Q9

(a) With a non-standard estimator it may be too difficult to derive an analytical expression for an estimate of the standard error. Or in some cases, it may not even be worth the effort.

Bootstrap estimate helps in this case. The bootstrap estimate of the standard error can be given by:

$$\hat{SD}_{*}\left[\hat{a}(S^*)\right] = \sqrt{\frac{\sum_{b=1}^{B}\left(a(S_b^*) - a^*\right)^2}{B}}$$

As you can see the only input it requires is the Bootstrap distribution which in turn only requires a sample of the population.

(b) The Bootstrap percentile method for constructing confidence intervals is the following:

- For a given sample S generate B bootstrap samples $S_1^*, \ldots, S_B^*$ by sampling with replacement from the sample S
- For the b^{th} bootstrap sample $(b = 1, \ldots, B)$ calculate $a(S_b^*)$
- From the values $a_1, \ldots, a_B$, find

$$a_{lower} = Q_a(P/2) \ \& \ a_{upper} = Q_a(1 - P/2)$$

- Then the $100(1-p)\%$ confidence interval is $[a_{lower}, a_{upper}]$

(c)

Suppose the regression model is

$$Y_i = \alpha + \beta(x_i - \bar{x}) + R_i$$

Where $R_i \sim F$, i.e. the errors come from some unknown density F. How might we estimate F?

- If we had α & β then $r_i = y_i - [\alpha + \beta(x_i - \bar{x})]$
$i = 1, \dots, n$

and $\{r_1, \dots, r_n\}$ would be a sample from F.

- We fit the model to find $\hat{\alpha}, \hat{\beta}$ and then obtain the residuals

$$\hat{r}_i = y_i - \hat{y}_i = y_i - [\hat{\alpha} + \hat{\beta}(x_i - \bar{x})]$$

- The sample of residuals or estimates of the errors is $\hat{R} = \{\hat{r}_1, \hat{r}_2, \dots, \hat{r}_n\}$

- We can then use the sample of residuals to estimate F using the empirical cdf

$$\hat{F}(t) = \frac{1}{n} \sum_{i=1}^{n} I(\hat{r}_i \leq t)$$

- We then perform the bootstrap using $\hat{F}$
- We generate a bootstrap sample of errors R_i^* by resampling with replacement from $\hat{R}$, then

$$y_i^* = \hat{\alpha} + \hat{\beta}(x_i - \bar{x}) + R_i^*$$

and then the bootstrap sample is

$$S_b^* = \{(x_1, y_1^*), (x_2, y_2^*), \dots, (x_n, y_n^*)\}$$