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HW1

ROBOTICS

CSE 5364

For this assignment an external library (GSL) has been utilized for matrix operations.

The GNU Scientific Library (GSL) is a numerical library for C and C++ programmers. It is free software under the GNU General Public License.

How to Install GSL on Linux:
https://coral.ise.lehigh.edu/jild13/2016/07/11/hello/

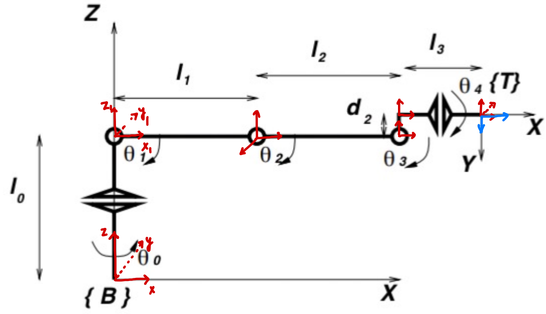
https://gist.github.com/TysonRayJones/af7bedcdb8dc59868c7966232b4da903

Running GSL
1. export LD_LIBRARY_PATH=/usr/local/lib
2. gcc -Wall -L/usr/local/lib/ kin_fncs.c -o output_name.o -lgsl -lgslcblas -lm
3. ./output_name.o

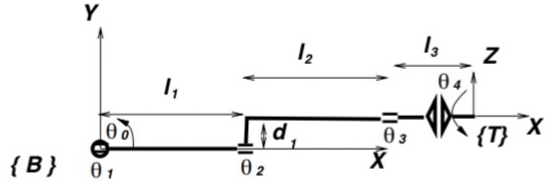
int gsl_matrix_memcpy(gsl_matrix *dest, const gsl_matrix *src)
This function copies the elements of the matrix src into the matrix dest. The two matrices must have the same size.

PART 1. Forward Kinematics

Side View:



Top View:



To determine the transformation from configuration space to Cartesian Location of Tool frame {T} in the base frame {B}, following are the series of rotational matrices used to express the frame {T} with respect to frame {B}.

$$R_z(\theta_0)\, D_z(l_0)\, R_y(\theta_1)\, D_x(l_1)\, R_y(\theta_2)\, D_y(d_1)\, D_x(l_2)\, R_y(\theta_3)\, D_z(d_2)\, R_x(\theta_4)\, D_x(l_3)$$

where,

$$R_z(\theta_0) = \begin{bmatrix} \cos\theta_0 & -\sin\theta_0 & 0 & 0 \\ \sin\theta_0 & \cos\theta_0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad R_y(\theta_1) = \begin{bmatrix} \cos\theta_1 & 0 & \sin\theta_1 & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta_1 & 0 & \cos\theta_1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_y(\theta_2) = \begin{bmatrix} \cos\theta_2 & 0 & \sin\theta_2 & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta_2 & 0 & \cos\theta_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad R_y(\theta_2) = \begin{bmatrix} \cos\theta_3 & 0 & \sin\theta_3 & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta_3 & 0 & \cos\theta_3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_x(\theta_4) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta_4 & -\sin\theta_4 & 0 \\ 0 & \sin\theta_4 & \cos\theta_4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad D_z(l_0) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$D_x(l_1) = \begin{bmatrix} 1 & 0 & 0 & l_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad D_y(d_1) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$D_x(l_2) = \begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

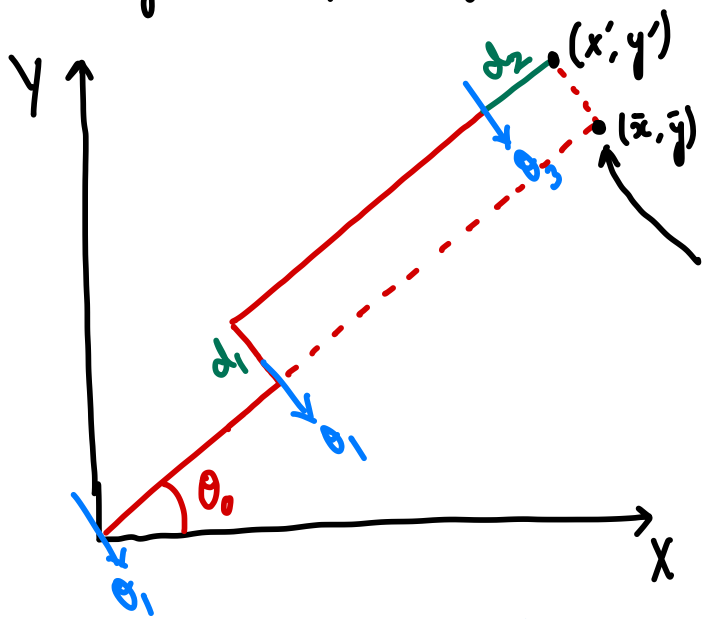
$$D_z(d_2) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$D_x(l_3) = \begin{bmatrix} 1 & 0 & 0 & l_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

PART 2. Inverse Kinematics

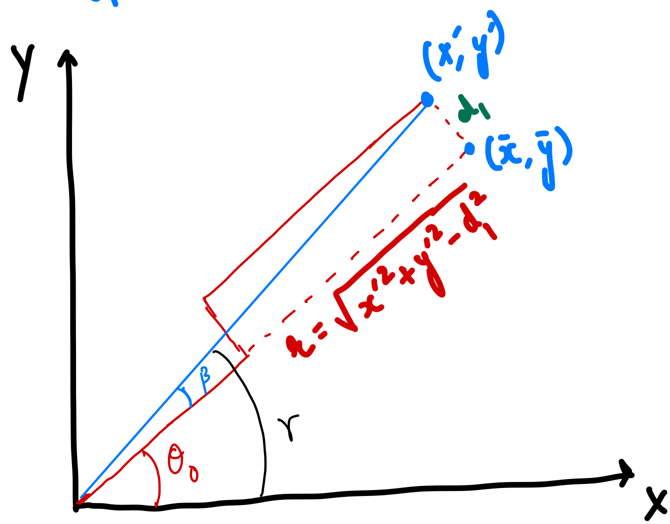
Evaluating θ_0

Considering the following XY view

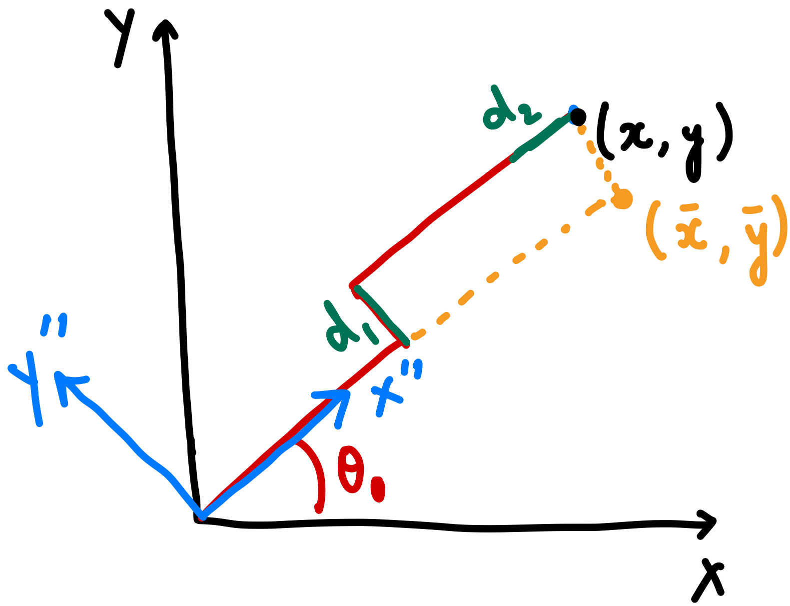


TOP VIEW

$$\bar{x} = d_1 \sin\theta_0 + x'$$
$$\bar{y} = d_1 \cos\theta_0 - y'$$



$$\gamma = \operatorname{atan}\left(\frac{y'}{x'}\right)$$
$$\beta = \operatorname{atan}\left(\frac{d_1}{r}\right)$$
$$\theta_0 = \gamma - \beta$$



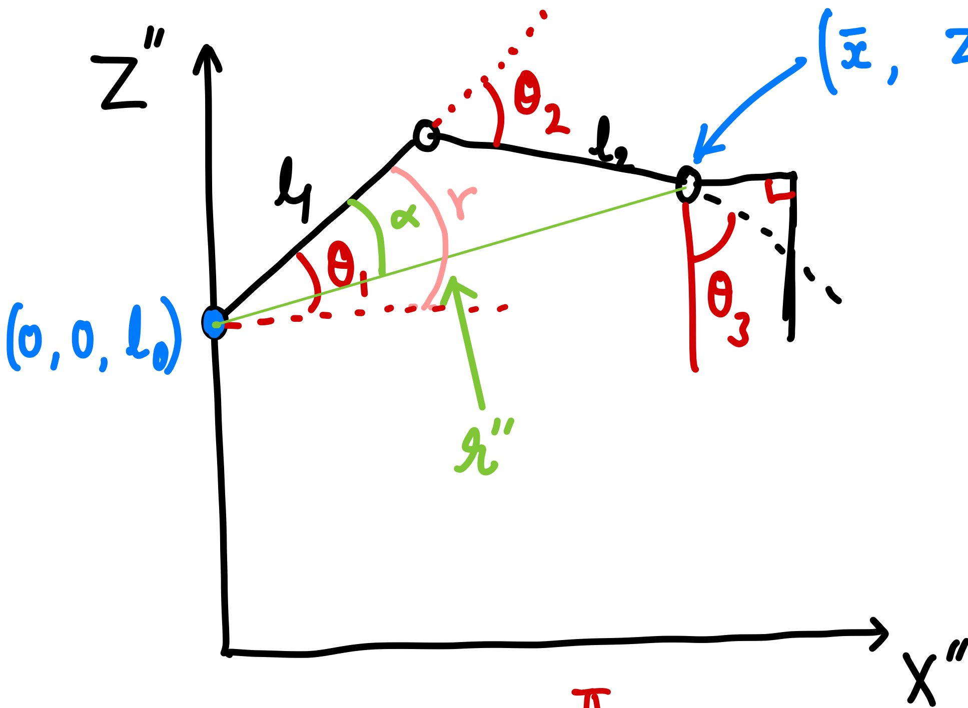
x in the frame $\{x''-y''\}$

$$x'' = \sqrt{(x - d_2\cos\theta_0)^2 + (y - d_2\sin\theta_0)^2}$$

$\bar{x}$ in the frame $\{x''-y''\}$

$$\bar{x} = \sqrt{(x - d_2\cos\theta_0 + d_1\sin\theta_0)^2 + (y - d_2\sin\theta_0 - d_1\cos\theta_0)^2}$$

Now, looking side-View in the $\{x''-y''\}$



$$\theta_1 + \theta_2 + \theta_3 = \frac{\pi}{2}$$

$$r'' = \sqrt{\bar{x}^2 + (z + l_3 - l_0)^2}$$

$$r'' = \sqrt{(x - d_2\cos\theta_0 + d_1\sin\theta_0)^2 + (y - d_2\sin\theta_0 - d_1\cos\theta_0)^2 + (z + l_3 - l_0)^2}$$

Using Cosine rule in the triangle,

$$\cos(\pi - \theta_2) = \frac{l_1^2 + l_2^2 - r''^2}{2 l_1 l_2}$$

$$\cos\alpha = \frac{l_1^2 + r''^2 - l_2^2}{2 l_1 r''}$$

$\theta_1 = \gamma + \alpha$ or $\theta_1 = \gamma - \alpha$
depending on the sign of θ_2

Once, we have θ_1 & θ_2 from this,

$$\theta_3 = \frac{\pi}{2} - \theta_1 - \theta_2$$

PART 3. Jacobian Matrix

In order to get Jacobian Matrix, series of rotational matrices are multiplied as done in PART 1 (kinematics).

Following is the matlab code snippet for multiplying the matrices using symbolic toolbox of MATLAB.

```
function mat = Jacobian()

    syms theta0 theta1 theta2 theta3 l0 l1 d1 l2 d2 l3

    R0 = [cos(theta0) -sin(theta0) 0 0; sin(theta0) cos(theta0) 0 0; 0 0 1 0; 0 0 0 1];

    R1 = [1 0 0 0; 0 1 0 0; 0 0 1 l0; 0 0 0 1];

    R2 = [cos(theta1) 0 sin(theta1) 0; 0 1 0 0; -sin(theta1) 0 cos(theta1) 0; 0 0 0 1];

    R3 = [1 0 0 l1; 0 1 0 0; 0 0 1 0; 0 0 0 1];

    R4 = [cos(theta2) 0 sin(theta2) 0; 0 1 0 0; -sin(theta2) 0 cos(theta2) 0; 0 0 0 1];

    R5 = [1 0 0 0; 0 1 0 d1; 0 0 1 0; 0 0 0 1];

    R6 = [1 0 0 l2; 0 1 0 0; 0 0 1 0; 0 0 0 1];

    R7 = [cos(theta3) 0 sin(theta3) 0; 0 1 0 0; -sin(theta3) 0 cos(theta3) 0; 0 0 0 1];

    R8 = [1 0 0 0; 0 1 0 0; 0 1 0 d2; 0 0 0 1];

    R8 = [1 0 0 l3; 0 1 0 0; 0 1 0 0; 0 0 0 1];

    R = R0*R1*R2*R3*R4*R5*R6*R7*R8;

    mat = R(1:end-1,end);

end
```

We are only concerned with the position in the matrix i.e. last column.

The Output from the above code is

$X \rightarrow$ l1*cos(theta0)*cos(theta1) - l3*(cos(theta3)*(cos(theta0)*sin(theta1)*sin(theta2) - cos(theta0)*cos(theta1)*cos(theta2)) + sin(theta3)*(cos(theta0)*cos(theta1)*sin(theta2) + cos(theta0)*cos(theta2)*sin(theta1))) - d1*sin(theta0) - l2*(cos(theta0)*sin(theta1)*sin(theta2) - cos(theta0)*cos(theta1)*cos(theta2))

$Y \rightarrow$ d1*cos(theta0) - l3*(cos(theta3)*(sin(theta0)*sin(theta1)*sin(theta2) - cos(theta1)*cos(theta2)*sin(theta0)) + sin(theta3)*(cos(theta1)*sin(theta0)*sin(theta2) + cos(theta2)*sin(theta0)*sin(theta1))) - l2*(sin(theta0)*sin(theta1)*sin(theta2) - cos(theta1)*cos(theta2)*sin(theta0)) + l1*cos(theta1)*sin(theta0)

$Z \rightarrow$ l0 - l2*(cos(theta1)*sin(theta2) + cos(theta2)*sin(theta1)) - l1*sin(theta1) - l3*(cos(theta3)*(cos(theta1)*sin(theta2) + cos(theta2)*sin(theta1)) + sin(theta3)*(cos(theta1)*cos(theta2) - sin(theta1)*sin(theta2)))

$$\begin{aligned}
\frac{\partial x}{\partial \theta_0} = {} & l_2\left[\sin\theta_0 \sin\theta_1 \sin\theta_2 - \cos\theta_1 \cos\theta_2 \sin\theta_0\right] \\
& - d_2\Big[\cos\theta_3\left(\cos\theta_1 \sin\theta_0 \sin\theta_2 + \cos\theta_2 \sin\theta_0 \sin\theta_1\right) \\
& \qquad - \sin\theta_3\left(\sin\theta_0 \sin\theta_1 \sin\theta_2 - \cos\theta_1 \cos\theta_2 \sin\theta_0\right)\Big] \\
& + l_3\Big[\cos\theta_3\left(\sin\theta_0 \sin\theta_1 \sin\theta_2 - \cos\theta_1 \cos\theta_2 \sin\theta_0\right) \\
& \qquad + \sin\theta_3\left(\cos\theta_1 \sin\theta_0 \sin\theta_2 + \cos\theta_2 \sin\theta_0 \sin\theta_1\right)\Big] \\
& - d_1 \cos\theta_0 - l_1 \cos\theta_1 \sin\theta_0
\end{aligned}$$

$$
\begin{aligned}
\frac{\partial x}{\partial \theta_0} = & -l_2 \sin\theta_0 \cos(\theta_1+\theta_2) \\
& - d_2 \cos\theta_3 \sin\theta_0 \sin(\theta_1+\theta_2) \\
& - d_2 \sin\theta_3 \sin\theta_0 \cos(\theta_1+\theta_2) \\
& - l_3 \cos\theta_3 \sin\theta_0 \cos(\theta_1+\theta_2) \\
& + l_3 \sin\theta_3 \sin\theta_0 \sin(\theta_1+\theta_2) \\
& - d_1 \cos\theta_0 - l_1 \cos\theta_1 \sin\theta_0
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial x}{\partial \theta_1} = & -l_2 \left(\cos\theta_0 \cos\theta_1 \sin\theta_2 + \cos\theta_0 \cos\theta_2 \sin\theta_1 \right) \\
& - d_2 \Big[\cos\theta_3 \left(\cos\theta_0 \sin\theta_1 \sin\theta_2 - \cos\theta_0 \cos\theta_1 \cos\theta_2 \right) \\
& \qquad + \sin\theta_3 \left(\cos\theta_0 \cos\theta_1 \sin\theta_2 + \cos\theta_0 \cos\theta_2 \sin\theta_1 \right) \Big] \\
& - l_3 \Big[\cos\theta_3 \left(\cos\theta_0 \cos\theta_1 \sin\theta_2 + \cos\theta_0 \cos\theta_2 \sin\theta_1 \right) \\
& \qquad - \sin\theta_3 \left(\cos\theta_0 \sin\theta_1 \sin\theta_2 - \cos\theta_0 \cos\theta_1 \cos\theta_2 \right) \Big] \\
& - l_1 \cos\theta_0 \sin\theta_1
\end{aligned}
$$

$$\frac{\partial x}{\partial \theta_1} = -l_2 \cos\theta_0 \sin(\theta_1+\theta_2)$$
$$+ d_2 \cos\theta_3 \cos\theta_0 \cos(\theta_1+\theta_2)$$
$$- d_2 \sin\theta_3 \cos\theta_0 \sin(\theta_1+\theta_2)$$
$$- l_3 \cos\theta_3 \cos\theta_0 \sin(\theta_1+\theta_2)$$
$$- l_3 \sin\theta_3 \cos\theta_0 \cos(\theta_1+\theta_2)$$
$$- l_1 \cos\theta_0 \sin\theta_1$$

$$\frac{\partial x}{\partial \theta_2} = -l_2 \left(\cos\theta_0 \cos\theta_1 \sin\theta_2 + \cos\theta_0 \cos\theta_2 \sin\theta_1\right)$$
$$- d_2 \left[\cos\theta_3 \left(\cos\theta_0 \sin\theta_1 \sin\theta_2 - \cos\theta_0 \cos\theta_1 \cos\theta_2\right) + \sin\theta_3 \left(\cos\theta_0 \cos\theta_1 \sin\theta_2 + \cos\theta_0 \cos\theta_2 \sin\theta_1\right)\right]$$
$$- l_3 \left[\cos\theta_3 \left(\cos\theta_0 \cos\theta_1 \sin\theta_2 + \cos\theta_0 \cos\theta_2 \sin\theta_1\right) - \sin\theta_3 \left(\cos\theta_0 \sin\theta_1 \sin\theta_2 - \cos\theta_0 \cos\theta_1 \cos\theta_2\right)\right]$$

$$\frac{\partial x}{\partial \theta_2} = -l_2 \cos\theta_0 \sin(\theta_1+\theta_2)$$

$$+ d_2 \cos\theta_3 \cos\theta_0 \cos(\theta_1+\theta_2)$$

$$- d_2 \sin\theta_3 \cos\theta_0 \sin(\theta_1+\theta_2)$$

$$- l_3 \cos\theta_3 \cos\theta_0 \sin(\theta_1+\theta_2)$$

$$- l_3 \sin\theta_3 \cos\theta_0 \cos(\theta_1+\theta_2)$$

$$\frac{\partial x}{\partial \theta_3} = -d_2 \Big[\cos\theta_3 \left(\cos\theta_0 \sin\theta_1 \sin\theta_2 - \cos\theta_0 \cos\theta_1 \cos\theta_2 \right)$$

$$+ \sin\theta_3 \left(\cos\theta_0 \cos\theta_1 \sin\theta_2 + \cos\theta_0 \cos\theta_2 \sin\theta_1 \right) \Big]$$

$$- l_3 \Big[\cos\theta_3 \left(\cos\theta_0 \cos\theta_1 \sin\theta_2 + \cos\theta_0 \cos\theta_2 \sin\theta_1 \right)$$

$$- \sin\theta_3 \left(\cos\theta_0 \sin\theta_1 \sin\theta_2 - \cos\theta_0 \cos\theta_1 \cos\theta_2 \right) \Big]$$

$$\frac{\partial x}{\partial \theta_3} = d_2 \cos\theta_3 \cos\theta_0 \cos(\theta_1+\theta_2)$$

$$- d_2 \sin\theta_3 \cos\theta_0 \sin(\theta_1+\theta_2)$$

$$- l_3 \cos\theta_3 \cos\theta_0 \sin(\theta_1+\theta_2)$$

$$- l_3 \sin\theta_3 \cos\theta_0 \cos(\theta_1+\theta_2)$$

$$\frac{\partial y}{\partial \theta_0} = d_2 \cos\theta_3 \cos\theta_0 \sin(\theta_1+\theta_2)$$

$$+ d_2 \sin\theta_3 \cos\theta_0 \cos(\theta_1+\theta_2)$$

$$+ l_2 \cos\theta_0 \cos(\theta_1+\theta_2)$$

$$+ l_3 \cos\theta_3 \cos\theta_0 \cos(\theta_1+\theta_2)$$

$$- l_3 \sin\theta_3 \cos\theta_0 \sin(\theta_1+\theta_2)$$

$$- d_1 \sin\theta_0 + l_1 \cos\theta_0 \cos\theta_1$$

$$\frac{\partial Y}{\partial \theta_1} = -l_2 \sin\theta_0 \sin(\theta_1+\theta_2)$$

$$+ d_2 \cos\theta_3 \sin\theta_0 \cos(\theta_1+\theta_2)$$

$$- d_2 \sin\theta_3 \sin\theta_0 \sin(\theta_1+\theta_2)$$

$$- l_3 \cos\theta_3 \sin\theta_0 \sin(\theta_1+\theta_2)$$

$$- l_3 \sin\theta_3 \sin\theta_0 \cos(\theta_1+\theta_2)$$

$$- l_1 \sin\theta_0 \sin\theta_1$$

$$\frac{\partial Y}{\partial \theta_2} = -l_2 \sin\theta_0 \sin(\theta_1+\theta_2)$$

$$+ d_2 \cos\theta_3 \sin\theta_0 \cos(\theta_1+\theta_2)$$

$$- d_2 \sin\theta_3 \sin\theta_0 \sin(\theta_1+\theta_2)$$

$$- l_3 \cos\theta_3 \sin\theta_0 \sin(\theta_1+\theta_2)$$

$$- l_3 \sin\theta_3 \sin\theta_0 \cos(\theta_1+\theta_2)$$

$$\frac{\partial y}{\partial \theta_3} = d_2 \cos\theta_3 \sin\theta_0 \cos(\theta_1+\theta_2)$$
$$- d_2 \sin\theta_3 \sin\theta_0 \sin(\theta_1+\theta_2)$$
$$- l_3 \cos\theta_3 \sin\theta_0 \sin(\theta_1+\theta_2)$$
$$- l_3 \sin\theta_3 \sin\theta_0 \cos(\theta_1+\theta_2)$$

$$\frac{\partial z}{\partial \theta_0} = 0$$

$$\frac{\partial z}{\partial \theta_1} = -l_2 \cos(\theta_1+\theta_2) - l_1 \cos\theta_1$$
$$- d_2 \cos\theta_3 \sin(\theta_1+\theta_2)$$
$$- d_2 \sin\theta_3 \cos(\theta_1+\theta_2)$$
$$- l_3 \cos\theta_3 \cos(\theta_1+\theta_2)$$
$$- l_3 \sin\theta_3 \sin(\theta_1+\theta_2)$$

$$\frac{\partial z}{\partial \theta_2} = -l_2 \cos(\theta_1+\theta_2) - d_2 \cos\theta_3 \sin(\theta_1+\theta_2)$$
$$- d_2 \sin\theta_3 \cos(\theta_1+\theta_2)$$
$$- l_3 \cos\theta_3 \cos(\theta_1+\theta_2)$$
$$+ l_3 \sin\theta_3 \sin(\theta_1+\theta_2)$$

$$\frac{\partial z}{\partial \theta_3} = -d_2 \cos\theta_3 \sin(\theta_1+\theta_2)$$
$$- d_2 \sin\theta_3 \cos(\theta_1+\theta_2)$$
$$- l_3 \cos\theta_3 \cos(\theta_1+\theta_2)$$
$$+ l_3 \sin\theta_3 \sin(\theta_1+\theta_2)$$

Hence Jacobian matrix is

$$J(\theta_0, \theta_1, \theta_2, \theta_3) = \begin{bmatrix} \frac{\partial x}{\partial \theta_0} & \frac{\partial x}{\partial \theta_1} & \frac{\partial x}{\partial \theta_2} & \frac{\partial x}{\partial \theta_3} \\ \frac{\partial y}{\partial \theta_0} & \frac{\partial y}{\partial \theta_1} & \frac{\partial y}{\partial \theta_2} & \frac{\partial y}{\partial \theta_3} \\ \frac{\partial z}{\partial \theta_0} & \frac{\partial z}{\partial \theta_1} & \frac{\partial z}{\partial \theta_2} & \frac{\partial z}{\partial \theta_3} \end{bmatrix}_{3\times 4}$$