

## Question 3

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(Content from 3.2.4)

### Question

Find the joint inclusion probability  $\pi_{uv}$  for a stratified random sampling. You need to find the  $\pi_{uv}$  for both cases:

- When  $u$  and  $v$  are in the same strata
- When  $u$  and  $v$  are in different stratas

[2 points]

### Solution:

Consider  $u, v$  to be in the same strata  $i$  (i.e  $u, v \in P_i$ ), the units will both be included in the sample  $S$  only if both are selected when  $P_i$  is sampled. Now strata  $i$  has  $N_i$  units and  $n_i$  units being selected at random and without replacement, we can find  $\pi_{uv}$  for  $u, v$  in the same sample as:

$$\pi_{uv} = P(u \in S_i, v \in S_i) = \frac{\binom{N_i-2}{n_i-2}}{\binom{N_i}{n_i}} = \frac{n_i(n_i-1)}{N_i(N_i-1)}$$

[1 point]

Now, consider  $u$  in strata  $i$  and  $v$  in strata  $j$ . In this case  $u, v$  will be included in the sample  $S$  only if  $u$  and  $v$  are selected when  $P_i$  and  $P_j$  are sampled respectively. Each strata is independent of the other and hence the joint inclusion probability in this case would be:

$$\pi_{uv} = P(u \in S_i, v \in S_j) = P(u \in S_i)P(v \in S_j) = \frac{n_i n_j}{N_i N_j}$$

[1 point]