Level M Regression Models Examples

Month

Consider the following Month dataset

Observation	Response	Month
1	0.690	1
2	1.028	3
3	0.507	2
4	1.689	3
5	-1.800	1
6	2.966	2
7	0.653	1
8	3.423	3
9	1.476	2
10	0.540	3

We decide to fit the regression model

$$Y_i = \alpha + \beta_1 I_{\text{Feb}_i} + \beta_2 I_{\text{Mar}_i} + \epsilon_i, \qquad i = 1, \ldots, n.$$

for response variable Y and

$$I_{\text{Mar}_i} = \begin{cases} 1 & \text{if observation } i \text{ was recored in March} \\ 0 & \text{if observation } i \text{ was not recored in March} \end{cases}$$

$$I_{\text{Feb}_i} = \begin{cases} 1 & \text{if observation } i \text{ was recored in February} \\ 0 & \text{if observation } i \text{ was not recored in February} \end{cases}$$

assuming

$$\text{E}(\epsilon_i) = 0 \quad \text{and} \quad \text{Var}(\epsilon_i) = \sigma^2.$$

Calculate the least squares estimates of α, β_1 and β_2 using the formula $\hat{\beta} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{Y}$.