Drill 4

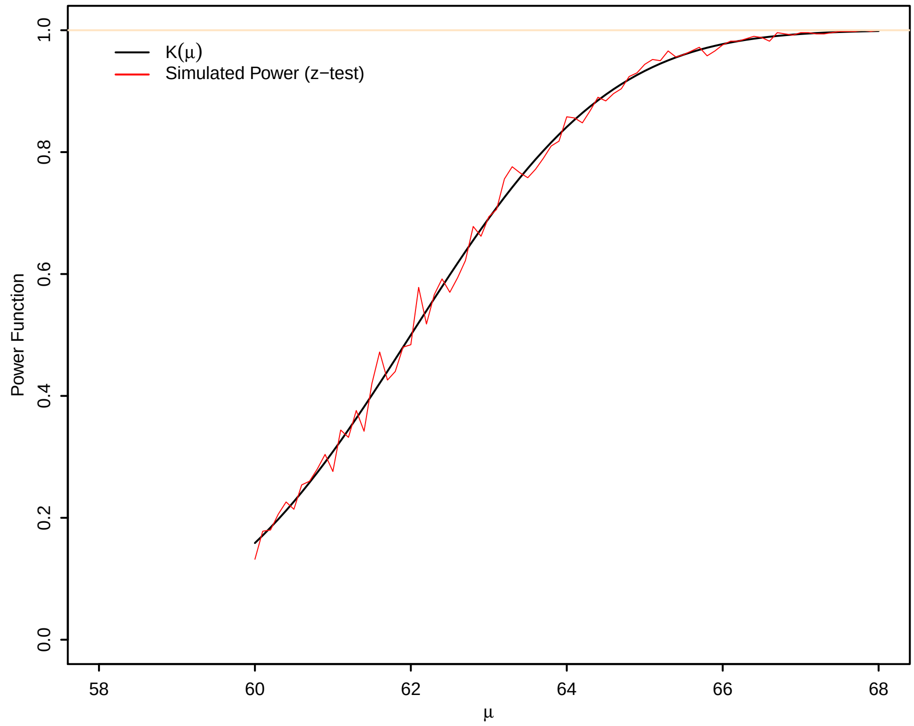
We consider Example 8.5-2 from Hogg et al. (2015). Let $X_1, X_2, \ldots, X_n$ $(n = 25)$ be a random sample from $N(\mu, 100)$. We want to test $H_0 : \mu = 60$ versus $H_1 : \mu > 60$. When we decide to reject H_0 if $\bar{X} > 62$ (critical/rejection region), the power function of μ can be derived as below.

Since $n = 25$ and $X_i \sim N(\mu, 100)$, we have $\bar{X} \sim N(\mu, 100/25)$. Then $Z = (\bar{X} - \mu)/\sqrt{100/25} = (\bar{X} - \mu)/2 \sim N(0, 1)$. Thus, under $H_1 : \mu$, the power function is given by

$$
\begin{aligned}
K(\mu) &= P[\bar{X} > 62 \mid \mu] \\
&= P\Big[\frac{\bar{X} - \mu}{2} > \frac{62 - \mu}{2}\Big] = P\Big[Z > \frac{62 - \mu}{2}\Big] \\
&= 1 - \Phi\Big[\frac{62 - \mu}{2}\Big].
\end{aligned}
$$

1. Example 8.5-2 states that the power at $\mu = 65$ is $K(65) = 0.9332$. Make a R function for the above theoretical power function. Calculate the power at $\mu = 65$ using the Monte Carlo simulation.

2. Figure 8.5-2 provides the power function. Make a R program to plot the theoretical and simulated power functions. The plot would be as below.



3. Example 8.5-2 pointed out that the significance level α is usually smaller than 0.1587 (usually $\alpha = 0.05$). We can generalize the above result for testing $H_0 : \mu = \mu_0$ versus $H_1 : \mu > \mu_0$ with the significance level α and sample size n under the assumption that σ is known. Then the rejection region is given by

$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} > z_\alpha.$$

Find the theoretical power function of this test.

Compare the theoretical power function with the simulated power function when we test $H_0 : \mu = 0$ versus $H_1 : \mu > 0$ with $n = 5$, $\sigma = 2$ and $\alpha = 0.05$.

4. The above z-test is useless because σ is unknown in practice. That is, we can not calculate $Z = (\bar{X} - \mu_0)/(\sigma/\sqrt{n})$ with $X_1, X_2, \ldots, X_n$. Thus, we use the

t-test and its rejection region is given by

$$T = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} > t_{\alpha,n-1}.$$

Find the theoretical power function of this test. Compare the theoretical power function with the simulated power function when we test $H_0 : \mu = 0$ versus $H_1 : \mu > 0$ with $n = 5$, $\sigma = 2$ and $\alpha = 0.05$.

5. Compare the power functions of the above z-test and t-test. Discuss the difference of the two power functions.

References

Hogg, R. V., Tanis, E. A., and Zimmerman, D. L. (2015). *Probability and Statistical Inference*. Pearson, 9th edition.