ECE 4960 Spring 2018: Computational and Software Engineering
Reading 2: Differentiation in Local Analysis

Deposit a pdf file of the two tables below to your Git directory before 11:59pm of 2/11

Document your programming environment: Language; development platform; operating system

Prob. 1. (Quadratic function to observe the tradeoffs between the truncation error and round-off error): For $f(x) = x^2$, we know the exact $f'(x=1) = 2$.

1.1 Use Eq. (1) below to estimate $f'(x=1)$ varying the value of h from 0.1 to 10^{-18} to observe the relative error in calculating $f'(x)$. Tabulate your results with sufficient precision in a table.

1.2 Repeat your calculation with $f(x) = x^2 + 10^8$. Add your results to the same table.

1.3 Repeat the above two procedures by using Eq. (2). Add your results to the same table.

$$f'(x) = \frac{f(x+h) - f(x)}{h} + O(h) \qquad (1)$$

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h} + O(h^2) \qquad (2)$$

h	Error in $f'(x=1)$ by Eq. (1) where $f(x) = x^2$	Error in $f'(x=1)$ by Eq. (1) where $f(x) = x^2 + 10^8$	Error in $f'(x=1)$ by Eq. (2) where $f(x) = x^2$	Error in $f'(x=1)$ by Eq. (2) where $f(x) = x^2 + 10^8$
10^{-1}	**1.000000e-01**	**1.000001e-01**	**0.000000e+00**	**0.000000e+00**
10^{-2}	**1.000000e-02**	**9.999722e-03**	**6.661338e-15**	**-4.470348e-07**
10^{-3}	**1.000000e-03**	**1.002401e-03**	**6.661338e-15**	**4.023313e-06**
10^{-4}	**1.000000e-04**	**3.382564e-05**	**8.393286e-13**	**-4.068017e-05**
10^{-5}	**1.000001e-05**	**-2.641976e-04**	**9.166001e-12**	**-2.641976e-04**
10^{-6}	**9.998760e-07**	**-3.244430e-03**	**-4.634537e-11**	**-3.244430e-03**
10^{-7}	**9.793083e-08**	**-6.284907e-02**	**-8.790126e-10**	**-6.284907e-02**
10^{-8}	**2.451656e-09**	**-5.098839e-01**	**-3.099459e-09**	**-5.098839e-01**
10^{-9}	**1.356784e-07**	**-2.000000e+00**	**2.465612e-08**	**-2.000000e+00**
10^{-10}	**1.356784e-07**	**-2.000000e+00**	**1.356784e-07**	**-2.000000e+00**
10^{-18}	**-2.000000e+00**	**-2.000000e+00**	**-2.000000e+00**	**-2.000000e+00**

Prob. 2. (Cubic function to observe the Richardson error estimation): For $f(x) = x^3$, we know the exact value of $f'(x=1) = 3$.

2.1 Use Eqs. (3) – (5) below to estimate $f'(x=1)$ varying the value of h from 2^{-4} to 2^{-40} to observe the relative error in calculating $f'(x)$. Tabulate your results with sufficient precision in a table.

2.2 Estimate η from Eqs. (6) and (7) for each choice of h. Add your results to the same table.

$$f'(x) = \frac{f(x+h) - f(x)}{h} + E(h); \qquad E(h) = O(h) = \frac{1}{2} h f''(x) + O(h^2) \qquad (3)$$

$$f'(x) = \frac{f(x+2h) - f(x)}{2h} + E(2h); \qquad E(2h) = O(h) = \frac{1}{2} 2h f''(x) + O(h^2) \qquad (4)$$

$$f'(x) = \frac{-1}{2h} f(x+2h) - \frac{3}{2h} f(x) + \frac{2}{h} f(x+h) + O(h^2) \qquad (5)$$

$$R(h) \equiv \frac{E(2h)}{E(h)} \cong \eta \qquad (6)$$

$$R(h) \cong \frac{\hat{A}(4h) - \hat{A}(2h)}{\hat{A}(2h) - \hat{A}(h)} \cong \eta \qquad (7)$$

h	Error in $f'(x=1)$ by Eq. (3)	Error in $f'(x=1)$ by Eq. (4)	Error in $f'(x=1)$ by Eq. (5)	η by Eq. (6)	η by Eq. (7)
2^{-4}	**3.000100e-04**	**6.000400e-04**	**-2.000161e-08**	**2.000067e+00**	**2.000200e+00**
2^{-5}	**3.000011e-05**	**6.000039e-05**	**-2.037268e-10**	**2.000006e+00**	**2.000022e+00**
2^{-6}	**2.999798e-06**	**6.000065e-06**	**-4.656613e-10**	**2.000156e+00**	**1.999778e+00**
2^{-7}	**3.015118e-07**	**5.979414e-07**	**7.450581e-09**	**1.983144e+00**	**2.031835e+00**
2^{-8}	**3.972047e-09**	**8.168766e-08**	**-8.940697e-08**	**2.056563e+01**	**4.285714e-01**
2^{-9}	**2.482211e-07**	**-8.484580e-08**	**7.152557e-07**	**-3.418154e-01**	**-5.000000e-01**
2^{-10}	**2.482211e-07**	**2.482211e-07**			
2^{-11}	**2.482211e-07**	**2.482211e-07**			
2^{-12}	**2.667017e-04**	**-6.636516e-05**			
2^{-13}	**-2.397834e-03**	**9.328356e-04**			
2^{-14}	**-2.397834e-03**	**-2.397834e-03**			
…					
2^{-40}					