Drill 5

In this exercise, we perform the two-sided hypothesis testing for $H_0 : \mu = \mu_0$ versus $H_1 : \mu \neq \mu_0$. We assume that $X_1, X_2, \ldots, X_n$ are from the normal with mean μ and variance σ^2. This test is well known as z-test (when σ is known) or t-test (when σ is unknown) in the statistics literature. Note that the rejection region of the z-test and t-test are given by

$$Z = \frac{|\bar{X} - \mu_0|}{\sigma/\sqrt{n}} > z_{\alpha/2} \quad \text{and} \quad T = \frac{|\bar{X} - \mu_0|}{S/\sqrt{n}} > t_{\alpha/2}.$$

1. (a) When the variance known, obtain the theoretical power function of the z-test

   (b) When the variance unknown, obtain the theoretical power function of the t-test

2. Obtain the simulated power functions of the z-test and t-test for testing $H_0 : \mu = 1/2$ versus $H_1 : \mu \neq 1/2$ with the significance level $\alpha = 0.05$. Generate a sample of size $n = 5$ from the normal distribution with mean μ and $\sigma = 1$, where μ varies from -1 to 2.

3. Compare the theoretical and simulated power functions of two tests. (The results should be similar to the following plot).