

## Design of Experiments: Chapter 1

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### 1 KEY CONCEPTS

- Difference between a designed experiment and an observational study;
- Definition and notation of key quantities (response, factor, level, design point and support, experimental unit);
- Principles of designed experiments (replication, randomisation, blocking);
- Mathematical definition of exact design;
- Basic concepts of linear models:
  - Mathematical definition;
  - Least square estimation;
  - Predicted values;
  - $R^2$  coefficient of model fit;
  - Analysis of variance.

### 2 EXERCISES

Let  $\mathbf{Y} = (Y_1, \dots, Y_N)$  be a  $N$ -dimensional vector of responses

1. Consider the null model  $Y_i = \beta_0 + \varepsilon_i$ , for  $i = 1, \dots, N$ . Demonstrate that the least square estimate  $\hat{\beta}_0 = \bar{Y} = \frac{1}{N} \sum_{i=1}^N Y_i$ ;
2. Consider again the null model. Demonstrate that the residual sum of square is  $\mathbf{Y}^T \mathbf{Y} - N\bar{Y}^2$ ;

3. Recall that in an Anova table the Regression sum of squares is equal to the total sum of squares  $(\mathbf{Y}^T \mathbf{Y} - N\bar{Y}^2)$  minus the regression sum of squares  $((\mathbf{Y} - X\hat{\boldsymbol{\beta}})^T (\mathbf{Y} - X\hat{\boldsymbol{\beta}}))$ . Demonstrate that the regression sum of squares is equal to  $\hat{\boldsymbol{\beta}}^T (X^T X) \hat{\boldsymbol{\beta}} - N\bar{Y}^2$ ;
4. Which components of an ANOVA table are not additive?
- a) degrees of freedom;
  - b) sums of squares;
  - c) mean square.