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1999 Europhys. Lett. 47 298

(http://iopscience.iop.org/0295-5075/47/3/298)

EUROPHYSICS LETTERS 1 August 1999
Europhys. Lett., **47** (3), pp. 298-303 (1999)

Noise-induced spiral dynamics in excitable media

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(received 11 February 1999; accepted in final form 2 June 1999)

PACS. 05.40−a – Fluctuation phenomena, random processes, noise, and Brownian motion.
PACS. 47.54+r – Pattern selection; pattern formation.
PACS. 87.18Hf – Spatiotemporal pattern formation in cellular populations.

Abstract. – We report the existence of complex spiral dynamics induced by external noise in a simple two-dimensional model of excitable media, with local dynamics of the FitzHugh-Nagumo type. Different dynamical regimes can be observed, depending on the value of the activator diffusion, including complex states resembling deterministic spiral turbulence, and spiral meandering. The complex dynamical behavior is seen to be driven by spiral breakup. The complexity of these noise-induced states can be characterized by the number and size distribution of coherent space-time clusters. In particular, the turbulent-like regime is characterized by a cluster-size distribution that displays power law scaling.

Many recent studies reveal the relevant influence of external noise on the spatiotemporal behaviour of excitable media [1-6]. Most of these investigations concentrate on the subexcitable regime, where by definition no structures can propagate under purely deterministic conditions. In that situation, an optimal amount of noise has been seen to support the propagation of structures such as spirals [1], irregular waves [2], travelling pulses [4] and pulsating spots [6]. The present work, on the other hand, directs its attention towards noise effects within the excitable region. We will show that, in this regime, parametric noise is able to induce complex spiral dynamics in a simple model of excitable media in which only stable (static or meandering) spirals exist in the absence of fluctuations [7]. Specifically, noise will be seen to induce turbulent-like states driven by spiral breakup.

Spiral breakup is a type of spatial instability leading to defect-mediated turbulence that has been recently observed in experiments with chemical excitable media [8]. Besides interest on the turbulent state by itself as a simple representation of spatiotemporal chaos [9, 10], the phenomenon of spiral breakup has attracted much attention due to its possible relation to the occurrence of ventricular fibrillation in human hearts [11]. Many questions remain open concerning the nature of the instability, such as for instance what is the minimal model in which it appears. Spiral breakup in homogeneous media has been reported so far in coupled-map lattices [12, 13], sophisticated models of cardiac tissue [11, 14], and generalizations of the standard FitzHugh-Nagumo model including delayed inhibitor production [15], modified inhibitor dynamics [16] and advective effects [17]. Here we consider a very simple model of excitable media, consisting of an array of excitable elements with a standard activator-inhibitor, FitzHugh-Nagumo–like dynamics, spatially coupled through diffusion of the activator variable. As far as we know, such a model has not been reported to exhibit spiral breakup, nor therefore spatiotemporal chaos. We will now show that a spatiotemporal parametric noise is able to induce breakup and lead to a complex dynamical state resembling the spiral turbulence exhibited by more complicated *deterministic* models, and characterized by a power law distribution of space-time cluster sizes.

The model that will be studied in what follows is defined by

$$\begin{aligned}
\frac{\mathrm{d}u_i}{\mathrm{d}t} &= \frac{1}{\varepsilon} u_i(1-u_i)\left(u_i - \frac{v_i+b}{a}\right) + D\sum_{j\in N(i)} c_{ij}u_j\,,\\
\frac{\mathrm{d}v_i}{\mathrm{d}t} &= \left[\gamma + \eta_i(t)\right] u_i - v_i\,,
\end{aligned} \tag{1}$$

where the activator and recovery variables, $u_i(t)$ and $v_i(t)$, respectively, are defined on a square two-dimensional lattice of $N \times N$ cells (only one index is used to label all sites: $i = 1, \ldots, N \times N$). The local dynamics of each element is of a standard excitable type, governed by a Z-shaped u-nullcline and a linear v-nullcline on the (v, u) phase plane [18], in such a way that in the rest state both u and v are zero, the excited state is characterized by $u \approx 1$, and the refractory state by $u \approx 0$. Activator diffusion is described by the sum term in eq. (1), which extends to the first three sets of neighbours of cell i. The weight coefficients c_{ij} correspond to the second-order discretization of the Laplacian operator [19]. The spatially discrete, temporally continuous character of this model makes it suitable for the description of excitable biological tissue, such as cell tissue in the cardiac muscle [12].

The parameters of the local dynamics are considered to be the same for all elements, except for the inhibitor-production coefficient, which is assumed to fluctuate in time and space around a mean value γ. These fluctuations are represented by a Gaussian noise, white in space and time with σ^2 being the noise intensity. The effect of the noise is to change locally the slope of the v-nullcline, and it disappears if the system reaches its rest state. Therefore, no new excitation can nucleate from the rest state. We have also investigated spatially correlated noise. If the correlation length exceeds the diffusion length of the activator u, the phenomena described later on disappear.

Model (1) is numerically integrated using a semi-implicit algorithm for the activator equation [20] and an explicit algorithm for the deterministic part of the inhibitor equation. The stochastic term in that equation is dealt with by means of a suitable extension of the explicit Euler method [21]. The following parameters are chosen in what follows: $N = 128$, $\varepsilon = 0.02$, $a = 0.85$, $b = 0.1$ and $\gamma = 1$. The strength D of the spatial coupling and the standard deviation σ of the fluctuations will be used as control parameters of the system. No-flux (Neumann) boundary conditions are considered, but results similar to those presented below have been obtained with periodic boundary conditions.

The behaviour of the system in the absence of noise is the following: for small coupling D, local initial perturbations are not able to propagate through the medium, and decay after a single excursion through the excited state. Beyond a certain threshold coupling ($D_{\mathrm{th}} \approx 0.64$ for the parameters chosen here), propagation of local perturbations is possible. In this situation, suitable initial conditions lead to stable spiral waves. However, for general (*e.g.*, random) initial conditions the system usually decays rapidly (typically in less than $t = 10$ time units) towards the stable (homogeneous) steady state of the deterministic dynamics ($u_i = v_i = 0,\ \forall i$), after all propagating structures have been annihilated by collisions between one another or

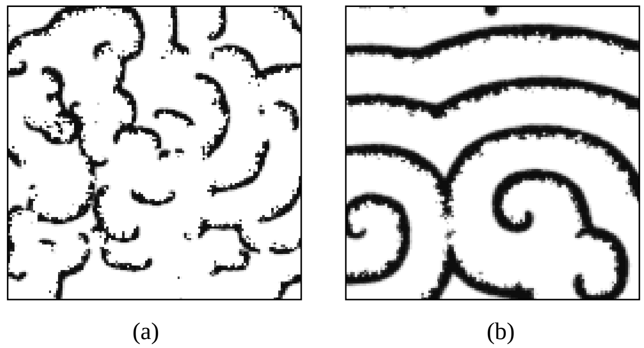

Fig. 1. – Spatial distribution of the activator at $t = 100$ time units in the presence of noise: (a) complex spiral dynamics for $D = 0.80$ and (b) spiral meandering for $D = 2.08$. Noise strength is given by $\sigma = 0.28$ in the two cases.

disappeared through the boundaries of the system. Therefore, the deterministic model either displays stable spiral waves or decays to the quiescent state after a short transient, depending on the initial conditions considered.

The situation is radically different in the presence of parametric noise. Two snapshots of the spatial distribution of the activator variable $u_i(t)$ taken at $t = 100$ time units are given in fig. 1, for two different values of the spatial coupling $D > D_{\mathrm{th}}$. In both cases the system evolves from random initial conditions. Close to the propagation threshold (fig. 1a) the system exhibits a long-lived complex state which closely resembles the spiral-chaos regime found in deterministic systems with more complicated local dynamics [10]. This state has been observed to survive for at least $t = 3000$ time units. Further above threshold (fig. 1b), the complexity of the dynamical state is reduced, consisting only of a small number of spirals, either single or coupled in pairs (as in the figure), whose core undergoes a slow meandering (drifting of spirals due to noise has already been observed and characterised in the complex Ginzburg-Landau

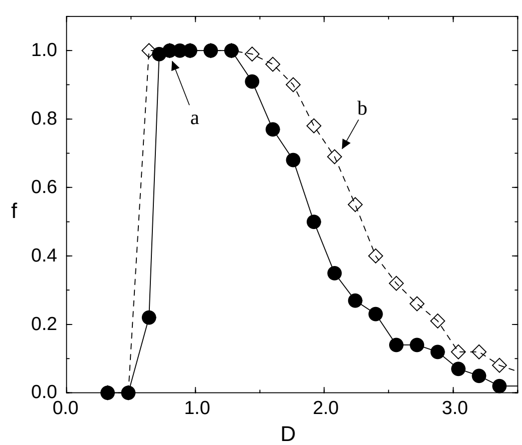

Fig. 2. – Fraction f of realizations (out of 100) that do not decay to the rest state after a reference time $t_{\mathrm{ref}} = 100$, for increasing values of the diffusion coefficient D. Two noise strengths are considered: $\sigma = 0.28$ (empty diamonds) and $\sigma = 0.40$ (solid circles). Labels a and b correspond to the non-decaying situations of fig. 1.

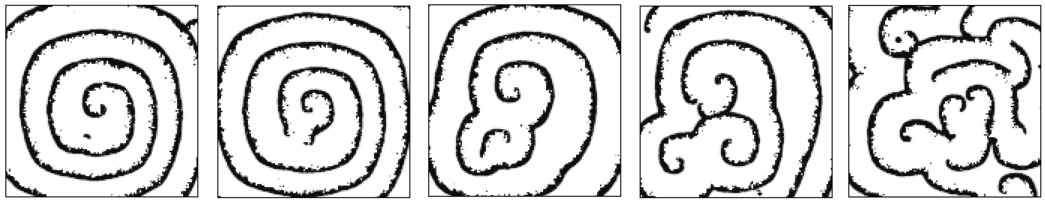

Fig. 3. – Time sequence showing spiral breakup leading to complex spiral dynamics. Time flows from left to right. Parameters are $\sigma = 0.28$ and $D = 1.12$.

equation [22]). Due to this drifting, this second regime eventually decays to the homogeneous rest state, soon after all spiral cores have disappeared through the system boundaries.

In order to characterise the complex dynamics described above, we perform a systematic analysis of the evolution of the model from random initial conditions, for different values of the diffusion coefficient D. We compute the number of evolutions, out of an ensemble of 100 different random initial conditions (with different realizations of the external noise), for which the system does *not* decay to the quiescent state up to a certain reference time ($t_{\rm ref} = 100$ time units here). The fraction f of non-decaying realizations (normalized to the total number of runs) *vs.* the diffusion coefficient D is plotted in fig. 2 for two different noise intensities. A plateau at $f = 1$ starting at $D = D_{\rm th}$ can be observed, indicating that all initial conditions tested remained in a dynamical state up to $t_{\rm ref}$. The plateau is followed by a smooth decay for increasing D. This decay is due to the meandering effect discussed in the previous paragraph, and indicates the increasing regularity of the dynamics as D becomes larger, as already shown in fig. 1 (whose two cases have been indicated in fig. 2 by their corresponding label).

The turbulent-like dynamics exhibited by the system at small values of the coupling strength (fig. 1a) can also be obtained from other initial states. In particular, when a deterministically stable spiral is subjected to the external fluctuations described above, it undergoes a process of breakup that leads again to a complex spiral dynamics, similarly to what happens in deterministic models [15]. A temporal sequence of this process is shown in fig. 3. Breakup occurs via a backfiring event (leftmost plot), which produces a pulse that travels towards the spiral centre, colliding with an inner arm and breaking it (second plot from left). The two resulting open ends become connected again, firing back new pulses that break the spiral again. The process is repeated until finally a turbulent-like state is generated (last plot of the figure).

In order to characterize the complexity of the structures that have been reported, we analyze the spatiotemporal evolution of the system in terms of coherent space-time clusters [2]. This is done by considering the system evolution in a three-dimensional space formed by the two spatial dimensions and the temporal axis. The activator dynamics is subjected to a threshold procedure that labels the state of each cell as *excited* or *quiescent*, depending on whether or not $u_i(t)$ exceeds a given reference value (here we used $u_{\rm th} = 0.5$). The evolution of this binary field is considered at discrete time intervals (not too sparse to ensure a smooth dynamics and not too dense to let the system evolve noticeably). A cluster is defined as a connected set of excited sites in the three-dimensional cubic grid, where connections are checked between nearest neighbours and diagonal next-nearest neighbours. According to this definition, a perfect spiral wave in an infinite spatial domain would correspond to a single space-time cluster, whereas a complex spiral state such as the one shown in fig. 1a would exhibit a large number of clusters. This method allows a quantitative measure of the complexity of a spatially evolving structure: the larger the number of clusters, the larger the complexity of the spatiotemporal dynamics.

We now apply the method described above to the spiral dynamical states induced by noise

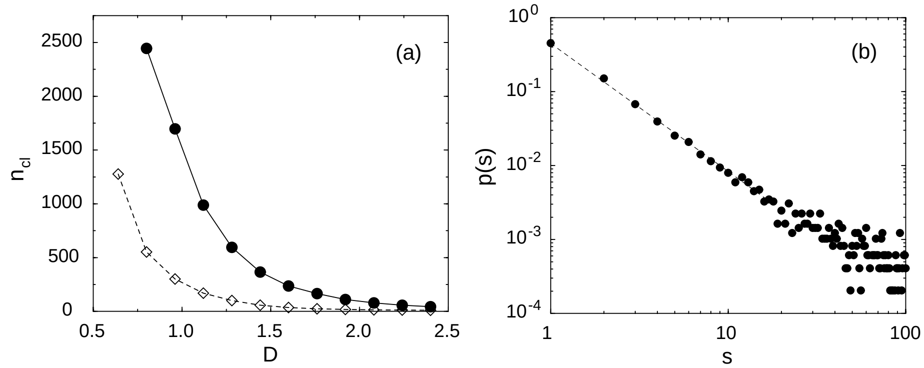

Fig. 4. – Statistics of coherent space-time clusters: (a) average number of clusters $n_{\rm cl}$ *vs.* coupling strength D for $\sigma = 0.28$ (empty diamonds) and $\sigma = 0.40$ (solid circles). (b) Cluster size distribution for $D = 0.10$ and $\sigma = 0.49$, the dashed line corresponds to a power law fitting with $\alpha = 1.73$. The dynamical evolution used to generate these was measured every 0.5 time units.

displayed by model (1). Figure 4a shows the number $n_{\rm cl}$ of coherent space-time clusters, averaged over all non-decaying realizations out of 100, for increasing values of the coupling and two fixed values of the noise strength. The figure shows that there is no sharp transition between the regimes of complex spiral dynamics (large number of clusters, fig. 1a) and spiral meandering (small number of clusters, fig. 1b), although $n_{\rm cl}$ decreases two orders of magnitude in a small range of D values. The number of clusters increases in any case with increasing noise level, as could be expected. This, however, does not imply an increase in the irregularity of the structures.

The distribution of coherent space-time cluster sizes has been used in the past to characterize the spatiotemporal behaviour of excitable media. It has been observed that such a distribution scales with a power law, in the form $p(s) \propto s^{-\alpha}$, where $p(s)$ is the distribution function and s the space-time size of the clusters. The exponent α has been identified to be approximately between 2 and 3 for noise-induced structures [2, 23]. Here we perform such an analysis for noise-induced complex dynamics (fig. 1a). The result is shown in fig. 4b, for parameters D and σ that ensure a large number of clusters (around 5000 in all cases examined) and thus good statistics. The measured distribution shown in fig. 4b obeys a power law (spanning 1.5 decades in s) with an exponent $\alpha \approx 1.75 \pm 0.05$, which is observed to depend neither on the coupling strength nor on the noise intensity, as long as these parameters ensure a turbulent-like state.

In conclusion, external spatiotemporal noise has been observed to induce spiral breakup and complex spiral dynamics in a simple spatially extended model of excitable media with local FitzHugh-Nagumo–like dynamics. This structured dynamical state resembles closely the spatiotemporal chaotic behavior observed in deterministic models with modified FitzHugh-Nagumo dynamics. This effect can be ultimately related to the parametric character of the noise, which makes the stochastic influence to depend on the local state of the system: noise only acts in those sites that are excited (*i.e.*, where the activator variable is non-zero). A non-parametric (additive) noise would arouse fluctuations both in the excited and in the non-excited regions, and hence it would not simply break the spiral up, but sweep it away and generate completely irregular, noisy dynamics. On the contrary, the noise-induced regimes reported here exhibit a certain degree of regularity, which can be quantitatively characterised by the number of space-time clusters. A large number of clusters corresponds to complex dynamics, and a small one to a few spirals that meander around the system. The number of clusters is seen to decrease with the strength of the activator diffusion and increase with noise intensity. A dynamical state with many clusters can be characterised by their size distribution, which is observed to scale with a power law, in agreement with recent results obtained for thermal waves in subexcitable media.

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We thank M. BÄR for suggestions, and P. JUNG for fruitful and enjoyable discussions on the dynamics of excitable media. We acknowledge the Deutsche Forschungsgemeinschaft (Sfb555) for financial support. JGO also thanks the A. von Humboldt-Stiftung and the Dirección General de Investigación Científica y Técnica (Spain, project PB96-0241).

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