

# DLFR

Demonstration Lead-cooled Fast Reactor

# S & U Analysis

Sensitivity & Uncertainty Analysis

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### Introduction



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**PART**

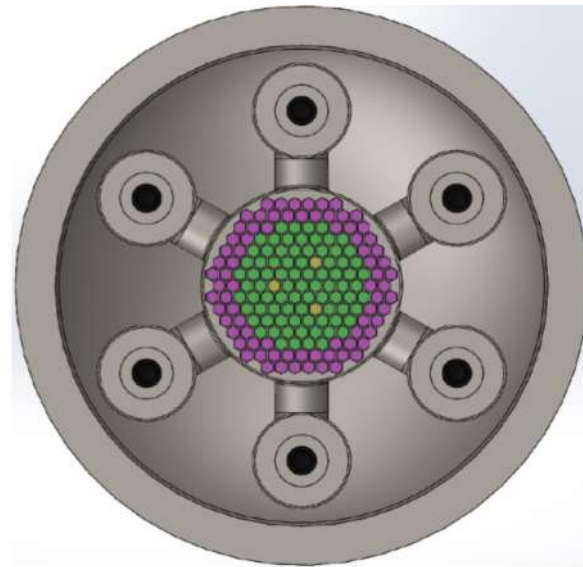
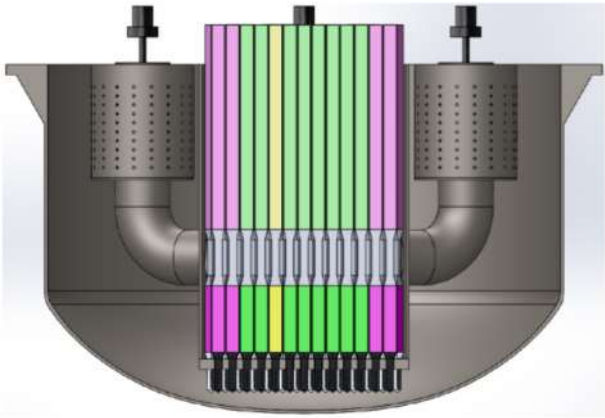
**01**

**Introduction**

# DLFR

By Westinghouse, U.S.

**Demonstration Lead-cooled Fast Reactor**



DLFR primary system layout, vertical and horizontal cross section  
(pre-conceptual, not in scale, DHRS not shown)

## Design Parameter

Power rate: 500MWt

Neutron flux: Peak  $\sim 2 \times 10^{15}$  n/cm<sup>2</sup>

## Reactor Type

Lead-cooled  
Pool-type  
Fast reactor

## Objective

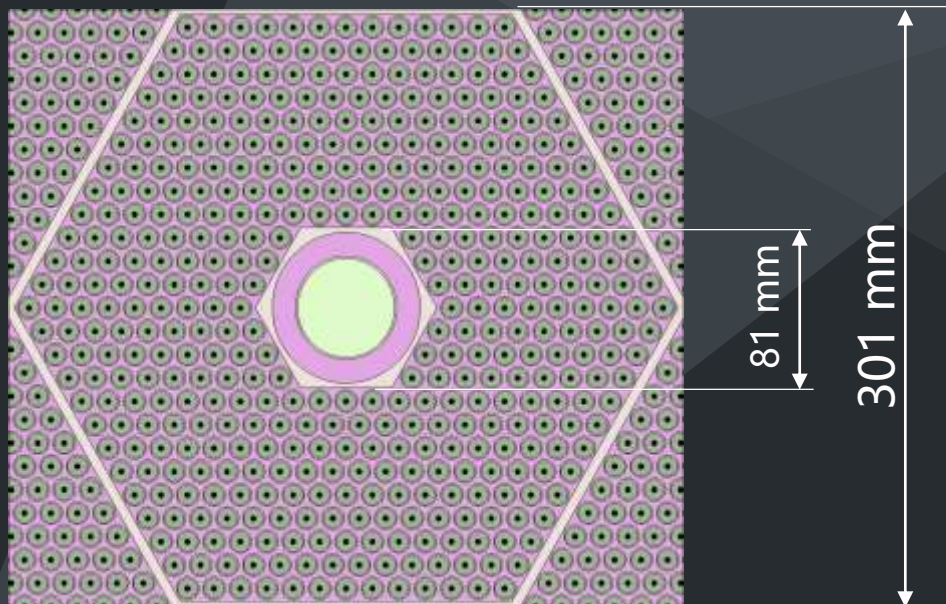
Feasibility  
Basic performance  
Upgrade





**PART**

**02**

**Modeling**

# Assembly



	Coolant	Pb
	Cladding	Steel
	Hollow	Void
	Fuel	

Boundary: Periodical

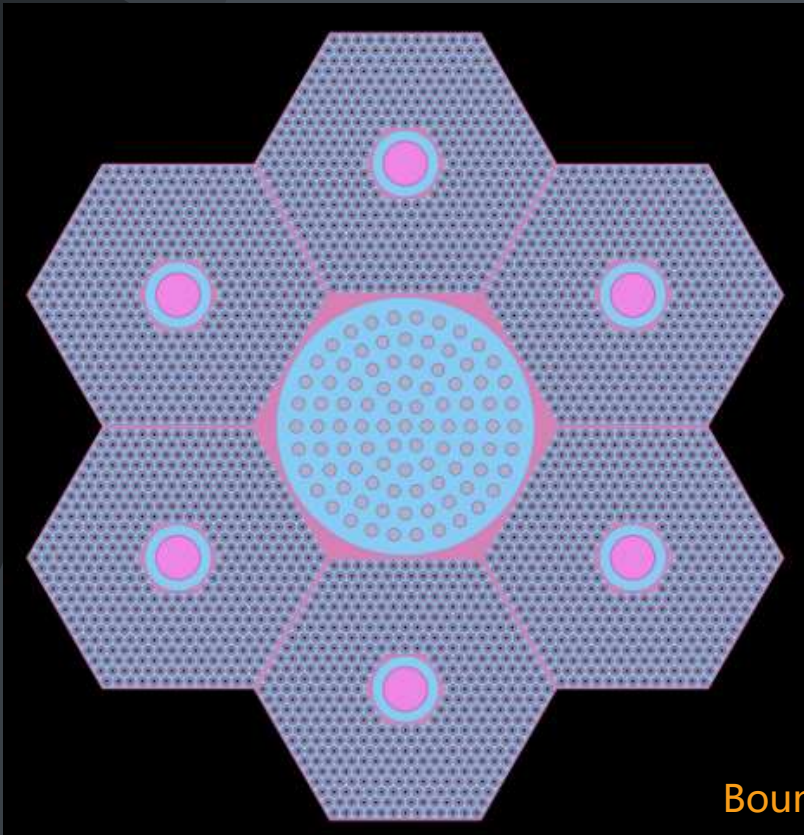
## Calculation:

Periodical Boundary

Inner Core Assembly		
Fuel	K-inf	Error
BoL	1.26791	0.00246
BoC	1.19807	0.00278
EoC	1.16523	0.00278
EoL	1.10776	0.00316

Outer Core Assembly		
Fuel	K-inf	Error
BoL	1.34074	0.00291
BoC	1.28491	0.00289
EoC	1.26066	0.00292
EoL	1.19467	0.00281

# Safety Rod



Boundary: Black

- Coolant -Pb
- Cladding -Steel
- Hollow -Void
- Inner Core Fuel
- Reflector -YSZ 8% mol
- Safety -B<sub>4</sub>C

## Calculation:

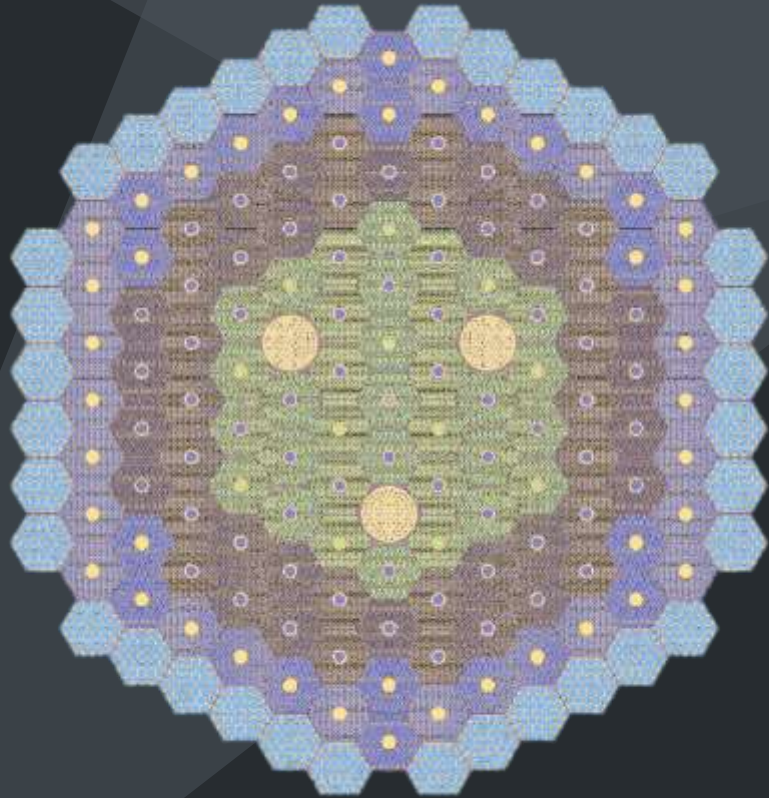
Black Boundary






Safety surrounded by Inner Core Assembly

Fuel	K-inf	Error
BoL	1.02864	0.00361
BoC	0.98266	0.00377
EoC	0.96995	0.00349
EoL	0.94755	0.00392



# Core

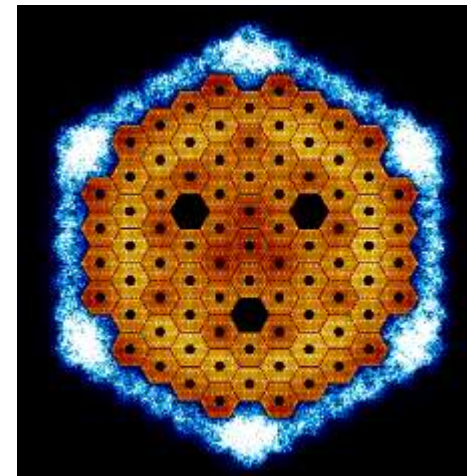


	Coolant	-Pb		Reflector	-YSZ 8% mol
	Cladding	-Steel		Shield	-Borated Steel
	Inner Fuel			Safety	$-B_4C$
	Outer Fuel			Plenum	

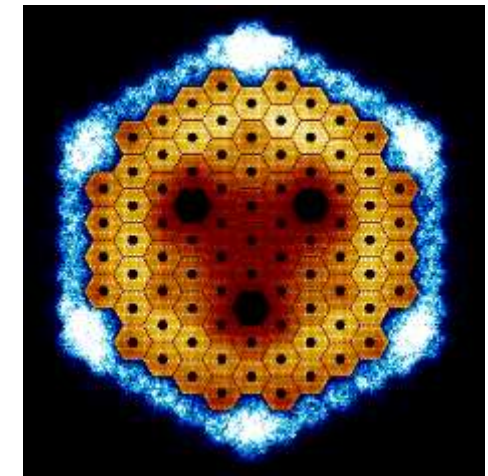
## Calculation:

Time	$k_{inf} \pm \text{error}$	
	Safety Rod out	Safety Rod in
BoL	$1.14892 \pm 0.00335$	$1.12366 \pm 0.00312$
BoC	$1.08653 \pm 0.00255$	$1.07694 \pm 0.00234$
EoC	$1.06150 \pm 0.00313$	$1.04444 \pm 0.00281$
EoL	$1.01073 \pm 0.00341$	$0.99147 \pm 0.00316$

## Power Distribution:



Safety Rod out



Safety Rod in



**PART**

**03**

**Calculation**

# Sensitivity



## Definition

$$S_x^R = \frac{\partial R/R}{\partial x/x}$$

$$R = \frac{\langle \Sigma_1, \Psi \rangle}{\langle \Sigma_2, \Psi \rangle}$$

- $R$  — Response function  
 $x$  — A certain perturbed parameter  
 $S_x^R$  — Sensitivity coefficient of  $R$  with respect to  $x$

$$S_x^R = \frac{\partial R/R}{\partial x/x} = \left\langle \frac{\frac{d\Sigma_1}{dx} \Psi x}{\Sigma_1 \Psi} - \frac{\frac{d\Sigma_2}{dx} \Psi x}{\Sigma_2 \Psi} + \frac{\partial R}{\partial \Psi} \frac{\partial \Psi}{\partial x} \frac{x}{R} \right\rangle$$

- $\langle \rangle$  — Inner product  
 $\Psi$  — Neutron flux  
 $\Sigma_1, \Sigma_2$  — Any kind of macroscopic cross section

# Sensitivity



## Calculation

$$S_x^R = \frac{\partial R/R}{\partial x/x} = \left\langle \overbrace{\frac{\frac{d\Sigma_1}{dx} \Psi x}{\Sigma_1 \Psi} - \frac{\frac{d\Sigma_2}{dx} \Psi x}{\Sigma_2 \Psi}}^{\text{Direct effect terms}} + \frac{\partial R}{\partial \Psi} \frac{\partial \Psi}{\partial x} \frac{x}{R} \right\rangle$$

Indirect effect terms

### Direct Terms

Describe impact on generalized response  
Relatively easy to compute

### Indirect Term

Describe impact on flux  
Complicated to compute

### Method for Indirect Term

- GEAR (Generalized Adjoint Response) method  
based on GPT (Generalized perturbation theory)  
used by TSUNAMI-3D
- Collision-based History method  
based on accepted and rejected events  
used by SERPENT2

# Uncertainty



## Definition

$$\vec{S} = (S_{x_1}^k, S_{x_2}^k, \dots, S_{x_n}^k)$$

$$n = \text{Nuclide} \sim \text{Reaction number} \times \text{Energy Bin number}$$

$$15543 = 471 \times 33$$

$$\text{cov}(x_i, x_j) = \int (x_i - E(x_i)) (x_j - E(x_j)) p(x_1, \dots, x_n) dx_1 \dots dx_n$$

## Sandwich Rule

$$r_k^2 = \vec{S} V \vec{S}^T$$

—  $V$  is (relative) covariance matrix

$$V = \begin{bmatrix} r_{x_1}^2 & \text{rcov}(x_1, x_2) & \dots & \text{rcov}(x_1, x_n) \\ \text{rcov}(x_2, x_1) & r_{x_1}^2 & \dots & \text{rcov}(x_2, x_n) \\ \vdots & \vdots & \ddots & \vdots \\ \text{rcov}(x_n, x_1) & \text{rcov}(x_n, x_2) & \dots & \text{rcov}(x_n, x_n) \end{bmatrix}$$

$$\text{rcov}(x_i, x_j) = \frac{\text{cov}(x_i, x_j)}{x_i x_j}$$

$$r_{x_i}^2 = \frac{\sigma_{x_i}^2}{x_i^2}$$

—  $r_{x_i}^2$  is relative variance

# COMMARA-2.0

**Released** by BNL & LANL in March 2011

**Based** on ENDF/B-VII.0

**Including 110 Nuclides:**

12 Light Nuclei (Coolant & Moderator)  
78 Structural Materials & Fission products  
20 Actinides

## Reaction Channels

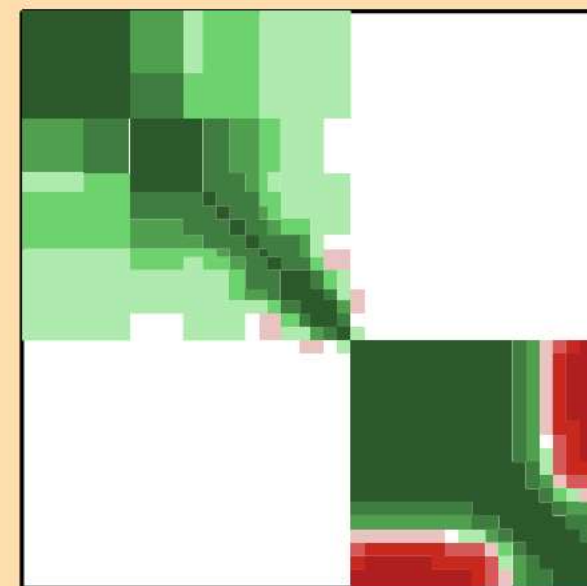
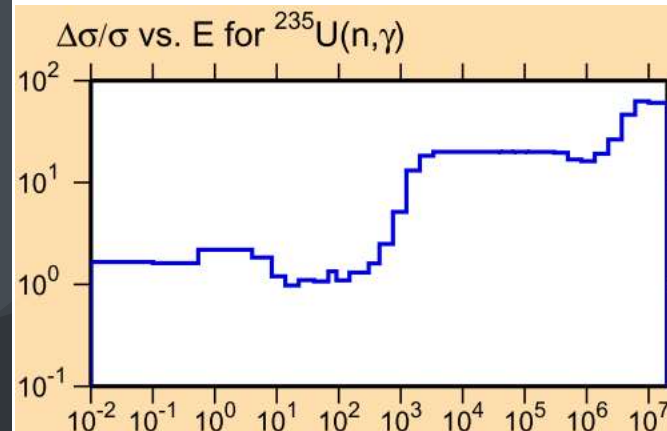
Elastic/Inelastic Scattering  $(n, n)/(n, n')$   
Capture  $(n, \gamma)$   
Neutron Multiplication  $(n, xn)$   
Fission  $(n, f)$ ,  $\bar{\nu}$ ,  $\chi$

## Total Files (Nuclide-Reaction)

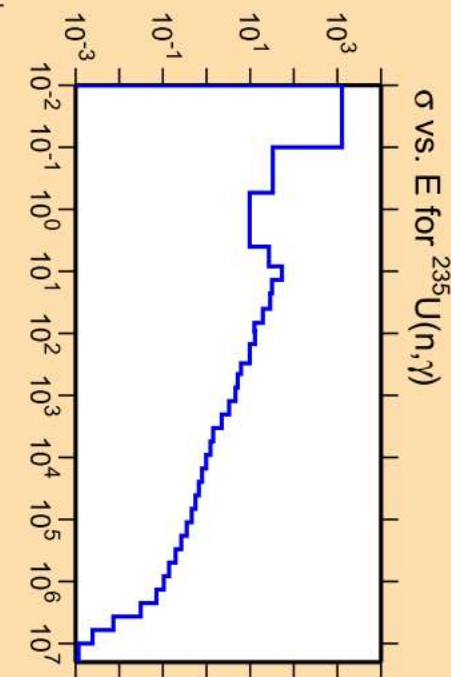
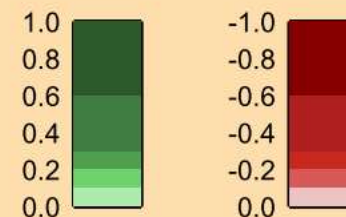
569

## Energy

$10^{-5}$  eV – 19.6 MeV  
33 Groups



Correlation Matrix





## Calculation

# Covariance Matrix

NEA provides COMMARA-2.0 correlation matrices and relative uncertainty:

<https://www.oecd-nea.org/science/wpec/sg33/benchmark/results/data.html>

$$C = \begin{bmatrix} 1 & \text{cor}(x_1, x_2) & \cdots & \text{cor}(x_1, x_n) \\ \text{cor}(x_2, x_1) & 1 & \cdots & \text{cor}(x_2, x_n) \\ \vdots & \vdots & \ddots & \vdots \\ \text{cor}(x_n, x_1) & \text{cor}(x_n, x_2) & \cdots & 1 \end{bmatrix}$$

$$r_{x_i} = \frac{\sigma_{x_i}}{x_i}$$

—  $r_{x_i}$  is relative uncertainty

Obtain relative covariance matrix from correlation matrices and relative uncertainty:

$$\text{rcov}(x_i, x_j) = r_{x_i} r_{x_j} \text{cor}(x_i, x_j)$$

$$V = \begin{bmatrix} r_{x_1}^2 & \text{rcov}(x_1, x_2) & \cdots & \text{rcov}(x_1, x_n) \\ \text{rcov}(x_2, x_1) & r_{x_1}^2 & \cdots & \text{rcov}(x_2, x_n) \\ \vdots & \vdots & \ddots & \vdots \\ \text{rcov}(x_n, x_1) & \text{rcov}(x_n, x_2) & \cdots & \text{rcov}(x_n, x_n) \end{bmatrix}$$



# Relative Uncertainty



## Definition

$$U_{total} = r_k^2 = \vec{S}V\vec{S}^T$$
$$= \sum_i S_i V_{ii} S_i + \sum_i \sum_{j \neq i} S_i V_{ij} S_j$$

Sensitivity Index:

$$SI_i = \frac{U_i}{U_{total}}$$

The conservative estimate of uncertainty:

$$\widetilde{U}_i = S_i V_{ii} S_i + 2 \times \sum_{j \neq i} S_i V_{ij} S_j$$

Conservative Sensitivity Index:

$$\widetilde{SI}_i = \frac{\widetilde{U}_i}{U_{total}}$$

# Calculation

## Serpent Output Files



- \* **\_res.m** – General Results
- \* **\_sens.m** – Sensitivity Data

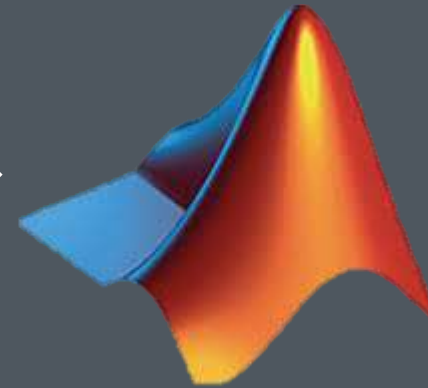
## C++



- Output File Preprocess
- Extract Information
- Generate MATLAB Script

## MATLAB

Mixed Programming!



- Arrange Covariance Matrix
- Calculate Uncertainty
- Plot Distribution Graph

## Results

Sensitivity  
&  
Uncertainty Distribution

- Nuclide-Reaction List
- Uncertainty Value
- Stairs Graph

# C++



- Read Serpent Output File
- Generate Nuclide-Reaction Index
- Match Sensitivity with Covariance

Generate MATLAB Scripts to :

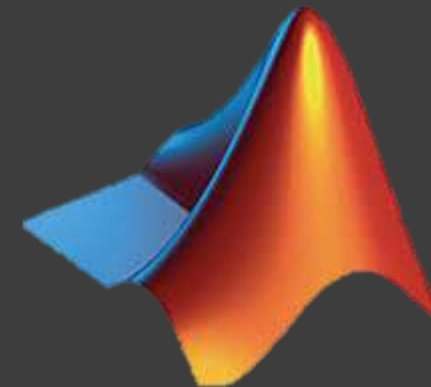
Arrange Covariance Matrix  
Calculate Uncertainty  
Sort Data  
Plot Stairs Graph  
.....

## Interface

**MATLAB Engine**  
**Dynamic-link Library**

**Index Files**  
**Temporary Files**  
**MATLAB Scripts**

# MATLAB



- Read Covariance Matrix
- Calculate Uncertainty
- Sort Data by Importance
- Plot Stairs Graph

Generate Temporary Files to :

Match Data by Name-Value  
Extract Calculation Results  
Transfer Data  
.....

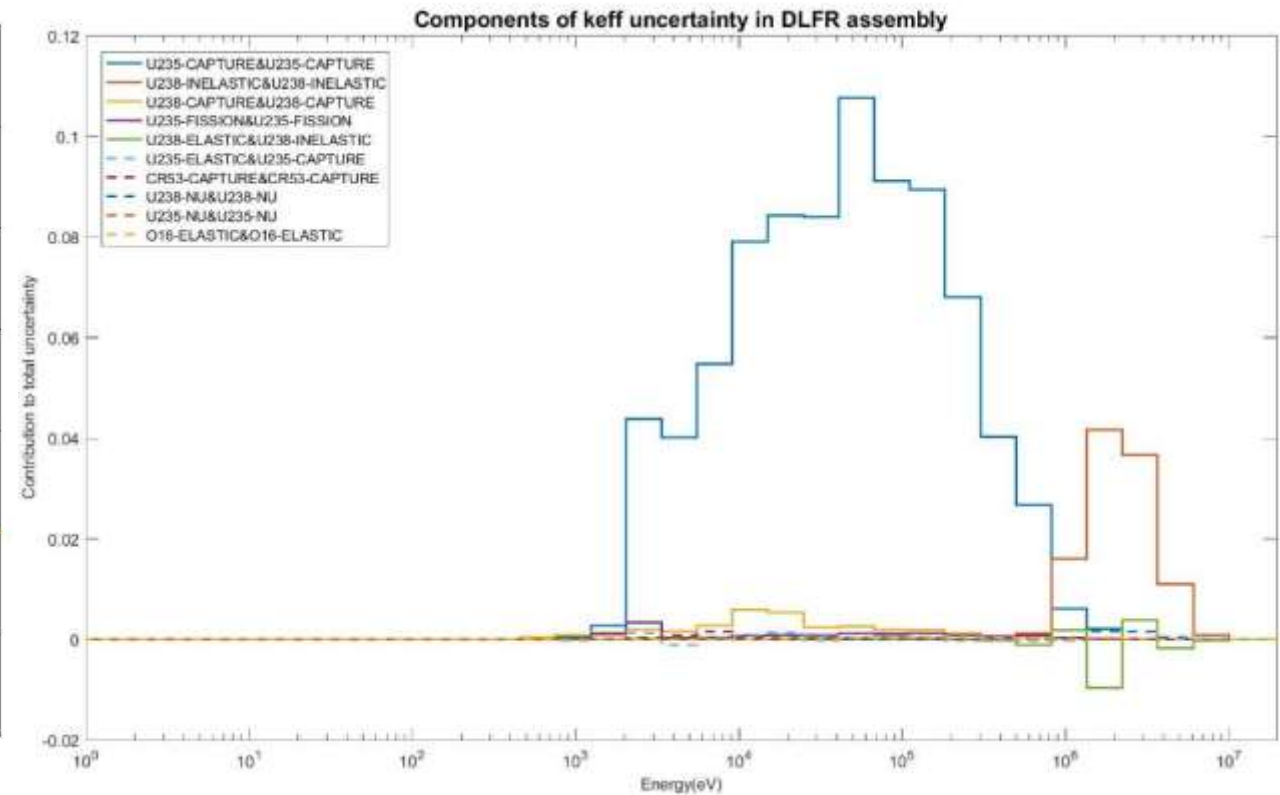
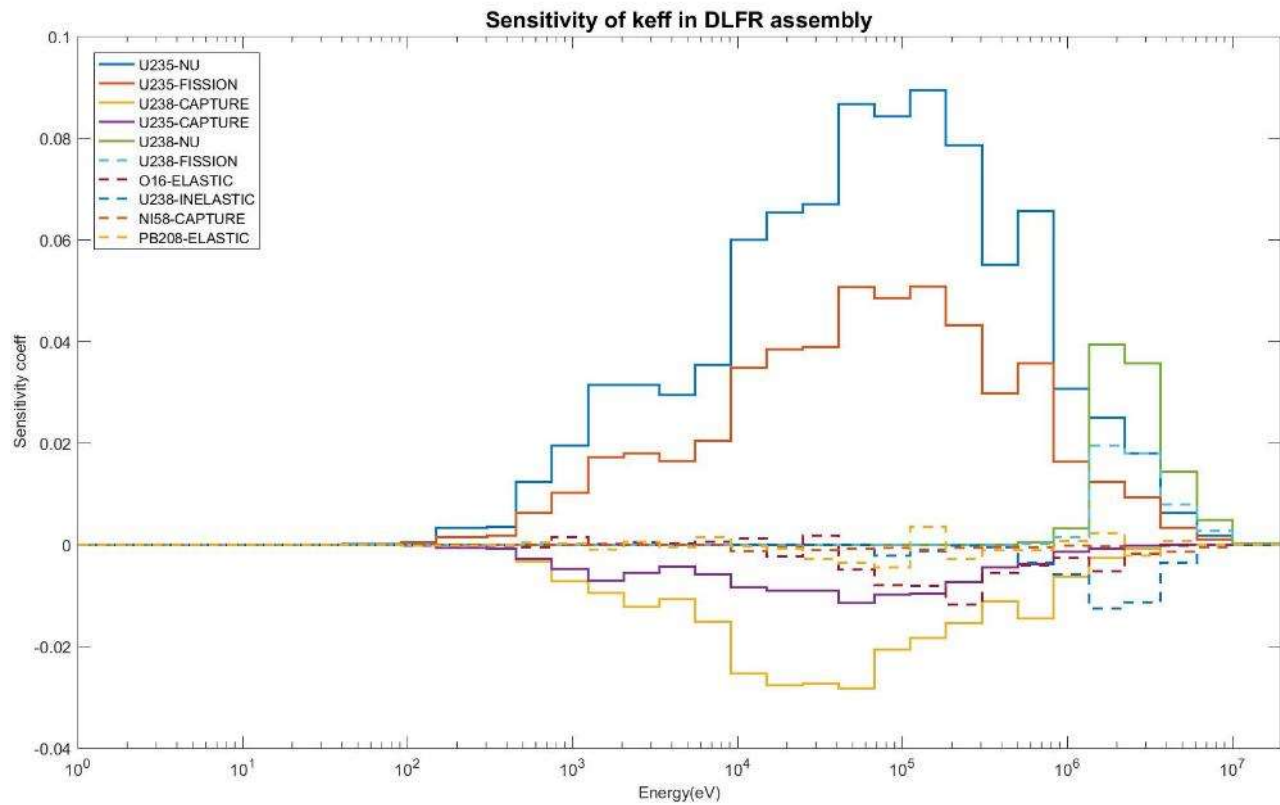
**PART**

**3.1**

**Fuel Assembly**

# Distribution of S/U Corresponding to the Most Important 20 Parameters

## Inner Core Fuel Assembly at Beginning of Life



## BoL inner Assembly Uncertainty Analysis:

$K_{eff}$  Standard Deviation: 0.019226

Parameter Pair	Contribution to Total Uncertainty (%)	Parameter	Conservatively Estimated Relative Uncertainty	Sensitivity
$^{235}\text{U} - \sigma_{n,\gamma}$	82.1235	$^{235}\text{U} - \sigma_{n,\gamma}$	1.56768	-0.107003
$^{238}\text{U} - \sigma_{n,n'}$	10.8820	$^{238}\text{U} - \sigma_{n,n'}$	0.17516	-0.041029
$^{238}\text{U} - \sigma_{n,\gamma}$	3.2416	$^{238}\text{U} - \sigma_{n,\gamma}$	0.05615	-0.257976
$^{235}\text{U} - \sigma_f$	1.4514	$^{235}\text{U} - \sigma_f$	0.02490	0.506052
$^{238}\text{U} - (\sigma_{n,n}, \sigma_{n,n'})$	-0.7076	$^{53}\text{Cr} - \sigma_{n,\gamma}$	0.00684	0.007763

## BoL outer Assembly Uncertainty Analysis:

$K_{eff}$  Standard Deviation: 0.019821

Parameter Pair	Contribution to Total Uncertainty (%)	Parameter	Conservatively Estimated Relative Uncertainty	Sensitivity
$^{235}\text{U} - \sigma_{n,\gamma}$	86.7332	$^{235}\text{U} - \sigma_{n,\gamma}$	1.64995	-0.110411
$^{238}\text{U} - \sigma_{n,n'}$	7.0166	$^{238}\text{U} - \sigma_{n,n'}$	0.122642	-0.041853
$^{238}\text{U} - \sigma_{n,\gamma}$	2.4011	$^{238}\text{U} - \sigma_{n,\gamma}$	0.041575	-0.232542
$^{235}\text{U} - \sigma_f$	1.2345	$^{235}\text{U} - \sigma_f$	0.021593	0.488544
$^{238}\text{U} - (\sigma_{n,n}, \sigma_{n,n'})$	0.4458	$^{53}\text{Cr} - \sigma_{n,\gamma}$	0.005432	0.007141



## BoC inner Assembly Uncertainty Analysis:

$K_{eff}$  Standard Deviation: 0.015357

Parameter Pair	Contribution to Total Uncertainty (%)	Parameter	Conservatively Estimated Relative Uncertainty	Sensitivity
$^{235}\text{U} - \sigma_{n,\gamma}$	65.8074	$^{235}\text{U} - \sigma_{n,\gamma}$	1.25340	-0.078050
$^{238}\text{U} - \sigma_{n,n'}$	19.0909	$^{238}\text{U} - \sigma_{n,n'}$	0.33066	-0.051604
$^{238}\text{U} - \sigma_{n,\gamma}$	5.1079	$^{238}\text{U} - \sigma_{n,\gamma}$	0.08877	-0.256832
$^{235}\text{U} - \sigma_f$	1.7412	$^{235}\text{U} - \sigma_f$	0.02919	0.431738
$^{56}\text{Fe} - \sigma_{n,n}$	1.7377	$^{56}\text{Fe} - \sigma_{n,n}$	0.02974	-0.023070

## BoC outer Assembly Uncertainty Analysis:

$K_{eff}$  Standard Deviation: 0.016577

Parameter Pair	Contribution to Total Uncertainty (%)	Parameter	Conservatively Estimated Relative Uncertainty	Sensitivity
$^{235}\text{U} - \sigma_{n,\gamma}$	86.7332	$^{235}\text{U} - \sigma_{n,\gamma}$	1.513810	-0.090276
$^{238}\text{U} - \sigma_{n,n'}$	7.0166	$^{238}\text{U} - \sigma_{n,n'}$	0.204594	-0.044881
$^{238}\text{U} - \sigma_{n,\gamma}$	2.4011	$^{238}\text{U} - \sigma_{n,\gamma}$	0.062313	-0.235775
$^{235}\text{U} - \sigma_f$	1.2345	$^{235}\text{U} - \sigma_f$	0.027417	0.454963
$^{238}\text{U} - (\sigma_{n,n}, \sigma_{n,n'})$	0.4458	$^{16}\text{O} - \sigma_{n,n}$	0.011894	-0.067184

## EoC inner Assembly Uncertainty Analysis:

$K_{eff}$  Standard Deviation: 0.014014

Parameter Pair	Contribution to Total Uncertainty (%)	Parameter	Conservatively Estimated Relative Uncertainty	Sensitivity
$^{235}\text{U} - \sigma_{n,\gamma}$	54.7410	$^{235}\text{U} - \sigma_{n,\gamma}$	1.04262	-0.065688
$^{238}\text{U} - \sigma_{n,n'}$	27.9248	$^{238}\text{U} - \sigma_{n,n'}$	0.48038	-0.050797
$^{238}\text{U} - \sigma_{n,\gamma}$	5.8622	$^{238}\text{U} - \sigma_{n,\gamma}$	0.10185	-0.250996
$^{235}\text{U} - \sigma_f$	1.8119	$^{235}\text{U} - \sigma_f$	0.02997	0.397506
$^{16}\text{O} - \sigma_{n,n}$	1.5251	$^{16}\text{O} - \sigma_{n,n}$	0.02818	-0.087629

## EoC outer Assembly Uncertainty Analysis:

$K_{eff}$  Standard Deviation: 0.015726

Parameter Pair	Contribution to Total Uncertainty (%)	Parameter	Conservatively Estimated Relative Uncertainty	Sensitivity
$^{235}\text{U} - \sigma_{n,\gamma}$	76.2761	$^{235}\text{U} - \sigma_{n,\gamma}$	1.451030	-0.084109
$^{238}\text{U} - \sigma_{n,n'}$	12.9362	$^{238}\text{U} - \sigma_{n,n'}$	0.226971	-0.044512
$^{238}\text{U} - \sigma_{n,\gamma}$	4.0763	$^{238}\text{U} - \sigma_{n,\gamma}$	0.070649	-0.236758
$^{235}\text{U} - \sigma_f$	1.6353	$^{235}\text{U} - \sigma_f$	0.027646	0.433899
$^{238}\text{U} - (\sigma_{n,n}, \sigma_{n,n'})$	0.9269	$^{53}\text{Cr} - \sigma_{n,\gamma}$	0.010715	-0.007833

## EoL inner Assembly Uncertainty Analysis:

$K_{eff}$  Standard Deviation: 0.012044

Parameter Pair	Contribution to Total Uncertainty (%)	Parameter	Conservatively Estimated Relative Uncertainty	Sensitivity
$^{238}\text{U} - \sigma_{n,n'}$	38.9070	$^{238}\text{U} - \sigma_{n,n'}$	0.68714	-0.059305
$^{235}\text{U} - \sigma_{n,\gamma}$	33.2131	$^{235}\text{U} - \sigma_{n,\gamma}$	0.63359	-0.044578
$^{238}\text{U} - \sigma_{n,\gamma}$	7.3113	$^{238}\text{U} - \sigma_{n,\gamma}$	0.12726	-0.238941
$^{238}\text{U} - (\sigma_{n,n}, \sigma_{n,n'})$	3.9664	$^{16}\text{O} - \sigma_{n,n}$	0.05135	0.100010
$^{16}\text{O} - \sigma_{n,n}$	2.7469	$^{238}\text{U} - \sigma_{n,n}$	0.04130	-0.005715

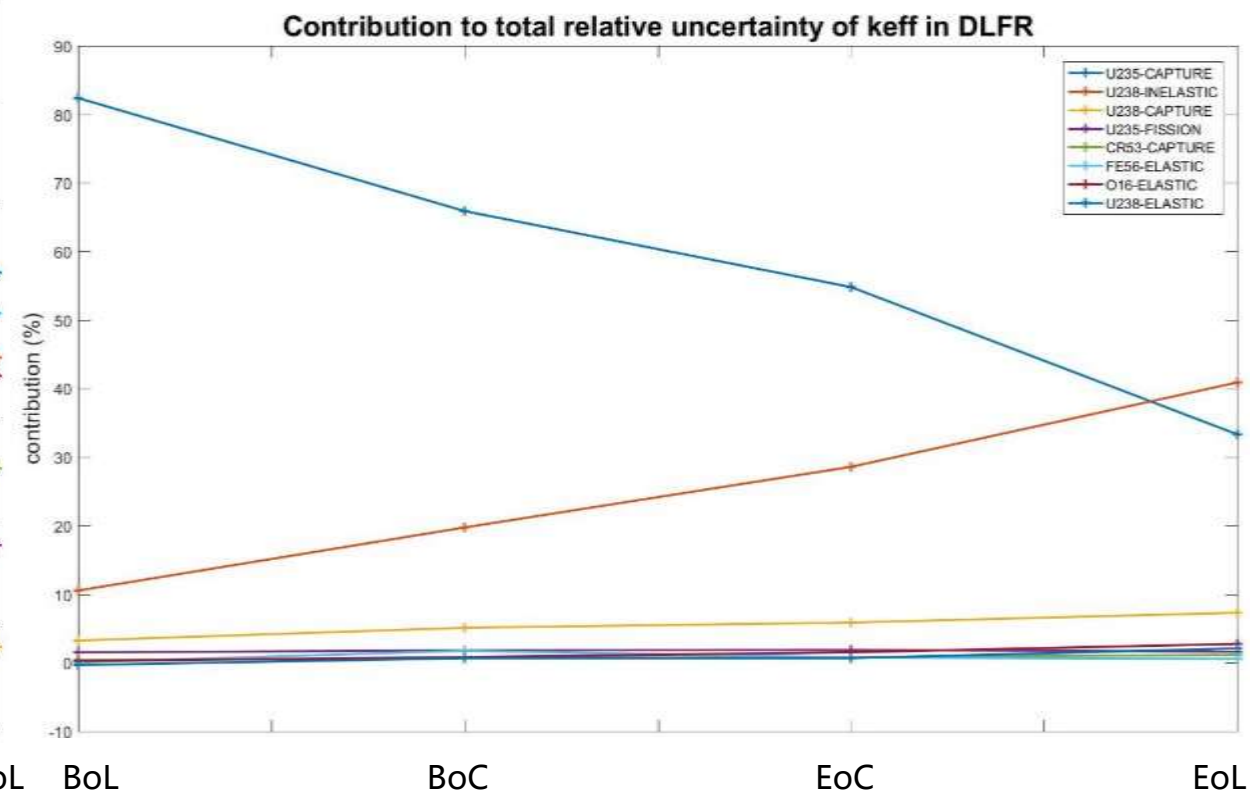
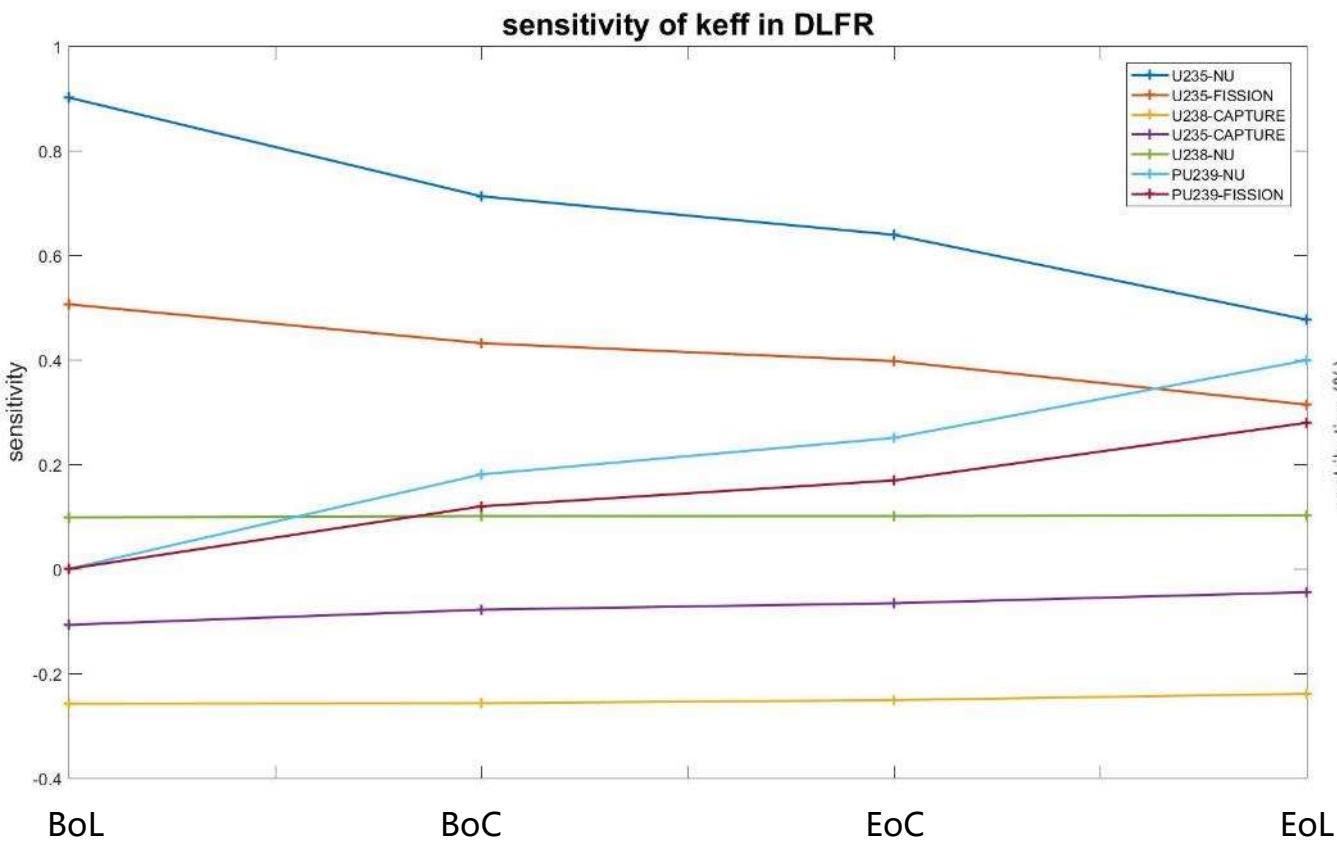
## EoL outer Assembly Uncertainty Analysis:

$K_{eff}$  Standard Deviation: 0.013416

Parameter Pair	Contribution to Total Uncertainty (%)	Parameter	Conservatively Estimated Relative Uncertainty	Sensitivity
$^{235}\text{U} - \sigma_{n,\gamma}$	62.8492	$^{235}\text{U} - \sigma_{n,\gamma}$	1.201250	-0.066350
$^{238}\text{U} - \sigma_{n,n'}$	21.0056	$^{238}\text{U} - \sigma_{n,n'}$	0.360621	-0.047198
$^{238}\text{U} - \sigma_{n,\gamma}$	5.5305	$^{238}\text{U} - \sigma_{n,\gamma}$	0.095879	-0.234376
$^{235}\text{U} - \sigma_f$	1.8597	$^{235}\text{U} - \sigma_f$	0.031221	0.392085
$^{16}\text{O} - \sigma_{n,n}$	1.0447	$^{16}\text{O} - \sigma_{n,n}$	0.018994	-0.070047

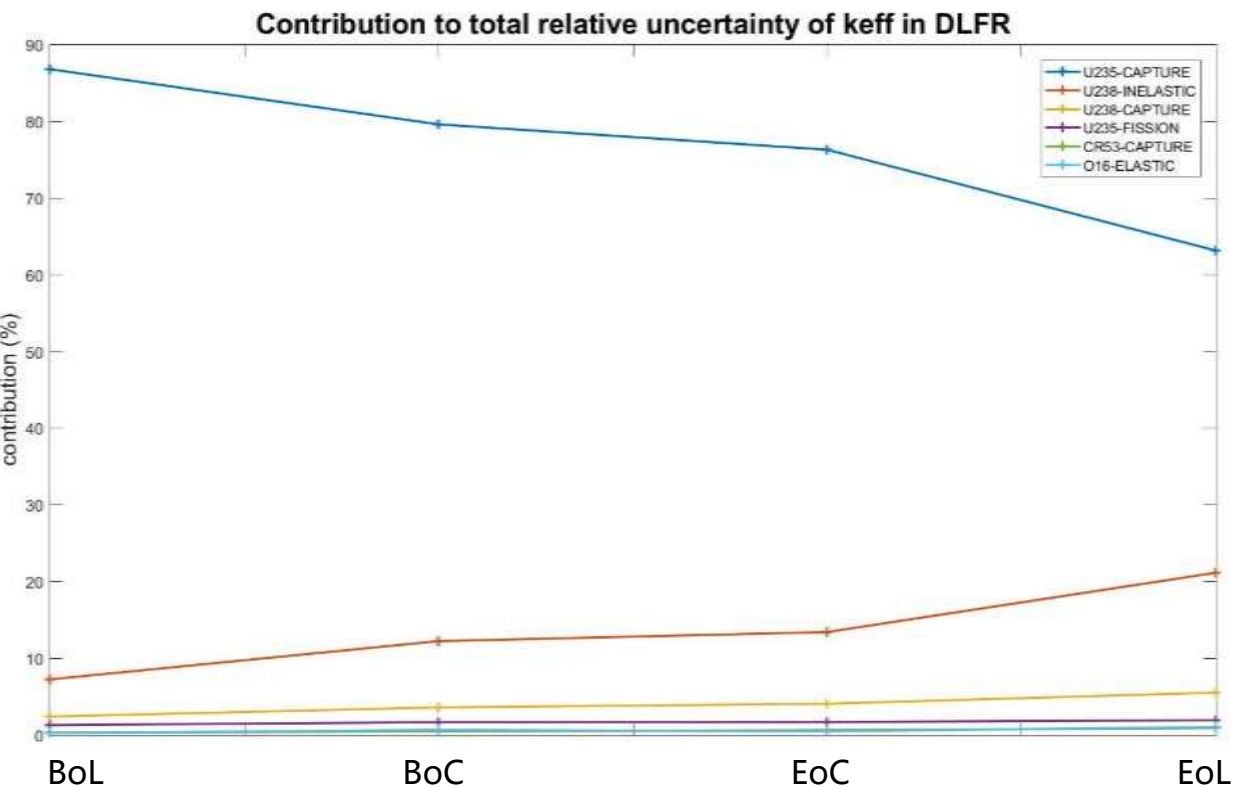
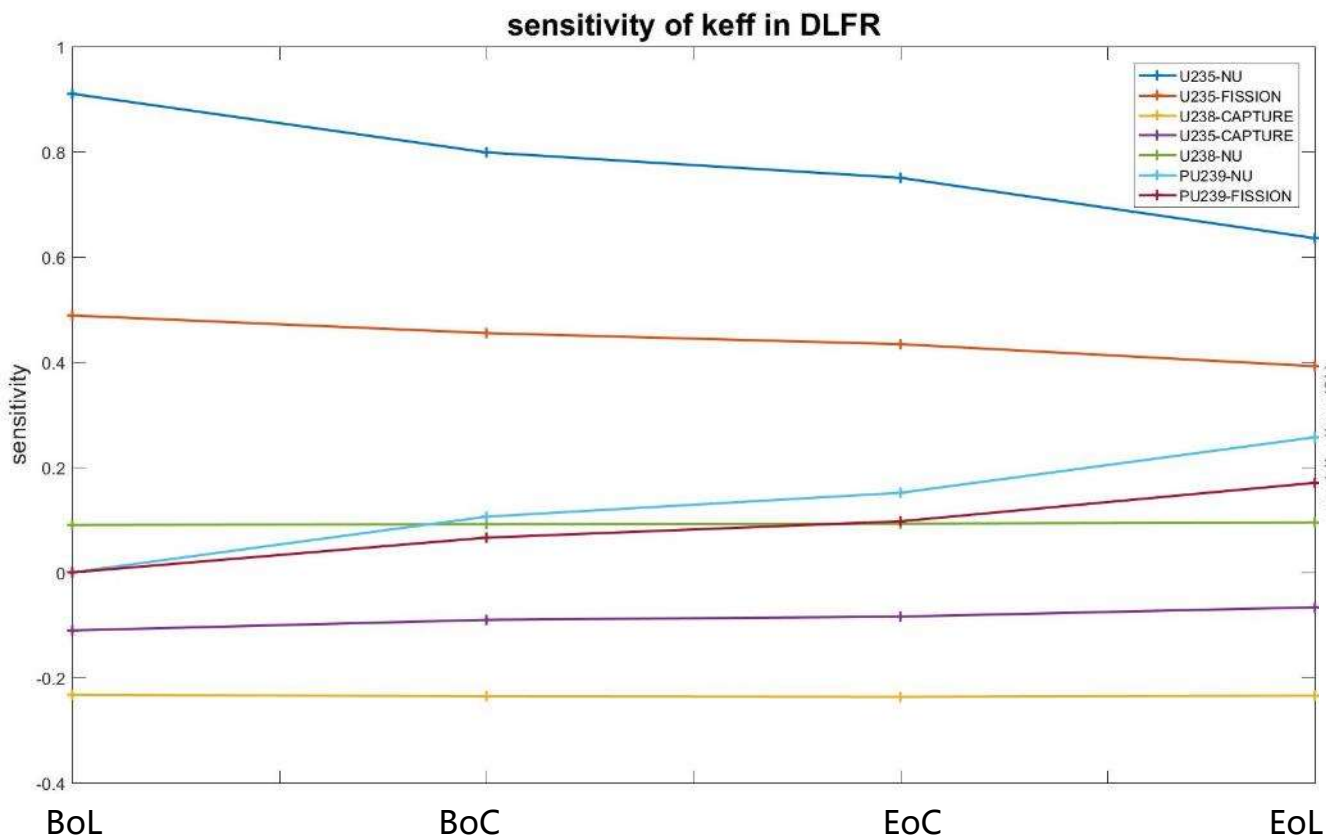
# Sensitivity and Uncertainty Contributed to Total in Different Periods

## Inner Core Fuel Assembly



# Sensitivity and Uncertainty Contributed to Total in Different Periods

## Outer Core Fuel Assembly



**PART**

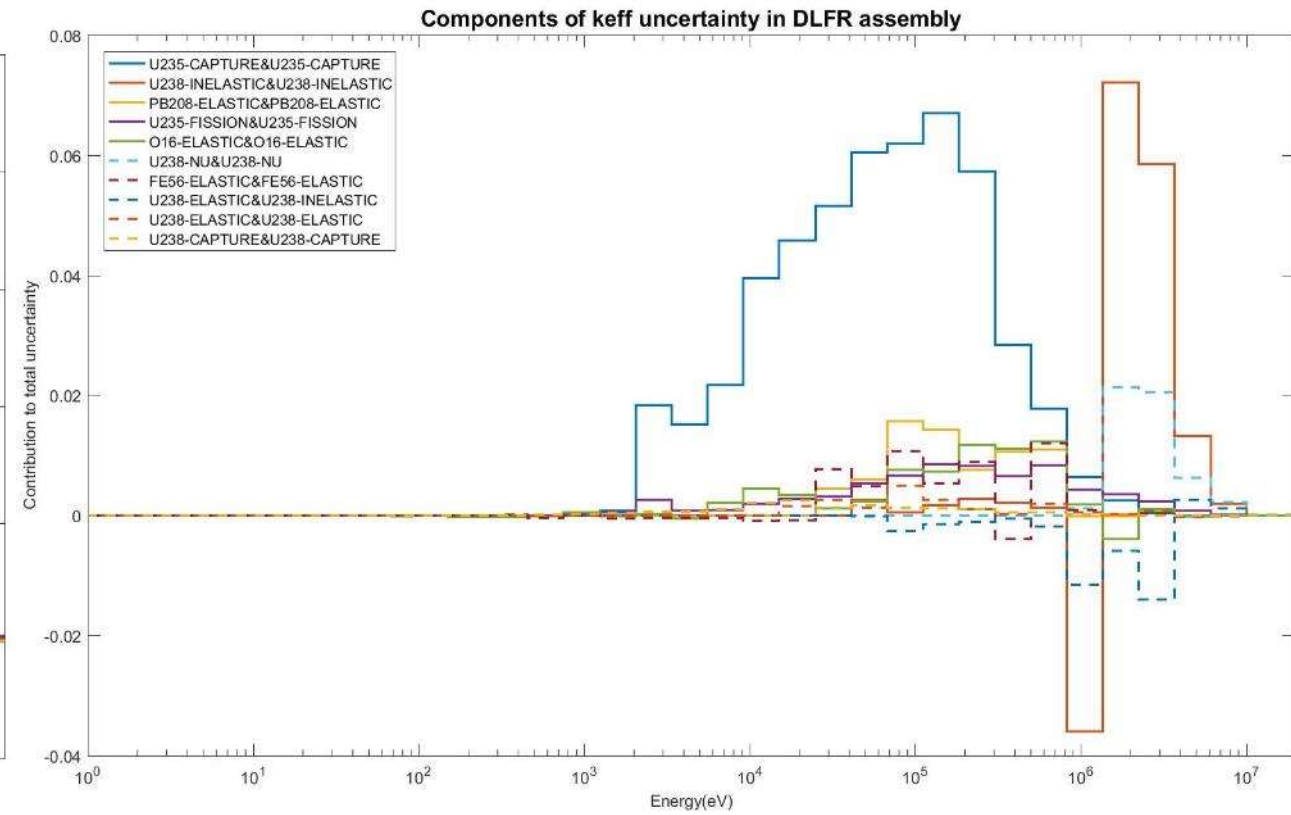
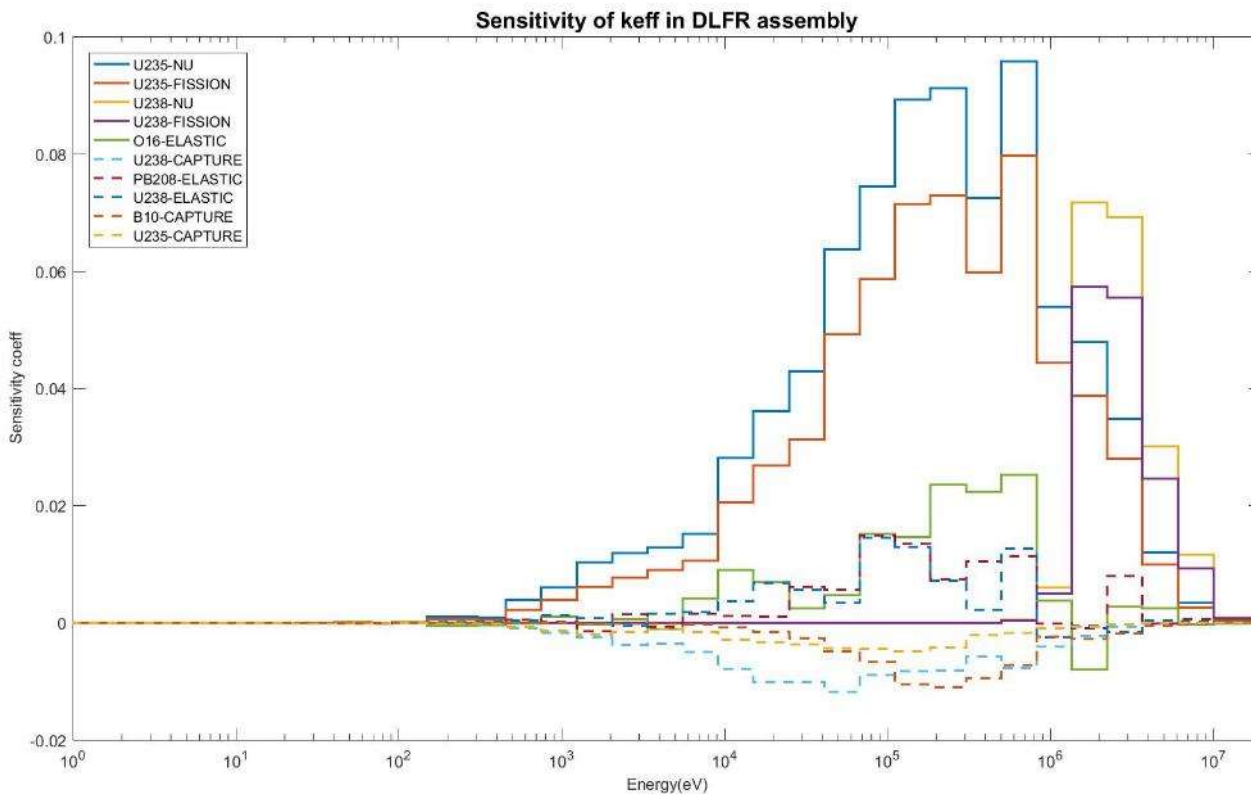
**3.2**

**Safety Rod**



# Distribution of S/U Corresponding to the Most Important 20 Parameters

Safety Rod surrounded by Inner Core Fuel Assembly at Beginning of Life



Safety Rod surrounded by BoL inner Assembly:

$K_{eff}$  Standard Deviation: 0.009983

Parameter Pair	Contribution to Total Uncertainty (%)	Parameter	Conservatively Estimated Relative Uncertainty	Sensitivity
$^{235}\text{U} - \sigma_{n,\gamma}$	49.6019	$^{235}\text{U} - \sigma_{n,\gamma}$	0.94729	-0.041557
$^{238}\text{U} - \sigma_{n,n'}$	12.1277	$^{208}\text{Pb} - \sigma_{n,n}$	0.12561	0.081090
$^{208}\text{Pb} - \sigma_{n,n}$	7.0904	$^{235}\text{U} - \sigma_f$	0.12502	0.635583
$^{235}\text{U} - \sigma_f$	6.8442	$^{16}\text{O} - \sigma_{n,n}$	0.11627	0.129220
$^{16}\text{O} - \sigma_{n,n}$	6.2960	$^{238}\text{U} - \sigma_{n,n'}$	0.09061	-0.102597

Safety Rod surrounded by BoC inner Assembly:

$K_{eff}$  Standard Deviation: 0.010595

Parameter Pair	Contribution to Total Uncertainty (%)	Parameter	Conservatively Estimated Relative Uncertainty	Sensitivity
$^{235}\text{U} - \sigma_{n,\gamma}$	20.9232	$^{235}\text{U} - \sigma_{n,\gamma}$	0.405438	-0.028904
$^{56}\text{Fe} - \sigma_{n,n}$	19.4188	$^{56}\text{Fe} - \sigma_{n,n}$	0.340385	0.057541
$^{238}\text{U} - \sigma_{n,n'}$	14.8347	$^{238}\text{U} - \sigma_{n,n'}$	0.298071	-0.006460
$^{238}\text{U} - (\sigma_{n,n}, \sigma_{n,n'})$	9.8891	$^{238}\text{U} - \sigma_{n,n}$	0.155563	0.074835
$^{16}\text{O} - \sigma_{n,n}$	6.8959	$^{16}\text{O} - \sigma_{n,n}$	0.129392	0.142088

Safety Rod surrounded by EoC inner Assembly:

$K_{eff}$  Standard Deviation: 0.009616

Parameter Pair	Contribution to Total Uncertainty (%)	Parameter	Conservatively Estimated Relative Uncertainty	Sensitivity
$^{238}\text{U} - \sigma_{n,n'}$	27.4812	$^{238}\text{U} - \sigma_{n,n'}$	0.44838	-0.006568
$^{235}\text{U} - \sigma_{n,\gamma}$	16.2871	$^{235}\text{U} - \sigma_{n,\gamma}$	0.31823	-0.023141
$^{56}\text{Fe} - \sigma_{n,n}$	15.8962	$^{56}\text{Fe} - \sigma_{n,n}$	0.28609	0.049147
$^{238}\text{U} - (\sigma_{n,n}, \sigma_{n,n'})$	7.1493	$^{238}\text{U} - \sigma_{n,n}$	0.11081	0.075307
$^{238}\text{U} - \bar{\nu}$	5.9046	$^{238}\text{U} - \bar{\nu}$	0.09529	0.194659

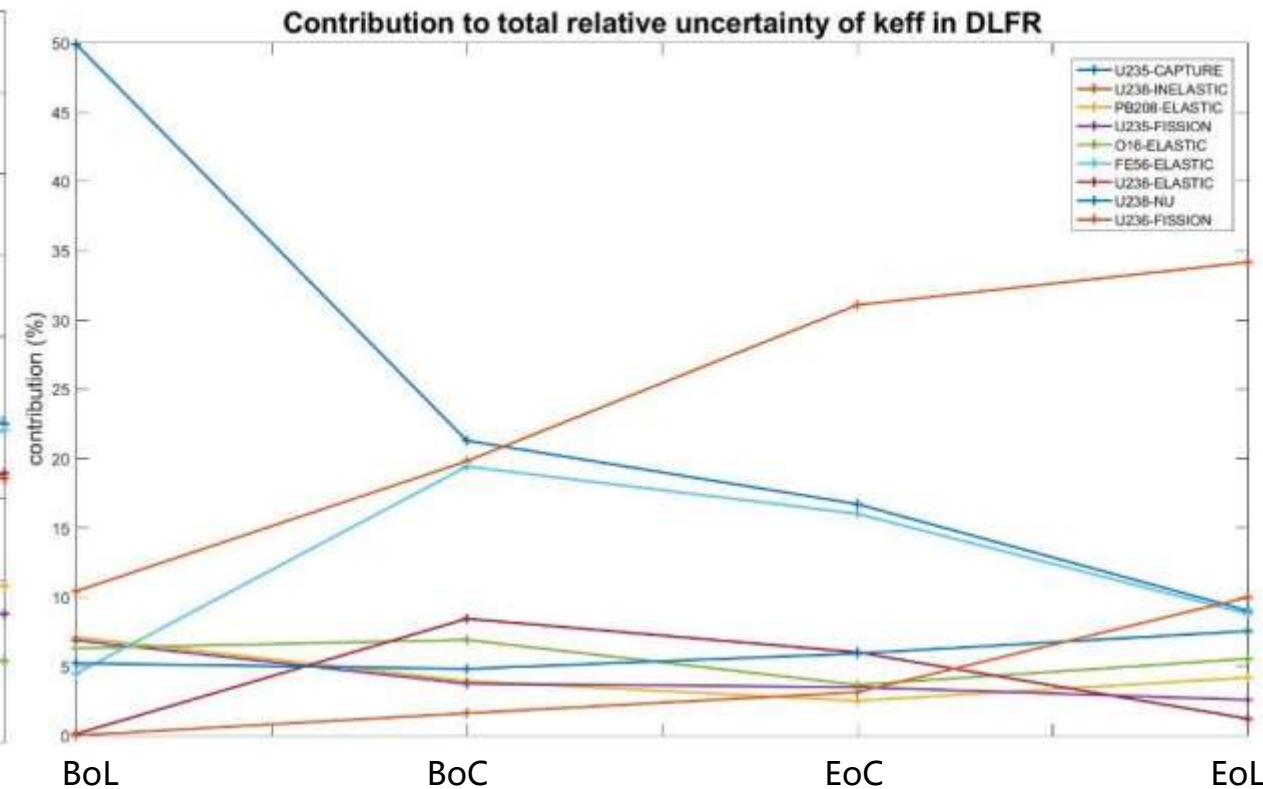
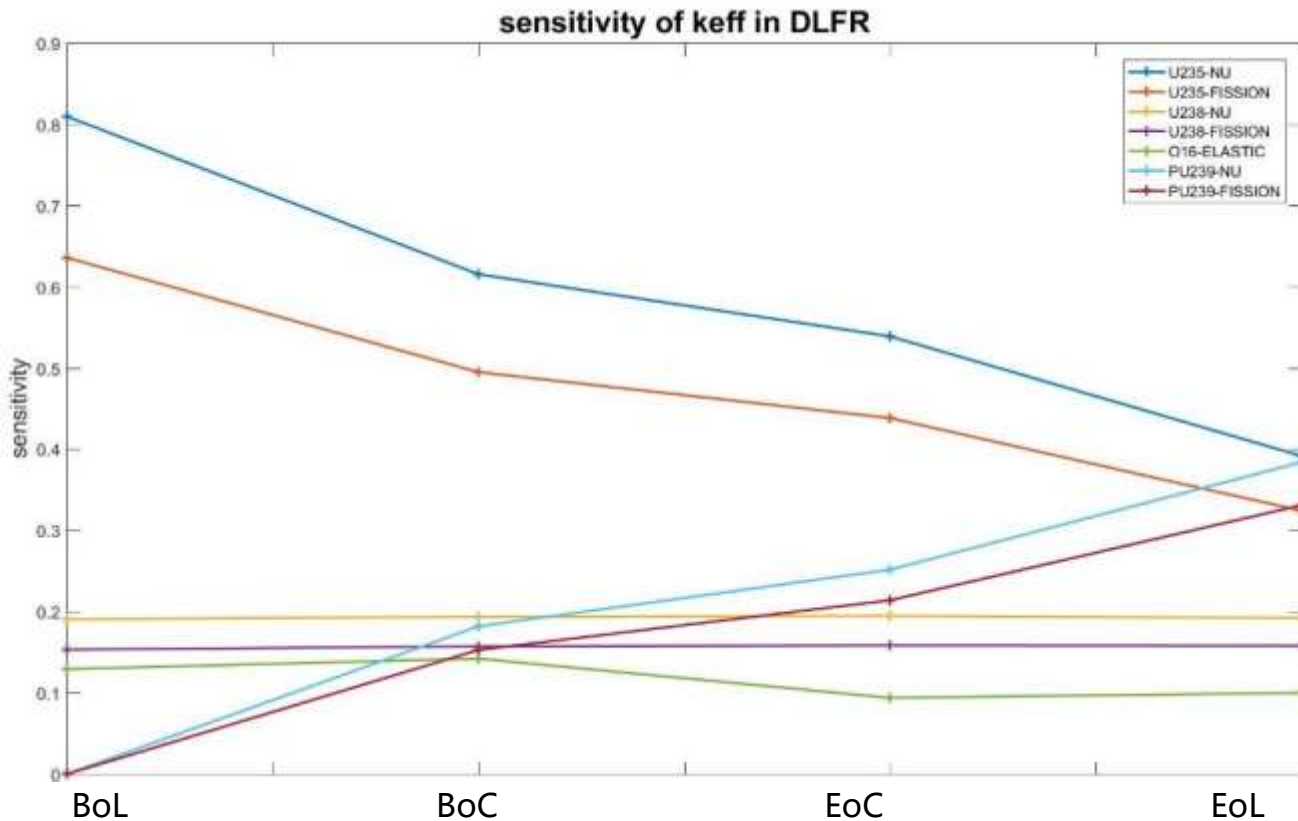
Safety Rod surrounded by EoL inner Assembly:

$K_{eff}$  Standard Deviation: 0.008373

Parameter Pair	Contribution to Total Uncertainty (%)	Parameter	Conservatively Estimated Relative Uncertainty	Sensitivity
$^{238}\text{U} - \sigma_{n,n'}$	34.8277	$^{238}\text{U} - \sigma_{n,n'}$	0.465302	-0.017072
$^{236}\text{U} - \sigma_f$	9.9612	$^{235}\text{U} - \sigma_{n,\gamma}$	0.171462	-0.015318
$^{235}\text{U} - \sigma_{n,\gamma}$	8.8226	$^{236}\text{U} - \sigma_f$	0.170007	0.010268
$^{56}\text{Fe} - \sigma_{n,n}$	8.7255	$^{56}\text{Fe} - \sigma_{n,n}$	0.139694	0.028763
$^{238}\text{U} - \bar{\nu}$	7.5280	$^{238}\text{U} - \bar{\nu}$	0.121730	0.191956

# Sensitivity and Uncertainty Contributed to Total in Different Periods

## Safety Rod



**PART**

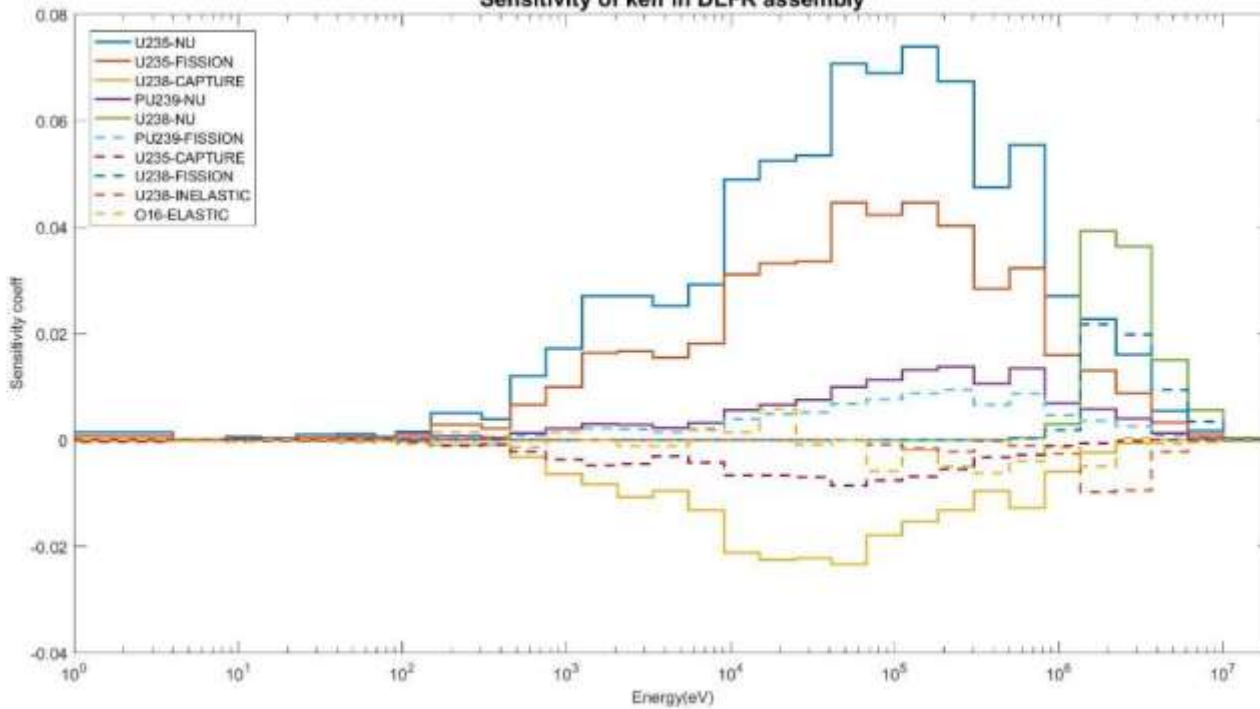
**3.3**

**2D Whole Core**

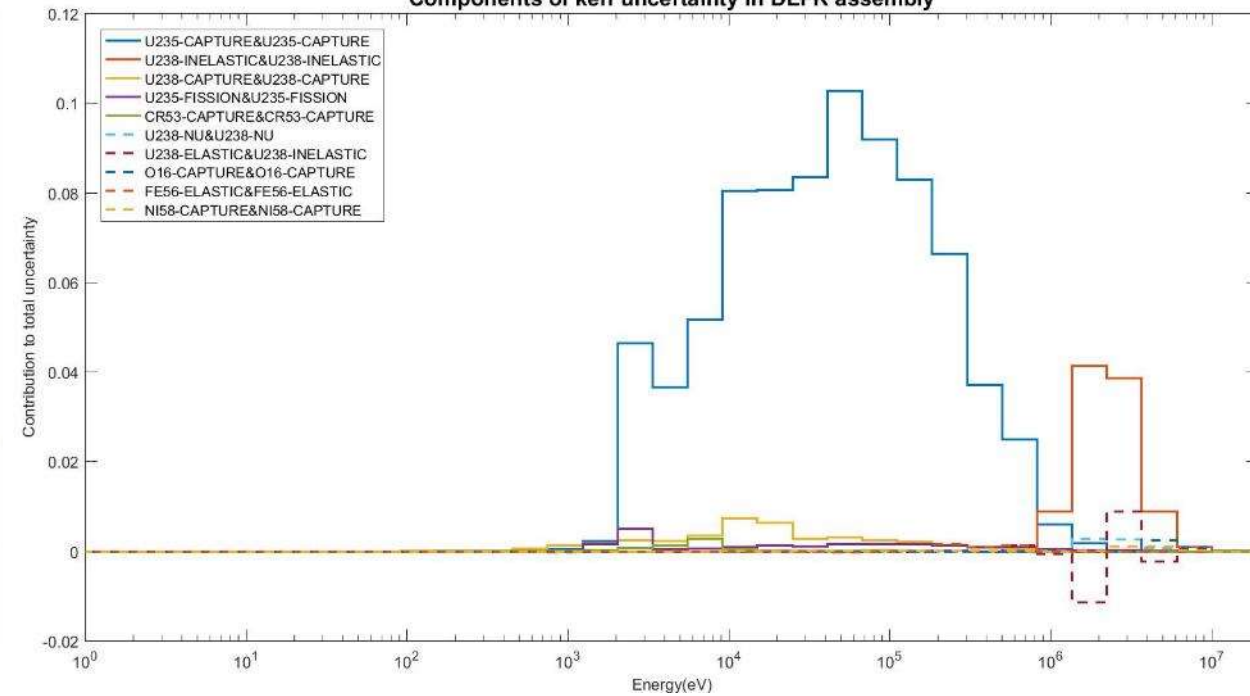
# Distribution of S/U Corresponding to the Most Important 10 Parameters

2D Whole Core with Safety Rod out at Beginning of Life

Sensitivity of  $k_{eff}$  in DLFR assembly



Components of  $k_{eff}$  uncertainty in DLFR assembly





## 2D Whole Core with Safety Rod out at BoL:

$K_{eff}$  Standard Deviation: 0.017122

Parameter Pair	Contribution to Total Uncertainty (%)	Parameter	Conservatively Estimated Relative Uncertainty	Sensitivity
$^{235}\text{U} - \sigma_{n,\gamma}$	84.7604	$^{235}\text{U} - \sigma_{n,\gamma}$	1.61504	-0.098547
$^{238}\text{U} - \sigma_{n,n'}$	9.2950	$^{238}\text{U} - \sigma_{n,n'}$	0.13958	-0.034373
$^{238}\text{U} - \sigma_{n,\gamma}$	2.7068	$^{238}\text{U} - \sigma_{n,\gamma}$	0.04700	-0.213675
$^{235}\text{U} - \sigma_f$	1.8171	$^{235}\text{U} - \sigma_f$	0.03180	0.526743
$^{238}\text{U} - (\sigma_{n,n}, \sigma_{n,n'})$	-1.3867	$^{238}\text{U} - \sigma_{n,n}$	-0.01317	0.007237

## 2D Whole Core with Safety Rod in at BoL:

$K_{eff}$  Standard Deviation: 0.016627

Parameter Pair	Contribution to Total Uncertainty (%)	Parameter	Conservatively Estimated Relative Uncertainty	Sensitivity
$^{235}\text{U} - \sigma_{n,\gamma}$	84.8102	$^{235}\text{U} - \sigma_{n,\gamma}$	1.607250	-0.096055
$^{238}\text{U} - \sigma_{n,n'}$	8.3663	$^{238}\text{U} - \sigma_{n,n'}$	0.139454	-0.029788
$^{238}\text{U} - \sigma_{n,\gamma}$	2.6780	$^{238}\text{U} - \sigma_{n,\gamma}$	0.046401	-0.207413
$^{235}\text{U} - \sigma_f$	1.9128	$^{235}\text{U} - \sigma_f$	0.033683	0.529179
$^{235}\text{U} - (\sigma_{n,n}, \sigma_{n,\gamma})$	-0.0078	$^{238}\text{U} - \bar{\nu}$	0.009034	-0.103665

## 2D Whole Core with Safety Rod out at BoC:

$K_{eff}$  Standard Deviation: 0.014614

Parameter Pair	Contribution to Total Uncertainty (%)	Parameter	Conservatively Estimated Relative Uncertainty	Sensitivity
$^{235}\text{U} - \sigma_{n,\gamma}$	70.9466	$^{235}\text{U} - \sigma_{n,\gamma}$	1.35330	-0.077559
$^{238}\text{U} - \sigma_{n,n'}$	16.4545	$^{238}\text{U} - \sigma_{n,n'}$	0.29119	-0.042023
$^{238}\text{U} - \sigma_{n,\gamma}$	3.7255	$^{238}\text{U} - \sigma_{n,\gamma}$	0.06472	-0.212163
$^{235}\text{U} - \sigma_f$	2.0054	$^{235}\text{U} - \sigma_f$	0.03487	0.468207
$^{238}\text{U} - (\sigma_{n,n}, \sigma_{n,n'})$	1.6438	$^{238}\text{U} - \sigma_{n,n}$	0.01699	0.010483

## 2D Whole Core with Safety Rod in at BoC:

$K_{eff}$  Standard Deviation: 0.014419

Parameter Pair	Contribution to Total Uncertainty (%)	Parameter	Conservatively Estimated Relative Uncertainty	Sensitivity
$^{235}\text{U} - \sigma_{n,\gamma}$	70.3116	$^{235}\text{U} - \sigma_{n,\gamma}$	1.350580	-0.076630
$^{238}\text{U} - \sigma_{n,n'}$	14.6748	$^{238}\text{U} - \sigma_{n,n'}$	0.266992	-0.035847
$^{238}\text{U} - \sigma_{n,\gamma}$	3.6564	$^{238}\text{U} - \sigma_{n,\gamma}$	0.063624	-0.207103
$^{238}\text{U} - (\sigma_{n,n}, \sigma_{n,n'})$	3.2333	$^{235}\text{U} - \sigma_f$	0.036008	0.473907
$^{235}\text{U} - \sigma_f$	2.0707	$^{238}\text{U} - \sigma_{n,n}$	0.035553	0.023593

## 2D Whole Core with Safety Rod out at EoC:

$K_{eff}$  Standard Deviation: 0.013229

Parameter Pair	Contribution to Total Uncertainty (%)	Parameter	Conservatively Estimated Relative Uncertainty	Sensitivity
$^{235}\text{U} - \sigma_{n,\gamma}$	65.1362	$^{235}\text{U} - \sigma_{n,\gamma}$	1.23615	-0.068290
$^{238}\text{U} - \sigma_{n,n'}$	22.5490	$^{238}\text{U} - \sigma_{n,n'}$	0.36535	-0.038401
$^{238}\text{U} - \sigma_{n,\gamma}$	4.4631	$^{238}\text{U} - \sigma_{n,\gamma}$	0.07786	-0.210651
$^{235}\text{U} - \sigma_f$	2.2424	$^{235}\text{U} - \sigma_f$	0.03846	0.442943
$^{238}\text{U} - \bar{\nu}$	0.9324	$^{238}\text{U} - \bar{\nu}$	0.01503	0.106290

## 2D Whole Core with Safety Rod in at EoC:

$K_{eff}$  Standard Deviation: 0.013081

Parameter Pair	Contribution to Total Uncertainty (%)	Parameter	Conservatively Estimated Relative Uncertainty	Sensitivity
$^{235}\text{U} - \sigma_{n,\gamma}$	72.2246	$^{235}\text{U} - \sigma_{n,\gamma}$	1.372160	-0.071178
$^{238}\text{U} - \sigma_{n,n'}$	16.0571	$^{238}\text{U} - \sigma_{n,n'}$	0.258783	-0.032237
$^{238}\text{U} - \sigma_{n,\gamma}$	4.3330	$^{238}\text{U} - \sigma_{n,\gamma}$	0.075305	-0.204820
$^{235}\text{U} - \sigma_f$	2.3320	$^{235}\text{U} - \sigma_f$	0.040107	0.447814
$^{238}\text{U} - \bar{\nu}$	0.9553	$^{238}\text{U} - \bar{\nu}$	0.015403	0.106501

## 2D Whole Core with Safety Rod out at EoL:

$K_{eff}$  Standard Deviation: 0.010885

Parameter Pair	Contribution to Total Uncertainty (%)	Parameter	Conservatively Estimated Relative Uncertainty	Sensitivity
$^{235}\text{U} - \sigma_{n,\gamma}$	53.8269	$^{235}\text{U} - \sigma_{n,\gamma}$	1.01789	-0.051801
$^{238}\text{U} - \sigma_{n,n'}$	29.1961	$^{238}\text{U} - \sigma_{n,n'}$	0.43188	-0.046472
$^{238}\text{U} - \sigma_{n,\gamma}$	6.3703	$^{238}\text{U} - \sigma_{n,\gamma}$	0.11114	-0.205452
$^{238}\text{U} - (\sigma_{n,n}, \sigma_{n,n'})$	-4.3104	$^{235}\text{U} - \sigma_f$	0.04106	0.375887
$^{235}\text{U} - \sigma_f$	2.4146	$^{238}\text{U} - \sigma_{n,n}$	-0.03818	0.019358

## 2D Whole Core with Safety Rod in at EoL:

$K_{eff}$  Standard Deviation: 0.011287

Parameter Pair	Contribution to Total Uncertainty (%)	Parameter	Conservatively Estimated Relative Uncertainty	Sensitivity
$^{235}\text{U} - \sigma_{n,\gamma}$	53.0769	$^{235}\text{U} - \sigma_{n,\gamma}$	1.016330	-0.053765
$^{238}\text{U} - \sigma_{n,n'}$	27.3726	$^{238}\text{U} - \sigma_{n,n'}$	0.454822	-0.037794
$^{238}\text{U} - \sigma_{n,\gamma}$	5.8802	$^{238}\text{U} - \sigma_{n,\gamma}$	0.102275	-0.209787
$^{235}\text{U} - \sigma_f$	2.3522	$^{235}\text{U} - \sigma_f$	0.039774	0.384521
$^{238}\text{U} - \bar{\nu}$	1.3014	$^{238}\text{U} - \bar{\nu}$	0.020972	0.107249

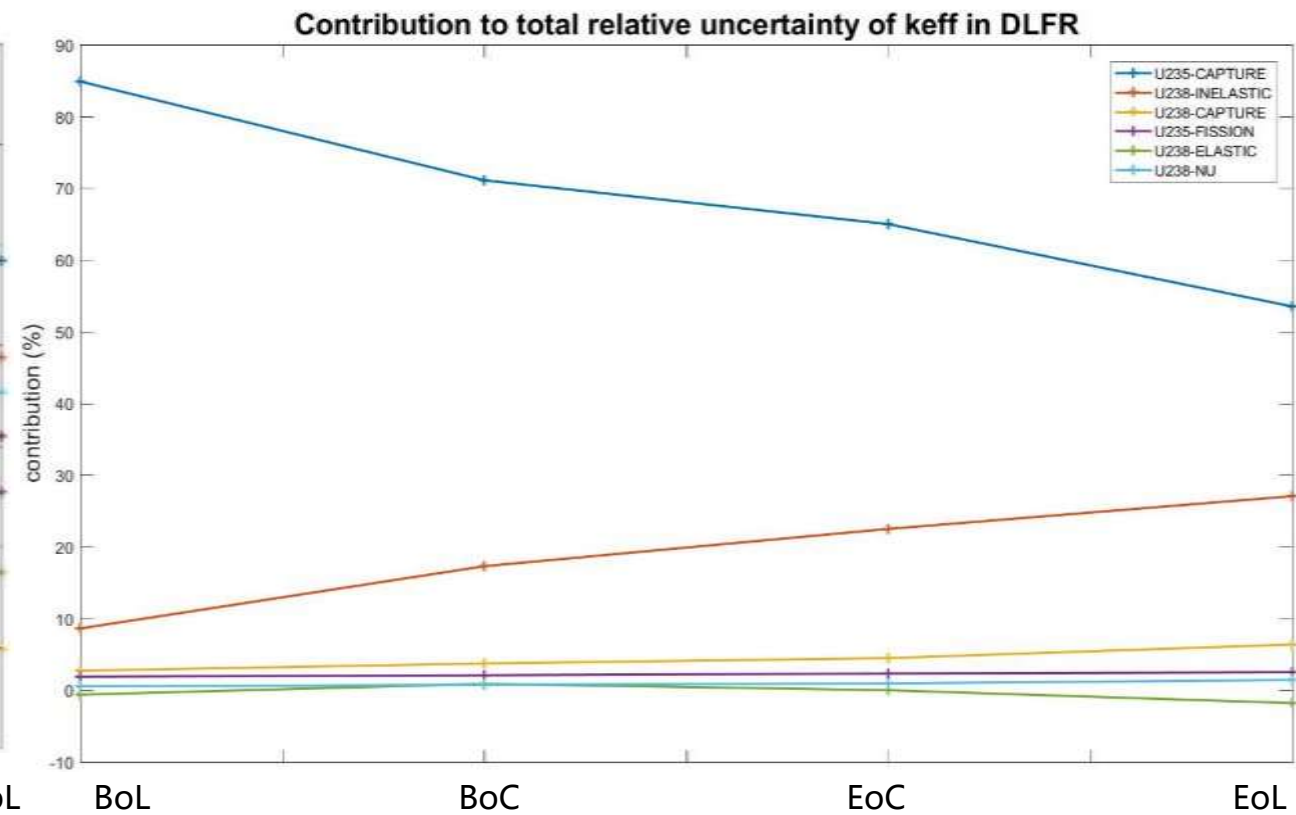
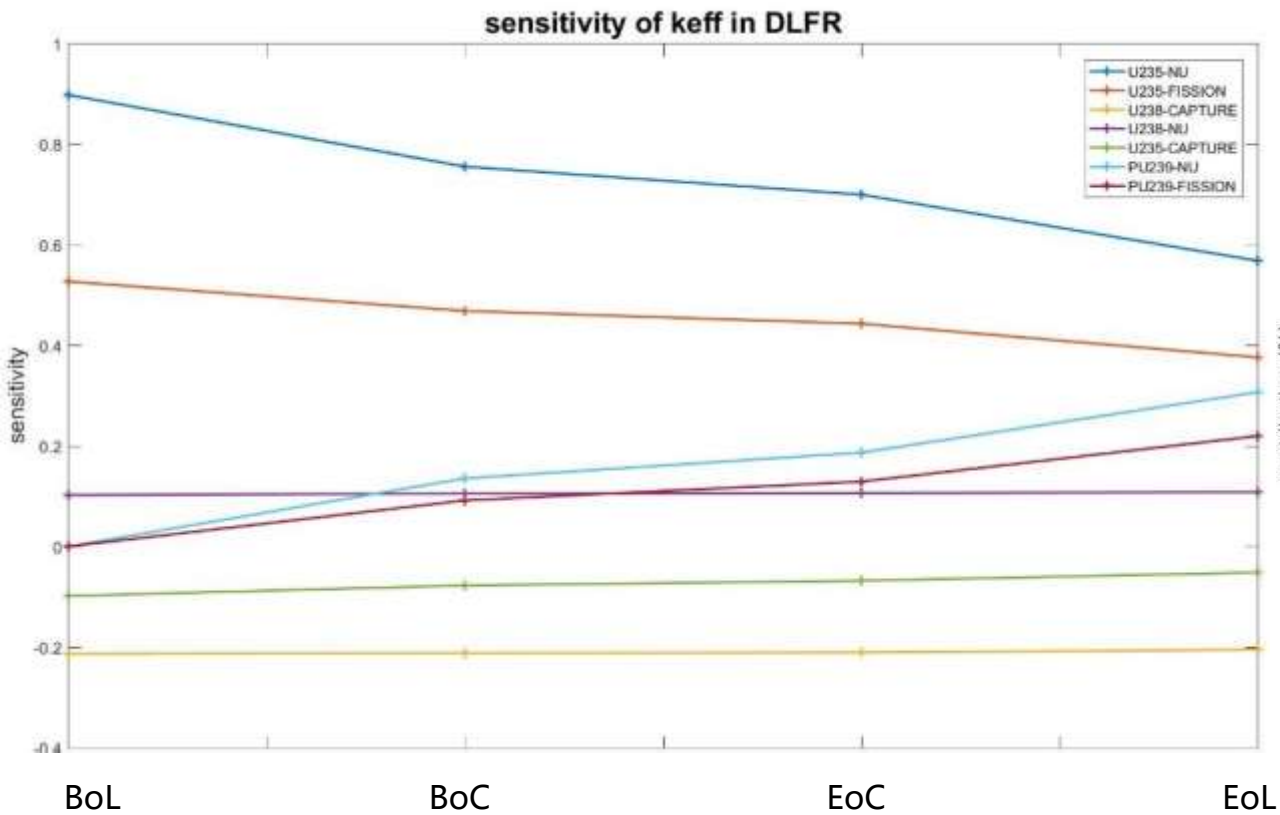
**PART**

**04**

**Conclusion**

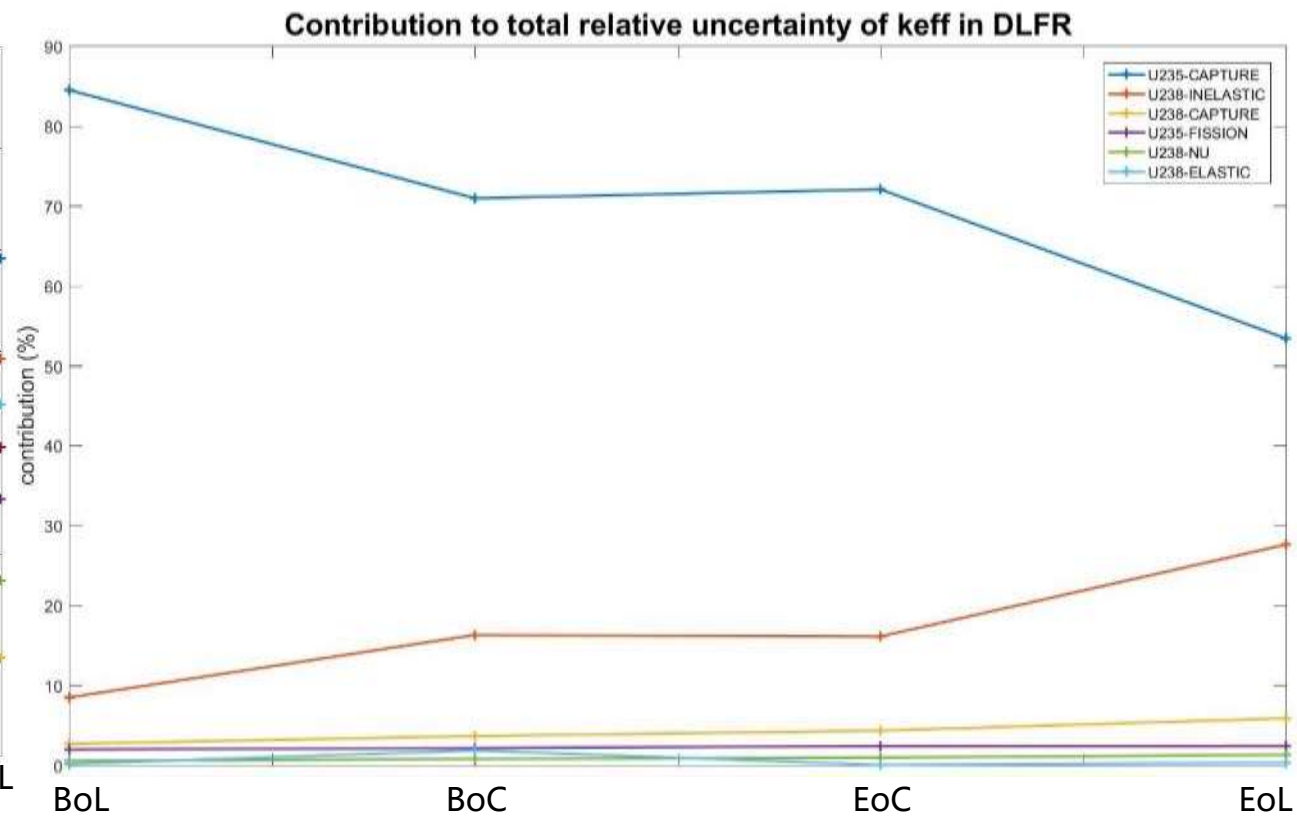
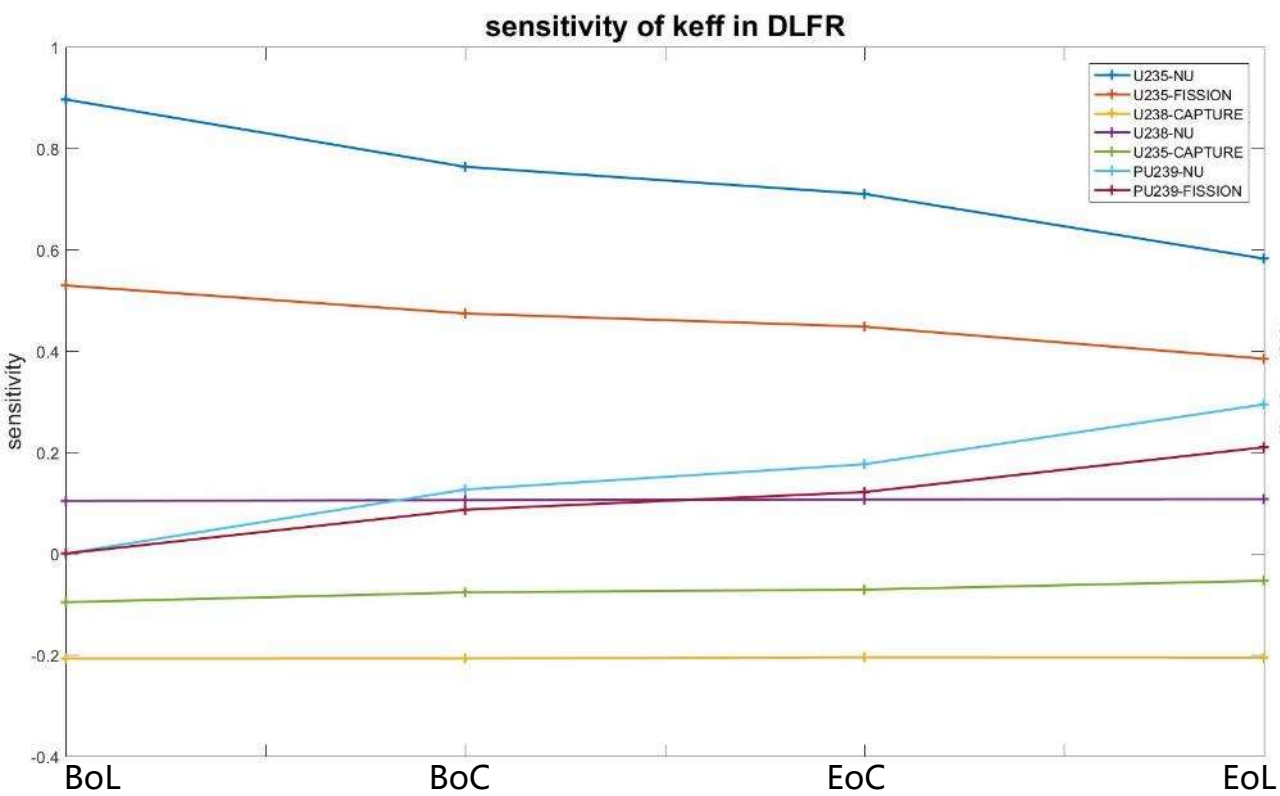
# Sensitivity and Uncertainty Contributed to Total in Different Periods

## 2D Whole Core with Safety Rod out



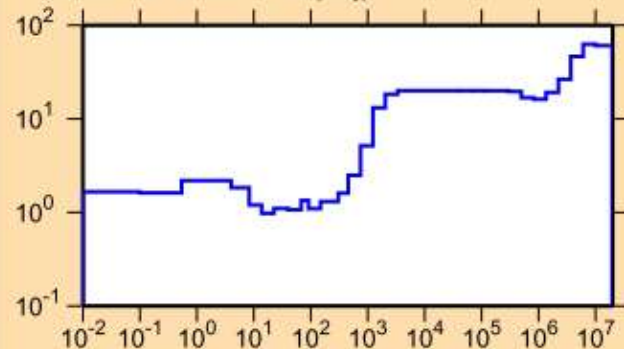
# Sensitivity and Uncertainty Contributed to Total in Different Periods

## 2D Whole Core with Safety Rod in



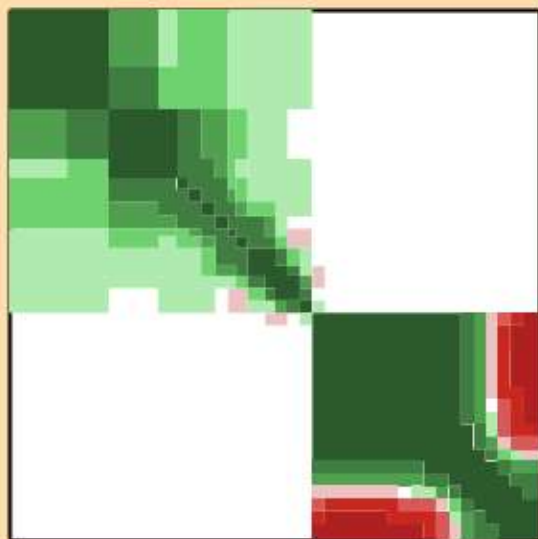
# Correlation Matrices

$\Delta\sigma/\sigma$  vs. E for  $^{235}\text{U}(n,\gamma)$

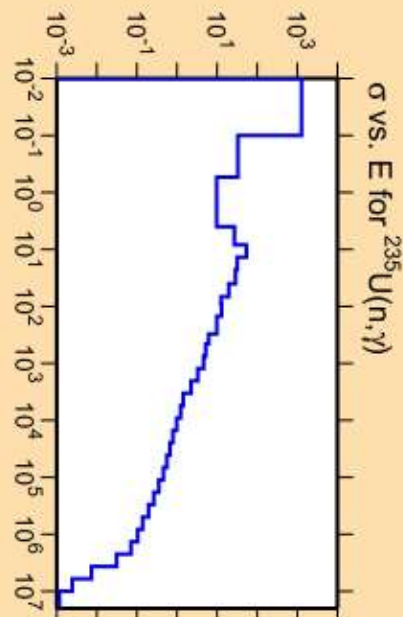
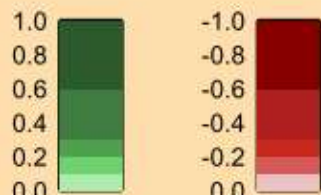


Ordinate scales are % relative standard deviation and barns.

Abscissa scales are energy (eV).

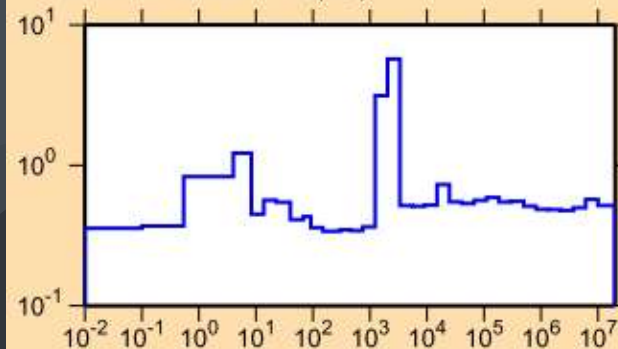


Correlation Matrix



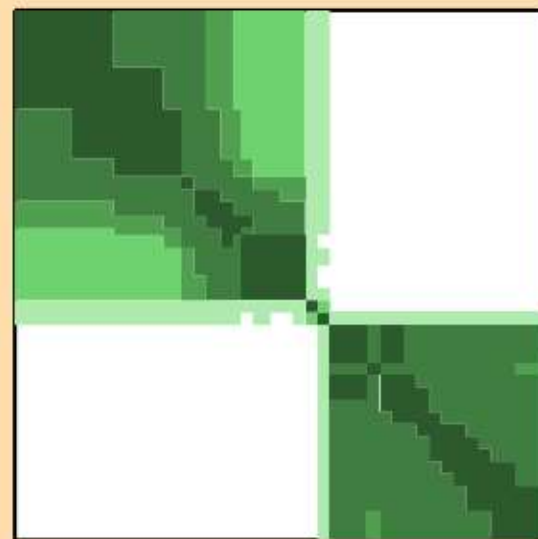
$\sigma$  vs. E for  $^{235}\text{U}(n,\gamma)$

$\Delta\sigma/\sigma$  vs. E for  $^{235}\text{U}(n,f)$

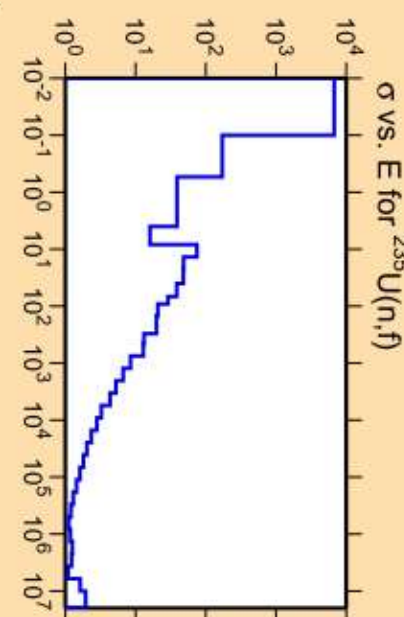
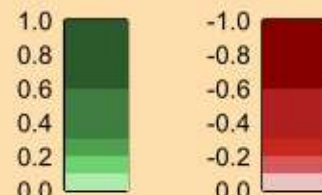


Ordinate scales are % relative standard deviation and barns.

Abscissa scales are energy (eV).



Correlation Matrix

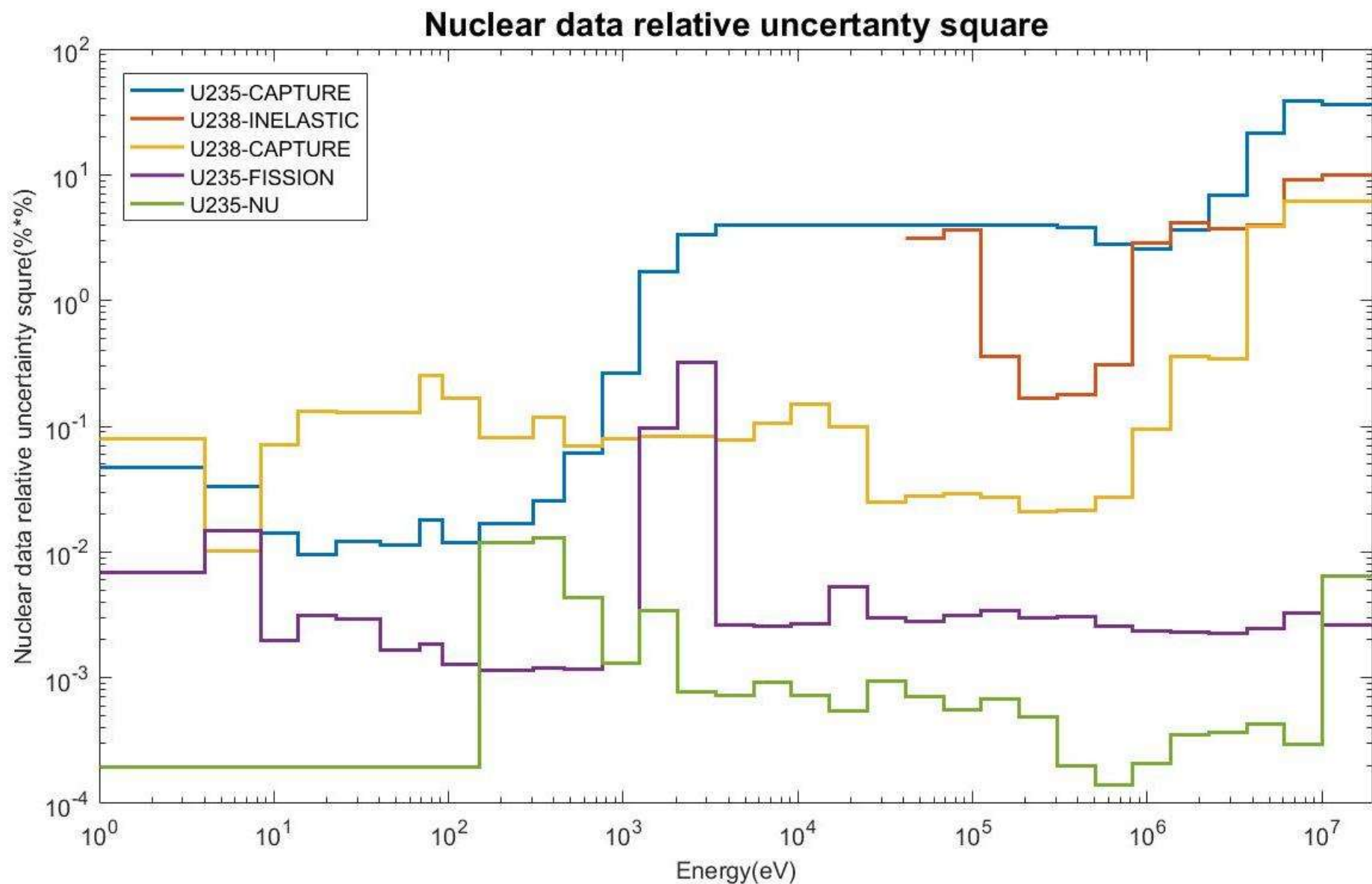


$\sigma$  vs. E for  $^{235}\text{U}(n,f)$



# Nuclear Data Relative Uncertainty

( Elements on diagonal of each relative covariance matrix )

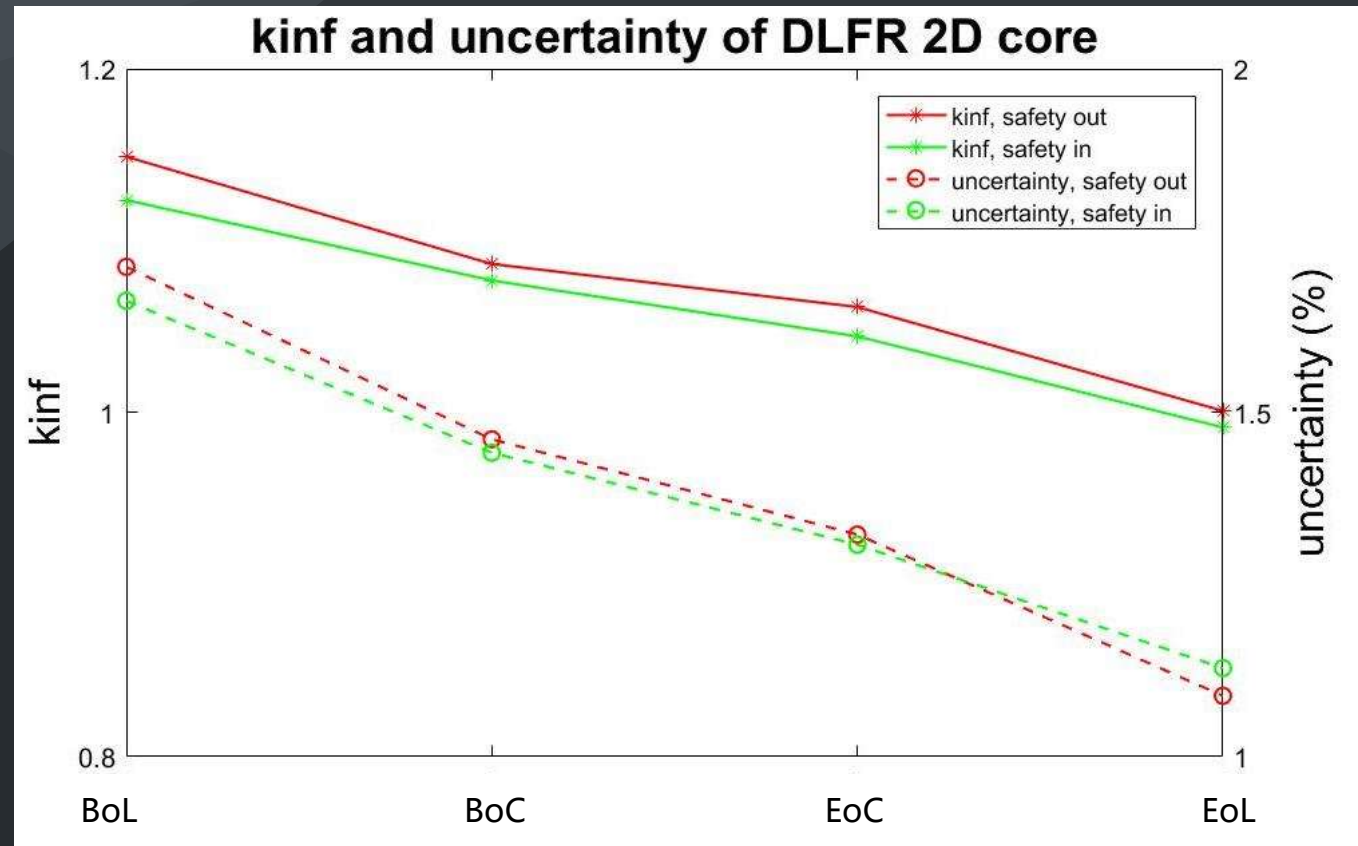


# Conclusion

Sensitivity	Uncertainty Contribution
$^{235}\text{U} - \bar{\nu}$	$^{235}\text{U} - \sigma_{n,\gamma}$
$^{235}\text{U} - \sigma_f$	$^{238}\text{U} - \sigma_{n,n'}$
$^{238}\text{U} - \sigma_{n,\gamma}$	$^{238}\text{U} - \sigma_{n,\gamma}$
$^{235}\text{U} - \sigma_{n,\gamma}$	$^{235}\text{U} - \sigma_f$
$^{238}\text{U} - \bar{\nu}$	$^{238}\text{U} - \sigma_{n,n}$
$^{239}\text{Pu} - \bar{\nu}$	$^{238}\text{U} - \bar{\nu}$
$^{239}\text{Pu} - \sigma_f$	$^{53}\text{Cr} - \sigma_{n,\gamma}$

Period	Relative Uncertainty
BoL	1.71%
BoC	1.46%
EoC	1.32%
EoL	1.09%

## k-inf & Uncertainty of 2D Whole Core

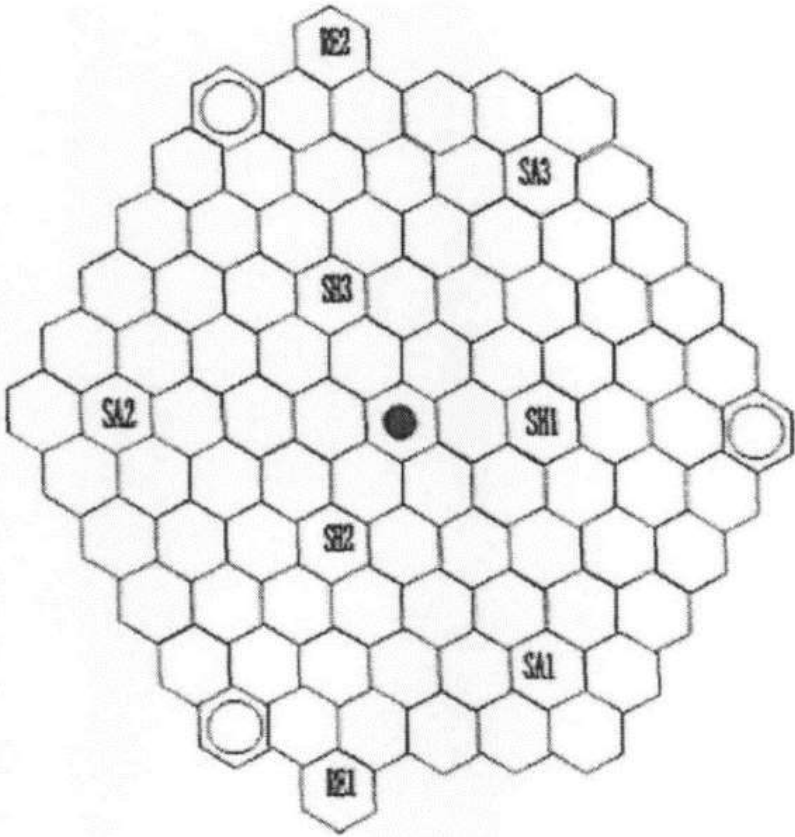


# Comparison<sup>[1]</sup>

**Reactor:** CEFR (China Experimental Fast Reactor)  
**Fuel:**  $UO_2$ ,  $^{235}U\% = 64.4\%$   
**Radius:** 30.2 cm  
**Covariance:** Based on Transportation Calculation via ANISN  
**Code:** SUCA1D  
(Sensitivity and Uncertainty Code of Analysis, one dimension)

Total Uncertainty	2.65%
Reference Total Uncertainty <sup>[2]</sup>	1.90%

Number	Parameter Pair	Uncertainty	Contribution to Total (%)
1	$^{235}U - \sigma_f$	1.27%	22.97
2	$^{235}U - \sigma_{n,\gamma}$	2.20%	68.92
3	$^{238}U - \sigma_f$	0.15%	0.32
4	$^{238}U - \sigma_{n,\gamma}$	0.73%	7.59
5	$^{56}Fe - \sigma_{n,\gamma}$	0.12%	0.21



- SA Safety Rod
- SH Compensation Control Rod
- RE Regulation Control Rod
- Fuel Assembly
- Neutron Source Assembly
- Stainless Steel Assembly

CEFR Core in Equilibrium State

[1] 刚直. 核截面引起积分参数 $k_{eff}$ 不确定度的一维分析程序开发[D]. 中国原子能科学研究院, 2006.  
[2] 俄罗斯技术报告. CEFR堆芯物理特性计算误差分析  
([2] is just mentioned but not cited by [1], reference is not given in [1].)

# Reference

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- [10] 刚直. 核截面引起积分参数 $k_{eff}$ 不确定度的一维分析程序开发[D]. 中国原子能科学研究院, 2006.

# Thanks for the help & guidance provided by :

<b>Yishu</b>	<b>QIU</b>	( Tsinghua, Department of Engineering Physics )
<b>Lidong</b>	<b>WANG</b>	( Tsinghua, Institute of Nuclear and New Energy Technology )
<b>Ishita</b>	<b>TRIVEDI</b>	( NCSU, Department of Nuclear Engineering )
<b>Kaiyue</b>	<b>ZENG</b>	( NCSU, Department of Nuclear Engineering )
<b>Jun</b>	<b>SHI</b>	( UCB, Engineering Research Centers )

**END | THANK YOU!**

PRESENTED BY LI JIN