

DLFR

Demonstration Lead-cooled Fast Reactor

S & U Analysis

Sensitivity & Uncertainty Analysis

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PART

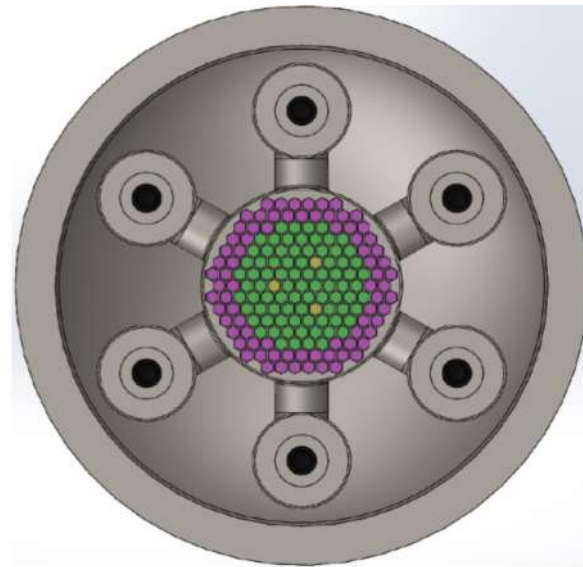
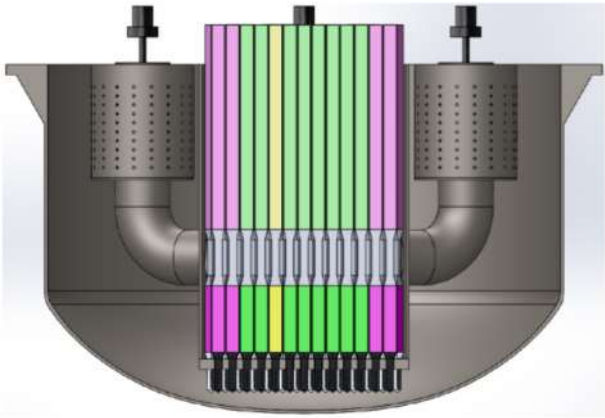
01

Introduction

DLFR

By Westinghouse, U.S.

Demonstration Lead-cooled Fast Reactor



DLFR primary system layout, vertical and horizontal cross section
(pre-conceptual, not in scale, DHRS not shown)

Design Parameter

Power rate: 500MWt

Neutron flux: Peak $\sim 2 \times 10^{15}$ n/cm²

Reactor Type

Lead-cooled
Pool-type
Fast reactor

Objective

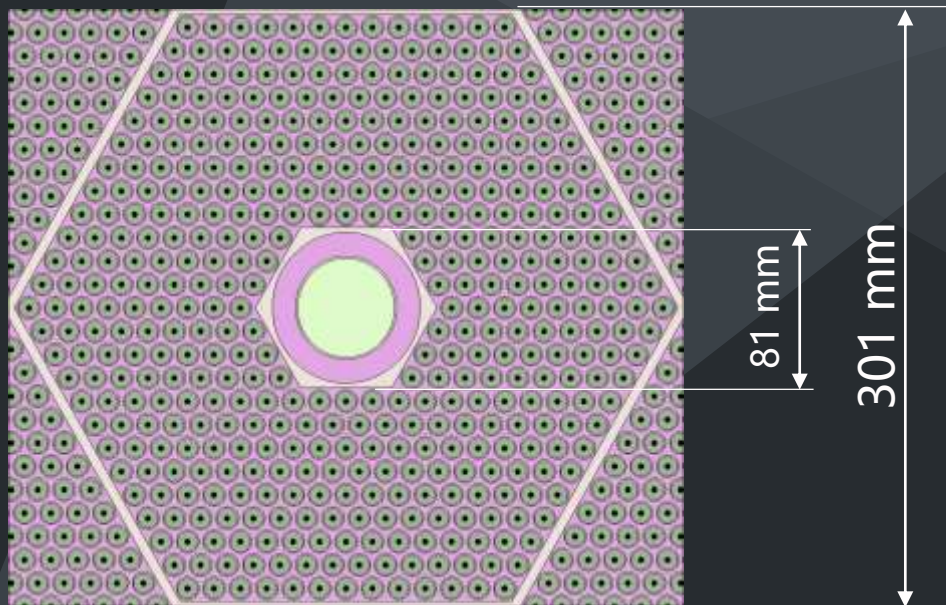
Feasibility
Basic performance
Upgrade





PART

02

Modeling

Assembly



| | | |
|---|----------|-------|
|  | Coolant | Pb |
|  | Cladding | Steel |
|  | Hollow | Void |
|  | Fuel | |

Boundary: Periodical

Calculation:

Periodical Boundary

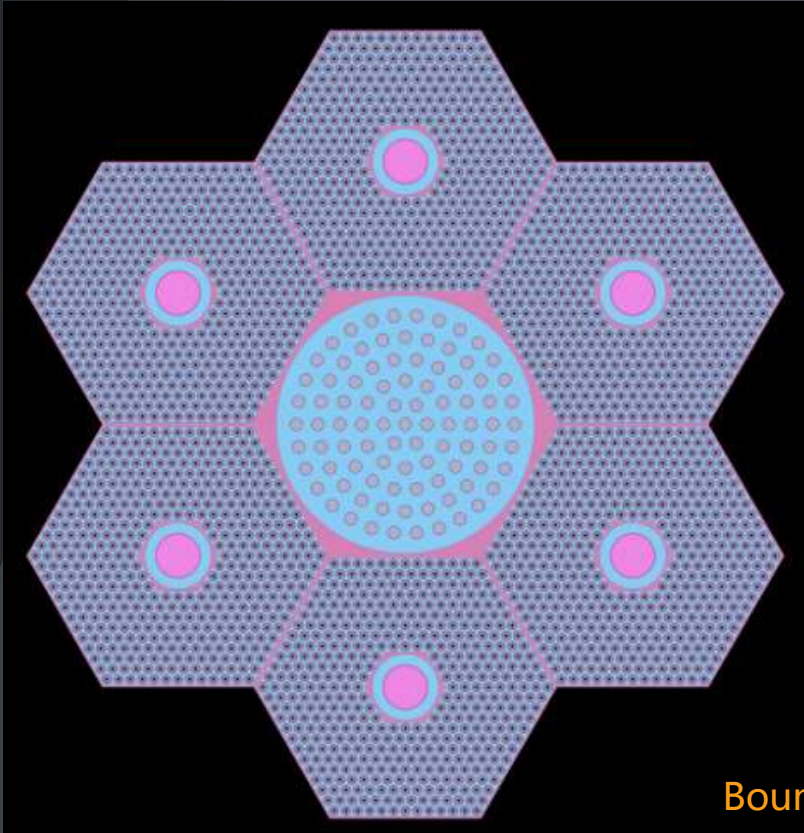
Inner Core Assembly

| Fuel | K-inf | Error |
|------|---------|---------|
| BoL | 1.26791 | 0.00246 |
| BoC | 1.19807 | 0.00278 |
| EoC | 1.16523 | 0.00278 |
| EoL | 1.10776 | 0.00316 |

Outer Core Assembly

| Fuel | K-inf | Error |
|------|---------|---------|
| BoL | 1.34074 | 0.00291 |
| BoC | 1.28491 | 0.00289 |
| EoC | 1.26066 | 0.00292 |
| EoL | 1.19467 | 0.00281 |

Safety Rod



- Coolant -Pb
- Cladding -Steel
- Hollow -Void
- Inner Core Fuel
- Reflector -YSZ 8% mol
- Safety - B_4C

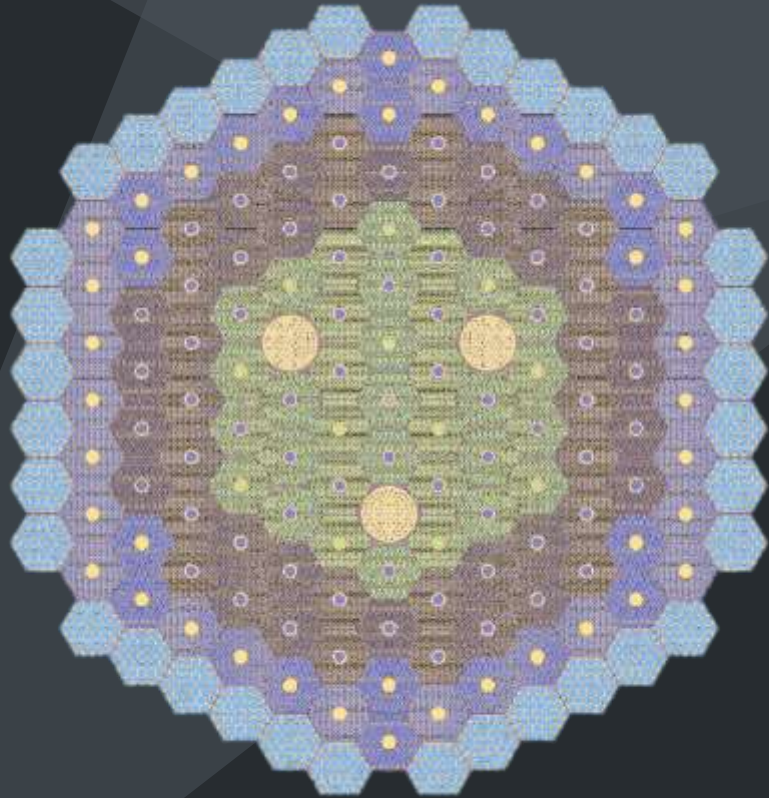
Calculation:






Black Boundary

Safety surrounded by Inner Core Assembly

| Fuel | K-inf | Error |
|------|---------|---------|
| BoL | 1.02864 | 0.00361 |
| BoC | 0.98266 | 0.00377 |
| EoC | 0.96995 | 0.00349 |
| EoL | 0.94755 | 0.00392 |

Core

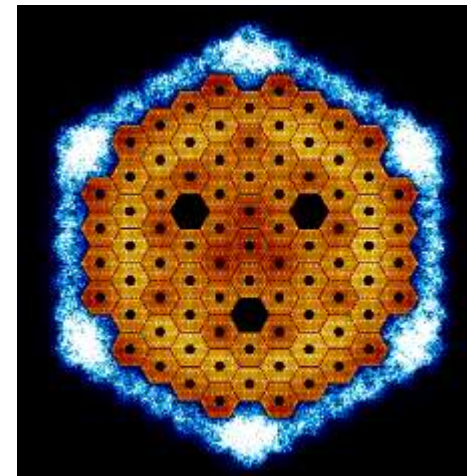


| | | | | | |
|---|------------|--------|---|-----------|----------------|
|  | Coolant | -Pb |  | Reflector | -YSZ 8% mol |
|  | Cladding | -Steel |  | Shield | -Borated Steel |
|  | Inner Fuel | |  | Safety | $-B_4C$ |
|  | Outer Fuel | |  | Plenum | |

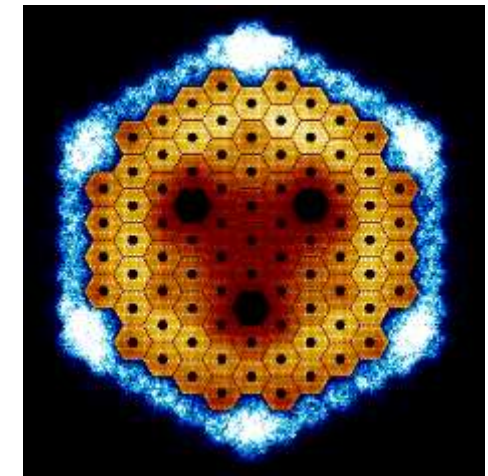
Calculation:

| Time | $k_{inf} \pm \text{error}$ | |
|------|----------------------------|-----------------------|
| | Safety Rod out | Safety Rod in |
| BoL | 1.14892 ± 0.00335 | 1.12366 ± 0.00312 |
| BoC | 1.08653 ± 0.00255 | 1.07694 ± 0.00234 |
| EoC | 1.06150 ± 0.00313 | 1.04444 ± 0.00281 |
| EoL | 1.01073 ± 0.00341 | 0.99147 ± 0.00316 |

Power Distribution:



Safety Rod out



Safety Rod in

PART

03

Calculation

Sensitivity



Definition

$$S_x^R = \frac{\partial R/R}{\partial x/x}$$

$$R = \frac{\langle \Sigma_1, \Psi \rangle}{\langle \Sigma_2, \Psi \rangle}$$

- R — Response function
 x — A certain perturbed parameter
 S_x^R — Sensitivity coefficient of R with respect to x

$$S_x^R = \frac{\partial R/R}{\partial x/x} = \left\langle \frac{\frac{d\Sigma_1}{dx} \Psi x}{\Sigma_1 \Psi} - \frac{\frac{d\Sigma_2}{dx} \Psi x}{\Sigma_2 \Psi} + \frac{\partial R}{\partial \Psi} \frac{\partial \Psi}{\partial x} \frac{x}{R} \right\rangle$$

- $\langle \rangle$ — Inner product
 Ψ — Neutron flux
 Σ_1, Σ_2 — Any kind of macroscopic cross section

Sensitivity



Calculation

$$S_x^R = \frac{\partial R/R}{\partial x/x} = \left\langle \overbrace{\frac{\frac{d\Sigma_1}{dx} \Psi x}{\Sigma_1 \Psi} - \frac{\frac{d\Sigma_2}{dx} \Psi x}{\Sigma_2 \Psi}}^{\text{Direct effect terms}} + \frac{\partial R}{\partial \Psi} \frac{\partial \Psi}{\partial x} \frac{x}{R} \right\rangle$$

Indirect effect terms

Direct Terms

Describe impact on generalized response
Relatively easy to compute

Indirect Term

Describe impact on flux
Complicated to compute

Method for Indirect Term

- GEAR (Generalized Adjoint Response) method
based on GPT (Generalized perturbation theory)
used by TSUNAMI-3D
- Collision-based History method
based on accepted and rejected events
used by SERPENT2

Uncertainty



Definition

$$\vec{S} = (S_{x_1}^k, S_{x_2}^k, \dots, S_{x_n}^k)$$

$$n = \text{Nuclide} \sim \text{Reaction number} \times \text{Energy Bin number}$$

$$15543 = 471 \times 33$$

$$\text{cov}(x_i, x_j) = \int (x_i - E(x_i)) (x_j - E(x_j)) p(x_1, \dots, x_n) dx_1 \dots dx_n$$

Sandwich Rule

$$r_k^2 = \vec{S} V \vec{S}^T$$

— V is (relative) covariance matrix

$$V = \begin{bmatrix} r_{x_1}^2 & \text{rcov}(x_1, x_2) & \dots & \text{rcov}(x_1, x_n) \\ \text{rcov}(x_2, x_1) & r_{x_1}^2 & \dots & \text{rcov}(x_2, x_n) \\ \vdots & \vdots & \ddots & \vdots \\ \text{rcov}(x_n, x_1) & \text{rcov}(x_n, x_2) & \dots & \text{rcov}(x_n, x_n) \end{bmatrix}$$

$$\text{rcov}(x_i, x_j) = \frac{\text{cov}(x_i, x_j)}{x_i x_j}$$

$$r_{x_i}^2 = \frac{\sigma_{x_i}^2}{x_i^2}$$

— $r_{x_i}^2$ is relative variance

COMMARA-2.0

Released by BNL & LANL in March 2011

Based on ENDF/B-VII.0

Including 110 Nuclides:

12 Light Nuclei (Coolant & Moderator)
78 Structural Materials & Fission products
20 Actinides

Reaction Channels

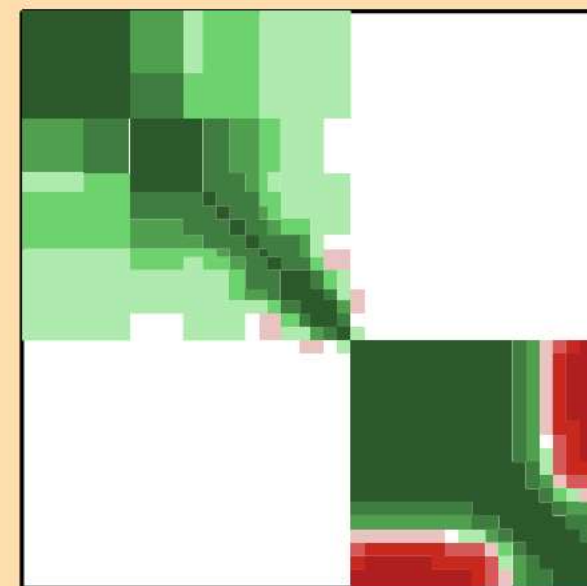
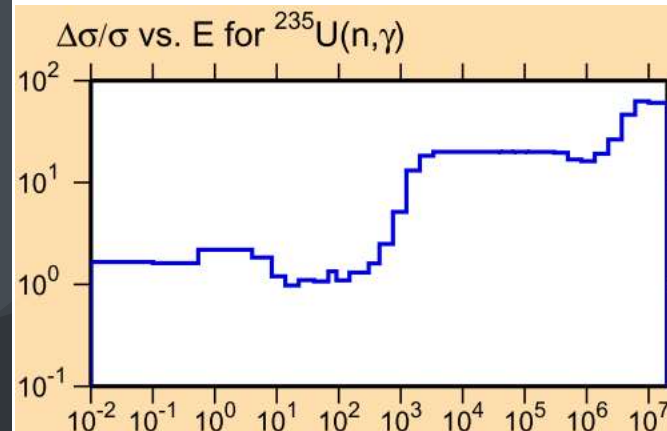
Elastic/Inelastic Scattering $(n, n)/(n, n')$
Capture (n, γ)
Neutron Multiplication (n, xn)
Fission (n, f) , $\bar{\nu}$, χ

Total Files (Nuclide-Reaction)

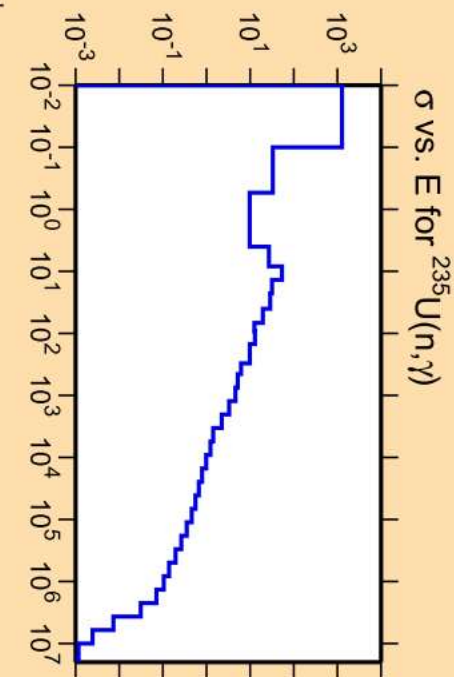
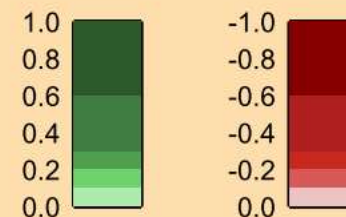
569

Energy

10^{-5} eV – 19.6 MeV
33 Groups



Correlation Matrix





Calculation

Covariance Matrix

NEA provides COMMARA-2.0 correlation matrices and relative uncertainty:

<https://www.oecd-nea.org/science/wpec/sg33/benchmark/results/data.html>

$$C = \begin{bmatrix} 1 & \text{cor}(x_1, x_2) & \cdots & \text{cor}(x_1, x_n) \\ \text{cor}(x_2, x_1) & 1 & \cdots & \text{cor}(x_2, x_n) \\ \vdots & \vdots & \ddots & \vdots \\ \text{cor}(x_n, x_1) & \text{cor}(x_n, x_2) & \cdots & 1 \end{bmatrix}$$

$$r_{x_i} = \frac{\sigma_{x_i}}{x_i}$$

— r_{x_i} is relative uncertainty

Obtain relative covariance matrix from correlation matrices and relative uncertainty:

$$\text{rcov}(x_i, x_j) = r_{x_i} r_{x_j} \text{cor}(x_i, x_j)$$

$$V = \begin{bmatrix} r_{x_1}^2 & \text{rcov}(x_1, x_2) & \cdots & \text{rcov}(x_1, x_n) \\ \text{rcov}(x_2, x_1) & r_{x_1}^2 & \cdots & \text{rcov}(x_2, x_n) \\ \vdots & \vdots & \ddots & \vdots \\ \text{rcov}(x_n, x_1) & \text{rcov}(x_n, x_2) & \cdots & \text{rcov}(x_n, x_n) \end{bmatrix}$$

Relative Uncertainty



Definition

$$U_{total} = r_k^2 = \vec{S}V\vec{S}^T$$
$$= \sum_i S_i V_{ii} S_i + \sum_i \sum_{j \neq i} S_i V_{ij} S_j$$

Sensitivity Index:

$$SI_i = \frac{U_i}{U_{total}}$$

The conservative estimate of uncertainty:

$$\widetilde{U}_i = S_i V_{ii} S_i + 2 \times \sum_{j \neq i} S_i V_{ij} S_j$$

Conservative Sensitivity Index:

$$\widetilde{SI}_i = \frac{\widetilde{U}_i}{U_{total}}$$

Calculation

Serpent Output Files



- * **_res.m** – General Results
- * **_sens.m** – Sensitivity Data

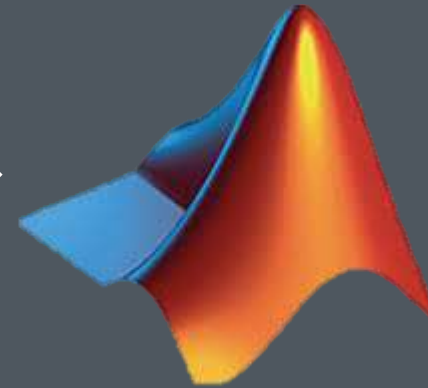
C++



- Output File Preprocess
- Extract Information
- Generate MATLAB Script

MATLAB

Mixed Programming!



- Arrange Covariance Matrix
- Calculate Uncertainty
- Plot Distribution Graph

Results

Sensitivity
&
Uncertainty Distribution

- Nuclide-Reaction List
- Uncertainty Value
- Stairs Graph

C++



- Read Serpent Output File
- Generate Nuclide-Reaction Index
- Match Sensitivity with Covariance

Generate MATLAB Scripts to :

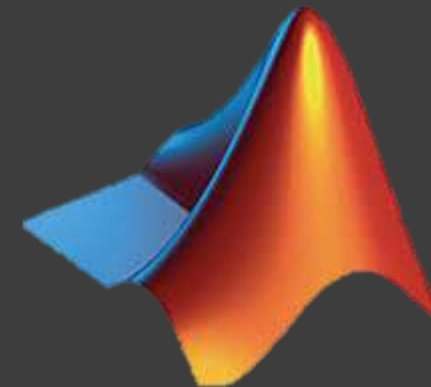
Arrange Covariance Matrix
Calculate Uncertainty
Sort Data
Plot Stairs Graph
.....

Interface

MATLAB Engine
Dynamic-link Library

Index Files
Temporary Files
MATLAB Scripts

MATLAB



- Read Covariance Matrix
- Calculate Uncertainty
- Sort Data by Importance
- Plot Stairs Graph

Generate Temporary Files to :

Match Data by Name-Value
Extract Calculation Results
Transfer Data
.....

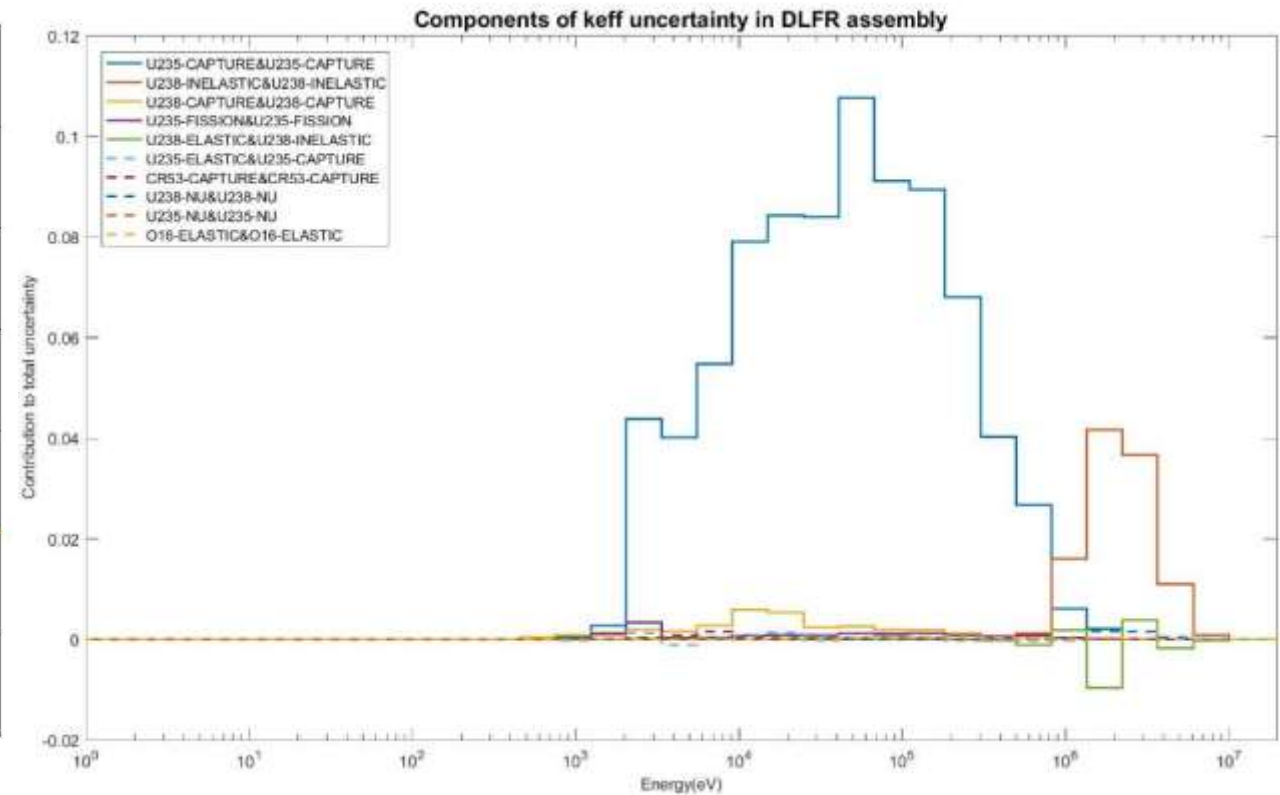
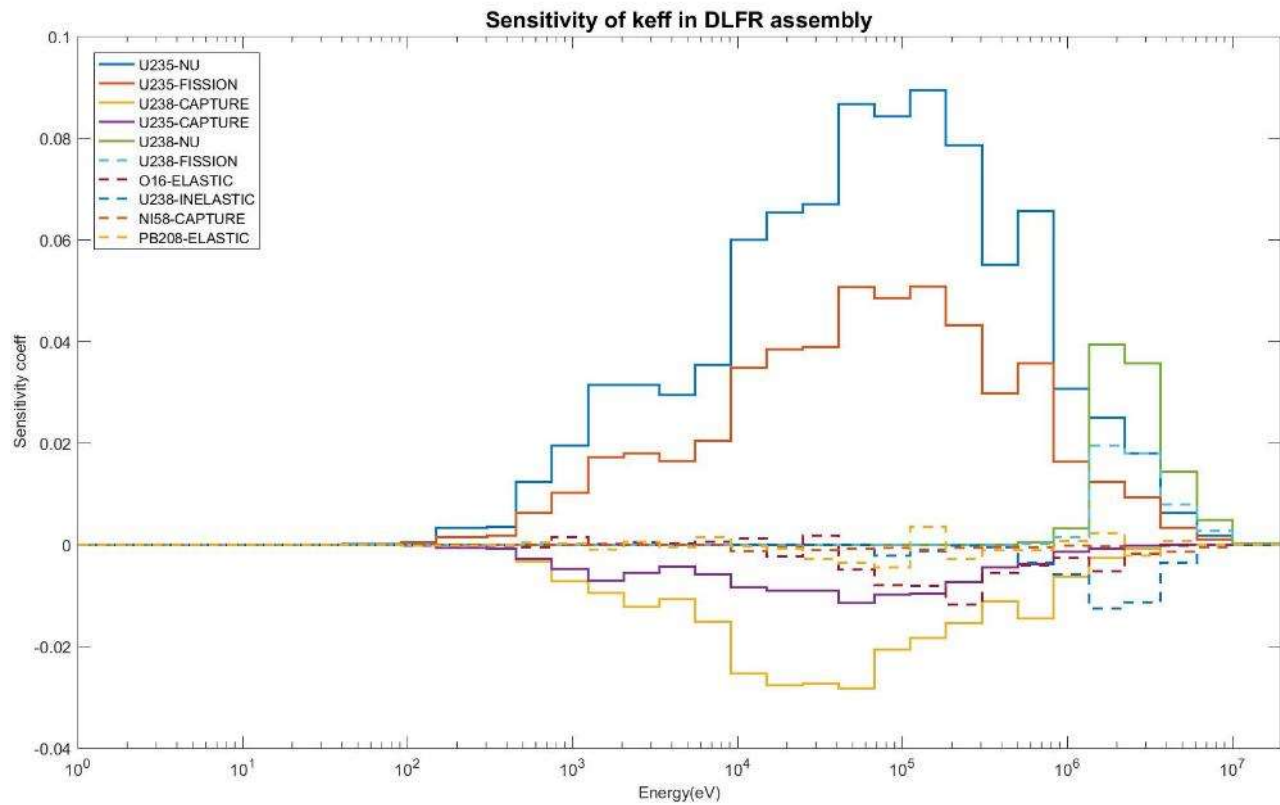
PART

3.1

Fuel Assembly

Distribution of S/U Corresponding to the Most Important 20 Parameters

Inner Core Fuel Assembly at Beginning of Life



BoL inner Assembly Uncertainty Analysis:

K_{eff} Standard Deviation: 0.019226

| Parameter Pair | Contribution to Total Uncertainty (%) | Parameter | Conservatively Estimated Relative Uncertainty | Sensitivity |
|--|---------------------------------------|--------------------------------------|---|-------------|
| $^{235}\text{U} - \sigma_{n,\gamma}$ | 82.1235 | $^{235}\text{U} - \sigma_{n,\gamma}$ | 1.56768 | -0.107003 |
| $^{238}\text{U} - \sigma_{n,n'}$ | 10.8820 | $^{238}\text{U} - \sigma_{n,n'}$ | 0.17516 | -0.041029 |
| $^{238}\text{U} - \sigma_{n,\gamma}$ | 3.2416 | $^{238}\text{U} - \sigma_{n,\gamma}$ | 0.05615 | -0.257976 |
| $^{235}\text{U} - \sigma_f$ | 1.4514 | $^{235}\text{U} - \sigma_f$ | 0.02490 | 0.506052 |
| $^{238}\text{U} - (\sigma_{n,n}, \sigma_{n,n'})$ | -0.7076 | $^{53}\text{Cr} - \sigma_{n,\gamma}$ | 0.00684 | 0.007763 |

BoL outer Assembly Uncertainty Analysis:

K_{eff} Standard Deviation: 0.019821

| Parameter Pair | Contribution to Total Uncertainty (%) | Parameter | Conservatively Estimated Relative Uncertainty | Sensitivity |
|--|---------------------------------------|--------------------------------------|---|-------------|
| $^{235}\text{U} - \sigma_{n,\gamma}$ | 86.7332 | $^{235}\text{U} - \sigma_{n,\gamma}$ | 1.64995 | -0.110411 |
| $^{238}\text{U} - \sigma_{n,n'}$ | 7.0166 | $^{238}\text{U} - \sigma_{n,n'}$ | 0.122642 | -0.041853 |
| $^{238}\text{U} - \sigma_{n,\gamma}$ | 2.4011 | $^{238}\text{U} - \sigma_{n,\gamma}$ | 0.041575 | -0.232542 |
| $^{235}\text{U} - \sigma_f$ | 1.2345 | $^{235}\text{U} - \sigma_f$ | 0.021593 | 0.488544 |
| $^{238}\text{U} - (\sigma_{n,n}, \sigma_{n,n'})$ | 0.4458 | $^{53}\text{Cr} - \sigma_{n,\gamma}$ | 0.005432 | 0.007141 |

BoC inner Assembly Uncertainty Analysis:

K_{eff} Standard Deviation: 0.015357

| Parameter Pair | Contribution to Total Uncertainty (%) | Parameter | Conservatively Estimated Relative Uncertainty | Sensitivity |
|--------------------------------------|---------------------------------------|--------------------------------------|---|-------------|
| $^{235}\text{U} - \sigma_{n,\gamma}$ | 65.8074 | $^{235}\text{U} - \sigma_{n,\gamma}$ | 1.25340 | -0.078050 |
| $^{238}\text{U} - \sigma_{n,n'}$ | 19.0909 | $^{238}\text{U} - \sigma_{n,n'}$ | 0.33066 | -0.051604 |
| $^{238}\text{U} - \sigma_{n,\gamma}$ | 5.1079 | $^{238}\text{U} - \sigma_{n,\gamma}$ | 0.08877 | -0.256832 |
| $^{235}\text{U} - \sigma_f$ | 1.7412 | $^{235}\text{U} - \sigma_f$ | 0.02919 | 0.431738 |
| $^{56}\text{Fe} - \sigma_{n,n}$ | 1.7377 | $^{56}\text{Fe} - \sigma_{n,n}$ | 0.02974 | -0.023070 |

BoC outer Assembly Uncertainty Analysis:

K_{eff} Standard Deviation: 0.016577

| Parameter Pair | Contribution to Total Uncertainty (%) | Parameter | Conservatively Estimated Relative Uncertainty | Sensitivity |
|--|---------------------------------------|--------------------------------------|---|-------------|
| $^{235}\text{U} - \sigma_{n,\gamma}$ | 86.7332 | $^{235}\text{U} - \sigma_{n,\gamma}$ | 1.513810 | -0.090276 |
| $^{238}\text{U} - \sigma_{n,n'}$ | 7.0166 | $^{238}\text{U} - \sigma_{n,n'}$ | 0.204594 | -0.044881 |
| $^{238}\text{U} - \sigma_{n,\gamma}$ | 2.4011 | $^{238}\text{U} - \sigma_{n,\gamma}$ | 0.062313 | -0.235775 |
| $^{235}\text{U} - \sigma_f$ | 1.2345 | $^{235}\text{U} - \sigma_f$ | 0.027417 | 0.454963 |
| $^{238}\text{U} - (\sigma_{n,n}, \sigma_{n,n'})$ | 0.4458 | $^{16}\text{O} - \sigma_{n,n}$ | 0.011894 | -0.067184 |

EoC inner Assembly Uncertainty Analysis:

K_{eff} Standard Deviation: 0.014014

| Parameter Pair | Contribution to Total Uncertainty (%) | Parameter | Conservatively Estimated Relative Uncertainty | Sensitivity |
|--------------------------------------|---------------------------------------|--------------------------------------|---|-------------|
| $^{235}\text{U} - \sigma_{n,\gamma}$ | 54.7410 | $^{235}\text{U} - \sigma_{n,\gamma}$ | 1.04262 | -0.065688 |
| $^{238}\text{U} - \sigma_{n,n'}$ | 27.9248 | $^{238}\text{U} - \sigma_{n,n'}$ | 0.48038 | -0.050797 |
| $^{238}\text{U} - \sigma_{n,\gamma}$ | 5.8622 | $^{238}\text{U} - \sigma_{n,\gamma}$ | 0.10185 | -0.250996 |
| $^{235}\text{U} - \sigma_f$ | 1.8119 | $^{235}\text{U} - \sigma_f$ | 0.02997 | 0.397506 |
| $^{16}\text{O} - \sigma_{n,n}$ | 1.5251 | $^{16}\text{O} - \sigma_{n,n}$ | 0.02818 | -0.087629 |

EoC outer Assembly Uncertainty Analysis:

K_{eff} Standard Deviation: 0.015726

| Parameter Pair | Contribution to Total Uncertainty (%) | Parameter | Conservatively Estimated Relative Uncertainty | Sensitivity |
|--|---------------------------------------|--------------------------------------|---|-------------|
| $^{235}\text{U} - \sigma_{n,\gamma}$ | 76.2761 | $^{235}\text{U} - \sigma_{n,\gamma}$ | 1.451030 | -0.084109 |
| $^{238}\text{U} - \sigma_{n,n'}$ | 12.9362 | $^{238}\text{U} - \sigma_{n,n'}$ | 0.226971 | -0.044512 |
| $^{238}\text{U} - \sigma_{n,\gamma}$ | 4.0763 | $^{238}\text{U} - \sigma_{n,\gamma}$ | 0.070649 | -0.236758 |
| $^{235}\text{U} - \sigma_f$ | 1.6353 | $^{235}\text{U} - \sigma_f$ | 0.027646 | 0.433899 |
| $^{238}\text{U} - (\sigma_{n,n}, \sigma_{n,n'})$ | 0.9269 | $^{53}\text{Cr} - \sigma_{n,\gamma}$ | 0.010715 | -0.007833 |

EoL inner Assembly Uncertainty Analysis:

K_{eff} Standard Deviation: 0.012044

| Parameter Pair | Contribution to Total Uncertainty (%) | Parameter | Conservatively Estimated Relative Uncertainty | Sensitivity |
|--|---------------------------------------|--------------------------------------|---|-------------|
| $^{238}\text{U} - \sigma_{n,n'}$ | 38.9070 | $^{238}\text{U} - \sigma_{n,n'}$ | 0.68714 | -0.059305 |
| $^{235}\text{U} - \sigma_{n,\gamma}$ | 33.2131 | $^{235}\text{U} - \sigma_{n,\gamma}$ | 0.63359 | -0.044578 |
| $^{238}\text{U} - \sigma_{n,\gamma}$ | 7.3113 | $^{238}\text{U} - \sigma_{n,\gamma}$ | 0.12726 | -0.238941 |
| $^{238}\text{U} - (\sigma_{n,n}, \sigma_{n,n'})$ | 3.9664 | $^{16}\text{O} - \sigma_{n,n}$ | 0.05135 | 0.100010 |
| $^{16}\text{O} - \sigma_{n,n}$ | 2.7469 | $^{238}\text{U} - \sigma_{n,n}$ | 0.04130 | -0.005715 |

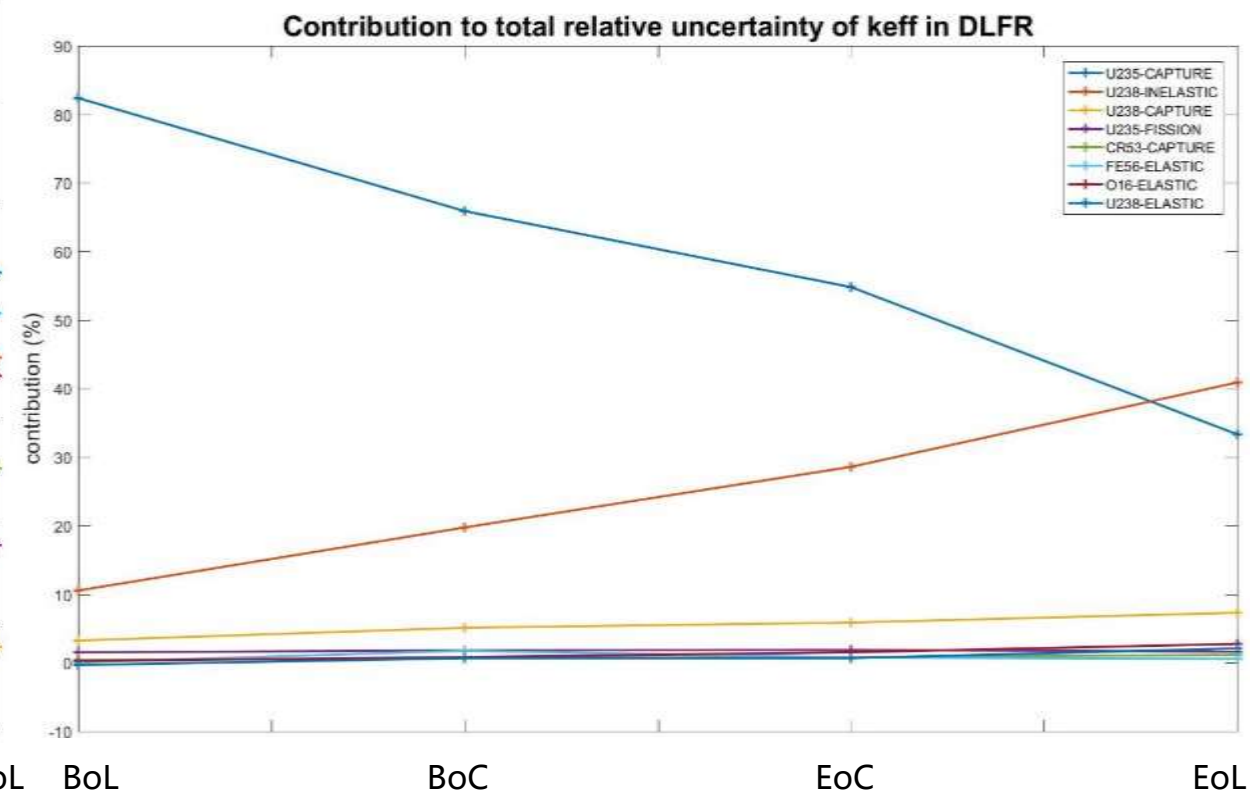
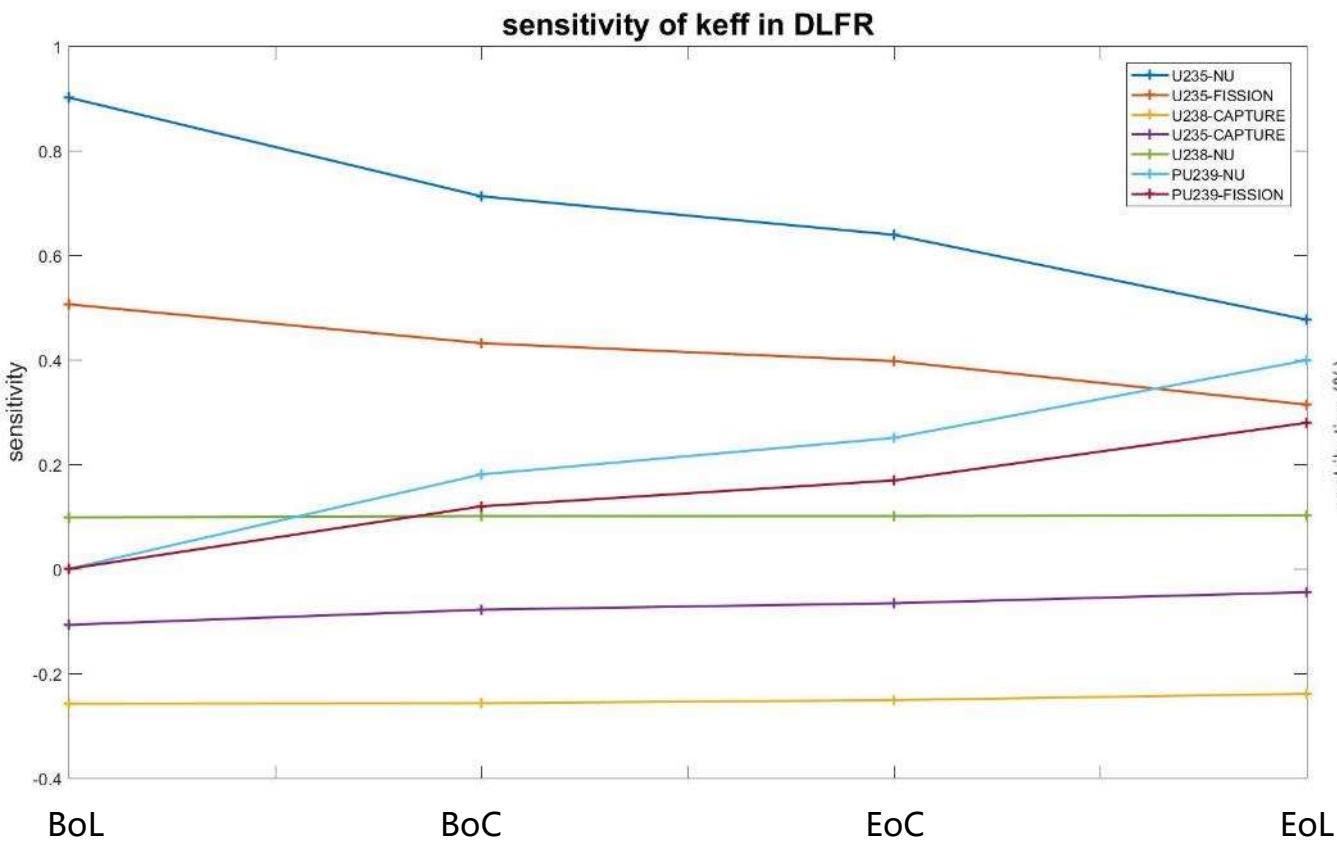
EoL outer Assembly Uncertainty Analysis:

K_{eff} Standard Deviation: 0.013416

| Parameter Pair | Contribution to Total Uncertainty (%) | Parameter | Conservatively Estimated Relative Uncertainty | Sensitivity |
|--------------------------------------|---------------------------------------|--------------------------------------|---|-------------|
| $^{235}\text{U} - \sigma_{n,\gamma}$ | 62.8492 | $^{235}\text{U} - \sigma_{n,\gamma}$ | 1.201250 | -0.066350 |
| $^{238}\text{U} - \sigma_{n,n'}$ | 21.0056 | $^{238}\text{U} - \sigma_{n,n'}$ | 0.360621 | -0.047198 |
| $^{238}\text{U} - \sigma_{n,\gamma}$ | 5.5305 | $^{238}\text{U} - \sigma_{n,\gamma}$ | 0.095879 | -0.234376 |
| $^{235}\text{U} - \sigma_f$ | 1.8597 | $^{235}\text{U} - \sigma_f$ | 0.031221 | 0.392085 |
| $^{16}\text{O} - \sigma_{n,n}$ | 1.0447 | $^{16}\text{O} - \sigma_{n,n}$ | 0.018994 | -0.070047 |

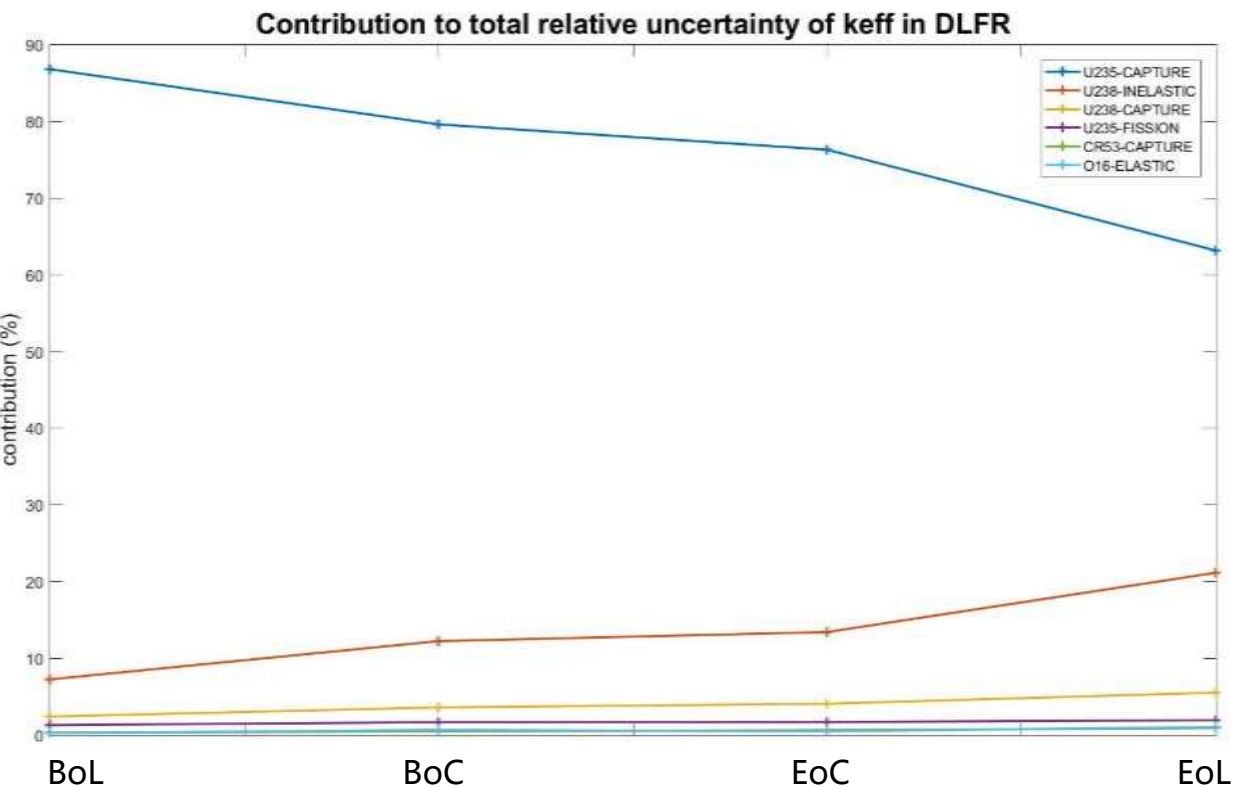
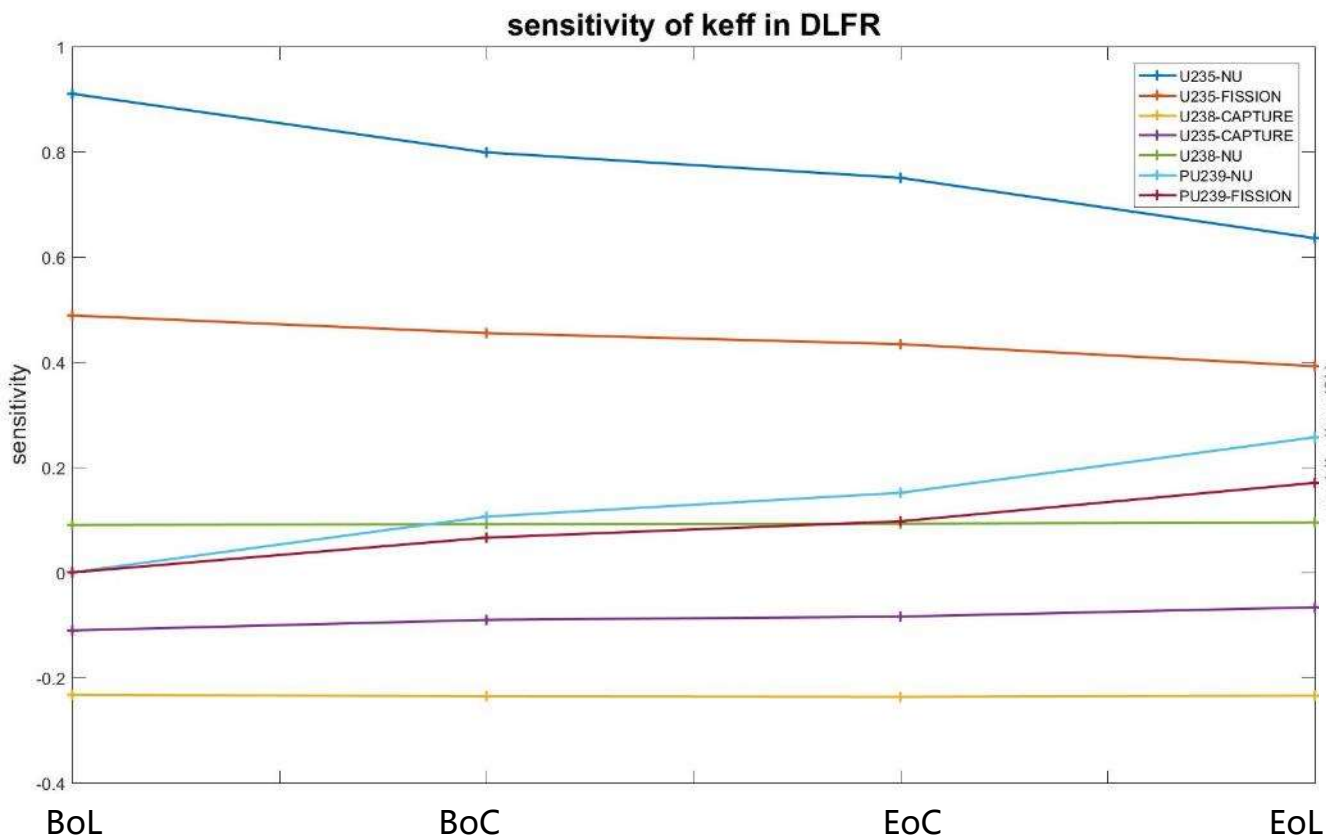
Sensitivity and Uncertainty Contributed to Total in Different Periods

Inner Core Fuel Assembly



Sensitivity and Uncertainty Contributed to Total in Different Periods

Outer Core Fuel Assembly



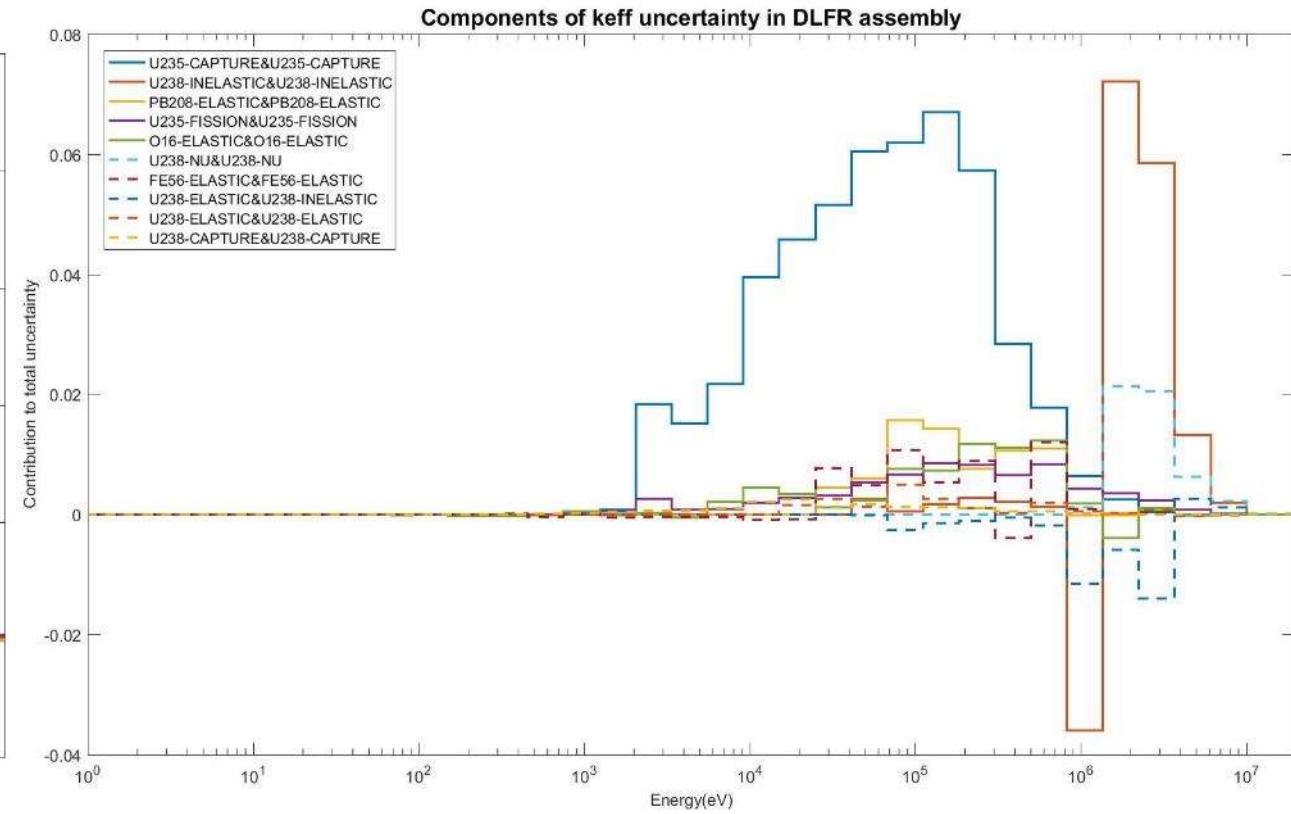
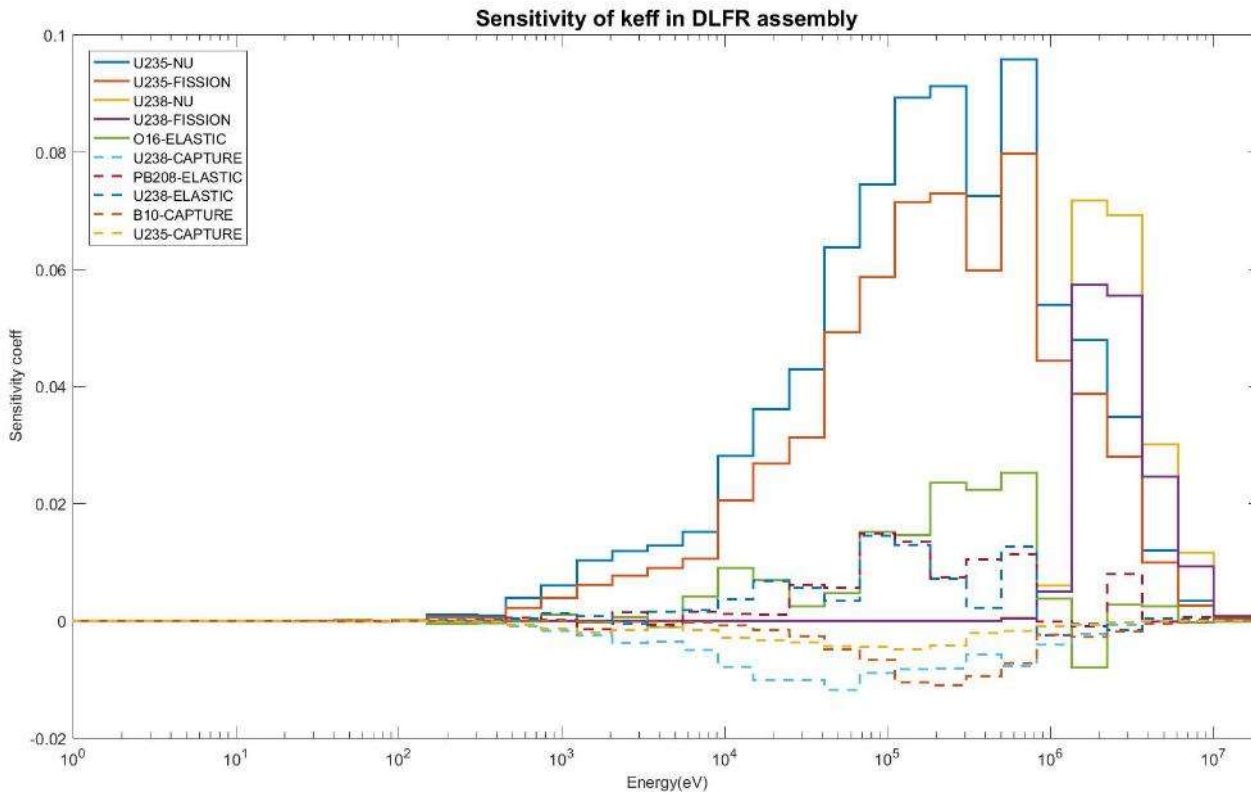
PART

3.2

Safety Rod

Distribution of S/U Corresponding to the Most Important 20 Parameters

Safety Rod surrounded by Inner Core Fuel Assembly at Beginning of Life



Safety Rod surrounded by BoL inner Assembly:

K_{eff} Standard Deviation: 0.009983

| Parameter Pair | Contribution to Total Uncertainty (%) | Parameter | Conservatively Estimated Relative Uncertainty | Sensitivity |
|--------------------------------------|---------------------------------------|--------------------------------------|---|-------------|
| $^{235}\text{U} - \sigma_{n,\gamma}$ | 49.6019 | $^{235}\text{U} - \sigma_{n,\gamma}$ | 0.94729 | -0.041557 |
| $^{238}\text{U} - \sigma_{n,n'}$ | 12.1277 | $^{208}\text{Pb} - \sigma_{n,n}$ | 0.12561 | 0.081090 |
| $^{208}\text{Pb} - \sigma_{n,n}$ | 7.0904 | $^{235}\text{U} - \sigma_f$ | 0.12502 | 0.635583 |
| $^{235}\text{U} - \sigma_f$ | 6.8442 | $^{16}\text{O} - \sigma_{n,n}$ | 0.11627 | 0.129220 |
| $^{16}\text{O} - \sigma_{n,n}$ | 6.2960 | $^{238}\text{U} - \sigma_{n,n'}$ | 0.09061 | -0.102597 |

Safety Rod surrounded by BoC inner Assembly:

K_{eff} Standard Deviation: 0.010595

| Parameter Pair | Contribution to Total Uncertainty (%) | Parameter | Conservatively Estimated Relative Uncertainty | Sensitivity |
|--|---------------------------------------|--------------------------------------|---|-------------|
| $^{235}\text{U} - \sigma_{n,\gamma}$ | 20.9232 | $^{235}\text{U} - \sigma_{n,\gamma}$ | 0.405438 | -0.028904 |
| $^{56}\text{Fe} - \sigma_{n,n}$ | 19.4188 | $^{56}\text{Fe} - \sigma_{n,n}$ | 0.340385 | 0.057541 |
| $^{238}\text{U} - \sigma_{n,n'}$ | 14.8347 | $^{238}\text{U} - \sigma_{n,n'}$ | 0.298071 | -0.006460 |
| $^{238}\text{U} - (\sigma_{n,n}, \sigma_{n,n'})$ | 9.8891 | $^{238}\text{U} - \sigma_{n,n}$ | 0.155563 | 0.074835 |
| $^{16}\text{O} - \sigma_{n,n}$ | 6.8959 | $^{16}\text{O} - \sigma_{n,n}$ | 0.129392 | 0.142088 |

Safety Rod surrounded by EoC inner Assembly:

K_{eff} Standard Deviation: 0.009616

| Parameter Pair | Contribution to Total Uncertainty (%) | Parameter | Conservatively Estimated Relative Uncertainty | Sensitivity |
|--|---------------------------------------|--------------------------------------|---|-------------|
| $^{238}\text{U} - \sigma_{n,n'}$ | 27.4812 | $^{238}\text{U} - \sigma_{n,n'}$ | 0.44838 | -0.006568 |
| $^{235}\text{U} - \sigma_{n,\gamma}$ | 16.2871 | $^{235}\text{U} - \sigma_{n,\gamma}$ | 0.31823 | -0.023141 |
| $^{56}\text{Fe} - \sigma_{n,n}$ | 15.8962 | $^{56}\text{Fe} - \sigma_{n,n}$ | 0.28609 | 0.049147 |
| $^{238}\text{U} - (\sigma_{n,n}, \sigma_{n,n'})$ | 7.1493 | $^{238}\text{U} - \sigma_{n,n}$ | 0.11081 | 0.075307 |
| $^{238}\text{U} - \bar{\nu}$ | 5.9046 | $^{238}\text{U} - \bar{\nu}$ | 0.09529 | 0.194659 |

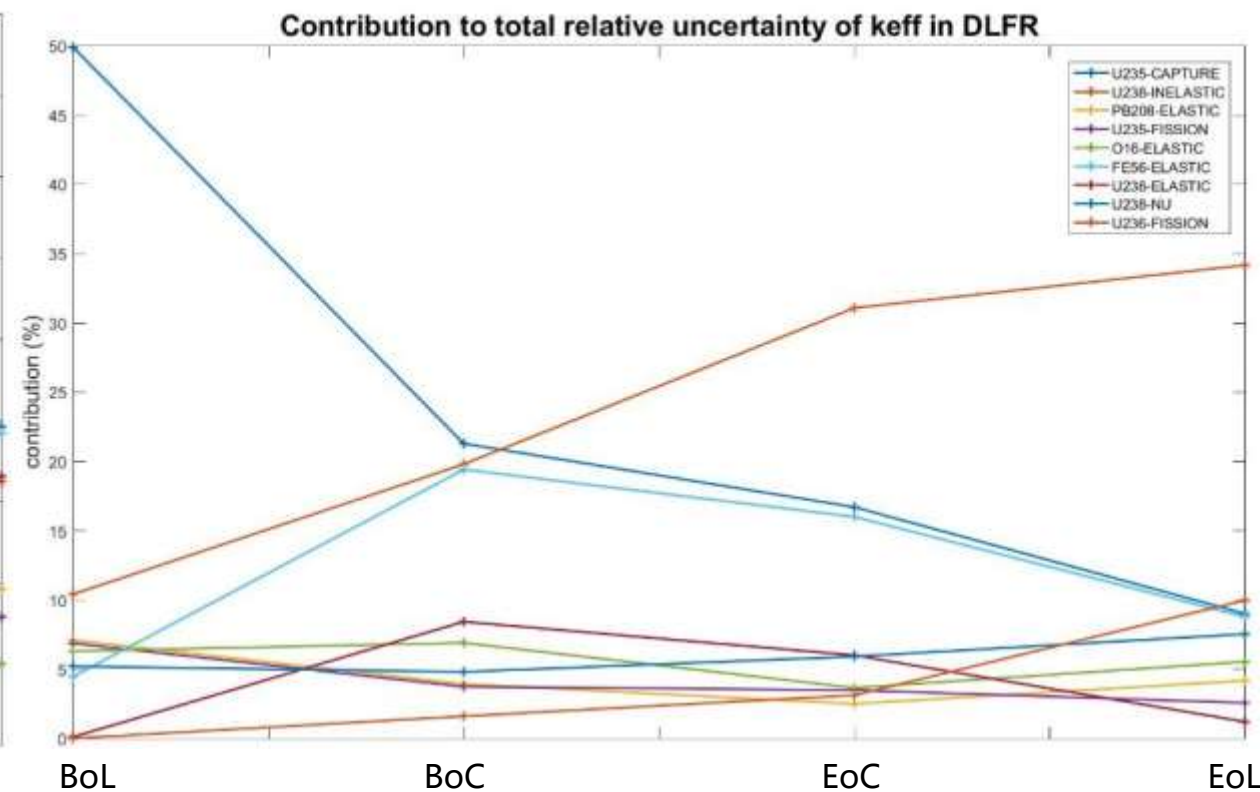
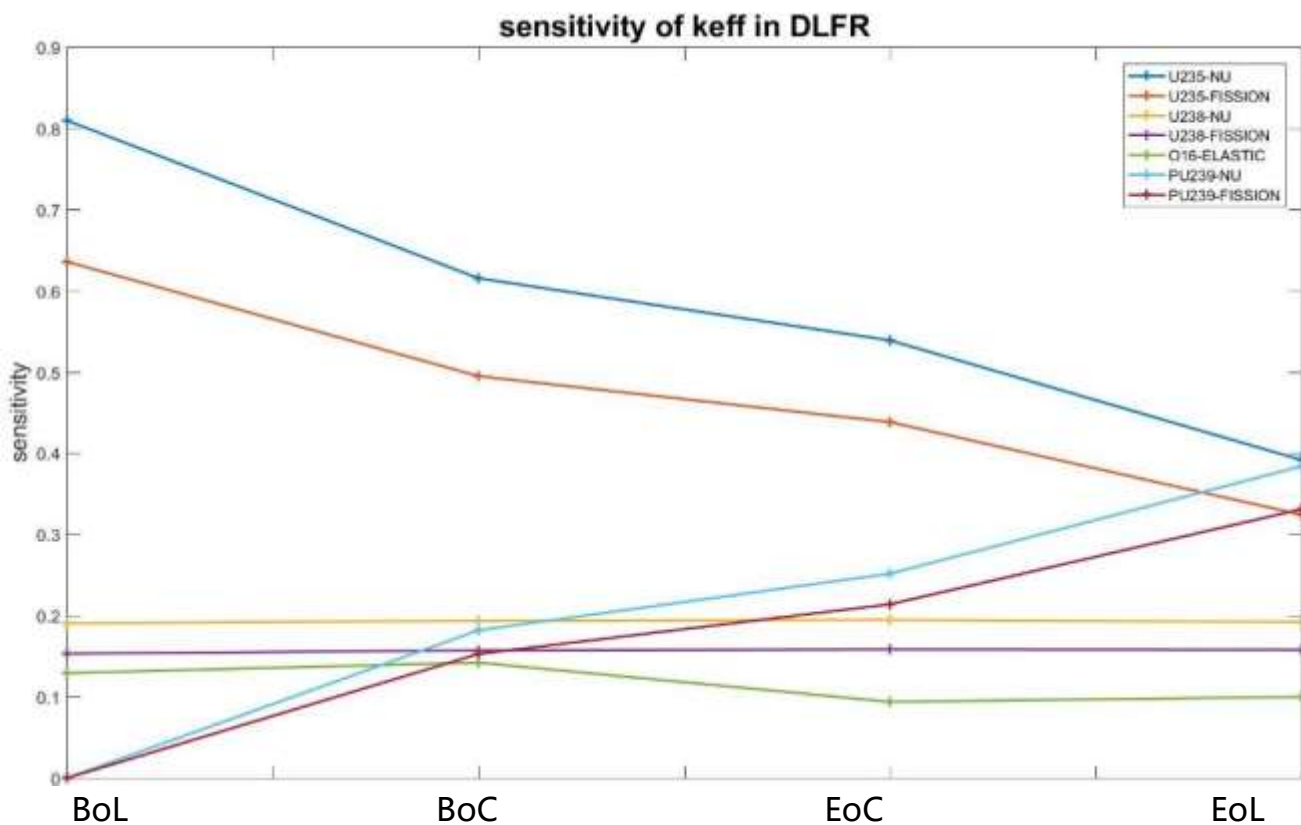
Safety Rod surrounded by EoL inner Assembly:

K_{eff} Standard Deviation: 0.008373

| Parameter Pair | Contribution to Total Uncertainty (%) | Parameter | Conservatively Estimated Relative Uncertainty | Sensitivity |
|--------------------------------------|---------------------------------------|--------------------------------------|---|-------------|
| $^{238}\text{U} - \sigma_{n,n'}$ | 34.8277 | $^{238}\text{U} - \sigma_{n,n'}$ | 0.465302 | -0.017072 |
| $^{236}\text{U} - \sigma_f$ | 9.9612 | $^{235}\text{U} - \sigma_{n,\gamma}$ | 0.171462 | -0.015318 |
| $^{235}\text{U} - \sigma_{n,\gamma}$ | 8.8226 | $^{236}\text{U} - \sigma_f$ | 0.170007 | 0.010268 |
| $^{56}\text{Fe} - \sigma_{n,n}$ | 8.7255 | $^{56}\text{Fe} - \sigma_{n,n}$ | 0.139694 | 0.028763 |
| $^{238}\text{U} - \bar{\nu}$ | 7.5280 | $^{238}\text{U} - \bar{\nu}$ | 0.121730 | 0.191956 |

Sensitivity and Uncertainty Contributed to Total in Different Periods

Safety Rod



PART

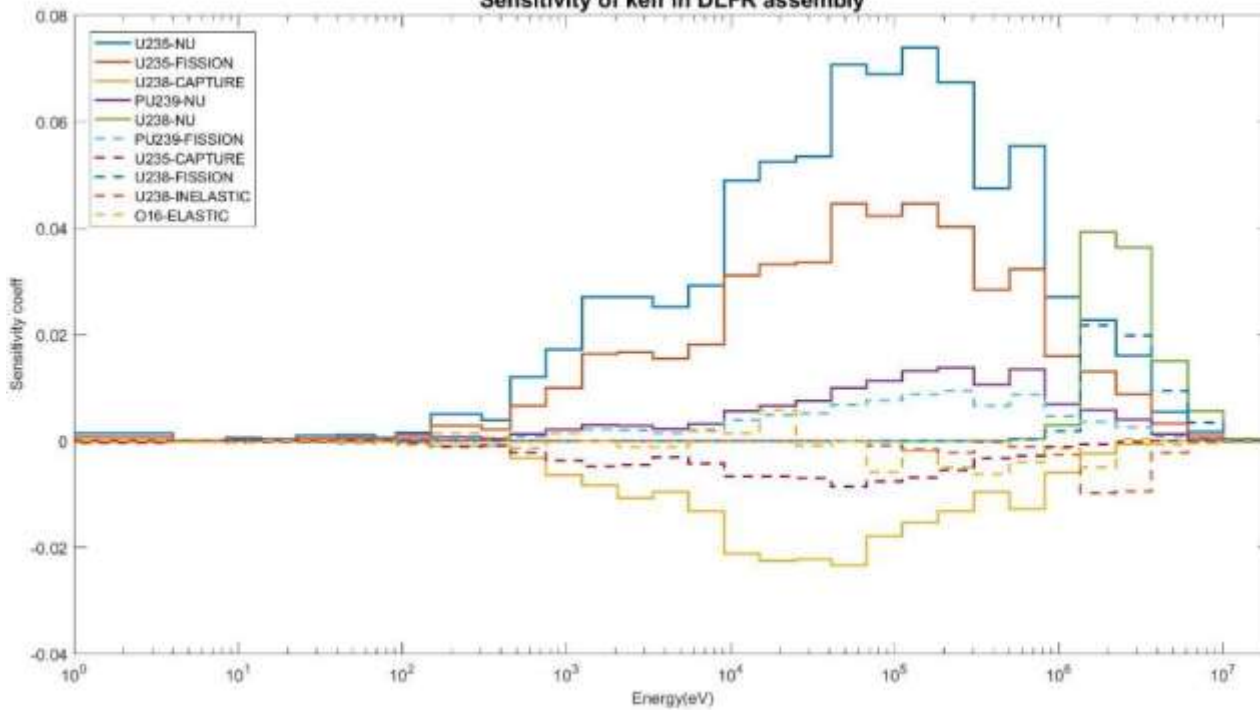
3.3

2D Whole Core

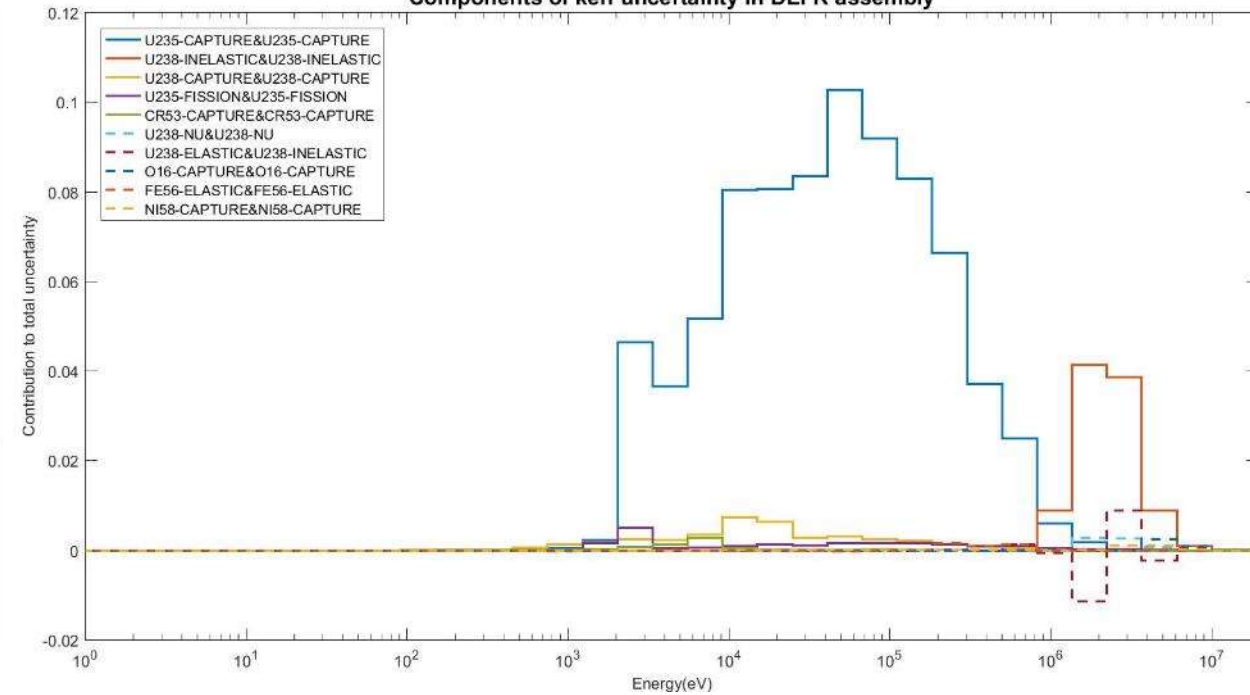
Distribution of S/U Corresponding to the Most Important 10 Parameters

2D Whole Core with Safety Rod out at Beginning of Life

Sensitivity of k_{eff} in DLFR assembly



Components of k_{eff} uncertainty in DLFR assembly



2D Whole Core with Safety Rod out at BoL:

K_{eff} Standard Deviation: 0.017122

| Parameter Pair | Contribution to Total Uncertainty (%) | Parameter | Conservatively Estimated Relative Uncertainty | Sensitivity |
|--|---------------------------------------|--------------------------------------|---|-------------|
| $^{235}\text{U} - \sigma_{n,\gamma}$ | 84.7604 | $^{235}\text{U} - \sigma_{n,\gamma}$ | 1.61504 | -0.098547 |
| $^{238}\text{U} - \sigma_{n,n'}$ | 9.2950 | $^{238}\text{U} - \sigma_{n,n'}$ | 0.13958 | -0.034373 |
| $^{238}\text{U} - \sigma_{n,\gamma}$ | 2.7068 | $^{238}\text{U} - \sigma_{n,\gamma}$ | 0.04700 | -0.213675 |
| $^{235}\text{U} - \sigma_f$ | 1.8171 | $^{235}\text{U} - \sigma_f$ | 0.03180 | 0.526743 |
| $^{238}\text{U} - (\sigma_{n,n}, \sigma_{n,n'})$ | -1.3867 | $^{238}\text{U} - \sigma_{n,n}$ | -0.01317 | 0.007237 |

2D Whole Core with Safety Rod in at BoL:

K_{eff} Standard Deviation: 0.016627

| Parameter Pair | Contribution to Total Uncertainty (%) | Parameter | Conservatively Estimated Relative Uncertainty | Sensitivity |
|--|---------------------------------------|--------------------------------------|---|-------------|
| $^{235}\text{U} - \sigma_{n,\gamma}$ | 84.8102 | $^{235}\text{U} - \sigma_{n,\gamma}$ | 1.607250 | -0.096055 |
| $^{238}\text{U} - \sigma_{n,n'}$ | 8.3663 | $^{238}\text{U} - \sigma_{n,n'}$ | 0.139454 | -0.029788 |
| $^{238}\text{U} - \sigma_{n,\gamma}$ | 2.6780 | $^{238}\text{U} - \sigma_{n,\gamma}$ | 0.046401 | -0.207413 |
| $^{235}\text{U} - \sigma_f$ | 1.9128 | $^{235}\text{U} - \sigma_f$ | 0.033683 | 0.529179 |
| $^{235}\text{U} - (\sigma_{n,n}, \sigma_{n,\gamma})$ | -0.0078 | $^{238}\text{U} - \bar{\nu}$ | 0.009034 | -0.103665 |

2D Whole Core with Safety Rod out at BoC:

K_{eff} Standard Deviation: 0.014614

| Parameter Pair | Contribution to Total Uncertainty (%) | Parameter | Conservatively Estimated Relative Uncertainty | Sensitivity |
|--|---------------------------------------|--------------------------------------|---|-------------|
| $^{235}\text{U} - \sigma_{n,\gamma}$ | 70.9466 | $^{235}\text{U} - \sigma_{n,\gamma}$ | 1.35330 | -0.077559 |
| $^{238}\text{U} - \sigma_{n,n'}$ | 16.4545 | $^{238}\text{U} - \sigma_{n,n'}$ | 0.29119 | -0.042023 |
| $^{238}\text{U} - \sigma_{n,\gamma}$ | 3.7255 | $^{238}\text{U} - \sigma_{n,\gamma}$ | 0.06472 | -0.212163 |
| $^{235}\text{U} - \sigma_f$ | 2.0054 | $^{235}\text{U} - \sigma_f$ | 0.03487 | 0.468207 |
| $^{238}\text{U} - (\sigma_{n,n}, \sigma_{n,n'})$ | 1.6438 | $^{238}\text{U} - \sigma_{n,n}$ | 0.01699 | 0.010483 |

2D Whole Core with Safety Rod in at BoC:

K_{eff} Standard Deviation: 0.014419

| Parameter Pair | Contribution to Total Uncertainty (%) | Parameter | Conservatively Estimated Relative Uncertainty | Sensitivity |
|--|---------------------------------------|--------------------------------------|---|-------------|
| $^{235}\text{U} - \sigma_{n,\gamma}$ | 70.3116 | $^{235}\text{U} - \sigma_{n,\gamma}$ | 1.350580 | -0.076630 |
| $^{238}\text{U} - \sigma_{n,n'}$ | 14.6748 | $^{238}\text{U} - \sigma_{n,n'}$ | 0.266992 | -0.035847 |
| $^{238}\text{U} - \sigma_{n,\gamma}$ | 3.6564 | $^{238}\text{U} - \sigma_{n,\gamma}$ | 0.063624 | -0.207103 |
| $^{238}\text{U} - (\sigma_{n,n}, \sigma_{n,n'})$ | 3.2333 | $^{235}\text{U} - \sigma_f$ | 0.036008 | 0.473907 |
| $^{235}\text{U} - \sigma_f$ | 2.0707 | $^{238}\text{U} - \sigma_{n,n}$ | 0.035553 | 0.023593 |

2D Whole Core with Safety Rod out at EoC:

K_{eff} Standard Deviation: 0.013229

| Parameter Pair | Contribution to Total Uncertainty (%) | Parameter | Conservatively Estimated Relative Uncertainty | Sensitivity |
|--------------------------------------|---------------------------------------|--------------------------------------|---|-------------|
| $^{235}\text{U} - \sigma_{n,\gamma}$ | 65.1362 | $^{235}\text{U} - \sigma_{n,\gamma}$ | 1.23615 | -0.068290 |
| $^{238}\text{U} - \sigma_{n,n'}$ | 22.5490 | $^{238}\text{U} - \sigma_{n,n'}$ | 0.36535 | -0.038401 |
| $^{238}\text{U} - \sigma_{n,\gamma}$ | 4.4631 | $^{238}\text{U} - \sigma_{n,\gamma}$ | 0.07786 | -0.210651 |
| $^{235}\text{U} - \sigma_f$ | 2.2424 | $^{235}\text{U} - \sigma_f$ | 0.03846 | 0.442943 |
| $^{238}\text{U} - \bar{\nu}$ | 0.9324 | $^{238}\text{U} - \bar{\nu}$ | 0.01503 | 0.106290 |

2D Whole Core with Safety Rod in at EoC:

K_{eff} Standard Deviation: 0.013081

| Parameter Pair | Contribution to Total Uncertainty (%) | Parameter | Conservatively Estimated Relative Uncertainty | Sensitivity |
|--------------------------------------|---------------------------------------|--------------------------------------|---|-------------|
| $^{235}\text{U} - \sigma_{n,\gamma}$ | 72.2246 | $^{235}\text{U} - \sigma_{n,\gamma}$ | 1.372160 | -0.071178 |
| $^{238}\text{U} - \sigma_{n,n'}$ | 16.0571 | $^{238}\text{U} - \sigma_{n,n'}$ | 0.258783 | -0.032237 |
| $^{238}\text{U} - \sigma_{n,\gamma}$ | 4.3330 | $^{238}\text{U} - \sigma_{n,\gamma}$ | 0.075305 | -0.204820 |
| $^{235}\text{U} - \sigma_f$ | 2.3320 | $^{235}\text{U} - \sigma_f$ | 0.040107 | 0.447814 |
| $^{238}\text{U} - \bar{\nu}$ | 0.9553 | $^{238}\text{U} - \bar{\nu}$ | 0.015403 | 0.106501 |

2D Whole Core with Safety Rod out at EoL:

K_{eff} Standard Deviation: 0.010885

| Parameter Pair | Contribution to Total Uncertainty (%) | Parameter | Conservatively Estimated Relative Uncertainty | Sensitivity |
|--|---------------------------------------|--------------------------------------|---|-------------|
| $^{235}\text{U} - \sigma_{n,\gamma}$ | 53.8269 | $^{235}\text{U} - \sigma_{n,\gamma}$ | 1.01789 | -0.051801 |
| $^{238}\text{U} - \sigma_{n,n'}$ | 29.1961 | $^{238}\text{U} - \sigma_{n,n'}$ | 0.43188 | -0.046472 |
| $^{238}\text{U} - \sigma_{n,\gamma}$ | 6.3703 | $^{238}\text{U} - \sigma_{n,\gamma}$ | 0.11114 | -0.205452 |
| $^{238}\text{U} - (\sigma_{n,n}, \sigma_{n,n'})$ | -4.3104 | $^{235}\text{U} - \sigma_f$ | 0.04106 | 0.375887 |
| $^{235}\text{U} - \sigma_f$ | 2.4146 | $^{238}\text{U} - \sigma_{n,n}$ | -0.03818 | 0.019358 |

2D Whole Core with Safety Rod in at EoL:

K_{eff} Standard Deviation: 0.011287

| Parameter Pair | Contribution to Total Uncertainty (%) | Parameter | Conservatively Estimated Relative Uncertainty | Sensitivity |
|--------------------------------------|---------------------------------------|--------------------------------------|---|-------------|
| $^{235}\text{U} - \sigma_{n,\gamma}$ | 53.0769 | $^{235}\text{U} - \sigma_{n,\gamma}$ | 1.016330 | -0.053765 |
| $^{238}\text{U} - \sigma_{n,n'}$ | 27.3726 | $^{238}\text{U} - \sigma_{n,n'}$ | 0.454822 | -0.037794 |
| $^{238}\text{U} - \sigma_{n,\gamma}$ | 5.8802 | $^{238}\text{U} - \sigma_{n,\gamma}$ | 0.102275 | -0.209787 |
| $^{235}\text{U} - \sigma_f$ | 2.3522 | $^{235}\text{U} - \sigma_f$ | 0.039774 | 0.384521 |
| $^{238}\text{U} - \bar{\nu}$ | 1.3014 | $^{238}\text{U} - \bar{\nu}$ | 0.020972 | 0.107249 |

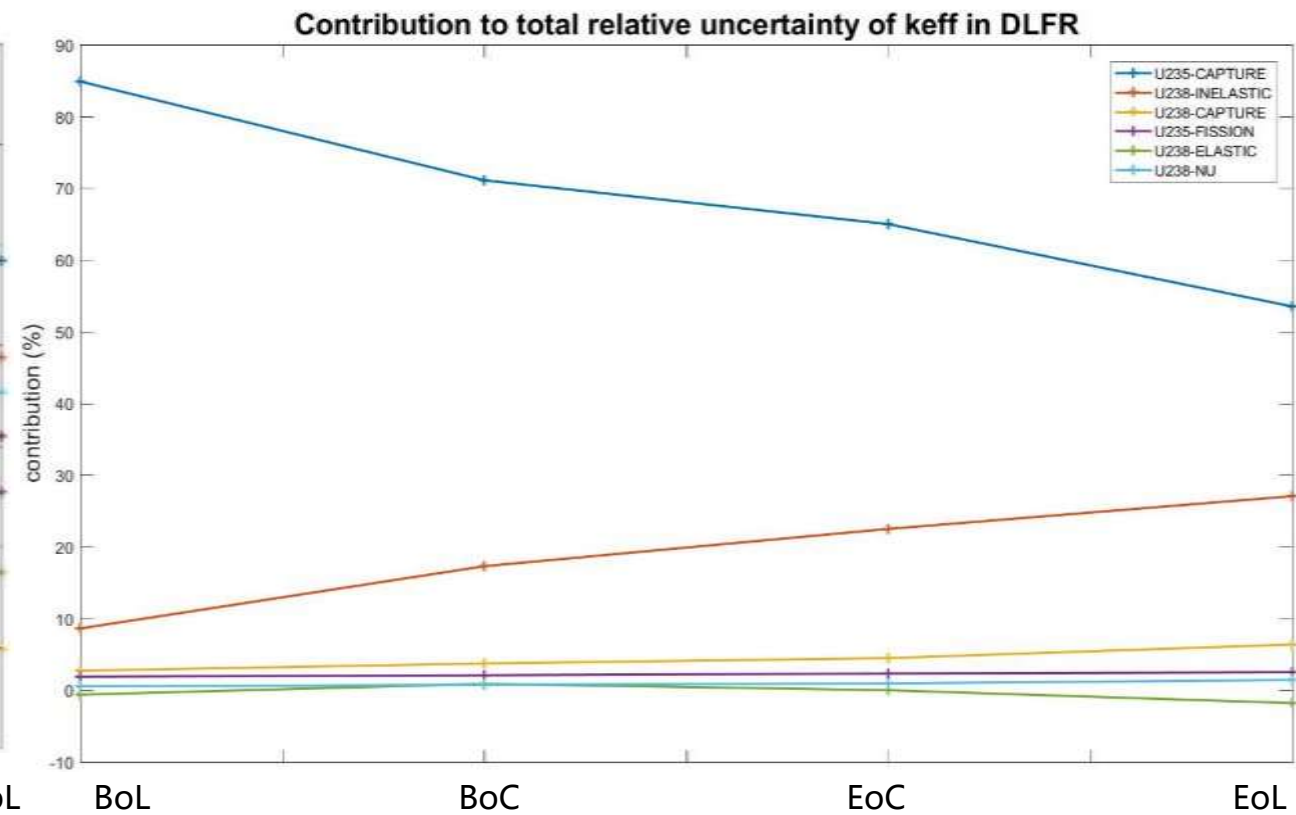
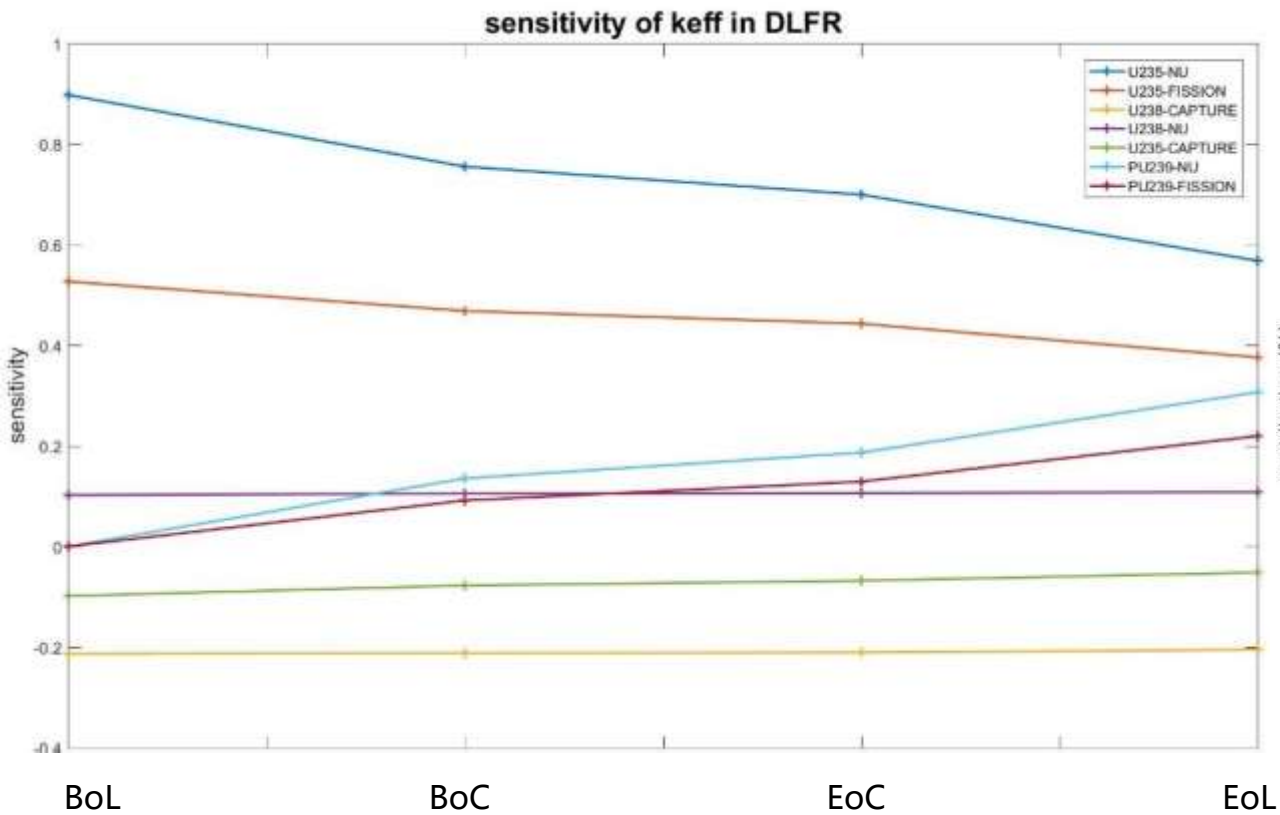
PART

04

Conclusion

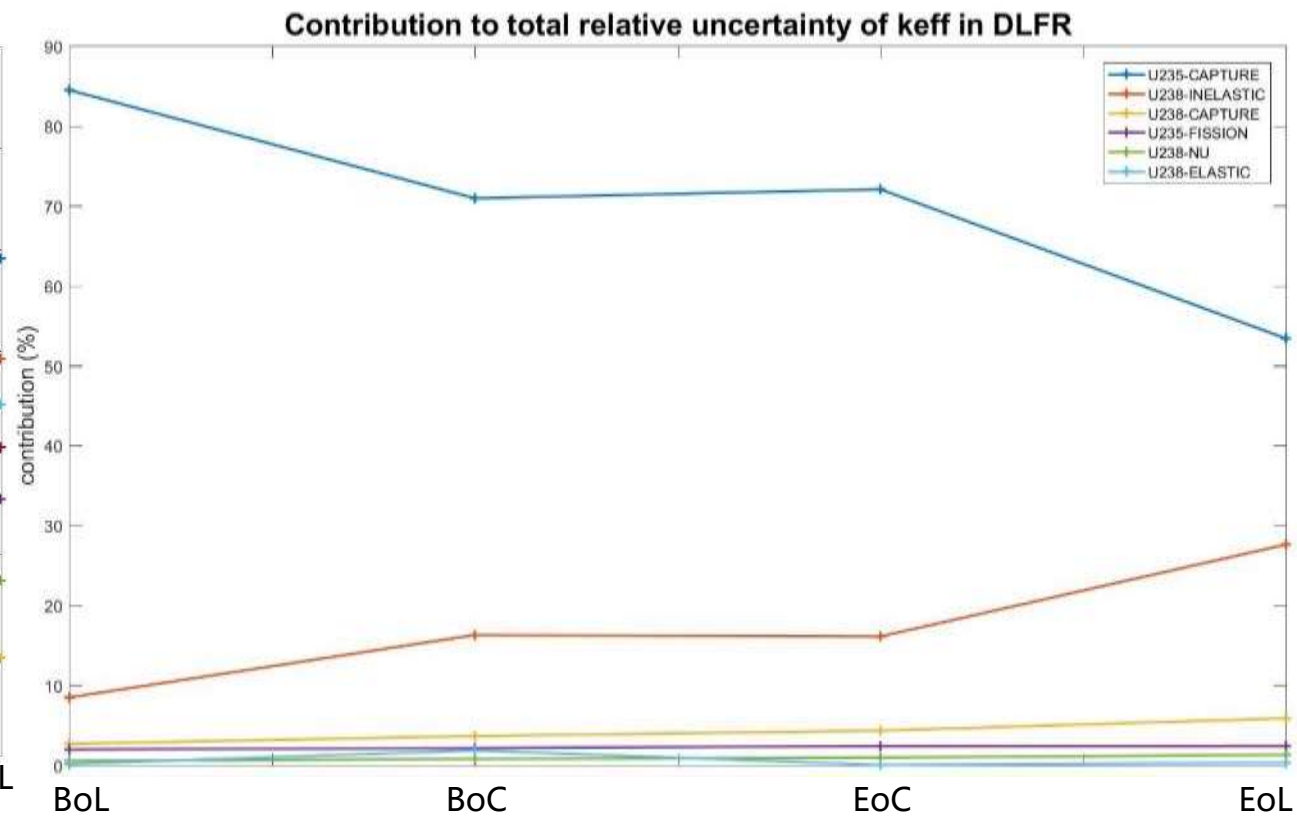
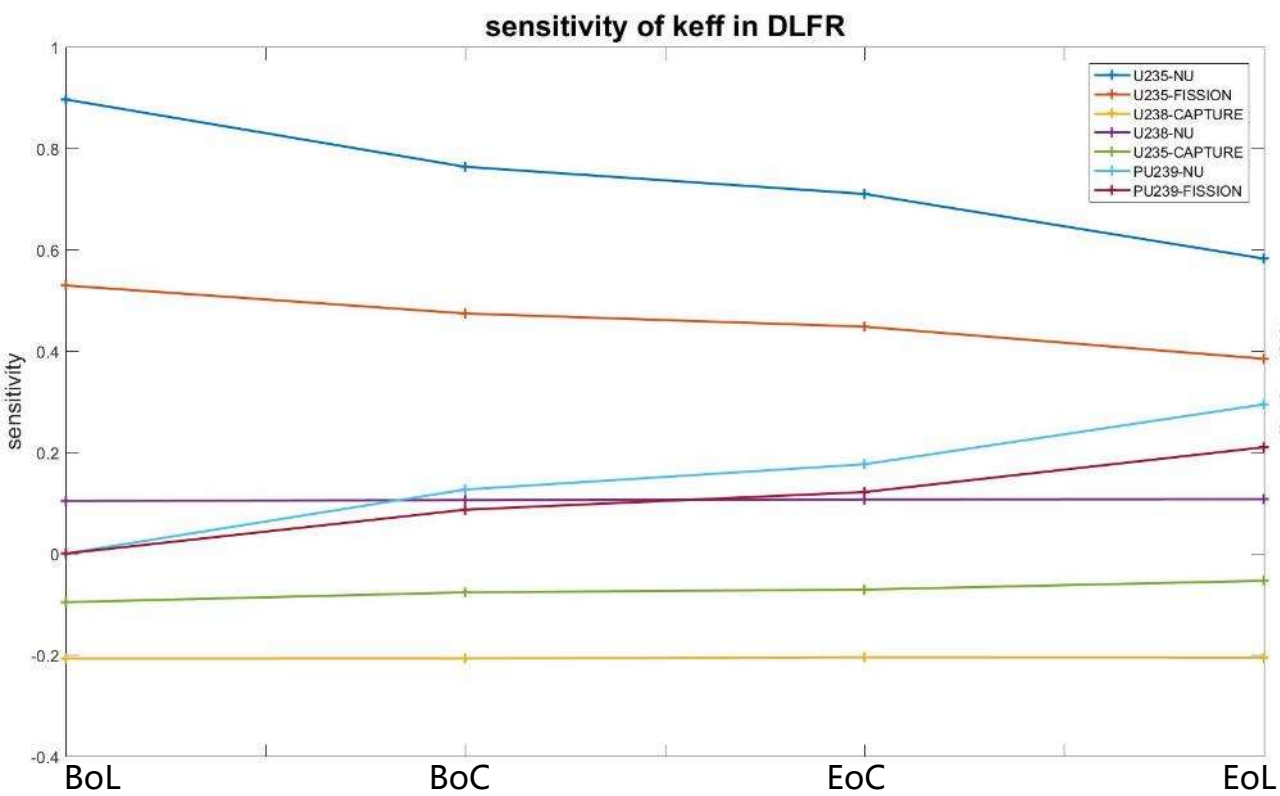
Sensitivity and Uncertainty Contributed to Total in Different Periods

2D Whole Core with Safety Rod out



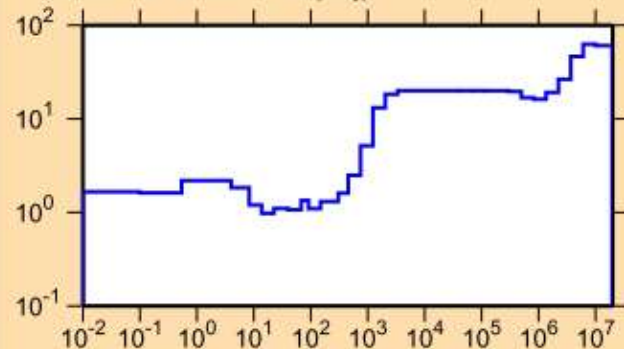
Sensitivity and Uncertainty Contributed to Total in Different Periods

2D Whole Core with Safety Rod in



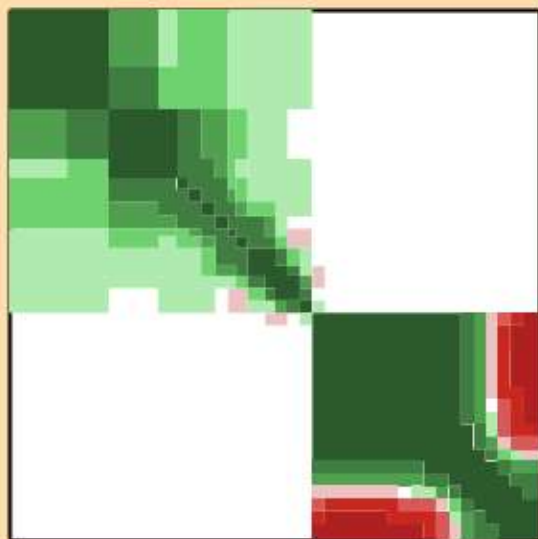
Correlation Matrices

$\Delta\sigma/\sigma$ vs. E for $^{235}\text{U}(n,\gamma)$

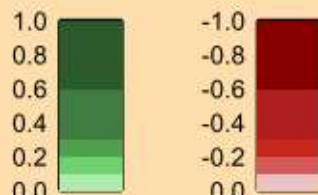


Ordinate scales are % relative standard deviation and barns.

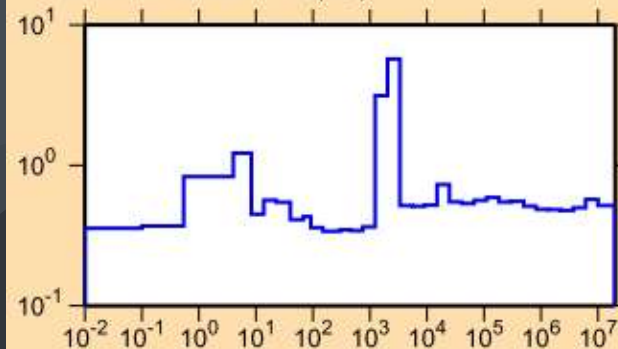
Abscissa scales are energy (eV).



Correlation Matrix

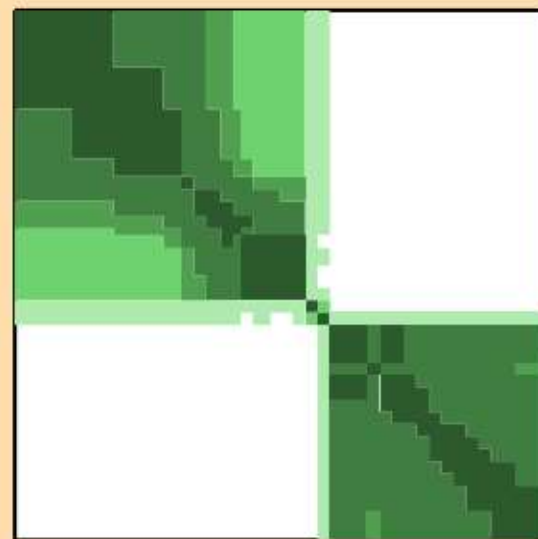


$\Delta\sigma/\sigma$ vs. E for $^{235}\text{U}(n,f)$

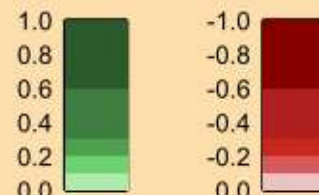


Ordinate scales are % relative standard deviation and barns.

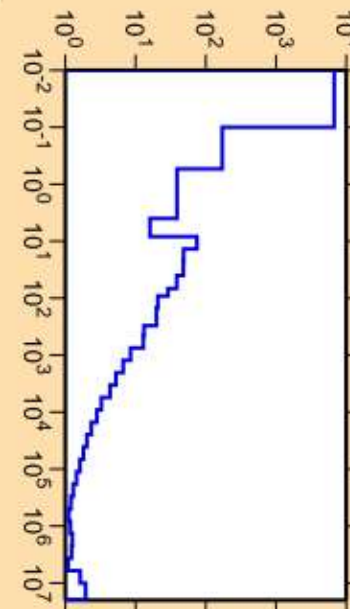
Abscissa scales are energy (eV).



Correlation Matrix

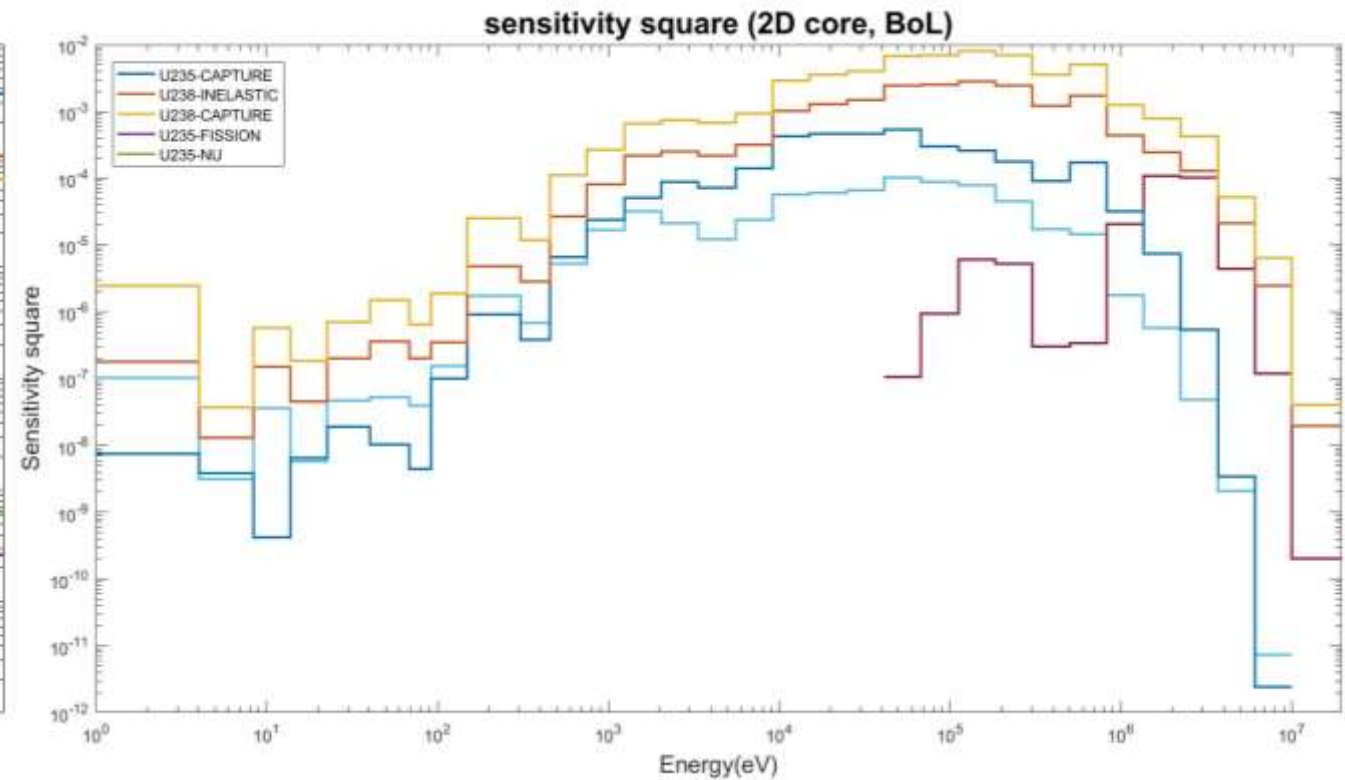
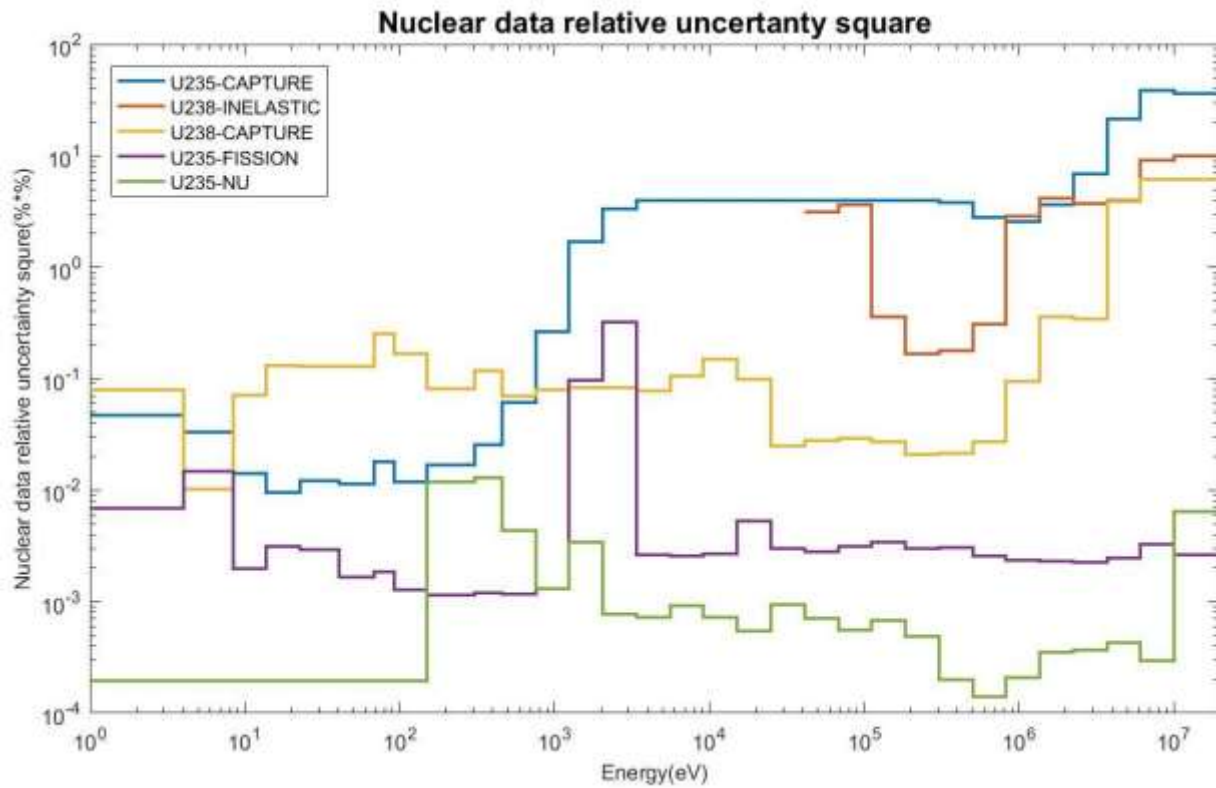


σ vs. E for $^{235}\text{U}(n,f)$

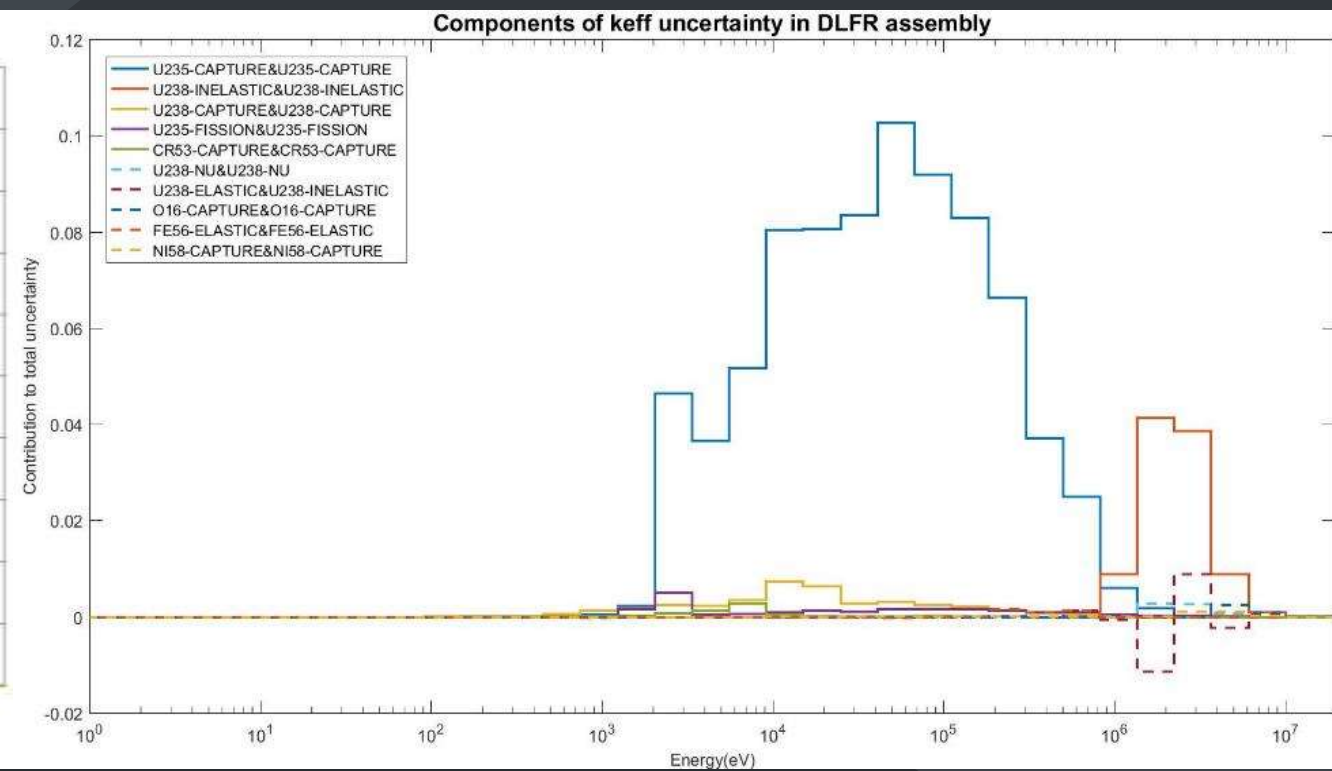
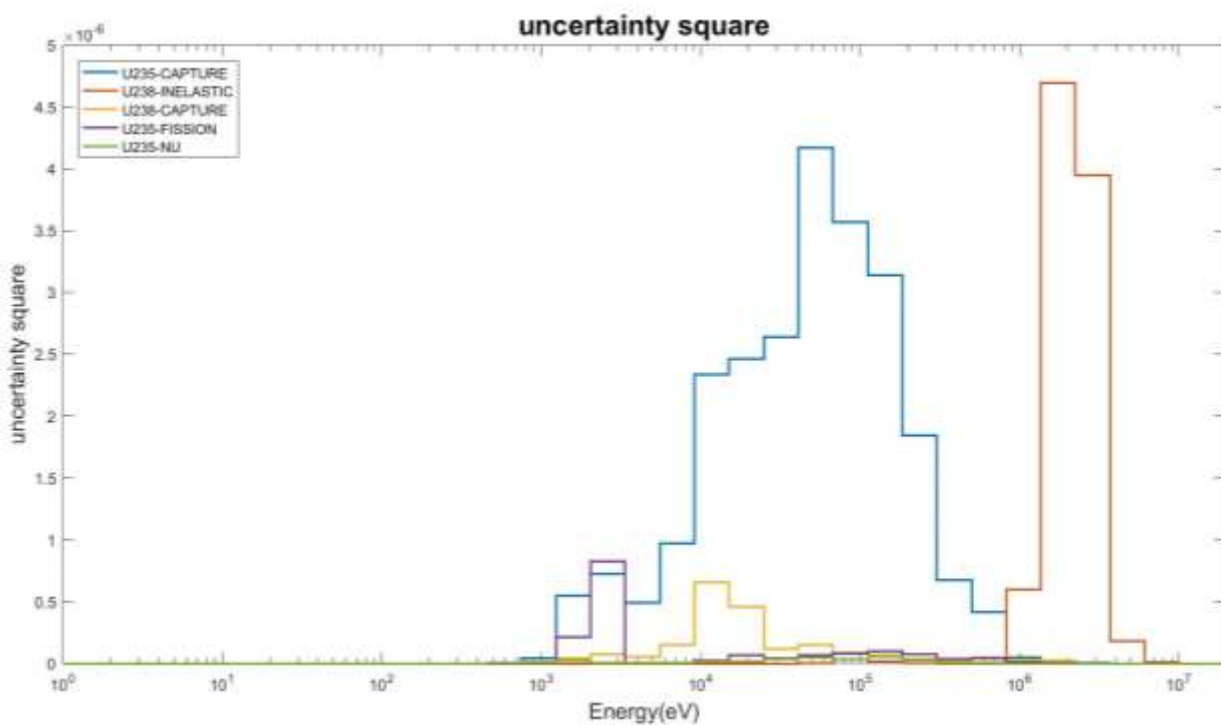


Nuclear Data Relative Uncertainty

(Elements on diagonal of each relative covariance matrix)



Uncertainty

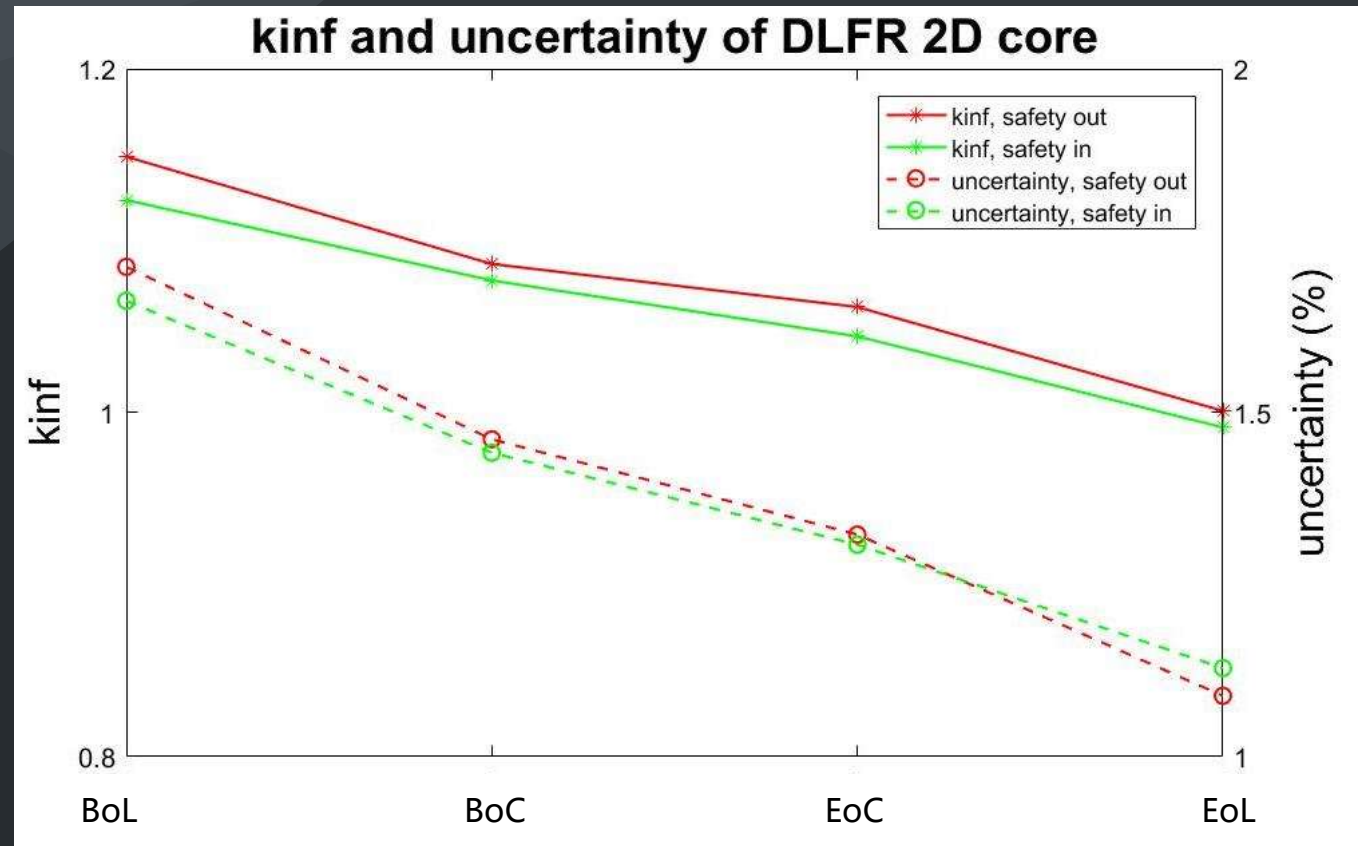


Conclusion

| Sensitivity | Uncertainty Contribution |
|--------------------------------------|--------------------------------------|
| $^{235}\text{U} - \bar{\nu}$ | $^{235}\text{U} - \sigma_{n,\gamma}$ |
| $^{235}\text{U} - \sigma_f$ | $^{238}\text{U} - \sigma_{n,n'}$ |
| $^{238}\text{U} - \sigma_{n,\gamma}$ | $^{238}\text{U} - \sigma_{n,\gamma}$ |
| $^{235}\text{U} - \sigma_{n,\gamma}$ | $^{235}\text{U} - \sigma_f$ |
| $^{238}\text{U} - \bar{\nu}$ | $^{238}\text{U} - \sigma_{n,n}$ |
| $^{239}\text{Pu} - \bar{\nu}$ | $^{238}\text{U} - \bar{\nu}$ |
| $^{239}\text{Pu} - \sigma_f$ | $^{53}\text{Cr} - \sigma_{n,\gamma}$ |

| Period | Relative Uncertainty |
|--------|----------------------|
| BoL | 1.71% |
| BoC | 1.46% |
| EoC | 1.32% |
| EoL | 1.09% |

k-inf & Uncertainty of 2D Whole Core

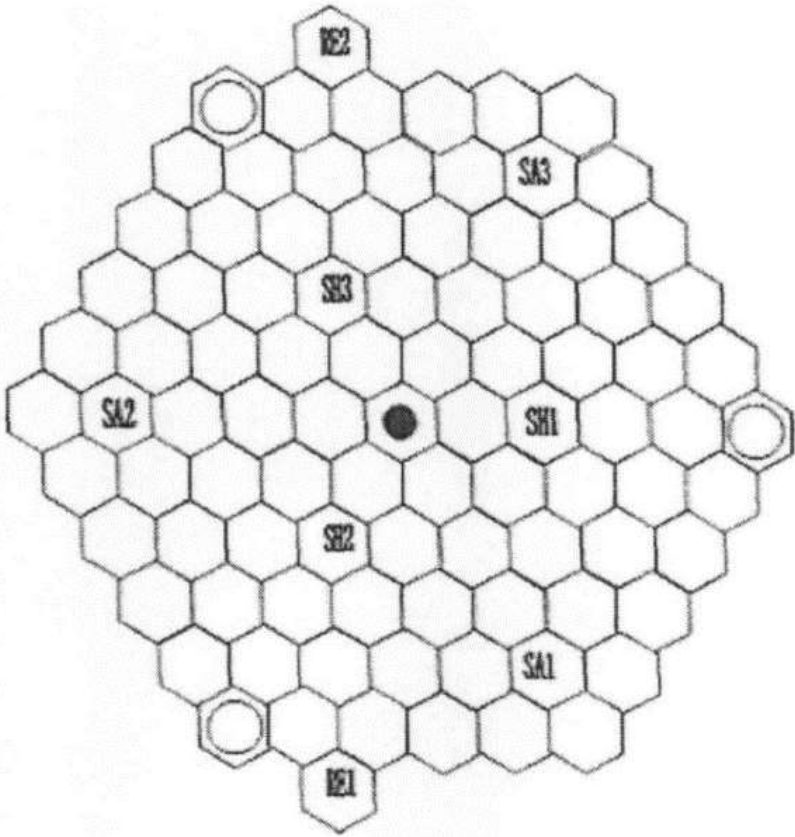


Comparison^[1]

Reactor: CEFR (China Experimental Fast Reactor)
Fuel: UO_2 , $^{235}U\% = 64.4\%$
Radius: 30.2 cm
Covariance: Based on Transportation Calculation via ANISN
Code: SUCA1D
(Sensitivity and Uncertainty Code of Analysis, one dimension)

| | |
|--|-------|
| Total Uncertainty | 2.65% |
| Reference Total Uncertainty ^[2] | 1.90% |

| Number | Parameter Pair | Uncertainty | Contribution to Total (%) |
|--------|-------------------------------|-------------|---------------------------|
| 1 | $^{235}U - \sigma_f$ | 1.27% | 22.97 |
| 2 | $^{235}U - \sigma_{n,\gamma}$ | 2.20% | 68.92 |
| 3 | $^{238}U - \sigma_f$ | 0.15% | 0.32 |
| 4 | $^{238}U - \sigma_{n,\gamma}$ | 0.73% | 7.59 |
| 5 | $^{56}Fe - \sigma_{n,\gamma}$ | 0.12% | 0.21 |



- SA Safety Rod
- SH Compensation Control Rod
- RE Regulation Control Rod
- Fuel Assembly
- Neutron Source Assembly
- Stainless Steel Assembly

CEFR Core in Equilibrium State

[1] 刚直. 核截面引起积分参数 k_{eff} 不确定度的一维分析程序开发[D]. 中国原子能科学研究院, 2006.
[2] 俄罗斯技术报告. CEFR堆芯物理特性计算误差分析
([2] is just mentioned but not cited by [1], reference is not given in [1].)

Reference

- [1] Westinghouse Electric Company LLC, Demonstration Lead-cooled Fast Reactor Details: Westinghouse Lead-cooled Fast Reactor. March 2016.
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| Ishita | TRIVEDI | (NCSU, Department of Nuclear Engineering) |
| Kaiyue | ZENG | (NCSU, Department of Nuclear Engineering) |
| Jun | SHI | (UCB, Engineering Research Centers) |

END | **THANK YOU!**

PRESENTED BY LI JIN