

Discrete univariate distributions

Distribution	Probability mass function (p.m.f.) $p(x)$	Range	Parameters	$\mathbb{E}(X)$	$\text{Var}(X)$	Moment-generating function $M(t)$	Comments
Binomial ^{1,2} $\text{Bi}(n, \theta)$	$\binom{n}{x} \theta^x (1 - \theta)^{n-x}$	$x \in \{0, 1, \dots, n\}$	$n \in \mathbb{N}$ $0 < \theta < 1$	$n\theta$	$n\theta(1 - \theta)$	$(1 - \theta + \theta \exp(t))^n$	No. of successes in n trials θ – probability of success
Geometric $\text{Geo}(\theta)$	$\theta^{x-1} (1 - \theta)$	$x \in \mathbb{N}$	$0 < \theta < 1$	$\frac{1}{1 - \theta}$	$\frac{\theta}{(1 - \theta)^2}$	$\frac{(1 - \theta) \exp(t)}{1 - \theta \exp(t)}$	No. of trials until (and including) first failure θ – probability of success
Hypergeometric $\text{HyGe}(n, N, M)$	$\frac{\binom{M}{x} \binom{N-M}{n-x}}{\binom{N}{n}}$	$x \in \{\max\{0, n - (N - M)\}, \dots, \min\{n, M\}\}$	$N, n \in \mathbb{N}$ $M \in \{0, \dots, N\}$	$n\theta$	$n\theta(1 - \theta) \cdot \frac{N - n}{N - 1}$ (with $\theta = \frac{M}{N}$)	— ³	No. of type I objects in a sample of size n , drawn <i>without</i> replacement from a population of size N , containing M type I objects.
Negative Binomial $\text{NeBi}(k, \theta)$	$\binom{x-1}{k-1} \theta^{x-k} (1 - \theta)^k$	$x \in \{k, k + 1, \dots\}$	$k \in \mathbb{N}$ $0 < \theta < 1$	$\frac{k}{1 - \theta}$	$\frac{k\theta}{(1 - \theta)^2}$	$\left(\frac{(1 - \theta) \exp(t)}{1 - \theta \exp(t)} \right)^k$	No. of trials until (and including) k^{th} failure θ – probability of success $\text{NeBi}(1, \theta) \equiv \text{Geo}(\theta)$
Poisson $\text{Poi}(\lambda)$	$\exp(-\lambda) \frac{\lambda^x}{x!}$	$x \in \mathbb{N}_0$	$\lambda > 0$	λ	λ	$\exp(\lambda(\exp(t) - 1))$	

¹ $\text{Bi}(n, \theta)$ can be approximated by $\text{Poi}(n\theta)$, if n large, θ small and $n\theta$ moderate.
² $\text{Bi}(n, \theta)$ can be approximated by $\text{N}(n\theta, n\theta(1 - \theta))$, if n large and θ not too close to 0 or 1.
³ No simple closed form expression exists.

Continuous univariate distributions

Distribution	Probability density function (p.d.f.) $f(x)$	Range	Parameters	$\mathbb{E}(X)$	$\text{Var}(X)$	Moment-generating function $M(t)$	Comments
Beta $\text{Be}(\alpha_1, \alpha_2)$	$\frac{x^{\alpha_1-1} (1 - x)^{\alpha_2-1}}{\text{B}(\alpha_1, \alpha_2)}$	$0 \leq x \leq 1$	$\alpha_1 > 0$ $\alpha_2 > 0$	$\frac{\alpha_1}{\alpha_1 + \alpha_2}$	$\frac{\alpha_1 \alpha_2}{(\alpha_1 + \alpha_2)^2 (\alpha_1 + \alpha_2 + 1)}$	— ³	$X_1 \sim \text{Ga}(\alpha_1, \theta)$ $X_2 \sim \text{Ga}(\alpha_2, \theta)$ independent $\Rightarrow \frac{X_1}{X_1 + X_2} \sim \text{Be}(\alpha_1, \alpha_2)$
Cauchy $\text{Ca}(\eta, \gamma)$	$\frac{1}{\pi \gamma \left(1 + \frac{(x-\eta)^2}{\gamma^2} \right)}$	$x \in \mathbb{R}$	$\eta \in \mathbb{R}$ $\gamma > 0$	— ⁴	— ⁴	— ⁴	$\text{Ca}(0, 1) \equiv \text{t}(1)$
Chi-Squared $\chi^2(\nu)$	$\frac{x^{\frac{\nu}{2}-1} \exp\left(-\frac{x}{2}\right)}{2^{\frac{\nu}{2}} \Gamma\left(\frac{\nu}{2}\right)}$	$x > 0$	$\nu \in \mathbb{N}$	ν	2ν	$\frac{1}{(1 - 2t)^{\frac{\nu}{2}}}$	$X_i \sim \text{N}(0, 1)$ independent $\Rightarrow \sum_{i=1}^{\nu} X_i^2 \sim \chi^2(\nu)$
Exponential $\text{Expo}(\theta)$	$\theta \exp(-\theta x)$	$x > 0$	$\theta > 0$	$\frac{1}{\theta}$	$\frac{1}{\theta^2}$	$\frac{1}{1 - \frac{t}{\theta}}$	
F $\text{F}(\nu_1, \nu_2)$	$\frac{\nu_1^{\frac{\nu_1}{2}} \nu_2^{\frac{\nu_2}{2}}}{\text{B}\left(\frac{\nu_1}{2}, \frac{\nu_2}{2}\right)} \frac{x^{\frac{\nu_1}{2}-1}}{(\nu_1 x + \nu_2)^{\frac{\nu_1+\nu_2}{2}}}$	$x > 0$	$\nu_1, \nu_2 \in \mathbb{N}$	$\frac{\nu_2}{\nu_2 - 2}$ (for $\nu_2 > 2$)	$\frac{2 \nu_2^2 (\nu_1 + \nu_2 - 2)}{\nu_1 (\nu_2 - 2)^2 (\nu_2 - 4)}$ (for $\nu_2 > 4$)	— ⁴	$X_1 \sim \chi^2(\nu_1)$ $X_2 \sim \chi^2(\nu_2)$ independent $\Rightarrow \frac{X_1/\nu_1}{X_2/\nu_2} \sim \text{F}(\nu_1, \nu_2)$

Distribution	Probability density function (p.d.f.) $f(x)$	Range	Parameters	$\mathbb{E}(X)$	$\text{Var}(X)$	Moment-generating function $M(t)$	Comments
Gamma $\text{Ga}(\alpha, \theta)$	$\frac{\theta^\alpha x^{\alpha-1} \exp(-\theta x)}{\Gamma(\alpha)}$	$x > 0$	$\theta > 0$ $\alpha > 0$	$\frac{\alpha}{\theta}$	$\frac{\alpha}{\theta^2}$	$\frac{1}{(1 - \frac{t}{\theta})^\alpha}$	$\text{Ga}(1, \theta) \equiv \text{Expo}(\theta)$ $\text{Ga}(\frac{\nu}{2}, \frac{1}{2}) \equiv \chi^2(\nu)$
Normal $\text{N}(\mu, \sigma^2)$	$\frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$	$x \in \mathbb{R}$	$\mu \in \mathbb{R}$ $\sigma^2 > 0$	μ	σ^2	$\exp\left(\mu t + \frac{\sigma^2 t^2}{2}\right)$	$\text{N}(0, 1)$ – standard normal $\frac{X-\mu}{\sigma} \sim \text{N}(0, 1)$
Pareto $\text{Pa}(k, \theta)$	$\frac{\theta k^\theta}{x^{\theta+1}}$	$x > k$	$k > 0$ $\theta > 0$	$\frac{\theta k}{\theta - 1}$ (for $\theta > 1$)	$\frac{\theta k^2}{(\theta - 1)^2(\theta - 2)}$ (for $\theta > 2$)	— ³	
Student's t $\text{t}(\nu)$	$\frac{1}{\sqrt{\nu} \text{B}\left(\frac{\nu}{2}, \frac{1}{2}\right) \left(1 + \frac{x^2}{\nu}\right)^{\frac{\nu+1}{2}}}$	$x \in \mathbb{R}$	$\nu \in \mathbb{N}$	0 (for $\nu > 1$)	$\frac{\nu}{\nu - 2}$ (for $\nu > 2$)	— ⁴	$X_1 \sim \text{N}(0, 1)$ $X_2 \sim \chi^2(\nu)$ independent $\Rightarrow \frac{X_1}{\sqrt{\frac{X_2}{\nu}}} \sim \text{t}(\nu)$
Uniform $\text{U}(a, b)$ c.d.f. for $a < x < b$:	$\frac{1}{b - a}$	$a \leq x \leq b$	$a, b \in \mathbb{R}$ $a < b$	$\frac{a + b}{2}$	$\frac{(b - a)^2}{12}$	$\frac{\exp(bt) - \exp(at)}{t(b - a)}$	$\text{U}(0, 1) \equiv \text{Be}(1, 1)$
Weibull $\text{We}(\alpha, \theta)$	$\alpha \theta x^{\alpha-1} \exp(-\theta x^\alpha)$	$x > 0$	$\alpha > 0$ $\theta > 0$	$\frac{\Gamma\left(1 + \frac{1}{\alpha}\right)}{\theta^{\frac{1}{\alpha}}}$	$\frac{\Gamma\left(1 + \frac{2}{\alpha}\right)}{\theta^{\frac{2}{\alpha}}} - (\mathbb{E}(X))^2$	— ³	$\text{We}(1, \theta) \equiv \text{Expo}(\theta)$

³ No simple closed form expression exists.

⁴ Does not exist.

Multivariate distributions

Distribution	Probability mass / density function $p(\mathbf{x}) = p(x_1, \dots, x_k)$ or $f(\mathbf{x}) = f(x_1, \dots, x_k)$	Range	Parameters	$\mathbb{E}(X_j)$	$\text{Var}(X_j)$	$\text{Cov}(X_i, X_j)$	Moment-generating function $M(\mathbf{t}) = M(t_1, \dots, t_k)$
Multinomial $\text{Mu}(n, \theta_1, \dots, \theta_k)$	$\frac{n!}{x_1! \dots x_k!} \theta_1^{x_1} \dots \theta_k^{x_k}$	$x \in \mathbb{N}_0$ $\sum_{j=1}^k x_j = n$	$n \in \mathbb{N}$ $0 < \theta_j < 1$ $\sum_{j=1}^k \theta_j = 1$	$n\theta_j$	$n\theta_j(1 - \theta_j)$	$-n\theta_i\theta_j$	$\left(\sum_{j=1}^k \theta_j \exp(t_j)\right)^n$
Normal $\text{N}_k(\boldsymbol{\mu}, \boldsymbol{\Sigma})$	$\frac{1}{(2\pi)^{\frac{k}{2}} \boldsymbol{\Sigma} ^{1/2}} \exp\left(-\frac{1}{2}(\mathbf{x} - \boldsymbol{\mu})^\top \boldsymbol{\Sigma}^{-1}(\mathbf{x} - \boldsymbol{\mu})\right)$	$x \in \mathbb{R}^k$	$\boldsymbol{\mu} \in \mathbb{R}^k$ $\boldsymbol{\Sigma}$ symm., pos. def.	μ_j	Σ_{jj}	Σ_{ij}	$\exp\left(\boldsymbol{\mu}^\top \mathbf{t} + \frac{1}{2} \mathbf{t}^\top \boldsymbol{\Sigma} \mathbf{t}\right)$
Special case: bivariate normal distribution with $\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$, $\boldsymbol{\mu} = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}$, $\boldsymbol{\Sigma} = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}$:							
	$\frac{1}{2\pi\sqrt{\sigma_1^2\sigma_2^2(1-\rho^2)}} \exp\left(-\frac{\sigma_2^2(x_1 - \mu_1)^2 - 2\rho\sigma_1\sigma_2(x_1 - \mu_1)(x_2 - \mu_2) + \sigma_1^2(x_2 - \mu_2)^2}{2\sigma_1^2\sigma_2^2(1-\rho^2)}\right)$		$\mu_1, \mu_2 \in \mathbb{R}$ $\sigma_1^2, \sigma_2^2 > 0$ $-1 < \rho < 1$	μ_j	σ_j^2	$\rho\sigma_1\sigma_2$	