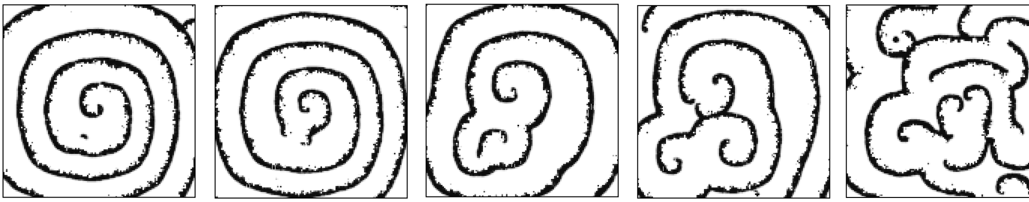

Fig. 3. – Time sequence showing spiral breakup leading to complex spiral dynamics. Time flows from left to right. Parameters are $\sigma = 0.28$ and $D = 1.12$.

equation [22]). Due to this drifting, this second regime eventually decays to the homogeneous rest state, soon after all spiral cores have disappeared through the system boundaries.

In order to characterise the complex dynamics described above, we perform a systematic analysis of the evolution of the model from random initial conditions, for different values of the diffusion coefficient D. We compute the number of evolutions, out of an ensemble of 100 different random initial conditions (with different realizations of the external noise), for which the system does *not* decay to the quiescent state up to a certain reference time ($t_{\rm ref} = 100$ time units here). The fraction f of non-decaying realizations (normalized to the total number of runs) *vs.* the diffusion coefficient D is plotted in fig. 2 for two different noise intensities. A plateau at $f = 1$ starting at $D = D_{\rm th}$ can be observed, indicating that all initial conditions tested remained in a dynamical state up to $t_{\rm ref}$. The plateau is followed by a smooth decay for increasing D. This decay is due to the meandering effect discussed in the previous paragraph, and indicates the increasing regularity of the dynamics as D becomes larger, as already shown in fig. 1 (whose two cases have been indicated in fig. 2 by their corresponding label).

The turbulent-like dynamics exhibited by the system at small values of the coupling strength (fig. 1a) can also be obtained from other initial states. In particular, when a deterministically stable spiral is subjected to the external fluctuations described above, it undergoes a process of breakup that leads again to a complex spiral dynamics, similarly to what happens in deterministic models [15]. A temporal sequence of this process is shown in fig. 3. Breakup occurs via a backfiring event (leftmost plot), which produces a pulse that travels towards the spiral centre, colliding with an inner arm and breaking it (second plot from left). The two resulting open ends become connected again, firing back new pulses that break the spiral again. The process is repeated until finally a turbulent-like state is generated (last plot of the figure).

In order to characterize the complexity of the structures that have been reported, we analyze the spatiotemporal evolution of the system in terms of coherent space-time clusters [2]. This is done by considering the system evolution in a three-dimensional space formed by the two spatial dimensions and the temporal axis. The activator dynamics is subjected to a threshold procedure that labels the state of each cell as *excited* or *quiescent*, depending on whether or not $u_i(t)$ exceeds a given reference value (here we used $u_{\rm th} = 0.5$). The evolution of this binary field is considered at discrete time intervals (not too sparse to ensure a smooth dynamics and not too dense to let the system evolve noticeably). A cluster is defined as a connected set of excited sites in the three-dimensional cubic grid, where connections are checked between nearest neighbours and diagonal next-nearest neighbours. According to this definition, a perfect spiral wave in an infinite spatial domain would correspond to a single space-time cluster, whereas a complex spiral state such as the one shown in fig. 1a would exhibit a large number of clusters. This method allows a quantitative measure of the complexity of a spatially evolving structure: the larger the number of clusters, the larger the complexity of the spatiotemporal dynamics.

We now apply the method described above to the spiral dynamical states induced by noise