Solution 1 of Homework1 Practice part IFT6135

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Coding Environment

python 3.7.1, numpy 1.15.2 matplotlib 2.2.0

Problem 1

In this problem, we will build a Multilayer Perceptron (MLP) and train it on the MNIST hand-written digit dataset.

Building the Model [35] Consider an MLP with two hidden layers with h^1 and h^2 hidden units. For the MNIST dataset, the number of features of the input data h^0 is 784. The output of the neural network is parameterized by a softmax of $h^3 = 10$ classes.

1. Build an MLP and choose the values of h1 and h2 such that the total number of parameters (including biases) falls within the range of [0.5M, 1.0M].

The total number of parameters including biases for the MLP are:

total number of parameters=(h1_hidden_units$*$features(784)+h1_hidden_units) + (h2_hidden_units $*$h1_hidden_units+h2_hidden_units) + (outputs(10)$*$h2_hidden_units+outputs(10)), it should be in range of [0.5M, 1.0M].

For example:

option1: h1_hidden_units = 512, h2_hidden_units = 256, total number of parameters = 535818

option2: h1_hidden_units = 600, h2_hidden_units = 600, total number of parameters = 837610

We could get the total number of parameters with below function:

In [31]:
```
def calculate_total_paramters(h1_n, h2_n):
    sum = (h1_n*784+h1_n) + ((h2_n*h1_n)+h2_n) + ((10*h2_n)+10)
    print('With h1 hidden units: {0}, h2 hidden units: {1}, total number of parame
ters: {2}'.format(h1_n, h2_n, sum))

calculate_total_paramters(512,256)
calculate_total_paramters(600,600)
calculate_total_paramters(710,610)
```

```
With h1 hidden units: 512, h2 hidden units: 256, total number of parameters: 5358
18
With h1 hidden units: 600, h2 hidden units: 600, total number of parameters: 8376
10
With h1 hidden units: 710, h2 hidden units: 610, total number of parameters: 9971
70
```

1. Implement the forward and backward propagation of the MLP in numpy without using any of the deep learning frameworks that provides automatic di erentiation. Use the class structure provided here.
2. Train the MLP using the probability loss (cross entropy) as training criterion. We minimize this criterion to optimize the model parameters using stochastic gradient descent.

In [32]:
```
import numpy as np
import pickle
import gzip
import matplotlib.pyplot as plt
import random
import time
from datetime import datetime
import os

import pdb
```

In [33]:
```
class MLP_NN(object):
    def initialize_weights(self, method):
        if method == 'zeros':
            self.w1 = np.zeros((self.h1, self.features))     # return h1 * feature
s matrix
            self.w2 = np.zeros((self.h2, self.h1))           # return h2 * h1 matr
ix
            self.w3 = np.zeros((self.outputs, self.h2))      # return outputs * h2
        elif method == 'normal':
            self.w1 = np.random.normal(0, 1, size=(self.h1, self.features))
            self.w2 = np.random.normal(0, 1, size=(self.h2, self.h1))
            self.w3 = np.random.normal(0, 1, size=(self.outputs, self.h2))
        elif method == 'glorot':
            d1 = np.sqrt(6/(self.features+self.h1))
            self.w1 = np.random.uniform(-d1, d1, size=(self.h1, self.features))
            d2 = np.sqrt(6/(self.h1+self.h2))
            self.w2 = np.random.uniform(-d2, d2, size=(self.h2, self.h1))
            d3 = np.sqrt(6/(self.h2+self.outputs))
            self.w3 = np.random.uniform(-d3, d3, size=(self.outputs, self.h2))
        return self.w1, self.w2, self.w3

    def initialize_biases(self):
        """
        Initialize biases at zeros.
        :return: biases for hidden layer and output layer
        """

        self.h1_b = np.zeros((self.h1, 1))
        self.h2_b = np.zeros((self.h2, 1))
        self.output_b = np.zeros((self.outputs, 1))

        return self.h1_b, self.h2_b, self.output_b

    def one_hot(self, y):
        z = []
        y = y.T
        for i in range(len(y)):
            z.append(np.eye(self.num_labels)[int(y[i])])
        z = np.matrix(z).T
        return z

    def import_data(self):
        f = gzip.open('./mnist.pkl.gz')
        # encoding='latin1' --> https://stackoverflow.com/a/41366785
        data = pickle.load(f, encoding='latin1')

        train_data = np.array(data[0][0], dtype=float)
        train_label = np.array(data[0][1], dtype=float)

        valid_data = np.array(data[1][0], dtype=float)
        valid_label = np.array(data[1][1], dtype=float)

        test_data = np.array(data[2][0], dtype=float)
        test_label = np.array(data[2][1], dtype=float)

        return train_data,train_label,valid_data,valid_label,test_data,test_label

    def activation(self, x, function):

        if function == 'sigmoid':
            z = 1/(1+np.exp(-x))

        elif function == 'relu':
            z = np.maximum(0, x)

        elif function == 'softmax':
            z = np.ones((x.shape[0], x.shape[1]))
            x_max = x.max(axis=0)
            a = x - x_max
            norm = np.sum(np.exp(a), axis=0)
            for i in range(x.shape[1]):
                z[:, i] = (np.exp(a[:, i]) / norm[i])
        return z

    def deactivation(self, A, function):
        if function == 'relu_backward':
            Z = np.int64(A > 0)
        elif function == 'sigmoid_backward':
            y = self.activation(A, 'sigmoid')
            Z = y * (1-y)
        return Z

    def forward(self, X, parameters):
        if X.shape[1] != self.features:
            raise ValueError('Inputs length does not match the input of the networ
k')

        self.w1 = parameters["W1"]
        self.b1 = parameters["b1"]
        self.w2 = parameters["W2"]
        self.b2 = parameters["b2"]
        self.w3 = parameters["W3"]
        self.b3 = parameters["b3"]

        self.a1 = np.dot(self.w1, X.T) + self.b1
        self.o1 = self.activation(self.a1, 'relu')
        self.a2 = np.dot(self.w2, self.o1) + self.b2
        self.o2 = self.activation(self.a2, 'relu')
        self.a3 = np.dot(self.w3, self.o2) + self.b3
        self.o3 = self.activation(self.a3, 'softmax')

        cache = (self.a1, self.o1, self.w1, self.b1, self.o2, self.a2, self.w2, se
lf.b2, self.a3, self.o3, self.w3, self.b3)

        return cache

    def loss(self, X, y, cache):
        # y is label, not onehot
        J = 0
        m = X.shape[0]
        a1, o1, w1, b1, o2, a2, w2, b2, a3, o3, w3, b3 = cache
        for i in range(m):
            loss = -np.log(o3[int(y[i])][i]+1e-10)
            J = J + loss
        J = J/m
        return J

    def back_propagation(self, X, y_onehot, cache):
        m = X.shape[0]
        a1, o1, w1, b1, o2, a2, w2, b2, a3, o3, w3, b3 = cache

        grad_a3 = o3 - y_onehot
        grad_w3 = np.dot(grad_a3, o2.T) / m
        grad_b3 = np.sum(grad_a3, axis=1) / m

        grad_o2 = np.dot(w3.T, grad_a3)
        d_a2 = self.deactivation(a2, 'relu_backward')
        grad_a2 = np.multiply(grad_o2, d_a2)
        grad_w2 = np.dot(grad_a2, o1.T) / m
        grad_b2 = np.sum(grad_a2, axis=1) / m


        grad_o1 = np.dot(w2.T, grad_a2)
        d_a1 = self.deactivation(a1, 'relu_backward')
        grad_a1 = np.multiply(grad_o1, d_a1)

        grad_w1 = np.dot(grad_a1, X) / m
        grad_b1 = np.sum(grad_a1, axis=1) / m

        gradients = {"dZ3": grad_a3, "dW3": grad_w3, "db3": grad_b3,
                     "dA2": grad_o2, "dZ2": grad_a2, "dW2": grad_w2, "db2": grad_b
2,
                     "dA1": grad_o1, "dZ1": grad_a1, "dW1": grad_w1, "db1": grad_b
1}
        return gradients

    def update(self, X, y_onehot, cache,learning_rate):
        gradients = self.back_propagation(X,y_onehot,cache)
        grad_w3, grad_w2, grad_w1, grad_b3, grad_b2, grad_b1 = gradients['dW3'], g
radients['dW2'], gradients['dW1'], gradients['db3'], gradients['db2'], gradients[
'db1']
        n = X.shape[0]

        self.w3 -= learning_rate * grad_w3
        self.w2 -= learning_rate * grad_w2
        self.w1 -= learning_rate * grad_w1

        self.b3 -= learning_rate * grad_b3
        self.b2 -= learning_rate * grad_b2
        self.b1 -= learning_rate * grad_b1
        return self.w3, self.w2, self.w1, self.b3, self.b2, self.b1

    def train(self, X, y_onehot, parameters, minibatches, lamb, p):
        n = minibatches
        set = X[p * n:p * n + n][:]
        #y1 = y[p * n:p * n + n][:]
        y1_onehot = y_onehot[:, p * n:p * n + n]

        batch_cache = self.forward(set, parameters)

        self.w3, self.w2, self.w1, self.b3, self.b2, self.b1 = self.update(set, y1
_onehot, batch_cache, lamb)
        update_parameters = {"W1": self.w1,
                             "b1": self.b1,
                             "W2": self.w2,
                             "b2": self.b2,
                             "W3": self.w3,
                             "b3": self.b3}
        return update_parameters

    def test(self, X, y, parameters):
        accuracy = 0
        cache = self.forward(X, parameters)
        a1, o1, w1, b1, o2, a2, w2, b2, a3, o3, w3, b3 = cache
        y1 = self.one_hot(y)
        k = np.argmax(o3, axis=0)
        idx = np.argmax(y1, axis=0).tolist()
        real_idx = np.array(idx[0])
        num = X.shape[0]
        for j in range(num):
            if k[j] == real_idx[j]:
                accuracy += 1
        accuracy /= num
        accuracy *= 100

        loss = self.loss(X, y, cache)
        print('The accuracy in validation set is: {0}% and loss: {1}'.format(accur
acy, loss))
        return accuracy, loss

    def __init__(self,
                 features=784,
                 h1=710,
                 h2=610,
                 outputs=10,
                 num_labels = 10,
                 epoch = 1000,
                 minibatches=8,
                 learning_rate = 0.001
                 ):
        self.features, self.h1, self.h2, self.outputs = features, h1, h2, outputs
        self.num_labels = num_labels
        self.epoch = epoch
        self.minibatches = minibatches
        self.learning_rate = learning_rate
```

In the following sub-questions, please specify the model architecture (number of hidden units per layer, and the total number of parameters), the nonlinearity chosen as neuron activation, learning rate, mini-batch size.

Initialization [10] In this sub-question, we consider di erent initial values for the weight parameters. Set the biases to be zeros, and consider the following settings for the weight parameters:

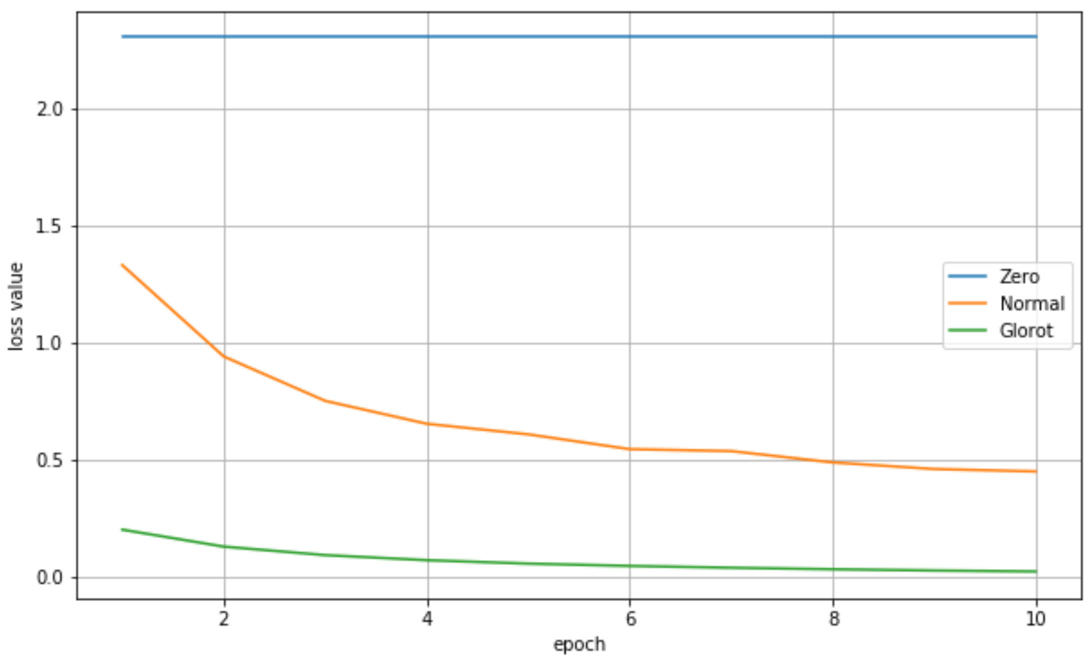
- **Zero**: all weight parameters are initialized to be zeros (like biases).
- **Normal**: sample the initial weight values from a standard Normal distribution; $w_{i,j} \sim N(w_{i,j}; 0, 1)$
- **Glorot**: sample the initial weight values from a uniform distribution; $w^l_{i,j} \sim U(w^l_{i,j}; -d^l, d^l)$ where $d^l = \sqrt{\frac{6}{h^{l-1}+h^l}}$

1. Train the model for 10 epochs using the initialization methods above and record the average loss measured on the training data at the end of each epoch (10 values for each setup).
2. Compare the three setups by plotting the losses against the training time (epoch) and comment on the result.

In [41]:
```
    def loss_with_initial(class_name, initial_method):
        '''
        :param class_name: P1_initial1
        :param initial_method: zeros, normal, golrot
        :return: loss list with 10 epochs
        '''
        w1, w2, w3 = class_name.initialize_weights(initial_method)
        b1, b2, b3 = class_name.initialize_biases()
        parameters = {"W1": w1,
                      "b1": b1,
                      "W2": w2,
                      "b2": b2,
                      "W3": w3,
                      "b3": b3}

        train_data, train_label, valid_data, valid_label, test_data, test_label = 
class_name.import_data()
        X = train_data
        y = train_label
        y_onehot = class_name.one_hot(y)
        m = X.shape[0]

        # setting the network structure
        h1 = 600
        h2 = 600
        minibatches = 8
        epoch = 10
        learning_rate = 0.01

        batch = int(m/minibatches)
        losslist = []

        print('NN structure: {0} * {1}, with epoch: {2}, minibatch: {3}, learning
 rate: {4}'.format(h1, h2, epoch, minibatches, learning_rate))
        print('active function in layer 1: relu')
        print('active function in layer 2: relu')

        for i in range(epoch+1):
            cache = class_name.forward(X, parameters)
            #print('Epoch: ', i, 'of ', epoch, ' training...')
            loss = class_name.loss(X, y, cache)
            losslist.append(loss)
            #print('The loss in training set is: {0}'.format(loss))

            for p in range(batch):
                parameters = class_name.train(X, y_onehot, parameters, minibatches
, learning_rate, p)
        print('Finished... Saved loss in list.')
        return losslist
```

In [42]:
```python
P1_initial1 = MLP_NN()
P1_initial2 = MLP_NN()
P1_initial3 = MLP_NN()

initial_zeros = []
initial_normal = []
initial_glorot = []

print('*** initial with zeros ***')
initial_zeros = loss_with_initial(P1_initial1, 'zeros')

print('*** Initial with normal ***')
initial_normal = loss_with_initial(P1_initial2, 'normal')

print('*** Initial with glorot ***')
initial_glorot = loss_with_initial(P1_initial3, 'glorot')

xaxis = np.linspace(1, 10, 10)
plt.figure(figsize=(10,6))
plt.xlabel("epoch")
plt.ylabel("loss value")
plt.plot(xaxis, initial_zeros[1:], '-')
plt.plot(xaxis, initial_normal[1:], '-')
plt.plot(xaxis, initial_glorot[1:], '-')
plt.grid()
plt.legend(('Zero', 'Normal', 'Glorot'))
plt.show()
```

```
*** initial with zeros ***
NN structure: 600 * 600, with epoch: 10, minibatch: 8, learning rate: 0.01
active function in layer 1: relu
active function in layer 2: relu
Finished... Saved loss in list.
*** Initial with normal ***
NN structure: 600 * 600, with epoch: 10, minibatch: 8, learning rate: 0.01
active function in layer 1: relu
active function in layer 2: relu
Finished... Saved loss in list.
*** Initial with glorot ***
NN structure: 600 * 600, with epoch: 10, minibatch: 8, learning rate: 0.01
active function in layer 1: relu
active function in layer 2: relu
Finished... Saved loss in list.
```



Hyperparameter Search From now on, use the Glorot initialization method.

1. Find out a combination of hyper-parameters (model architecture, learning rate, nonlinearity, etc.) such that the average accuracy rate on the validation set $(r^{(valid)})$ is at least 97%.
2. Report the hyper-parameters you tried and the corresponding $r^{(valid)}$

The hyper-parameters we tried:

h1	h2	h1_act	h2_act	batch_size	learning rate	epoch	train_accu	train_loss	valid_accu	valid_loss
512	256	sigmoid	relu	8	0.01	24	97.94%	0.0739	97.04%	0.1016
512	256	relu	relu	8	0.01	8	97.53%	0.0928	97.10%	0.1077
512	256	sigmoid	sigmoid	8	0.01	50	97.86%	0.0831	96.99%	0.1083
512	256	relu	relu	8	0.01	3	98.01%	0.0766	97.16%	0.0994
512	256	relu	relu	32	0.01	11	97.92%	0.0838	97.06%	0.1059
512	256	relu	relu	32	0.001	50	95.64%	0.1607	95.65%	0.1614
710	610	relu	relu	32	0.01	10	97.77%	0.0885	97.00%	0.1084
710	610	relu	relu	8	0.01	3	98.00%	0.0753	97.07%	0.0993
710	610	relu	relu	4	0.01	2	98.30%	0.0628	97.11%	0.0929

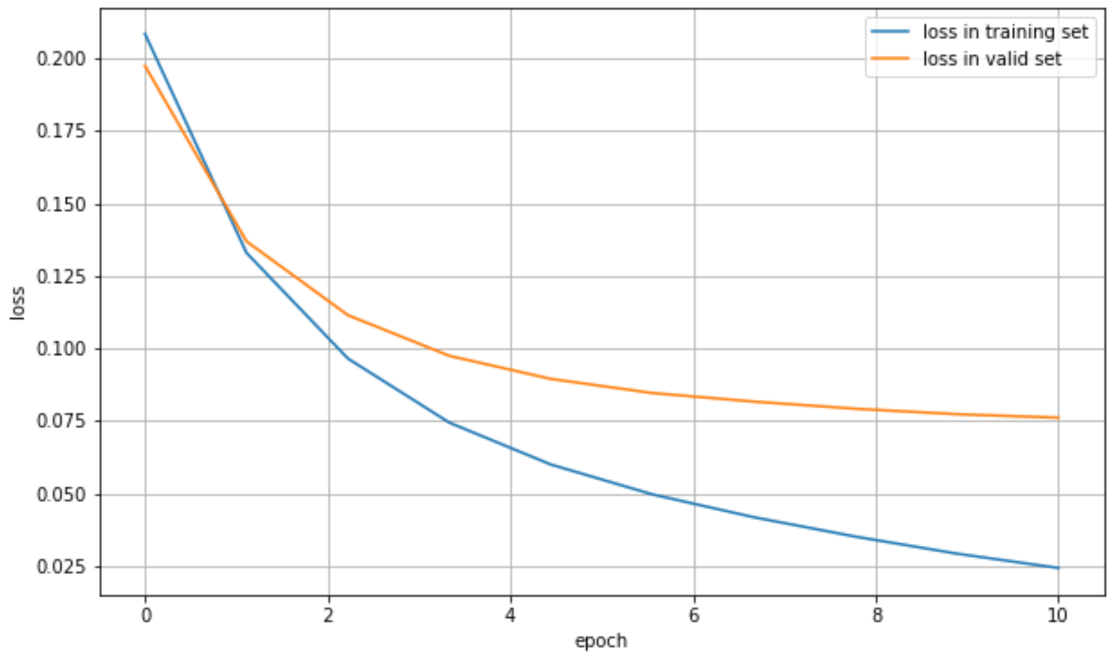
From these four sets of hyper-parameters, we choose:

- Relu as the activation function
- 0.01 as learning rate
- 8 or less than 8 as batch size
- 710 and 610 as hidden units for two layers

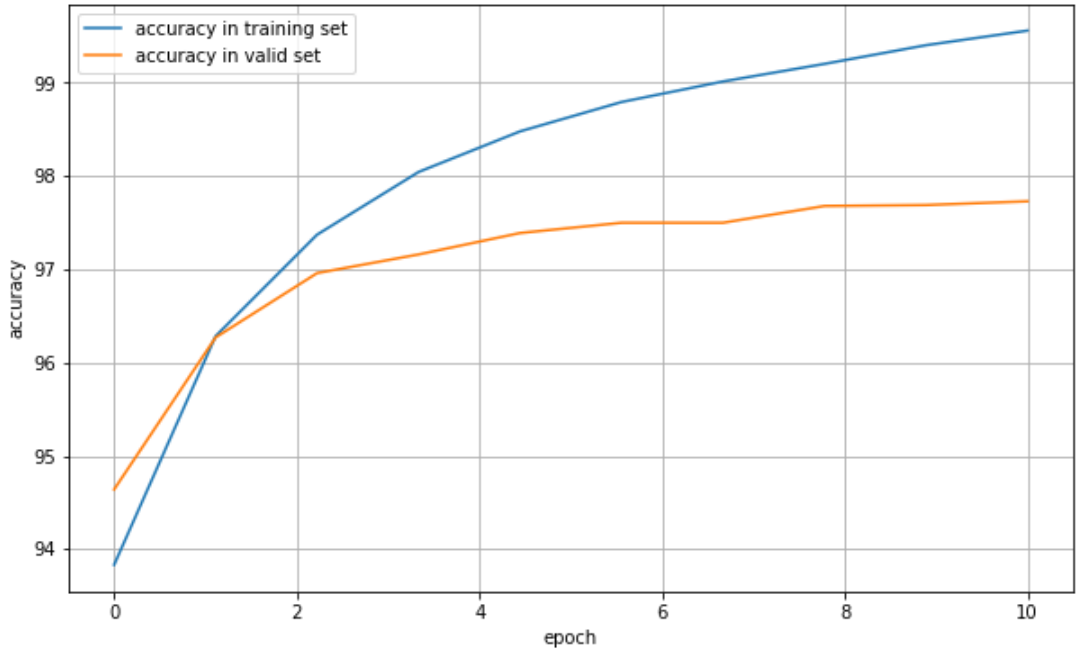
In [38]:
```
#=== Hyperparameter Search ===
P1_hyper = MLP_NN()

# setting the network structure
P1_hyper.h1 = 710
P1_hyper.h2 = 610
P1_hyper.minibatches = 8
P1_hyper.epoch = 10
P1_hyper.learning_rate = 0.01

w1, w2, w3 = P1_hyper.initialize_weights('glorot')
b1, b2, b3 = P1_hyper.initialize_biases()
parameters = {"W1": w1,
              "b1": b1,
              "W2": w2,
              "b2": b2,
              "W3": w3,
              "b3": b3}

train_data, train_label, valid_data, valid_label, test_data, test_label = P1_hyper
.import_data()
X = train_data  # X shape (50000, 784)
y = train_label
y_onehot = P1_hyper.one_hot(y)
m = X.shape[0]
batch = int(m / P1_hyper.minibatches)
accu = 0
loss_train = []
loss_valid = []
accu_train = []
accu_valid = []

print('NN structure: {0} * {1}, with epoch: {2}, minibatch: {3}, learning rate:
{4}'.format(P1_hyper.h1, P1_hyper.h2, P1_hyper.epoch, P1_hyper.minibatches, P1_hyp
er.learning_rate))
print('active function in layer 1: relu')
print('active function in layer 2: relu')

for i in range(P1_hyper.epoch):

    for p in range(batch):
        parameters = P1_hyper.train(X, y_onehot, parameters, P1_hyper.minibatches,
P1_hyper.learning_rate, p)

    # calculate the loss for whole training set, one epoch
    cache = P1_hyper.forward(X, parameters)
    print('Epoch: ', i, 'of ', P1_hyper.epoch, ' training...')
    loss = P1_hyper.loss(X, y, cache)
    a1, o1, w1, b1, o2, a2, w2, b2, a3, o3, w3, b3 = cache
    k = np.argmax(o3, axis=0)
    idx = np.argmax(y_onehot, axis=0).tolist()
    real_idx = np.array(idx[0])
    for j in range(m):
        if k[j] == y[j]:
            accu += 1
    accu /= m
    accu *= 100
    print('The accuracy in training set is: {0}% and loss: {1}'.format(accu, loss
))
    loss_train.append(loss)
    accu_train.append(accu)

    # get the accuracy in validation set
    accu_validation,loss_v = P1_hyper.test(valid_data, valid_label, parameters)
    loss_valid.append(loss_v)
    accu_valid.append(accu_validation)
```

```
NN structure: 710 * 610, with epoch: 10, minibatch: 8, learning rate: 0.01
active function in layer 1: relu
active function in layer 2: relu
Epoch:  0 of  10  training...
The accuracy in training set is: 93.83200000000001% and loss: 0.20851925400148397
The accuracy in validation set is: 94.64% and loss: 0.19748396142338534
Epoch:  1 of  10  training...
The accuracy in training set is: 96.283664% and loss: 0.1329672429817316
The accuracy in validation set is: 96.27% and loss: 0.13694195191203445
Epoch:  2 of  10  training...
The accuracy in training set is: 97.372567328% and loss: 0.09648028099117625
The accuracy in validation set is: 96.96000000000001% and loss: 0.111447552920769
91
Epoch:  3 of  10  training...
The accuracy in training set is: 98.044745134656% and loss: 0.07436021014927519
The accuracy in validation set is: 97.16% and loss: 0.09741803394865976
Epoch:  4 of  10  training...
The accuracy in training set is: 98.48008949026931% and loss: 0.05991765514734097
The accuracy in validation set is: 97.39% and loss: 0.089457474991266
Epoch:  5 of  10  training...
The accuracy in training set is: 98.79496017898055% and loss: 0.04961628212162004
5
The accuracy in validation set is: 97.5% and loss: 0.08459174606596892
Epoch:  6 of  10  training...
The accuracy in training set is: 99.01558992035795% and loss: 0.04172888360424107
5
The accuracy in validation set is: 97.5% and loss: 0.08163581303467568
Epoch:  7 of  10  training...
The accuracy in training set is: 99.20403117984073% and loss: 0.03505278853273965
The accuracy in validation set is: 97.68% and loss: 0.07917749221855722
Epoch:  8 of  10  training...
The accuracy in training set is: 99.40440806235968% and loss: 0.02917032146292939
The accuracy in validation set is: 97.69% and loss: 0.07728712923434336
Epoch:  9 of  10  training...
The accuracy in training set is: 99.56080881612472% and loss: 0.02423472689486299
8
The accuracy in validation set is: 97.72999999999999% and loss: 0.076093044876118
63
```

In [39]:
```
plt.figure(figsize=(10,6))
xaxis = np.linspace(0, 10, 10)
plt.xlabel("epoch")
plt.ylabel("loss")
plt.plot(xaxis, loss_train, '-')
plt.plot(xaxis, loss_valid, '-')
plt.legend(('loss in training set', 'loss in valid set'))
plt.grid()
plt.show()
```



In [40]:
```
plt.figure(figsize=(10,6))
xaxis = np.linspace(0, 10, 10)
plt.xlabel("epoch")
plt.ylabel("accuracy")
plt.plot(xaxis, accu_train, '-')
plt.plot(xaxis, accu_valid, '-')
plt.legend(('accuracy in training set', 'accuracy in valid set'))
plt.grid()
plt.show()
```



Validate Gradients using Finite Difference The finite difference gradient approximation of a scalar function $x \in R \rightarrow f(x) \in R$, of precision ϵ, is defined as $\frac{f(x+\epsilon)-f(x-\epsilon)}{2\epsilon}$. Consider the second layer weights of the MLP you built in the previous section, as a vector $\theta = (\theta_1, \ldots, \theta_m)$. We are interested in approximating the gradient of the loss function L, evaluated using one training sample, at the end of training, with respect to $\theta_{1:p}$, the first $p = min(10, m)$ elements of θ, using finite differences.

1. Evaluate the finite difference gradients $\nabla^N \in R^p$ using $\epsilon = \frac{1}{N}$ for different values of N

$$\nabla_i^N = \frac{L(\theta_1, \ldots, \theta_{i-1}, \theta_{i+\epsilon}, \ldots, \theta_p) - L(\theta_1, \ldots, \theta_{i-1}, \theta_{i-\epsilon}, \ldots, \theta_p)}{2\epsilon}$$

Using at least 5 values of N from the set $\{k10^i : i \in \{0, \ldots, 5\}, k \in \{1, 5\}\}$

1. Plot the maximum difference between the true gradient and the finite difference gradient ($max_{1 \leq i \leq p} |\nabla_i^N - \frac{\partial L}{\partial \theta_i}|$) as a function of N. Comment on the result.

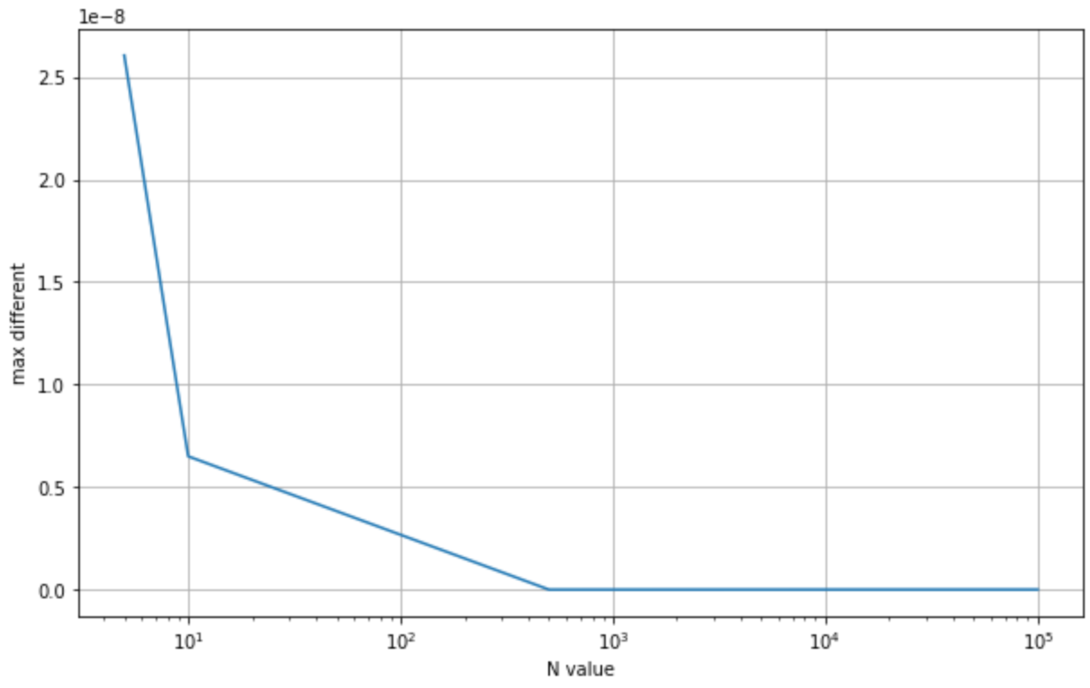
In [20]:
```
#=== Validate Gradients using Finite Difference ===

def dictionary_to_vector(parameters):
    """
    Roll all our parameters dictionary into a single vector satisfying our specifi
c required shape.
    """
    keys = []
    count = 0
    for key in ["W1", "b1", "W2", "b2", "W3", "b3"]:

        # flatten parameter
        new_vector = np.reshape(parameters[key], (-1, 1))
        keys = keys + [key] * new_vector.shape[0]

        if count == 0:
            theta = new_vector
        else:
            theta = np.concatenate((theta, new_vector), axis=0)
        count = count + 1
    return theta, keys


def vector_to_dictionary(theta, features, h1, h2, outputs):
    """
    Unroll all our parameters dictionary from a single vector satisfying our speci
fic required shape.
    network structure: 3,2,3,2
    """
    parameters = {}
    w1_t = h1 * features
    b1_t = w1_t + h1

    w2_t = b1_t + h2 * h1
    b2_t = w2_t + h2

    w3_t = b2_t + outputs * h2
    b3_t = w3_t + outputs

    parameters["W1"] = theta[:w1_t].reshape((h1, features))  # w1: h1*features=6
    parameters["b1"] = theta[w1_t:b1_t].reshape((h1, 1))  # b1: h1=2
    parameters["W2"] = theta[b1_t:w2_t].reshape((h2, h1))  # w2: h2*h1=6
    parameters["b2"] = theta[w2_t:b2_t].reshape((h2, 1))  # b2: h2*1=3
    parameters["W3"] = theta[b2_t:w3_t].reshape((outputs, h2))  # w3: outputs*h2=6
    parameters["b3"] = theta[w3_t:b3_t].reshape((outputs, 1))  # b3: outputs*1=2
    return parameters

def gradients_to_vector(gradients):
    """
    Roll all our gradients dictionary into a single vector satisfying our specific
required shape.
    """
    count = 0
    for key in ["dW1", "db1", "dW2", "db2", "dW3", "db3"]:
        # flatten parameter
        new_vector = np.reshape(gradients[key], (-1, 1))

        if count == 0:
            theta = new_vector
        else:
            theta = np.concatenate((theta, new_vector), axis=0)
        count = count + 1
    return theta
```

In [29]:
```
P1_valid = MLP_NN()

train_data, train_label, valid_data, valid_label, test_data, test_label = P1_valid
.import_data()
X = train_data[5,:] # choose the fifth sample
X = X.reshape((1, 784))

y = train_label[5]
y = y.reshape((1,1))
y_onehot = P1_valid.one_hot(y)

m = X.shape[0]

# setting the network structure
P1_valid.features = 784
P1_valid.outputs = 10
P1_valid.h1 = 600
P1_valid.h2 = 600
P1_valid.minibatches = 1
P1_valid.epoch = 1
P1_valid.learning_rate = 0.0001

w1_t = P1_valid.h1 * P1_valid.features
b1_t = w1_t + P1_valid.h1

w2_t = b1_t + P1_valid.h2 * P1_valid.h1
b2_t = w2_t + P1_valid.h2

p = min(10, b2_t-b1_t)

w1, w2, w3 = P1_valid.initialize_weights('glorot')
b1, b2, b3 = P1_valid.initialize_biases()
parameters = {"W1": w1,
              "b1": b1,
              "W2": w2,
              "b2": b2,
              "W3": w3,
              "b3": b3}

print('NN structure: {0} * {1}, with epoch: {2}, minibatch: {3}, learning rate:
{4}'.format(P1_valid.h1, P1_valid.h2, P1_valid.epoch, P1_valid.minibatches, P1_val
id.learning_rate))
print('active function in layer 1: relu')
print('active function in layer 2: relu')

cache = P1_valid.forward(X, parameters)
#loss = P1_valid.loss(X, y, cache)
#print('J orig: ', loss)

gradients = P1_valid.back_propagation(X, y_onehot, cache)

grad = gradients_to_vector(gradients)

grad_l2 = grad[b1_t:b1_t+p].T
#print('grad_l2: ', grad_l2)

grad_backpropogation = grad_l2.tolist()
#print('grad back: ', grad_backpropogation)

# compare grad list: grad[b1_t:b1_t+p]

# Create N set:
N = []
```

```python
N_set = []
e_set = []

for i in range(6):
    N.append(1*10**i)
    N.append(5*10**i)

random.shuffle(N)
N_set = N[0:5]

N_set.sort()

for r in range(5):
    e_set.append(1/N_set[r])

grad_finite = []
grad_finites = []
for l in range(p):
    for e in range(len(e_set)):
        parameters_values, _ = dictionary_to_vector(parameters)
        thetaplus = np.copy(parameters_values)  # Step 1
        thetaplus[b1_t+l][0] = thetaplus[b1_t+l][0] + e_set[e]  # Step 2, calculat
e the from and end index for each layer
        plus = vector_to_dictionary(thetaplus, P1_valid.features, P1_valid.h1, P1_
valid.h2, P1_valid.outputs)
        plus_cache = P1_valid.forward(X, plus)
        J_plus = P1_valid.loss(X, y, plus_cache)
        #print('J_plus: ', J_plus)

        thetaminus = np.copy(parameters_values)  # Step 1
        thetaminus[b1_t+l][0] = thetaminus[b1_t+l][0] - e_set[e]  # Step 2
        minus = vector_to_dictionary(thetaminus, P1_valid.features, P1_valid.h1, P
1_valid.h2, P1_valid.outputs)
        minus_cache = P1_valid.forward(X, minus)
        J_minus = P1_valid.loss(X, y, minus_cache)  # Step 3
        #print('J_minus: ', J_minus)

        gradapprox = (J_plus - J_minus) / e_set[e] / 2
        grad_finite.append(gradapprox)

grad_finites=np.array(grad_finite)
grad_finites = grad_finites.reshape((10, 5))

max_diffs = []
for q in range(5):
    diff = np.abs(grad_finites[:,q]-grad_backpropogation)
    max_diff = np.max(diff)
    max_diffs.append(max_diff)

print('N set: ', N_set)
print('e set: ', e_set)
print('max diffs: ', max_diffs)
```

```
NN structure: 600 * 600, with epoch: 1, minibatch: 1, learning rate: 0.0001
active function in layer 1: relu
active function in layer 2: relu
N set:  [5, 10, 500, 10000, 100000]
e set:  [0.2, 0.1, 0.002, 0.0001, 1e-05]
max diffs:  [2.604756871305014e-08, 6.504299487897214e-09, 7.589318062883876e-12,
1.0364875624446768e-11, 1.0364875624446768e-11]
```

In [30]:
```
plt.figure(figsize=(10,6))
plt.xlabel("N value")
plt.ylabel("max different")
plt.plot(N_set, max_diffs, '-')
plt.xscale('log')
plt.grid()
plt.show()
```

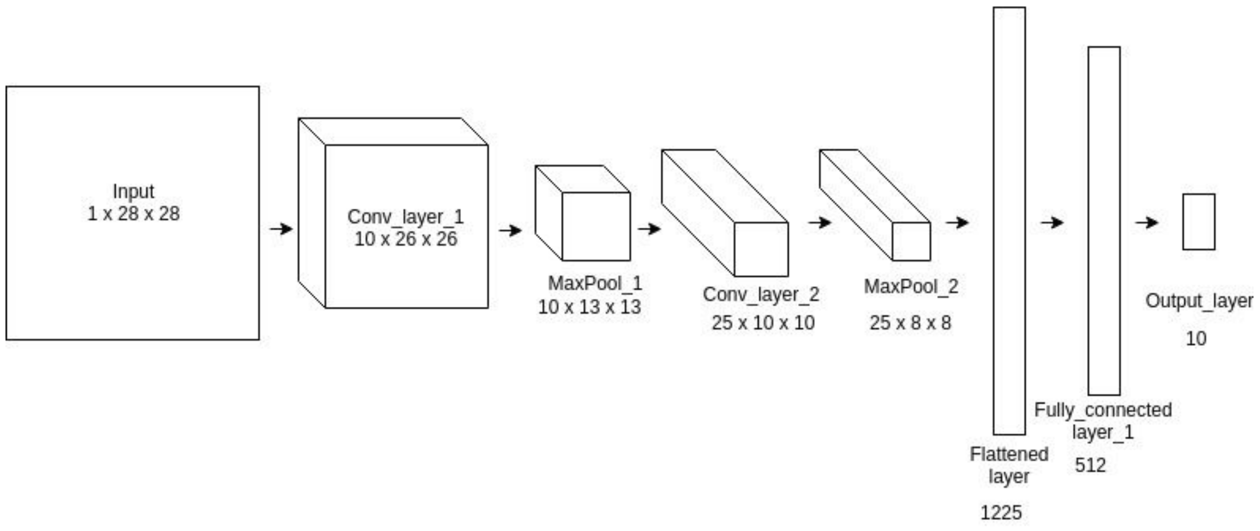


Conclustion: From the figure showed above, we know that as N value is larger, the ϵ would be smaller, then the difference between the gradient calculated by back propogation and calculated by finite difference would be smaller. The value of difference should be smaller than e^{-7}.

In [ ]:
```
```

Problem 2

1. The CNN has the following architecture:

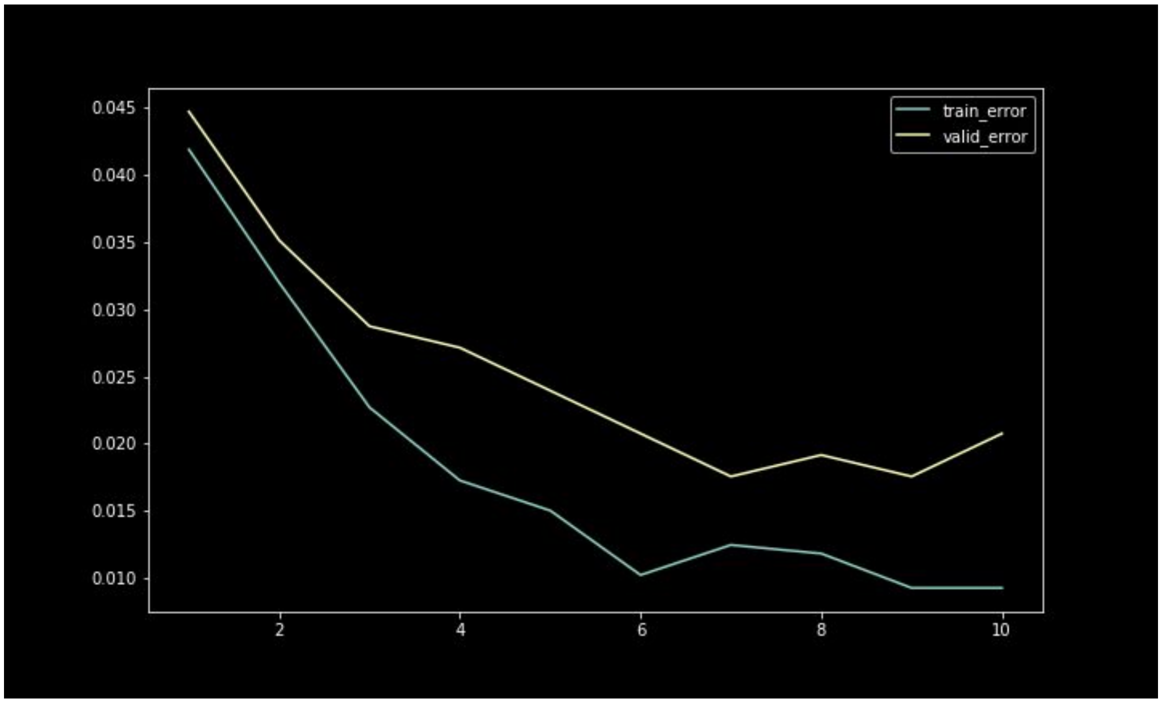


- Input image convolves on 10 3x3 kernels
- Conv_layer_1 go through a max pooling with window_size=2, stride=2
- Then it convolves on 25 4x4 kernels
- Conv_layer_2 go through a max pooling with window_size=3, stride=1
- Flatten the feature maps to 1-d vector
- The flattened layer then connects to one hidden fully-connected layer
- Fully_connected_layer_1 fully connects to the output layer

- Activation functions for hidden layers: ReLU
- Loss function: cross entropy

- # of parameters = 3x3x1x10 + 4x4x10x25 + 25x8x8x512 + 512x10 + 512 + 10
  = 828,932

2. Training and validation error curve



x-axis is the number of epoch, y-axis is the error

For the MLP and CNN we used the same hyper-parameters:

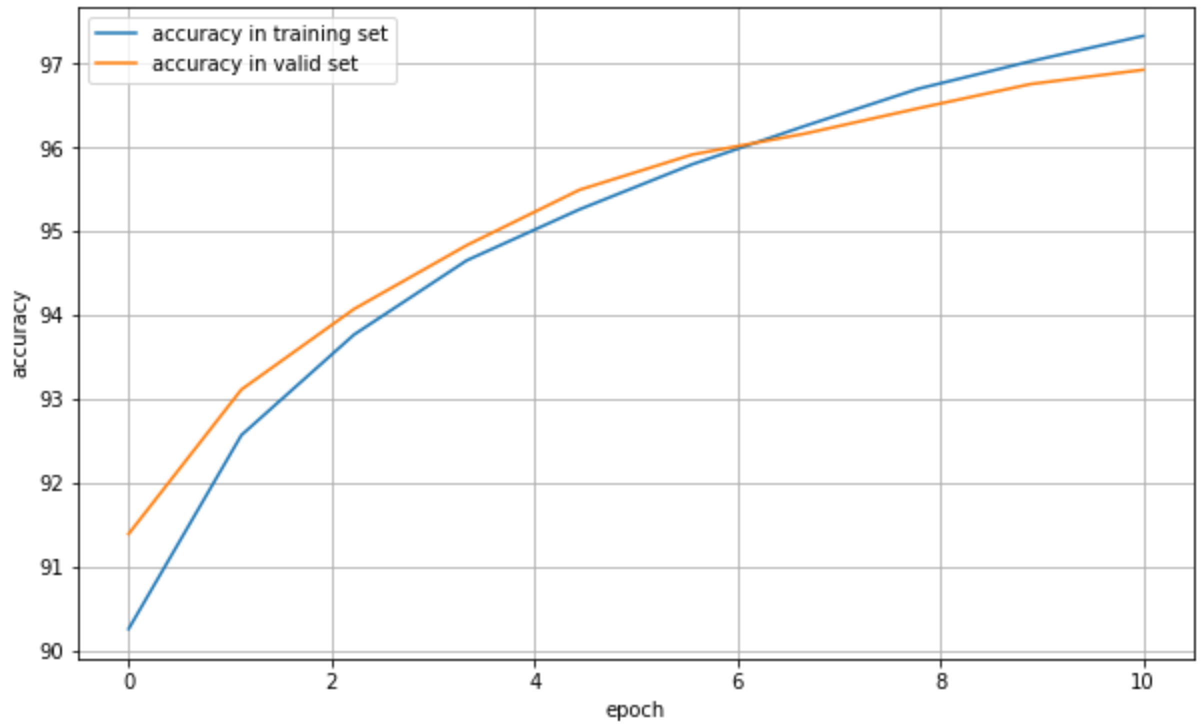
- learning_rate=0.01
- batch_size=32
- activation functions for hidden layers: ReLU
- loss function: cross entropy

As we can see in the graph, the errors on both training and validation set decrease very quickly with CNN. After 1 epoch of training, the validation error got 4.5%. After around 7 epochs, it reached the smallest validation error 1.76%. Then it starts to overfit after 7 epoches.

Compare to the MLP we did in Problem 1(NN structure: 512 * 256, with epoch: 10, minibatch: 32, learning rate: 0.01, activation function:ReLU), we see that CNN converges much faster than MLP. After the first training epoch, the validation error of MLP is 8.6%. The validation error of MLP at epoch 10 is 3.07% and it curve is still decreasing which means it still learns from the training set without overfitting. From this comparison we can see the advantage of using CNN over MLP for image classification problem, as the convolution kernel can preserve the structure of the image but MLP cannot.

Reference:

The validation accuracy curve of MLP


accuracy in training set
accuracy in valid set
accuracy
epoch