0.07	0.17	-0.07	0.43	0.14	0.15	0.16	0.04	0.15	-0.13	**-0.20**	-0.14	0.51	0.54	**0.54**	0.46	0.51	-0.29	-0.52	-0.35
	4	**0.53**	0.15	0.12	0.07	0.06	0.19	-0.02	0.47	-0.01	0.02	0.02	-0.01	-0.01	-0.02	**-0.08**	0.00	0.08	0.10	0.11	0.10	0.08	-0.19	**-0.26**	-0.06
	Avg.	**0.51**	0.15	0.10	0.19	0.06	0.19	-0.07	0.49	0.20	0.20	0.21	0.08	**0.22**	-0.18	**-0.25**	-0.19	0.47	**0.49**	0.48	0.42	0.47	-0.22	-0.43	-0.22

Classes

		Group Features vs. Group Persistence								Node Features vs. Group Persistence								Node Features vs. Node Persistence							
	Size of HOIs	$\#$	$\frac{\#}{\cup}$	$\frac{\Sigma}{\Sigma\cup}$	$\cap$	$\frac{\#}{\cap}$	$\frac{\Sigma}{\cap}$	$\frac{\Sigma}{\#}$	$\mathcal{H}$	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l
MI	2	0.13	0.10	**0.17**	0.03	0.07	0.09	0.07	0.14	0.04	0.08	0.05	0.03	**0.24**	0.24	0.24	0.23	0.43	0.48	0.36	0.38	**0.78**	0.70	0.73	0.51
	3	0.14	0.06	0.11	0.03	0.06	0.09	0.06	**0.15**	0.03	0.06	0.04	0.03	0.16	0.16	**0.16**	0.16	0.50	0.56	0.48	0.43	**0.77**	0.73	0.75	0.63
	4	**0.17**	0.05	0.09	0.05	0.07	0.11	0.08	0.17	0.03	0.06	0.04	0.06	0.11	0.11	**0.12**	0.11	0.51	0.57	0.51	0.44	**0.73**	0.70	0.71	0.64
	Avg.	0.15	0.07	0.12	0.04	0.07	0.10	0.07	**0.15**	0.03	0.06	0.04	0.04	0.17	0.17	**0.18**	0.17	0.48	0.53	0.45	0.42	**0.76**	0.71	0.73	0.59
CC	2	**0.54**	0.12	0.12	0.27	0.17	0.42	0.01	0.49	0.12	0.11	**0.12**	0.05	0.05	-0.07	-0.05	-0.03	0.04	**0.16**	0.14	0.01	0.06	-0.07	0.02	0.00
	3	**0.51**	0.04	0.02	0.13	0.19	0.40	-0.04	0.45	-0.02	0.18	0.12	-0.05	0.06	**-0.18**	-0.03	0.09	0.02	**0.11**	0.06	0.00	0.06	-0.02	0.10	0.04
	4	**0.47**	0.01	-0.02	0.09	0.20	0.39	-0.04	0.43	-0.18	0.31	0.20	-0.13	-0.01	**-0.31**	-0.01	0.27	-0.03	0.03	-0.02	-0.02	0.05	-0.02	**0.17**	0.12
	Avg.	**0.51**	0.06	0.04	0.16	0.19	0.40	-0.03	0.45	-0.03	**0.20**	0.15	-0.05	0.03	-0.19	-0.03	0.11	0.01	**0.10**	0.06	-0.01	0.06	-0.04	0.10	0.05

Substances

		Group Features vs. Group Persistence								Node Features vs. Group Persistence								Node Features vs. Node Persistence							
	Size of HOIs	#	$\frac{\#}{\cup}$	$\frac{\Sigma}{\Sigma\cup}$	$\cap$	$\frac{\#}{\cap}$	$\frac{\Sigma}{\cap}$	$\frac{\Sigma}{\#}$	$\mathcal{H}$	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l
MI	2	0.11	0.07	**0.12**	0.03	0.05	0.07	0.05	0.10	0.05	0.06	0.06	0.03	0.19	0.19	0.19	**0.19**	0.43	0.43	0.25	0.36	**0.64**	0.60	0.61	0.53
	3	**0.10**	0.03	0.05	0.03	0.03	0.04	0.03	0.09	0.02	0.03	0.03	0.02	0.09	0.09	0.09	**0.09**	0.46	0.47	0.31	0.40	**0.60**	0.59	0.59	0.56
	4	**0.07**	0.01	0.02	0.02	0.03	0.03	0.03	0.06	0.01	0.01	0.01	0.02	0.05	0.05	**0.05**	0.05	0.44	0.45	0.35	0.38	0.54	0.53	0.53	**0.55**
	Avg.	**0.09**	0.04	0.07	0.03	0.04	0.05	0.04	0.09	0.03	0.03	0.03	0.02	0.11	0.11	0.11	**0.11**	0.45	0.45	0.30	0.38	**0.59**	0.57	0.58	0.55
CC	2	0.45	-0.21	-0.24	0.13	0.04	0.18	-0.08	**0.45**	0.37	**0.41**	0.38	0.18	0.00	0.19	0.10	-0.29	0.08	0.12	**0.23**	0.02	0.03	0.09	0.08	-0.05
	3	**0.40**	-0.11	-0.15	-0.01	0.07	0.15	-0.12	0.39	0.22	**0.32**	0.29	0.07	-0.03	0.04	-0.04	-0.18	-0.01	0.02	**0.09**	-0.05	0.00	-0.01	0.02	-0.04
	4	**0.31**	-0.06	-0.10	-0.09	0.10	0.14	-0.14	0.28	0.11	**0.20**	0.18	0.00	-0.05	-0.02	-0.07	-0.11	-0.03	-0.01	0.04	-0.07	**0.07**	-0.05	-0.03	-0.02
	Avg.	**0.39**	-0.13	-0.16	0.01	0.07	0.16	-0.11	0.37	0.23	**0.31**	0.28	0.08	-0.03	0.07	0.00	-0.19	0.01	0.04	**0.12**	-0.04	0.03	0.01	0.03	-0.04

Math.sx (Tags)

		Group Features vs. Group Persistence								Node Features vs. Group Persistence								Node Features vs. Node Persistence							
	Size of HOIs	#	$\frac{\#}{\cup}$	$\frac{\Sigma}{\Sigma\cup}$	$\cap$	$\frac{\#}{\cap}$	$\frac{\Sigma}{\cap}$	$\frac{\Sigma}{\#}$	$\mathcal{H}$	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l
MI	2	**0.14**	0.05	0.08	0.05	0.05	0.06	0.05	0.12	0.02	0.04	0.03	0.01	0.15	0.15	**0.15**	0.15	0.53	0.55	0.48	0.48	**0.91**	0.88	0.90	0.68
	3	**0.11**	0.02	0.04	0.02	0.03	0.04	0.02	0.08	0.01	0.02	0.01	0.01	0.06	**0.06**	0.06	0.06	0.55	0.57	0.52	0.50	**0.80**	0.79	0.79	0.71
	4	**0.11**	0.01	0.01	0.01	0.01	0.02	0.01	0.06	0.01	0.01	0.01	0.00	0.02	0.02	0.02	**0.02**	0.48	0.50	0.51	0.41	0.45	0.45	0.45	**0.55**
	Avg.	**0.12**	0.03	0.04	0.03	0.03	0.04	0.03	0.09	0.01	0.02	0.02	0.01	0.08	0.08	**0.08**	0.08	0.52	0.54	0.50	0.46	**0.72**	0.71	0.71	0.65
CC	2	**0.58**	-0.02	-0.02	0.39	0.14	0.33	-0.04	0.47	0.19	0.09	0.09	-0.04	**0.52**	-0.41	-0.34	-0.45	**0.82**	0.75	0.75	0.81	0.82	-0.41	-0.34	-0.35
	3	**0.40**	-0.03	-0.03	0.12	0.09	0.14	-0.03	0.31	0.19	0.12	0.12	0.02	**0.31**	-0.26	-0.22	-0.29	0.77	0.69	0.69	**0.79**	0.78	-0.42	-0.34	-0.62
	4	**0.27**	-0.02	-0.03	0.03	0.05	0.07	-0.01	0.17	0.14	0.13	0.13	0.05	**0.16**	-0.13	-0.11	-0.15	0.69	0.66	0.66	0.58	**0.69**	-0.33	-0.27	-0.45
	Avg.	**0.42**	-0.02	-0.03	0.18	0.09	0.18	-0.03	0.32	0.17	0.11	0.11	0.01	**0.33**	-0.27	-0.22	-0.30	**0.76**	0.70	0.70	0.73	0.76	-0.38	-0.32	-0.47

Ubuntu (Tags)

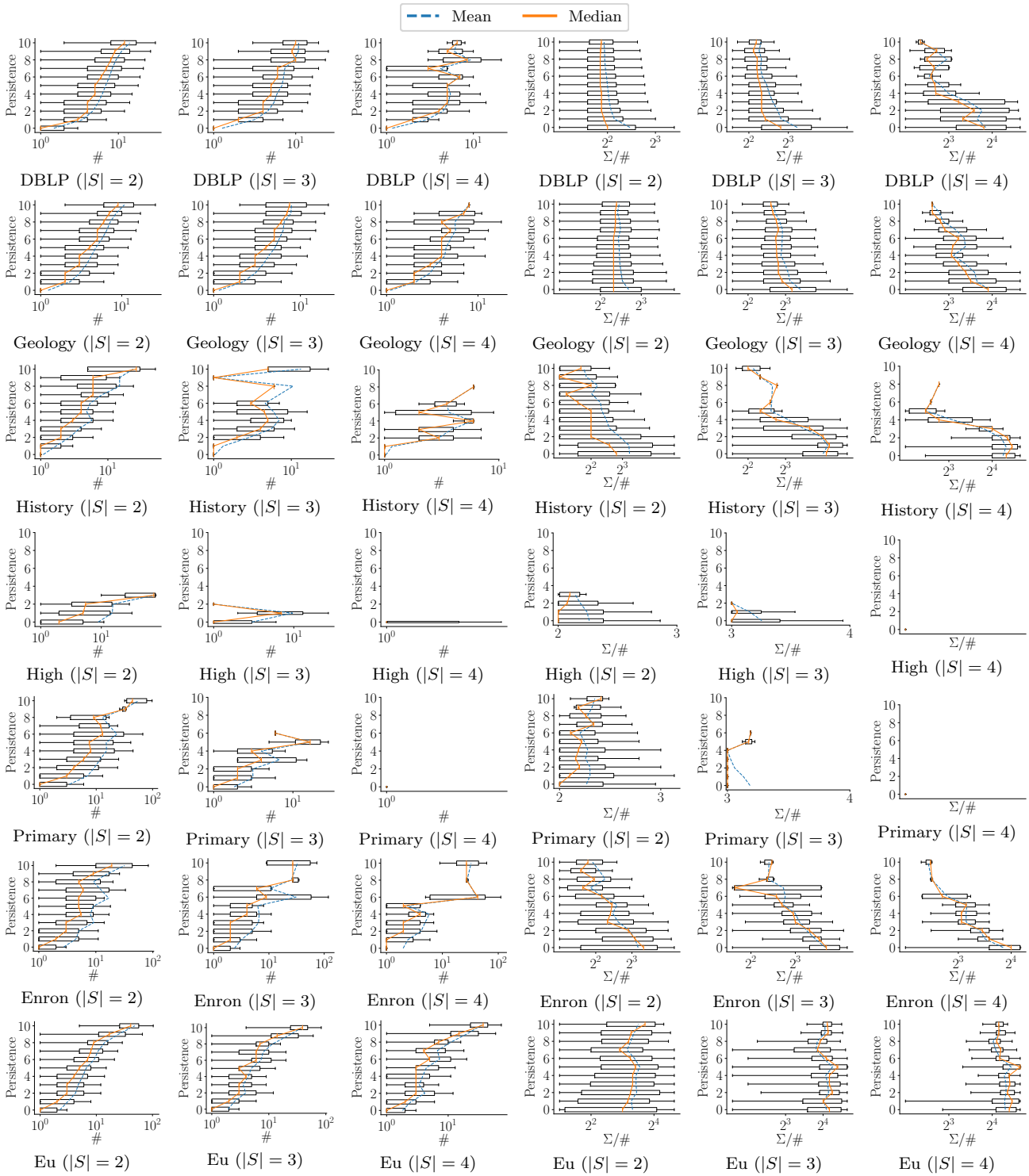
		Group Features vs. Group Persistence								Node Features vs. Group Persistence								Node Features vs. Node Persistence							
	Size of HOIs	#	$\frac{\#}{\cup}$	$\frac{\Sigma}{\Sigma\cup}$	$\cap$	$\frac{\#}{\cap}$	$\frac{\Sigma}{\cap}$	$\frac{\Sigma}{\#}$	$\mathcal{H}$	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l
MI	2	**0.15**	0.03	0.05	0.05	0.05	0.06	0.05	0.13	0.02	0.03	0.02	0.02	**0.11**	0.11	0.11	0.11	0.45	0.47	0.41	0.40	**0.78**	0.76	0.77	0.61
	3	**0.11**	0.01	0.02	0.02	0.02	0.02	0.02	0.08	0.01	0.01	0.01	0.01	**0.03**	0.03	0.03	0.03	0.50	0.51	0.49	0.43	0.55	0.54	0.55	**0.62**
	4	**0.11**	0.00	0.00	0.01	0.01	0.01	0.00	0.06	0.00	0.00	0.00	0.00	0.01	0.01	0.01	**0.01**	0.33	0.33	**0.37**	0.26	0.21	0.21	0.21	0.32
	Avg.	**0.12**	0.02	0.02	0.03	0.02	0.03	0.02	0.09	0.01	0.01	0.01	0.01	**0.05**	0.05	0.05	0.05	0.43	0.44	0.42	0.36	0.51	0.50	0.51	**0.52**
CC	2	**0.59**	-0.06	-0.06	0.43	0.08	0.28	-0.02	0.47	0.25	0.18	0.18	0.14	0.29	-0.33	-0.31	**-0.34**	0.76	0.68	0.68	**0.77**	0.24	-0.28	-0.28	-0.40
	3	**0.38**	-0.03	-0.03	0.10	0.04	0.08	-0.01	0.27	0.16	0.14	0.14	0.09	0.18	**-0.19**	-0.18	-0.19	**0.62**	0.58	0.58	0.60	0.17	-0.24	-0.24	-0.40
	4	**0.24**	-0.01	-0.01	0.01	0.02	0.03	-0.01	0.17	0.07	0.07	0.07	0.04	0.08	-0.08	**-0.08**	-0.08	0.22	**0.22**	0.22	0.18	0.07	-0.08	-0.08	-0.12
	Avg.	**0.40**	-0.03	-0.03	0.18	0.05	0.13	-0.01	0.30	0.16	0.13	0.13	0.09	**0.18**	-0.20	-0.19	**-0.20**	**0.53**	0.49	0.49	0.52	0.16	-0.20	-0.20	-0.30

Math.sx (Threads)

		Group Features vs. Group Persistence								Node Features vs. Group Persistence								Node Features vs. Node Persistence							
	Size of HOIs	#	$\frac{\#}{\cup}$	$\frac{\Sigma}{\Sigma\cup}$	$\cap$	$\frac{\#}{\cap}$	$\frac{\Sigma}{\cap}$	$\frac{\Sigma}{\#}$	$\mathcal{H}$	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l
MI	2	**0.11**	0.01	0.01	0.00	0.01	0.01	0.01	0.09	0.01	0.01	0.01	0.01	0.02	0.02	0.02	**0.02**	0.06	0.06	**0.14**	0.03	0.08	0.06	0.06	0.11
	3	**0.14**	0.00	0.00	0.00	0.00	0.00	0.00	0.11	0.00	0.00	**0.00**	0.00	0.00	0.00	0.00	0.00	0.01	0.01	**0.02**	0.00	0.00	0.00	0.00	0.02
	4	**1.00**	0.00	0.00	0.00	0.00	0.00	0.00	**1.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**
	Avg.	**0.41**	0.00	0.00	0.00	0.00	0.00	0.00	0.40	0.00	0.00	0.00	0.00	0.01	0.01	0.01	**0.01**	0.02	0.02	**0.05**	0.01	0.03	0.02	0.02	0.04
CC	2	**0.30**	-0.07	-0.07	0.03	0.02	0.06	-0.01	0.26	0.18	0.17	0.17	0.18	**0.21**	-0.03	-0.03	-0.12	0.13	0.13	0.18	0.04	**0.20**	0.00	0.00	-0.02
	3	**0.21**	0.00	0.00	0.00	0.00	0.00	0.00	0.17	0.04	0.04	0.04	0.02	**0.04**	0.00	-0.01	-0.02	0.01	0.02	0.02	0.00	**0.02**	0.02	0.01	-0.01
	Avg.	**0.25**	-0.04	-0.04	0.02	0.01	0.03	0.00	0.22	0.11	0.10	0.10	0.10	**0.12**	-0.02	-0.02	-0.07	0.07	0.07	0.10	0.02	**0.11**	0.01	0.01	-0.01

Ubuntu (Threads)

		Group Features vs. Group Persistence								Node Features vs. Group Persistence								Node Features vs. Node Persistence							
	Size of HOIs	$\#$	$\frac{\#}{\cup}$	$\frac{\Sigma}{\Sigma\cup}$	$\cap$	$\frac{\#}{\cap}$	$\frac{\Sigma}{\cap}$	$\frac{\Sigma}{\#}$	$\mathcal{H}$	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l
MI	2	**0.10**	0.01	0.01	0.00	0.00	0.00	0.00	0.07	0.00	0.00	0.00	0.01	0.01	0.01	0.01	**0.01**	0.03	0.03	**0.12**	0.01	0.01	0.02	0.02	0.05
	3	0.00	**0.00**	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**0.00**	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**0.00**	0.00	0.00	0.00	0.00	0.00
	4	**1.00**	0.00	0.00	0.00	0.00	0.00	0.00	**1.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**
	Avg.	**0.37**	0.00	0.00	0.00	0.00	0.00	0.00	0.36	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**0.00**	0.01	0.01	**0.04**	0.00	0.00	0.01	0.01	0.02
CC	2	**0.21**	-0.06	-0.07	0.01	0.01	0.03	0.00	0.17	0.14	**0.14**	0.14	0.10	0.04	0.01	0.01	-0.05	0.03	0.03	**0.05**	0.00	0.00	0.01	0.01	0.00
	Avg.	**0.21**	-0.03	-0.03	0.01	0.00	0.01	0.00	0.17	0.08	**0.08**	0.08	0.05	0.02	0.00	0.00	-0.03	0.08	0.08	**0.11**	0.00	0.00	0.00	0.00	-0.02


Mean
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Eu (|S| = 4)

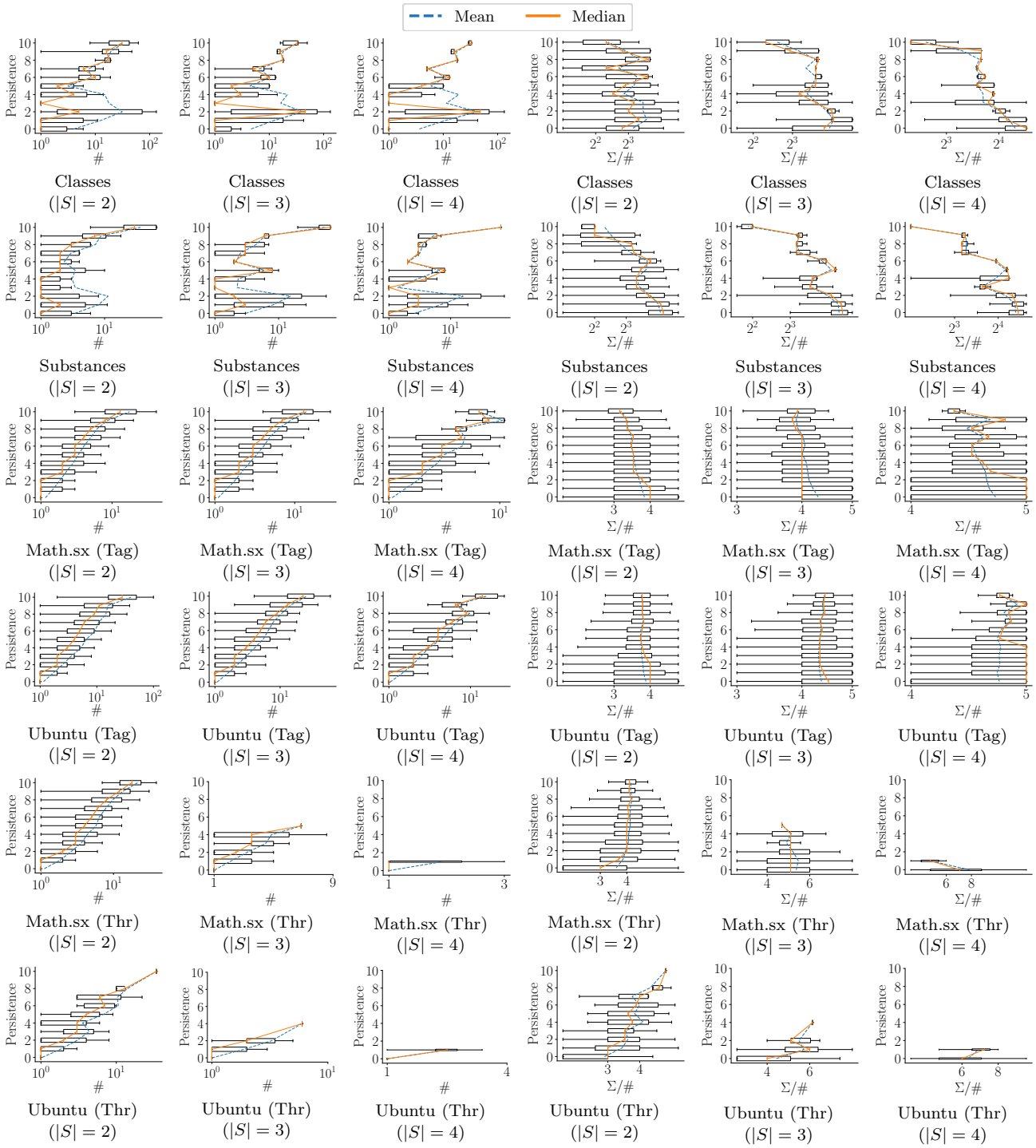
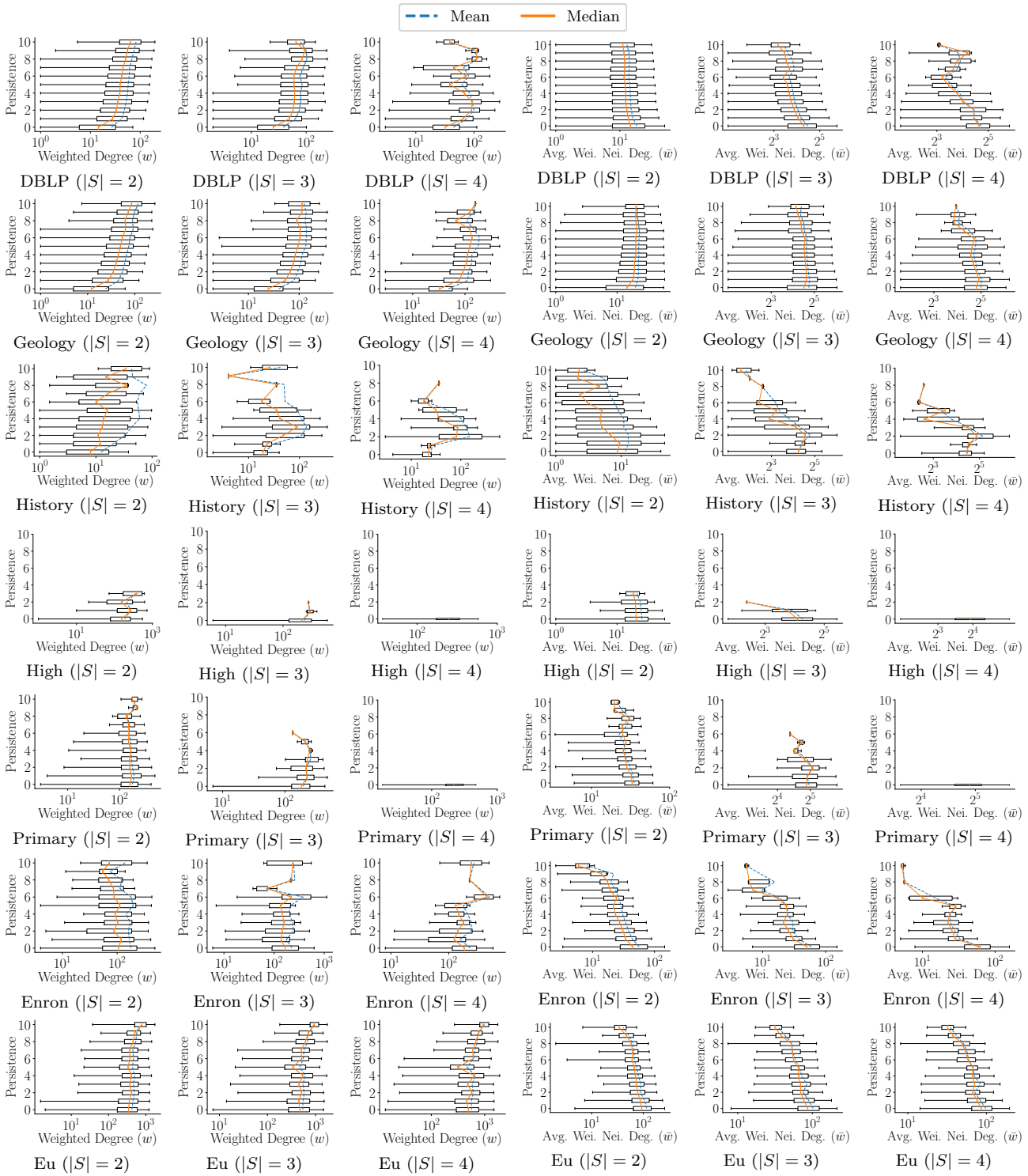


Figure 2: **Group Features vs. Group Persistence.** The distribution of # (i.e., the number of hyperedges containing each HOI) and Σ/# (i.e., the average size of the hyperedges containing each HOI) of HOIs with each level of persistence in all 13 real-world hypergraphs.


Mean
Median
Persistence
Weighted Degree (w)
Avg. Wei. Nei. Deg. ($\bar{w}$)
DBLP ($|S| = 2$)
DBLP ($|S| = 3$)
DBLP ($|S| = 4$)
Geology ($|S| = 2$)
Geology ($|S| = 3$)
Geology ($|S| = 4$)
History ($|S| = 2$)
History ($|S| = 3$)
History ($|S| = 4$)
High ($|S| = 2$)
High ($|S| = 3$)
High ($|S| = 4$)
Primary ($|S| = 2$)
Primary ($|S| = 3$)
Primary ($|S| = 4$)
Enron ($|S| = 2$)
Enron ($|S| = 3$)
Enron ($|S| = 4$)
Eu ($|S| = 2$)
Eu ($|S| = 3$)
Eu ($|S| = 4$)

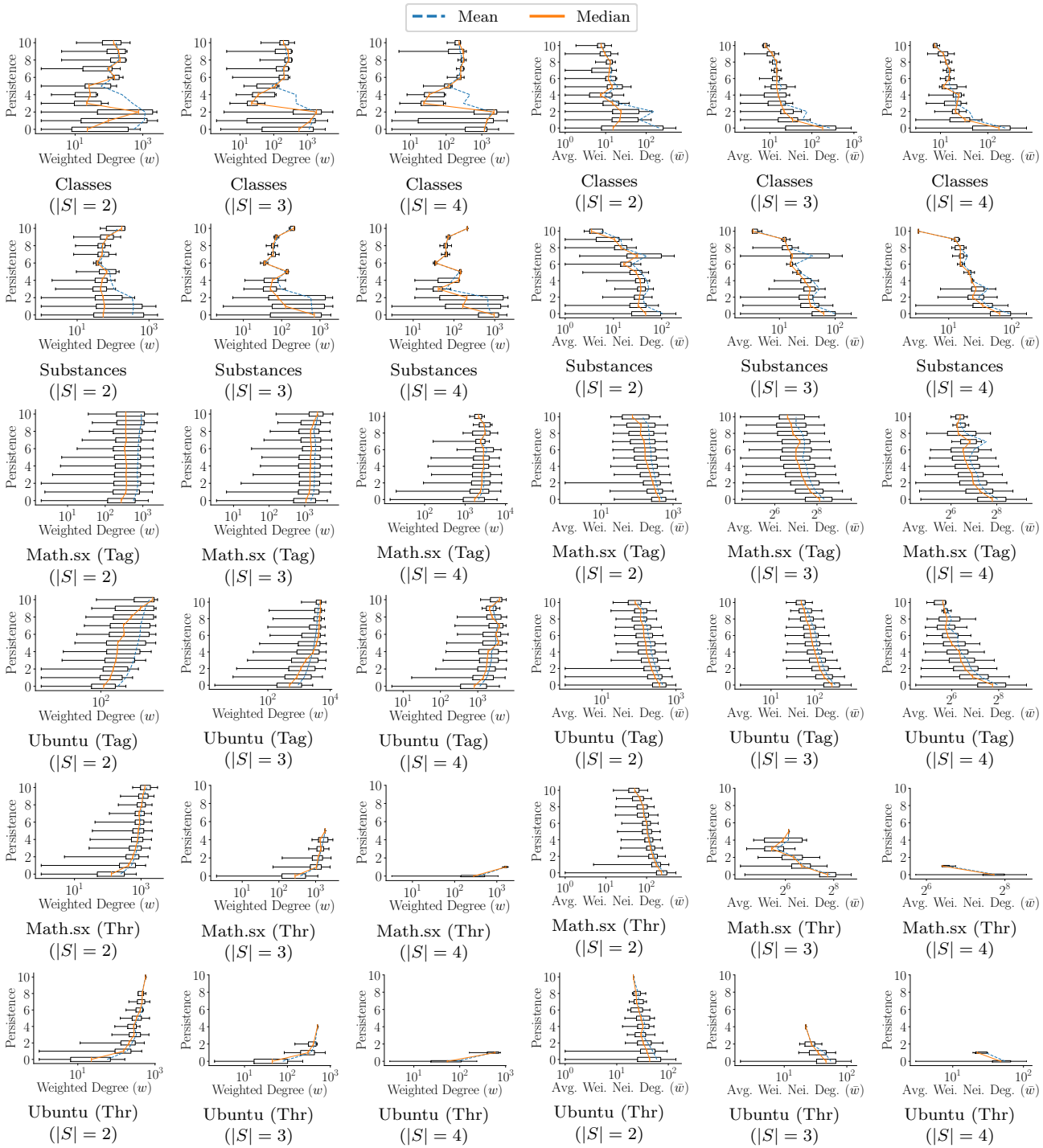
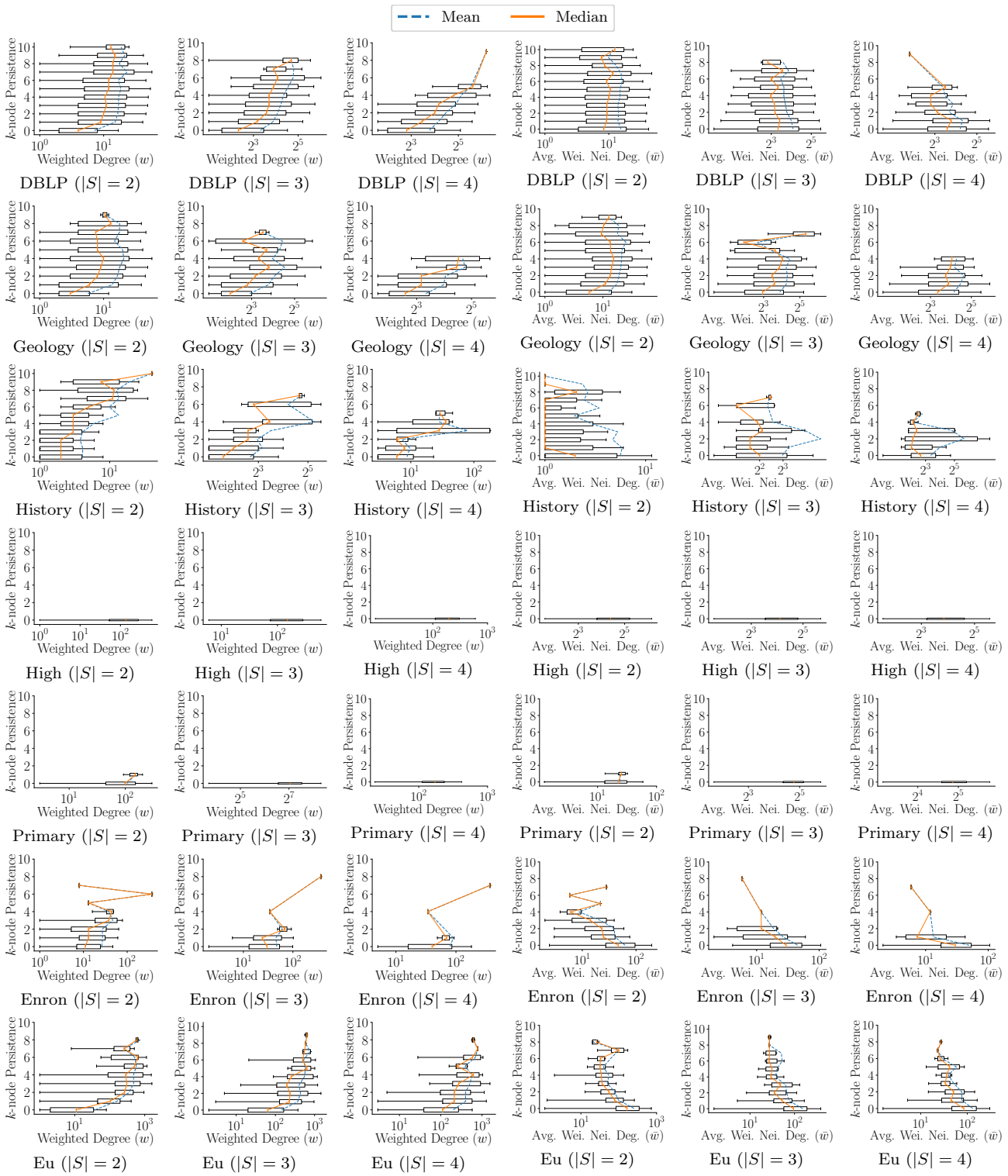


Figure 3: **Node Features vs. Group Persistence.** The distribution of averaged w (i.e., weighted degree) and $\bar{w}$ (i.e., the average weighted degree of neighbors) of HOIs with each level of persistence in all 13 real-world hypergraphs.


Mean
Median
k-node Persistence
Weighted Degree (w)
Avg. Wei. Nei. Deg. (w̄)
DBLP (|S| = 2)
DBLP (|S| = 3)
DBLP (|S| = 4)
DBLP (|S| = 2)
DBLP (|S| = 3)
DBLP (|S| = 4)
Geology (|S| = 2)
Geology (|S| = 3)
Geology (|S| = 4)
Geology (|S| = 2)
Geology (|S| = 3)
Geology (|S| = 4)
History (|S| = 2)
History (|S| = 3)
History (|S| = 4)
History (|S| = 2)
History (|S| = 3)
History (|S| = 4)
High (|S| = 2)
High (|S| = 3)
High (|S| = 4)
High (|S| = 2)
High (|S| = 3)
High (|S| = 4)
Primary (|S| = 2)
Primary (|S| = 3)
Primary (|S| = 4)
Primary (|S| = 2)
Primary (|S| = 3)
Primary (|S| = 4)
Enron (|S| = 2)
Enron (|S| = 3)
Enron (|S| = 4)
Enron (|S| = 2)
Enron (|S| = 3)
Enron (|S| = 4)
Eu (|S| = 2)
Eu (|S| = 3)
Eu (|S| = 4)
Eu (|S| = 2)
Eu (|S| = 3)
Eu (|S| = 4)

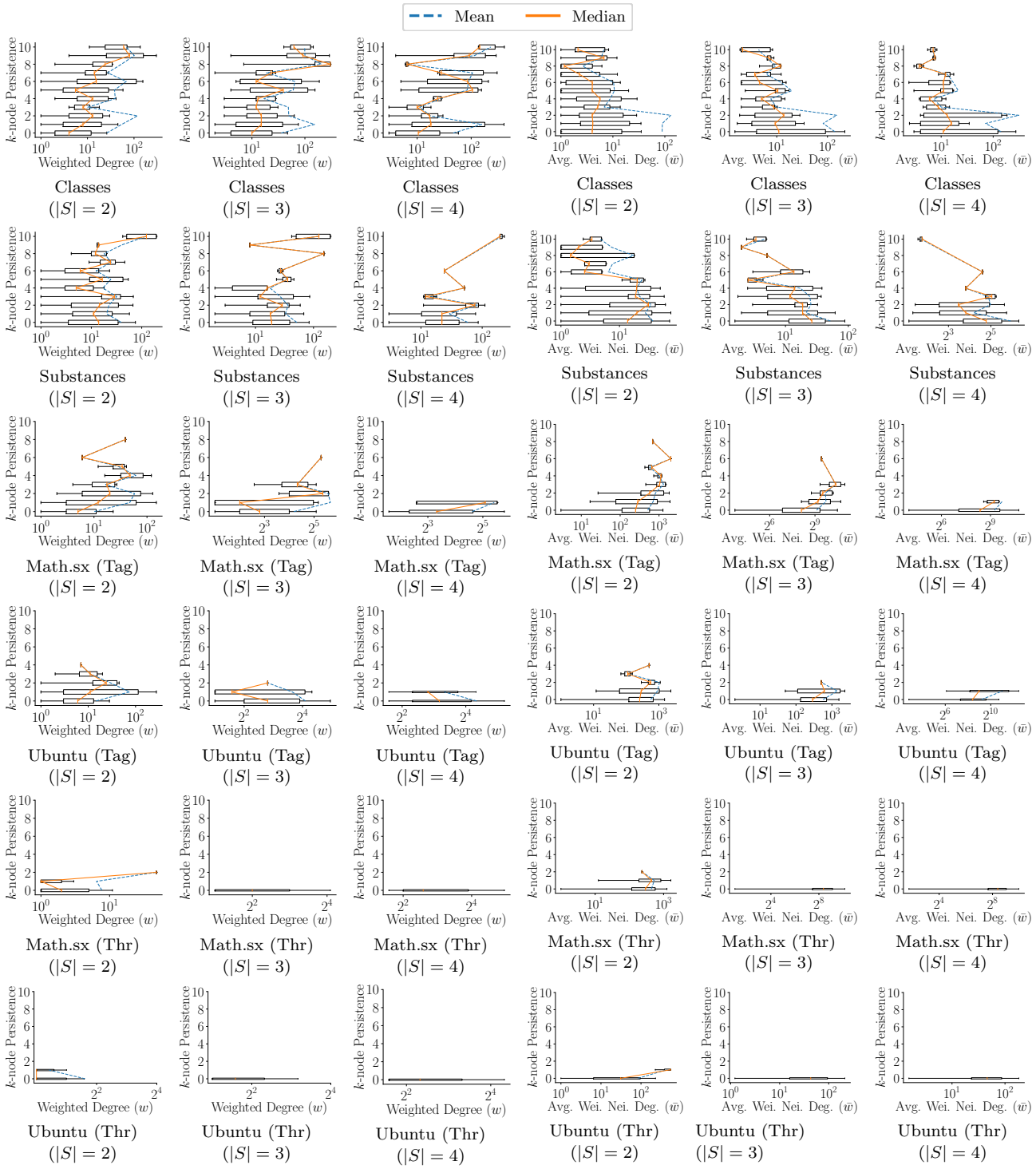


Figure 4: **Node Features vs. Node Persistence.** The distribution of w (i.e., weighted degree) and $\bar{w}$ (i.e., the average weighted degree of neighbors) of nodes with each level of k-node persistence in all 13 real-world hypergraphs.

Table 6: The average coefficient, standard error, and p-value of each structural feature obtained by linear regression analysis of each dataset. In each case, p-values smaller than 0.05 and 0.01 are in **bold** and underlined, respectively.

DBLP

Size of HOIs		$\#$	$\frac{\#}{\cup}$	$\frac{\Sigma}{\Sigma\cup}$	$\cap$	$\frac{\#}{\cap}$	$\frac{\Sigma}{\cap}$	$\frac{\Sigma}{\#}$	$\mathcal{H}$	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l
2	**Coef.**	0.25	0.09	-0.13	0.01	0.31	-0.17	-0.01	0.17	0.00	0.00	0.01	0.00	-3.62	-0.01	0.00	-0.06
	Std. Err.	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	3.39	0.00	0.00	0.00
	p-**value**	**0**	**1.2e-10**	**3.2e-18**	**4.7e-37**	**0**	**0**	**1.2e-05**	**0**	**0**	**0**	**0**	0.57	0.30	**0**	**4.0e-96**	**2.4e-67**
3	**Coef.**	0.17	0.01	0.01	0.00	0.36	-0.15	0.00	-0.03	0.00	0.00	0.00	0.00	-40.00	0.00	0.00	-0.06
	Std. Err.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	11.26	0.00	0.00	0.00
	p-**value**	**0**	**0.01**	**1.8e-04**	**3.7e-42**	**0**	**0**	**1.0e-07**	**9.8e-88**	0.23	**9.2e-25**	**7.2e-21**	0.45	**6.3e-04**	**1.9e-64**	**4.3e-32**	**0**
4	**Coef.**	0.09	0.00	0.00	0.01	0.27	-0.08	-0.01	-0.04	0.00	0.00	0.00	0.00	26.99	0.00	0.00	0.02
	Std. Err.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	4.03	0.00	0.00	0.00
	p-**value**	**0**	0.61	**0.02**	**0**	**0**	**0**	**0**	**0**	**1.6e-29**	**0**	**0**	**7.4e-19**	**6.2e-11**	0.26	**8.3e-05**	**0**

Geology

Size of HOIs		$\#$	$\frac{\#}{\cup}$	$\frac{\Sigma}{\Sigma\cup}$	$\cap$	$\frac{\#}{\cap}$	$\frac{\Sigma}{\cap}$	$\frac{\Sigma}{\#}$	$\mathcal{H}$	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l
2	Coeff.	0.34	0.04	-0.06	0.00	0.46	-0.34	-0.01	0.28	0.01	0.00	0.01	0.01	-1.15	0.00	0.00	-0.01
	Std. Err.	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.71	0.00	0.00	0.00
	p-value	**0**	**3.2e-05**	**1.1e-11**	**2.1e-06**	**0**	**0**	**1.6e-56**	**0**	**0**	**0**	**0**	**4.1e-46**	0.11	0.73	**6.0e-08**	0.12
3	Coeff.	0.28	0.00	0.01	-0.01	0.63	-0.29	0.01	0.08	0.00	0.00	0.00	0.00	-0.10	0.00	0.00	-0.02
	Std. Err.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.19	0.00	0.00	0.00
	p-value	**0**	0.35	**0.04**	**0**	**0**	**0**	**0**	**0**	**2.6e-88**	**4.1e-08**	**3.0e-11**	**1.3e-14**	0.62	**4.1e-10**	**4.8e-54**	**1.7e-69**
4	Coeff.	0.19	0.00	0.01	-0.01	0.64	-0.20	0.01	0.07	0.00	0.00	0.00	0.00	0.09	0.00	0.00	-0.02
	Std. Err.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00	0.00	0.00
	p-value	**0**	0.07	**2.0e-14**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0.04**	**5.4e-58**	**0**	**0**

History

Size of HOIs		$\#$	$\frac{\#}{\cup}$	$\frac{\Sigma}{\Sigma\cup}$	$\cap$	$\frac{\#}{\cap}$	$\frac{\Sigma}{\cap}$	$\frac{\Sigma}{\#}$	$\mathcal{H}$	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l
2	Coeff.	0.02	0.01	0.03	0.01	0.00	-0.01	-0.01	0.24	0.00	0.00	0.00	0.00	6.07	0.00	0.00	-0.03
	Std. Err.	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.39	0.00	0.00	0.00
	p-value	**0**	0.20	**2.4e-08**	**0**	0.22	**1.7e-22**	**9.7e-69**	**0**	**1.2e-59**	**0.03**	**7.3e-15**	**2.0e-13**	**0.01**	**3.7e-57**	**1.3e-57**	**8.2e-27**
3	Coeff.	0.03	0.02	-0.01	0.01	0.05	-0.02	-0.01	0.15	0.00	0.00	0.00	0.00	-3.75	0.00	0.00	0.01
	Std. Err.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.61	0.00	0.00	0.00
	p-value	**0**	**4.3e-36**	**7.2e-04**	**0**	**7.0e-67**	**5.1e-40**	**0**	**0**	**3.8e-50**	**1.7e-06**	**9.6e-21**	**1.5e-57**	**7.0e-08**	**0**	**0**	**0.00**
4	Coeff.	0.09	0.02	-0.01	0.00	0.31	-0.10	0.00	0.14	0.00	0.00	0.00	0.00	-8.51	0.00	0.00	0.01
	Std. Err.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.31	0.00	0.00	0.00
	p-value	**0**	**0**	**3.4e-21**	**1.2e-06**	**0**	**0**	**1.3e-11**	**0**	**0**	**1.8e-04**	**2.5e-49**	**6.5e-15**	**0**	**0**	**0**	0.26

High

Size of HOIs		$\#$	$\frac{\#}{\cup}$	$\frac{\Sigma}{\Sigma\cup}$	$\cap$	$\frac{\#}{\cap}$	$\frac{\Sigma}{\cap}$	$\frac{\Sigma}{\#}$	$\mathcal{H}$	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l
2	Coeff.	0.00	0.13	0.00	0.01	-0.01	0.00	-0.03	0.01	0.00	0.00	0.00	0.00	1.71	0.00	0.00	0.09
	Std. Err.	0.00	0.96	0.96	0.01	0.01	0.00	0.01	0.02	0.00	0.00	0.00	0.00	3.06	0.00	0.00	0.03
	p-value	0.34	0.25	0.30	0.33	0.21	0.22	**0.00**	0.55	0.42	**8.3e-05**	**6.3e-06**	0.54	0.51	0.67	0.37	**0.01**
3	Coeff.	0.00	-1.10	0.67	-0.01	0.01	0.00	0.00	0.01	0.00	0.00	0.00	0.00	-1.76	0.00	0.00	0.03
	Std. Err.	0.00	2.07	1.46	0.01	0.01	0.00	0.01	0.01	0.00	0.00	0.00	0.00	2.41	0.00	0.00	0.02
	p-value	0.28	0.57	0.61	0.63	0.25	0.26	0.62	0.32	0.69	0.47	0.42	0.66	0.46	0.47	0.75	0.31

Primary

Size of HOIs		$\#$	$\frac{\#}{\cup}$	$\frac{\Sigma}{\Sigma\cup}$	$\cap$	$\frac{\#}{\cap}$	$\frac{\Sigma}{\cap}$	$\frac{\Sigma}{\#}$	$\mathcal{H}$	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l
2	Coeff.	0.00	12.80	-5.61	0.18	0.48	-0.22	-0.04	0.37	0.04	0.00	0.01	-0.04	-111.39	-0.03	0.00	1.90
	Std. Err.	0.01	9.64	9.38	0.03	0.10	0.05	0.05	0.09	0.00	0.00	0.00	0.01	16.15	0.01	0.00	0.16
	p-value	0.49	0.29	0.53	**1.1e-06**	**1.8e-05**	**5.4e-05**	0.40	**1.9e-04**	**4.2e-15**	0.09	**0.01**	**0.00**	**1.5e-11**	**1.2e-06**	0.14	**1.1e-28**
3	Coeff.	0.01	-35.04	30.43	0.07	0.61	-0.20	-0.04	-0.02	0.00	0.00	0.00	0.01	-21.58	0.00	0.00	-0.02
	Std. Err.	0.01	17.24	11.83	0.03	0.12	0.04	0.03	0.05	0.00	0.00	0.00	0.00	6.18	0.00	0.00	0.06
	p-value	0.27	0.32	0.21	0.13	**3.1e-05**	**9.0e-05**	0.15	0.41	0.12	0.34	0.16	0.22	**0.01**	0.40	0.78	0.71

Enron

Size of HOIs		$\#$	$\frac{\#}{\cup}$	$\frac{\Sigma}{\Sigma\cup}$	$\cap$	$\frac{\#}{\cap}$	$\frac{\Sigma}{\cap}$	$\frac{\Sigma}{\#}$	$\mathcal{H}$	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l
2	Coeff.	0.02	5.38	-2.56	0.01	0.08	0.02	-0.04	1.58	0.04	-0.01	0.01	-0.18	40.24	0.01	0.00	2.82
	Std. Err.	0.01	1.47	1.13	0.03	0.07	0.03	0.03	0.20	0.02	0.00	0.00	0.06	6.39	0.03	0.00	0.52
	p-value	**0.04**	**0.00**	**0.05**	0.45	0.19	0.32	0.23	**1.3e-12**	**0.01**	**8.1e-06**	**3.4e-05**	**0.01**	**3.8e-09**	0.76	0.40	**5.0e-06**
3	Coeff.	0.04	3.63	-1.52	-0.02	0.41	-0.05	0.01	0.95	0.00	0.00	0.01	-0.07	21.52	0.02	0.00	0.86
	Std. Err.	0.01	0.99	0.52	0.01	0.05	0.01	0.01	0.07	0.00	0.00	0.00	0.02	1.93	0.01	0.00	0.19
	p-value	**2.0e-05**	**0.00**	**0.02**	0.12	**8.7e-09**	0.11	0.34	**6.8e-40**	0.52	**3.1e-08**	**6.7e-09**	**1.6e-04**	**4.3e-27**	**0.02**	**0.02**	**1.1e-05**
4	Coeff.	0.03	3.43	-1.30	-0.03	0.40	-0.05	0.03	0.75	-0.01	0.00	0.00	-0.03	17.28	0.02	0.00	0.23
	Std. Err.	0.00	0.96	0.35	0.00	0.03	0.01	0.01	0.03	0.00	0.00	0.00	0.01	0.73	0.00	0.00	0.08
	p-value	**4.3e-16**	**0.00**	**3.6e-04**	**2.3e-08**	**1.1e-29**	**1.5e-16**	**2.0e-05**	**0**	**7.2e-05**	**1.2e-07**	**5.3e-07**	**5.1e-08**	**0**	**1.1e-10**	**5.2e-08**	**0.01**

Eu

Size of HOIs		$\#$	$\frac{\#}{\cup}$	$\frac{\Sigma}{\Sigma\cup}$	$\cap$	$\frac{\#}{\cap}$	$\frac{\Sigma}{\cap}$	$\frac{\Sigma}{\#}$	$\mathcal{H}$	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l
2	Coeff.	0.03	-0.89	3.01	0.09	-0.13	0.11	-0.11	1.19	-0.06	0.00	0.00	0.03	1586.84	0.00	0.00	0.63
	Std. Err.	0.00	0.58	0.39	0.00	0.01	0.01	0.01	0.05	0.00	0.00	0.00	0.01	104.74	0.00	0.00	0.15
	p-value	**1.8e-22**	0.24	**6.9e-12**	**9.4e-64**	**5.1e-09**	**1.5e-65**	**3.7e-75**	**0**	**7.6e-46**	0.33	0.84	**1.4e-06**	**9.4e-51**	0.44	**4.7e-19**	**2.7e-05**
3	Coeff.	0.06	-8.02	7.77	0.05	0.00	0.07	-0.06	0.48	-0.03	0.00	0.00	0.02	830.95	0.00	0.00	0.35
	Std. Err.	0.00	0.46	0.24	0.00	0.02	0.00	0.00	0.02	0.00	0.00	0.00	0.00	40.26	0.00	0.00	0.07
	p-value	**3.1e-76**	**1.4e-58**	**0**	**1.0e-57**	0.25	**1.4e-42**	**7.3e-74**	**0**	**4.5e-73**	**0**	**0**	**1.1e-05**	**5.5e-91**	**0.02**	**0**	**1.1e-05**
4	Coeff.	0.15	-13.15	10.26	0.00	-0.19	0.00	-0.02	0.38	-0.02	0.00	0.00	0.01	570.27	0.00	0.00	0.15
	Std. Err.	0.00	0.38	0.16	0.00	0.02	0.00	0.00	0.01	0.00	0.00	0.00	0.00	17.81	0.00	0.00	0.04
	p-value	**0**	**0**	**0**	**0.01**	**3.5e-05**	**2.7e-06**	**4.5e-09**	**0**	**0**	**0**	**0**	**4.2e-14**	**0**	**6.5e-10**	**0**	**9.4e-05**

Classes

Size of HOIs		$\#$	$\frac{\#}{\cup}$	$\frac{\Sigma}{\Sigma\cup}$	$\cap$	$\frac{\#}{\cap}$	$\frac{\Sigma}{\cap}$	$\frac{\Sigma}{\#}$	$\mathcal{H}$	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l
2	Coeff.	0.00	-3.39	4.46	-0.25	0.39	-0.04	-0.01	2.79	0.18	0.00	0.02	0.21	-25.32	-0.13	0.00	2.02
	Std. Err.	0.01	0.81	0.86	0.08	0.05	0.02	0.06	0.29	0.02	0.00	0.00	0.07	4.18	0.02	0.00	0.35
	p-value	0.67	**1.5e-04**	**5.9e-07**	**0.00**	**1.7e-09**	0.22	0.63	**1.3e-07**	**3.8e-16**	**5.6e-15**	**8.0e-12**	**0.01**	**1.6e-08**	**1.0e-08**	0.71	**1.2e-07**
3	Coeff.	-0.05	-1.25	-0.51	0.04	0.03	0.07	-0.18	0.92	0.15	0.00	0.01	0.17	-31.37	-0.28	0.00	5.66
	Std. Err.	0.01	0.31	0.33	0.04	0.04	0.01	0.03	0.13	0.01	0.00	0.00	0.04	3.19	0.01	0.00	0.22
	p-value	**1.3e-10**	**9.2e-04**	0.25	0.39	0.49	**1.1e-10**	**2.5e-08**	**4.2e-11**	**4.6e-68**	**4.3e-52**	**3.8e-44**	**2.3e-05**	**5.5e-22**	**0**	0.50	**0**
4	Coeff.	-0.09	1.03	-3.86	0.21	-0.43	0.16	-0.33	-0.12	0.15	0.00	0.01	0.19	-30.78	-0.39	0.00	7.88
	Std. Err.	0.00	0.14	0.13	0.02	0.02	0.01	0.02	0.06	0.00	0.00	0.00	0.02	2.57	0.01	0.00	0.11
	p-value	**0**	**4.3e-12**	**0**	**4.8e-25**	**1.0e-57**	**0**	**0**	0.16	**0**	**0**	**0**	**2.3e-22**	**1.5e-29**	**0**	0.19	**0**

Substances

Size of HOIs		$\#$	$\frac{\#}{\cup}$	$\frac{\Sigma}{\Sigma\cup}$	$\cap$	$\frac{\#}{\cap}$	$\frac{\Sigma}{\cap}$	$\frac{\Sigma}{\#}$	$\mathcal{H}$	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l
2	Coeff.	0.04	-0.13	0.58	-0.05	0.51	-0.07	0.02	0.83	0.06	0.00	0.02	-0.08	-37.92	0.01	0.00	1.24
	Std. Err.	0.01	0.28	0.30	0.01	0.06	0.02	0.01	0.08	0.00	0.00	0.00	0.01	4.93	0.00	0.00	0.12
	p-value	**2.1e-06**	0.50	0.21	**4.3e-13**	**5.4e-10**	0.07	0.19	**2.6e-18**	**0**	**9.2e-25**	**4.4e-07**	**2.3e-22**	**3.2e-04**	0.30	**0.00**	**5.1e-17**
3	Coeff.	0.05	-0.36	0.25	-0.03	0.72	-0.11	0.01	0.61	0.04	0.00	-0.01	-0.05	-131.63	-0.02	0.00	2.13
	Std. Err.	0.00	0.07	0.07	0.00	0.03	0.01	0.00	0.02	0.00	0.00	0.00	0.00	3.56	0.00	0.00	0.04
	p-value	**6.2e-76**	**1.2e-05**	**0.01**	**0**	**0**	**1.6e-55**	**2.5e-08**	**0**	**0**	0.25	**9.8e-07**	**0**	**0**	**9.6e-93**	**0.00**	**0**
4	Coeff.	0.05	-0.26	-0.01	-0.02	0.92	-0.10	0.01	0.36	0.02	0.00	0.00	-0.03	-126.63	-0.02	0.00	1.91
	Std. Err.	0.00	0.02	0.02	0.00	0.02	0.00	0.00	0.01	0.00	0.00	0.00	0.00	1.48	0.00	0.00	0.01
	p-value	**0**	**1.1e-29**	0.62	**0**	**0**	**0**	**4.0e-38**	**0**	**0**	**8.2e-42**	**1.4e-32**	**0**	**0**	**0**	**4.9e-18**	**0**

Math.sx (Tags)

Size of HOIs		$\#$	$\frac{\#}{\cup}$	$\frac{\Sigma}{\Sigma\cup}$	$\cap$	$\frac{\#}{\cap}$	$\frac{\Sigma}{\cap}$	$\frac{\Sigma}{\#}$	$\mathcal{H}$	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l
2	Coeff.	0.08	0.02	-0.21	0.29	-0.67	0.76	-0.37	0.52	0.00	0.00	0.00	0.01	188.79	0.00	0.00	0.19
	Std. Err.	0.01	1.01	0.91	0.01	0.07	0.03	0.01	0.03	0.00	0.00	0.00	0.00	5.32	0.00	0.00	0.07
	p-value	**8.4e-21**	0.73	0.62	**0**	**1.2e-15**	**0**	**0**	**1.1e-59**	**5.6e-33**	0.28	0.41	**3.8e-07**	**0**	**9.6e-06**	0.42	**0.04**
3	Coeff.	0.48	2.25	-3.08	0.11	0.99	-0.35	-0.07	0.41	0.00	0.00	0.00	0.00	47.36	0.00	0.00	-0.06
	Std. Err.	0.01	3.17	2.36	0.01	0.04	0.02	0.01	0.01	0.00	0.00	0.00	0.00	2.64	0.00	0.00	0.05
	p-value	**0**	0.36	0.30	**1.1e-38**	**3.9e-83**	**5.4e-66**	**4.7e-14**	**0**	**1.2e-43**	**2.7e-06**	**3.7e-15**	**1.1e-06**	**3.9e-65**	**2.5e-05**	**6.3e-76**	0.20
4	Coeff.	0.08	-6.08	0.56	0.42	1.33	-0.25	-0.16	0.52	0.00	0.00	0.00	0.00	15.14	0.00	0.00	-0.13
	Std. Err.	0.01	13.93	8.97	0.01	0.06	0.01	0.01	0.02	0.00	0.00	0.00	0.00	2.98	0.00	0.00	0.05
	p-value	**2.5e-10**	0.56	0.57	**0**	**2.1e-83**	**8.5e-37**	**6.4e-67**	**0**	**4.6e-33**	0.18	**7.0e-06**	**9.4e-05**	**3.6e-06**	**1.2e-07**	**4.5e-58**	**0.03**

Ubuntu (Tags)

Size of HOIs		$\#$	$\frac{\#}{\cup}$	$\frac{\Sigma}{\Sigma\cup}$	$\cap$	$\frac{\#}{\cap}$	$\frac{\Sigma}{\cap}$	$\frac{\Sigma}{\#}$	$\mathcal{H}$	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l
2	Coeff.	-0.07	-2.03	1.68	0.17	-1.61	1.41	-0.25	0.54	0.00	0.00	0.00	0.01	11.16	0.00	0.00	-0.40
	Std. Err.	0.00	0.37	0.35	0.00	0.05	0.03	0.01	0.02	0.00	0.00	0.00	0.00	1.92	0.00	0.00	0.04
	p-value	**0**	**2.5e-07**	**5.5e-06**	**0**	**0**	**0**	**0**	**0**	**0**	**3.3e-38**	**7.8e-10**	**5.4e-41**	**7.3e-07**	**6.4e-72**	**4.5e-13**	**2.1e-21**
3	Coeff.	0.07	-0.34	0.50	0.21	-1.21	0.60	-0.23	0.06	0.00	0.00	0.00	0.00	-17.16	0.00	0.00	-0.16
	Std. Err.	0.00	0.68	0.56	0.00	0.04	0.01	0.00	0.01	0.00	0.00	0.00	0.00	1.87	0.00	0.00	0.03
	p-value	**1.6e-60**	0.62	0.39	**0**	**0**	**0**	**0**	**0.00**	**1.2e-66**	**1.0e-48**	**3.4e-07**	**1.3e-08**	**2.4e-16**	0.16	**5.9e-42**	**9.4e-09**
4	Coeff.	0.38	-0.16	-0.26	0.17	-0.23	0.10	-0.14	-0.06	0.00	0.00	0.00	0.00	-7.39	0.00	0.00	-0.21
	Std. Err.	0.01	2.06	1.53	0.01	0.06	0.02	0.01	0.02	0.00	0.00	0.00	0.00	2.43	0.00	0.00	0.03
	p-value	**0**	0.72	0.63	**4.2e-29**	0.12	**1.1e-04**	**0**	**0.02**	**2.2e-22**	**3.4e-34**	**3.2e-10**	0.18	**0.00**	**1.3e-05**	**1.1e-27**	**3.0e-11**

Math.sx (Threads)

Size of HOIs		$\#$	$\frac{\#}{\cup}$	$\frac{\Sigma}{\Sigma\cup}$	$\cap$	$\frac{\#}{\cap}$	$\frac{\Sigma}{\cap}$	$\frac{\Sigma}{\#}$	$\mathcal{H}$	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l
2	Coeff.	0.15	-0.29	0.42	0.04	0.06	-0.07	-0.04	-0.01	0.00	0.00	0.00	0.00	49.09	0.00	0.00	-0.06
	Std. Err.	0.00	0.03	0.03	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.67	0.00	0.00	0.00
	p-value	**0**	**8.7e-27**	**1.4e-57**	**0**	**3.7e-14**	**3.8e-38**	**0**	**0.01**	**0**	**1.3e-39**	**0**	**0**	**0**	**0**	**3.9e-59**	**1.0e-64**
3	Coeff.	0.30	-0.02	0.03	-0.02	0.67	-0.34	0.02	-0.08	0.00	0.00	0.00	0.00	2.38	0.00	0.00	0.00
	Std. Err.	0.00	0.01	0.01	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.11	0.00	0.00	0.00
	p-value	**0**	**0.01**	**2.3e-06**	**1.5e-07**	**0**	**0**	**2.5e-07**	**4.7e-95**	0.60	**3.7e-38**	**1.1e-54**	0.24	**3.7e-82**	**0.01**	**9.6e-13**	**0.00**
4	Coeff.	0.15	0.00	0.00	-0.04	0.78	-0.26	0.04	0.04	0.00	0.00	0.00	0.00	-0.04	0.00	0.00	0.00
	Std. Err.	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00
	p-value	0.32	0.76	0.65	0.33	0.32	0.32	0.32	0.32	**0.00**	**4.8e-04**	0.29	0.68	0.20	0.20	0.07	0.71

Ubuntu (Threads)

Size of HOIs		$\#$	$\frac{\#}{\cup}$	$\frac{\Sigma}{\Sigma\cup}$	$\cap$	$\frac{\#}{\cap}$	$\frac{\Sigma}{\cap}$	$\frac{\Sigma}{\#}$	$\mathcal{H}$	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l
2	Coeff.	0.09	-0.02	0.04	0.05	0.03	-0.03	-0.06	-0.05	0.00	0.00	0.00	0.00	1.72	0.00	0.00	-0.01
	Std. Err.	0.00	0.02	0.01	0.00	0.01	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.45	0.00	0.00	0.00
	p-value	**4.4e-09**	0.16	**0.01**	**1.2e-17**	**1.2e-14**	**4.1e-20**	**2.1e-25**	0.19	**0.00**	**2.6e-15**	**1.4e-16**	**2.1e-25**	0.07	**0.04**	0.06	**6.4e-05**
3	Coeff.	0.12	0.00	0.01	0.01	0.18	-0.09	-0.01	0.07	0.00	0.00	0.00	0.00	-0.98	0.00	0.00	0.00
	Std. Err.	0.01	0.01	0.01	0.00	0.03	0.02	0.00	0.01	0.00	0.00	0.00	0.00	0.34	0.00	0.00	0.00
	p-value	**5.7e-06**	0.68	0.51	**7.1e-17**	**0.03**	**0.03**	**5.5e-16**	**1.7e-04**	0.29	**2.2e-21**	**1.5e-33**	0.07	**0.02**	0.31	0.34	**0.03**
4	Coeff.	0.00	0.00	0.00	0.02	-0.15	0.05	-0.02	0.06	0.00	0.00	0.00	0.00	-0.23	0.00	0.00	0.00
	Std. Err.	0.01	0.00	0.00	0.00	0.04	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.19	0.00	0.00	0.00
	p-value	0.33	0.89	0.90	0.31	0.33	0.33	0.32	0.32	0.32	0.06	0.08	0.64	0.30	**0.04**	**0.05**	0.59

Table 7: **Statistical significance of structural features.** We report the number of datasets where each feature is significant with a given p-value in linear regression analysis.

		Persistence of HOIs																k-Node Persistence of Nodes							
Size of HOIs	p-**value**	$\#$	$\frac{\#}{\cup}$	$\frac{\Sigma}{\Sigma\cup}$	$\cap$	$\frac{\#}{\cap}$	$\frac{\Sigma}{\cap}$	$\frac{\Sigma}{\#}$	$\mathcal{H}$	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l	d	w	o	c	r	$\bar{d}$	$\bar{w}$	l
2	$\leq$ **0.05**	10	6	9	11	10	9	9	11	12	10	11	11	9	8	7	12	8	7	9	8	7	10	9	9
	$\leq$ **0.01**	9	6	7	11	10	9	9	10	11	9	11	11	9	7	7	11	8	7	9	8	5	10	7	8
	$\leq$ **0.001**	9	5	7	10	10	9	8	10	10	9	10	8	8	7	6	10	7	6	7	8	4	6	5	7
	$\leq$ **0.0001**	9	4	7	10	10	9	8	9	10	9	10	8	7	7	6	10	7	6	6	8	4	4	4	6
3	$\leq$ **0.05**	11	7	7	9	10	11	10	11	7	10	11	8	11	9	9	10	9	9	6	7	8	6	5	10
	$\leq$ **0.01**	11	6	5	9	9	10	10	11	7	10	11	8	10	7	8	9	8	9	5	7	4	5	3	10
	$\leq$ **0.001**	11	4	4	9	9	10	10	10	7	10	11	8	9	6	7	7	7	8	5	7	2	3	2	8
	$\leq$ **0.0001**	11	3	2	9	9	10	10	9	7	10	11	7	8	6	7	7	5	8	5	7	2	3	2	6
4	$\leq$ **0.05**	9	5	6	9	8	9	9	8	10	9	9	8	9	9	9	8	8	10	7	8	6	4	3	8
	$\leq$ **0.01**	9	5	5	8	8	9	9	7	10	9	9	8	8	8	8	6	8	8	7	5	3	2	3	6
	$\leq$ **0.001**	9	4	5	8	8	9	9	7	9	9	9	8	7	8	8	6	7	8	5	4	3	2	2	6
	$\leq$ **0.0001**	9	4	4	8	8	8	9	7	9	7	9	8	7	8	8	6	7	8	5	4	1	2	2	5
Avg.		9.8	4.9	5.7	9.3	9.1	9.3	9.2	9.2	9.1	9.3	10.2	8.4	8.5	7.5	7.5	8.5	7.4	7.8	6.3	6.8	4.1	4.8	3.9	7.4

References

[1] F. Chung and L. Lu, *The average distances in random graphs with given expected degrees*, PNAS, 99 (2002), pp. 15879–15882.