$H_0: \mu = \mu_0$, vs $H_1: \mu > \mu_0$ (σ = known).

$\boxed{\text{under } H_0}$

$$\frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \sim N(0,1)$$

$\boxed{\text{under } H_1}$

$$\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0,1)$$

But the rejection region is

$$\frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} > Z_\alpha \qquad \boxed{\text{under } H_1}$$

$$\bar{X} > Z_\alpha \frac{\sigma}{\sqrt{n}} + \mu_0$$

$$\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} > Z_\alpha + \frac{\mu_0 - \mu}{\sigma/\sqrt{n}}.$$

$H_0: \mu = \mu_0$ vs $H_1: \mu > \mu_0$ (σ = Unknown)

Under H_0

$$T = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} \sim t\,(df = n-1)$$

under H_1

$$T = \frac{\bar{X} - \mu}{S/\sqrt{n}} \sim t\,(df = n-1)$$

Rejection region is

$$T = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} > t_{\alpha, n-1} \quad \text{under } H_1$$

Let $Z \sim N(0,1)$

$$\frac{Z + \delta}{\sqrt{V/r}} \sim t\text{-dist}\,(df = r, \text{ non-centrality } \delta).$$

$V \sim \chi^2 (df = r)$.

$$V = \frac{(n-1)s^2}{\sigma^2} \sim \chi^2(df=n-1)$$

$$T = \frac{\bar{X}-\mu_0}{s/\sqrt{n}} = \frac{\bar{X}-\mu_0}{\sqrt{\frac{\sigma^2 V}{n-1}}\Big/\sqrt{n}}$$

$$= \frac{\sqrt{n}\,(\bar{X}-\mu_0)}{\sigma\sqrt{V/(n-1)}} = \frac{\frac{\sqrt{n}}{\sigma}(\bar{X}-\mu_0)}{\sqrt{V/(n-1)}}$$

$$= \frac{\frac{\sqrt{n}}{\sigma}(\bar{X}-\mu+\mu-\mu_0)}{\sqrt{V/r}} \qquad (r=n-1)$$

$$= \frac{\frac{(\bar{X}-\mu)}{\sigma/\sqrt{n}} + \frac{(\mu-\mu_0)}{\sigma/\sqrt{n}}}{\sqrt{V/r}} \longrightarrow Z \sim N(0,1) \text{ under } H_1$$

$$= \frac{Z + \frac{(\mu-\mu_0)}{\sigma/\sqrt{n}}}{\sqrt{V/r}} = \frac{Z+\delta}{\sqrt{V/r}}$$

$$\sim \text{t-dist}(df=n-1,\ \delta).$$