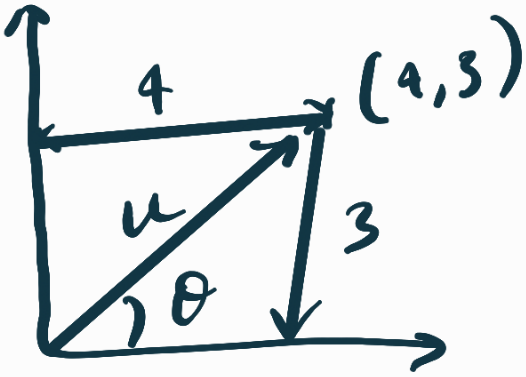
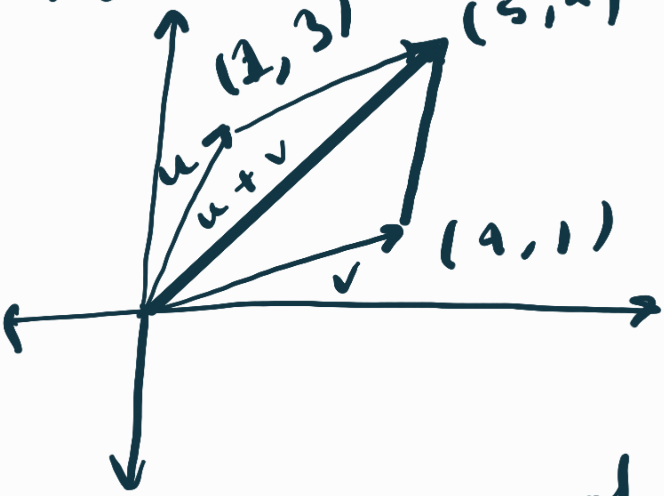
Vectors :



L2 norm

$$\tan(\theta) = \frac{3}{4} \Rightarrow \theta = \arctan\left(\frac{3}{4}\right) = 0.64 = 36.87^\circ$$

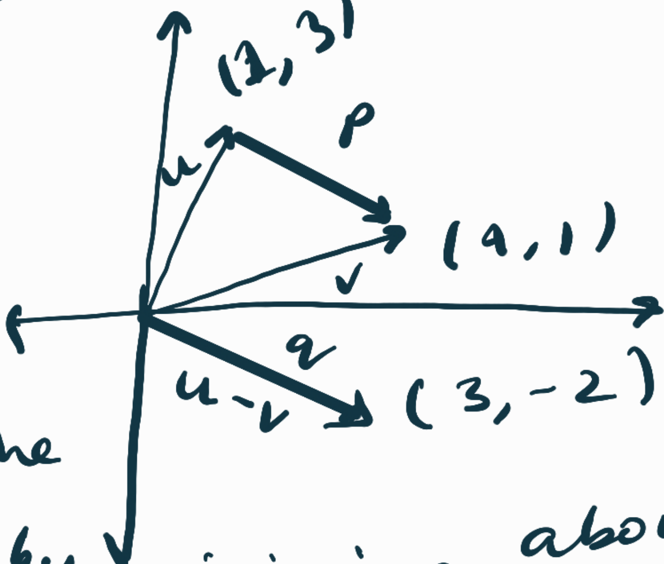
Sum of vectors :
Add cthe directional components



$$u+v = (4+1, 1+3) = (5,4)$$

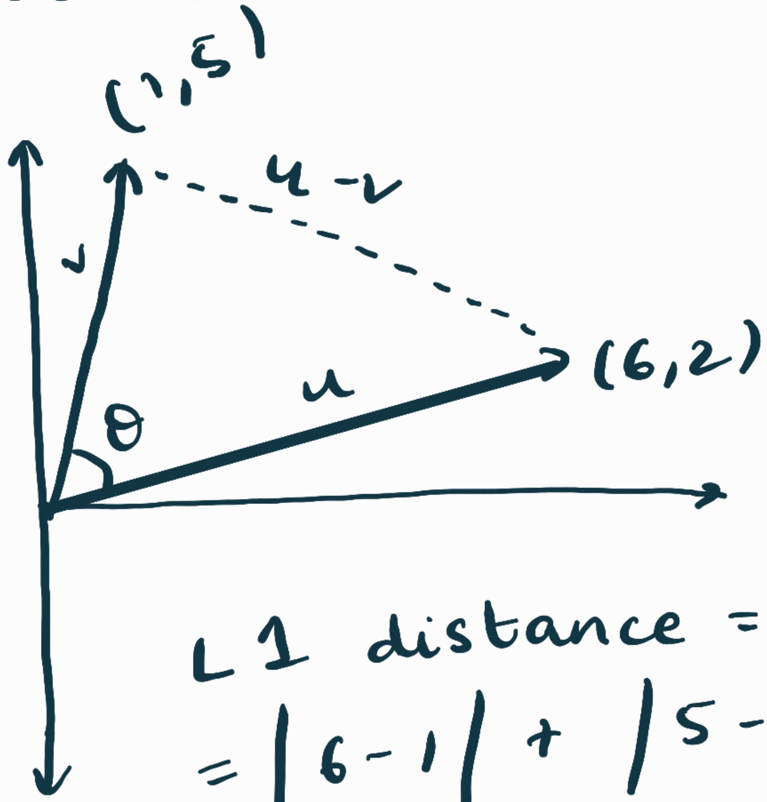
Subtraction of vectors

$$u-v = (4-1, 1-3) = (3,-2)$$



Vector matches the points obtained by joining above vectors i.e $p == q$

Distance between 2 vectors:



L1 distance $= |u-v|_1$

$= |6-1| + |5-2| = 5+3 = 8$

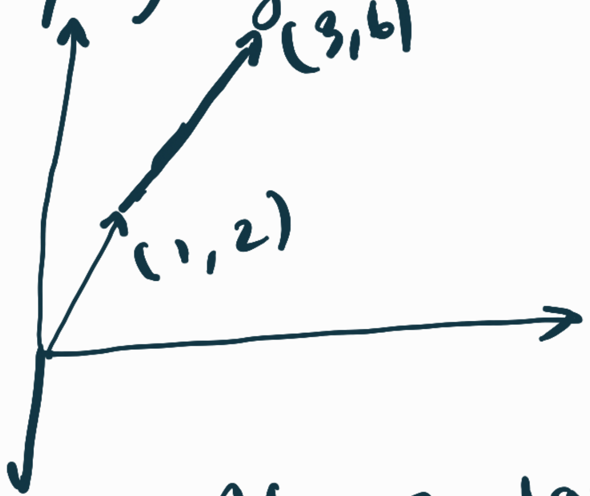
L2 distance $= |u-v|_2$

$= \sqrt{5^2+3^2} = 5.83$

Cosine distance

$= \cos(\theta)$

Multiplying a vector by scalar



$u = (1,2)$

$\lambda = 3$

$\lambda u = (3,6)$

If scalar is positive

Multiply by negative if scalar is negative

Dot product

e.g.

Quantities
2 apples
4 bananas
1 cherry

Prices
apple = 3
bananas = 5
cherries = 2

Total
$2 \times 3 = 6$
$5 \times 4 = 20$
$2 \times 1 = 2$
$\overline{28}$

$$\Rightarrow \begin{bmatrix} 2 & 4 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} = 2 \times 3 + 4 \times 5 + 2 \times 1$$

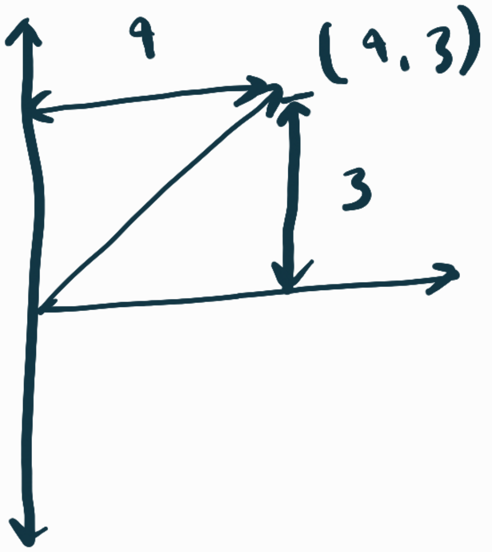
$$= \begin{bmatrix} 28 \end{bmatrix}$$

$1 \times \cancel{3} \quad \cancel{3} \times 1$

Relation between norm & dot-product

$$L2\text{-norm} = \sqrt{\text{dotprod}(u, u)}$$

$$|u|_2 = \sqrt{\langle u, u \rangle}$$

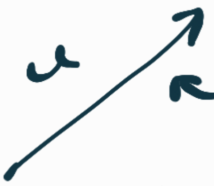


$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

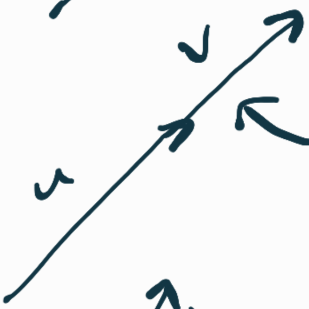
$$\begin{bmatrix} 4 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 25 \end{bmatrix}$$

Geometric dot product :

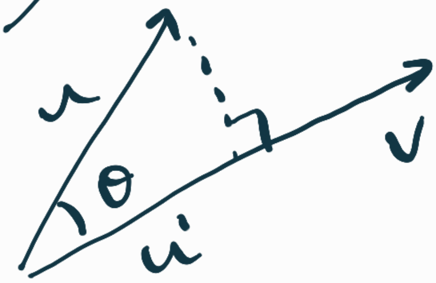
Two vectors are said to be orthogonal if their dot prod is 0 (orthogonal == perpendicular)



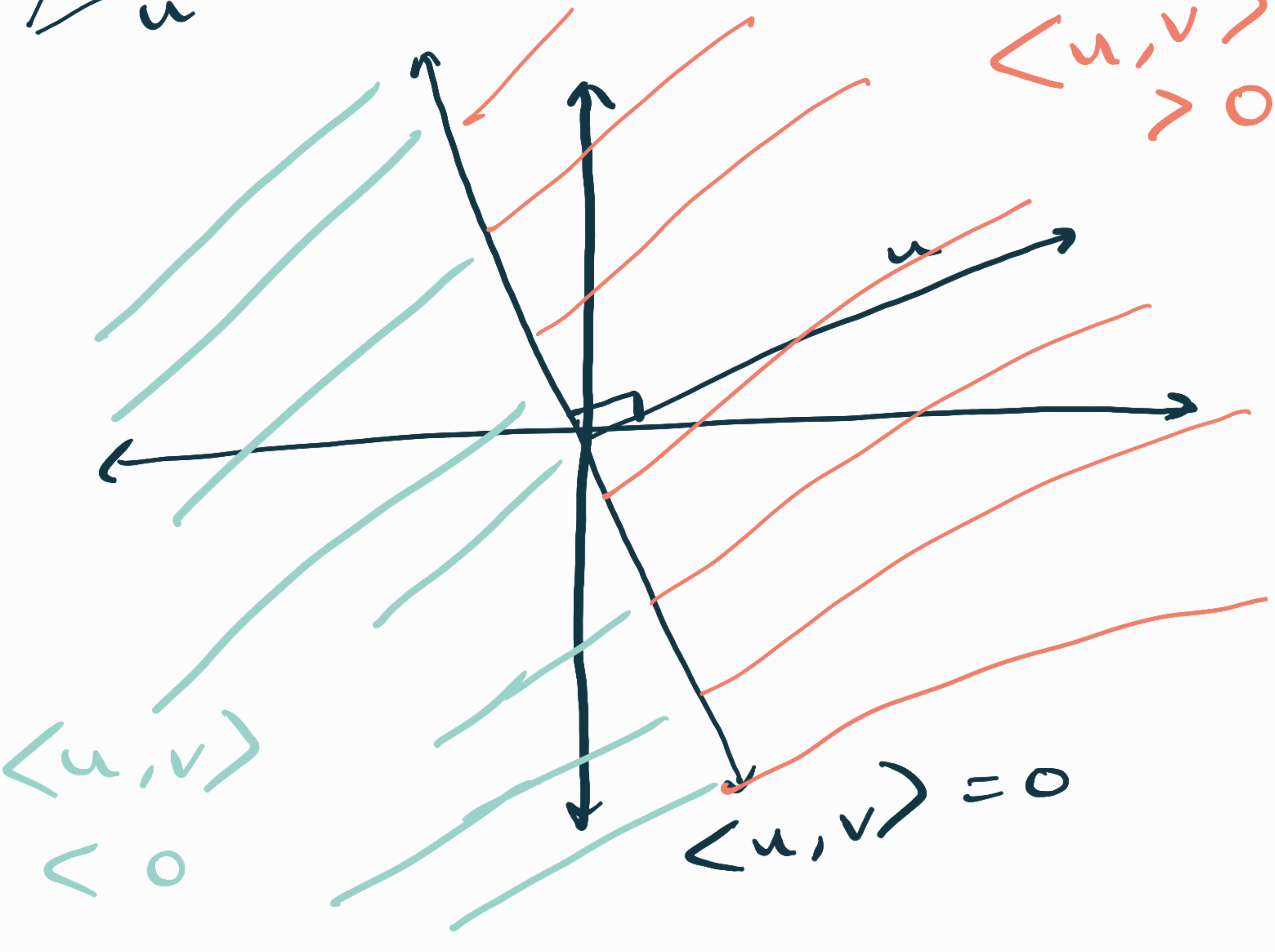
$$\langle u,u\rangle = |u|^2 = |u|\cdot|u|$$



$$\langle u,v\rangle = |u|\cdot|v|$$



$$\langle u,v\rangle = |u'|\cdot|v|$$
$$= |u|\cdot|v|\cos\theta$$



Multiplying matrix by vector

System of equations

$$a + b + c = 10$$
$$a + 2b + c = 15$$
$$a + b + 2c = 12$$

Matrix product

$$\Rightarrow \underset{3 \times \cancel{3}}{\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}} \underset{\cancel{3} \times 1}{\begin{bmatrix} a \\ b \\ c \end{bmatrix}} = \underset{3 \times 1}{\begin{bmatrix} 10 \\ 15 \\ 12 \end{bmatrix}}$$

Vector operations Quiz:

(2) Compute sum of $\vec{u}, \vec{v}$

$\vec{u} = (1,3)$, $\vec{v} = (6,2)$

$\Rightarrow \vec{u} + \vec{v} = (7, 5)$

(3) Difference

$\vec{u} - \vec{v} = (-5, 1)$

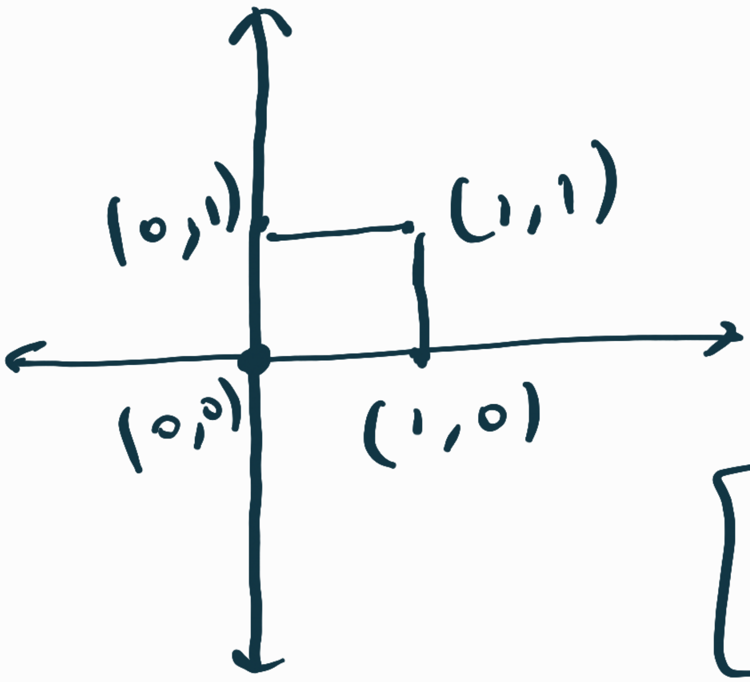
(4) Calculate dot product

$$\vec{a} = \begin{bmatrix} -1 \\ 5 \\ 2 \end{bmatrix} \qquad b = \begin{bmatrix} -3 \\ 6 \\ -4 \end{bmatrix}$$

$$\vec{a} \cdot \vec{b} = -1 \times -3 + 5 \times 6 + 2 \times -4$$

$$= 3 + 30 - 8 \qquad = 25$$

Matrices as linear transformati



$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

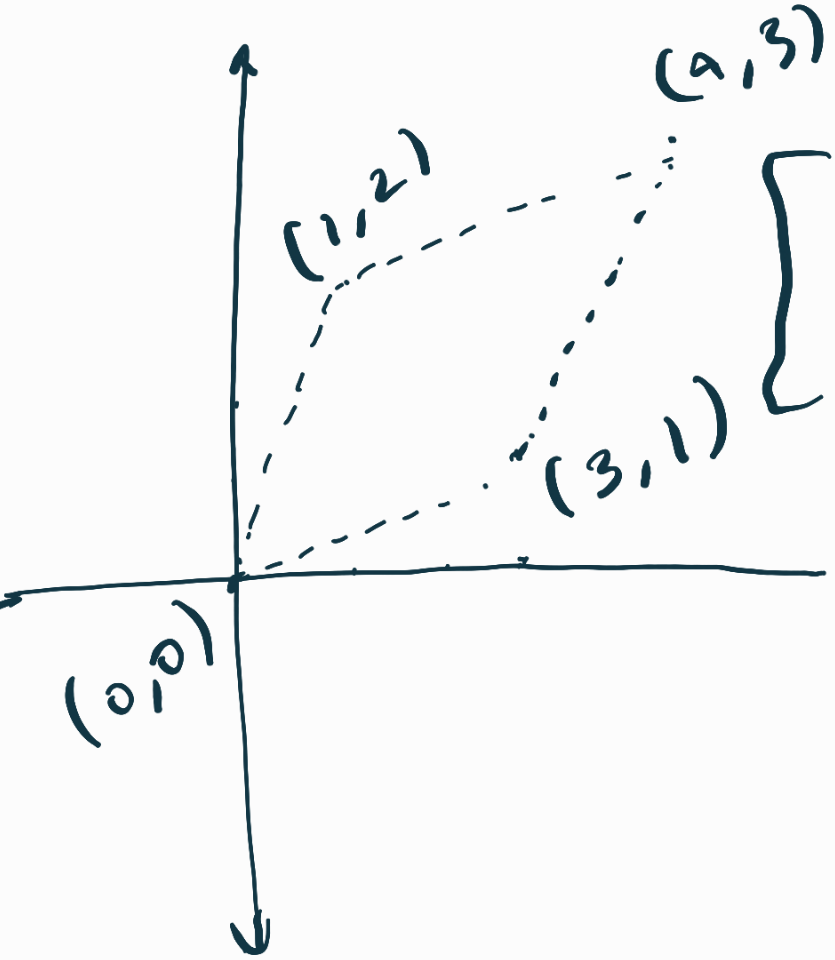
$\Rightarrow (0,0) = (0,0)$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$(1,0) = (3,1)$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$(0,1) = (1,2)$



$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

$(1,1) = (4,3)$

Identity matrix:

Has 1's in its diagonal and 0's everywhere else.

e.g $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Find the inverse of the matrix:

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\Rightarrow 5a + 2c = 1$ — 1

$5b + 2d = 0$ — 2

$a + 2c = 0$ — 3

$b + 2d = 1$ — 4

Solving for 3

$a = -2c$

Substitute in ①

$5(-2c) + 2c = 1$

$-10c + 2c = 1$

$c = +1/8$

$\Rightarrow a = +1/4$

$b = 1 - 2d$

Substitute in ②,

$5(1-2d) + 2d = 0$

$5 - 10d + 2d = 0$

$d = 5/8$

$\Rightarrow b = 1 - 5/4$

$= -1/4$

Q2. Find inverse of

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow a + c = 1$$
$$b + d = 0$$
$$2a + 2c = 0$$
$$2b + 2d = 1$$

$$a = 1 - c$$
$$2(1-c) + 2c = 0$$
$$2 - \cancel{2c} + \cancel{2c} = 0$$
$$2 \neq 0$$

$\therefore$ Non-singular matrices are invertible. Singular matrices are not (Determinant should be 0 for matrix to be singular)

Quiz :

i) Find distance between vectors $\vec{v}(1, 0, 7)$ and $\vec{w}(0, -1, 2)$

$\Rightarrow \sqrt{|1-0| + |0+1| + |7-2|}$

$= \sqrt{1+1+25} = \sqrt{27}$

ii) What is magnitude of vector from P to Q for $P(1, 0, -3)$ $Q(-1, 0, -3)$

$= \sqrt{(-1-1)^2 + (0-0)^2 + (-3+3)^2}$

$= \sqrt{4 + 0 + 0} = 2$

iii) Calculate norm of the vector $\vec{v}(1, -5, 2, 0, 3)$

$= \sqrt{(1)^2 + (-5)^2 + (2)^2 + (0)^2 + (3)^2}$

$= \sqrt{1+25+4+9}$

$= \sqrt{39}$

5) which vector has greater norm ?

$\rightarrow \sqrt{(2)^2+(5)^2} = \sqrt{29} \rightarrow$ highest

$\rightarrow \sqrt{4+4+4+4} = \sqrt{16} = 4$

$\rightarrow \sqrt{1+4+9} = \sqrt{14}$

$\rightarrow \sqrt{1+4+1} = \sqrt{6}$

6 Calculate the dot prod

$\vec{a} = \begin{bmatrix} -1 \\ 5 \\ 2 \end{bmatrix} \quad \vec{b} = \begin{bmatrix} -3 \\ 6 \\ -4 \end{bmatrix}$

$= -1 \times -3 + 5 \times 6 + 2 \times -4$

$= 3 + 30 - 8 = 25$

① Find result of $M_1 \times M_2$

$$M_1 = \begin{bmatrix} 2 & -1 \\ 3 & -3 \end{bmatrix} \quad M_2 = \begin{bmatrix} 5 & -2 \\ 0 & 1 \end{bmatrix}$$

$$M_1 \times M_2 = \begin{bmatrix} 2 \times 5 + (-1)(0) & -4 -1 \\ 15 + (3)(0) & -6 - 3 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & -5 \\ 15 & -9 \end{bmatrix}$$

⑧ Calculate dot prod

$$\vec{w} = \begin{bmatrix} -9 \\ -1 \end{bmatrix}, \vec{z} = \begin{bmatrix} -3 \\ -5 \end{bmatrix}$$

$$= -9 \times -3 + (-1)(-5)$$

$$= 27 + 5 = 32$$