Drill 6

We consider the simple linear regression model

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i,$$

where we assume that ϵ_i are independent and normally distributed with mean zero and variance σ^2.

1. We assume that β_0 and β_1 are known and want to use the estimator below to estimate σ^2.

$$\hat{\sigma}^2 = \frac{1}{K_1(n)} \sum_{i=1}^{n} (Y_i - \beta_0 - \beta_1 X_i)^2.$$

   Using the Monte Carlo simulation (R program), obtain $K_1(n)$ satisfying $E(\hat{\sigma}^2) = \sigma^2$ (Set $n = 10$ for example).

2. We assume that β_0 and β_1 are not known so that we use the ML estimators of β_0 and β_1, which are given by

$$\hat{\beta}_1 = \frac{\sum (X_i - \bar{X})(Y_i - \bar{Y})}{\sum (X_i - \bar{X})^2} = \frac{\sum X_i Y_i - n\bar{X}\bar{Y}}{\sum X_i^2 - n\bar{X}^2} = \frac{S_{xy}}{S_{xx}}$$

$$\hat{\beta}_0 = \frac{1}{n}\Big\{ \sum_{i=1}^{n} Y_i - \hat{\beta}_1 \sum_{i=1}^{n} X_i \Big\} = \bar{Y} - \hat{\beta}_1 \bar{X},$$

   where $\bar{X} = (1/n)\sum_{i=1}^{n} X_i$ and $\bar{Y} = (1/n)\sum_{i=1}^{n} Y_i$. Then we want to use the estimator below to estimate σ^2.

$$S^2 = \frac{1}{K_2(n)} \sum_{i=1}^{n} (Y_i - \hat{\beta}_0 - \hat{\beta}_1 X_i)^2.$$

   Using the Monte Carlo simulation (R program), obtain $K_2(n)$ satisfying $E(S^2) = \sigma^2$ (Set $n = 12$ for example).