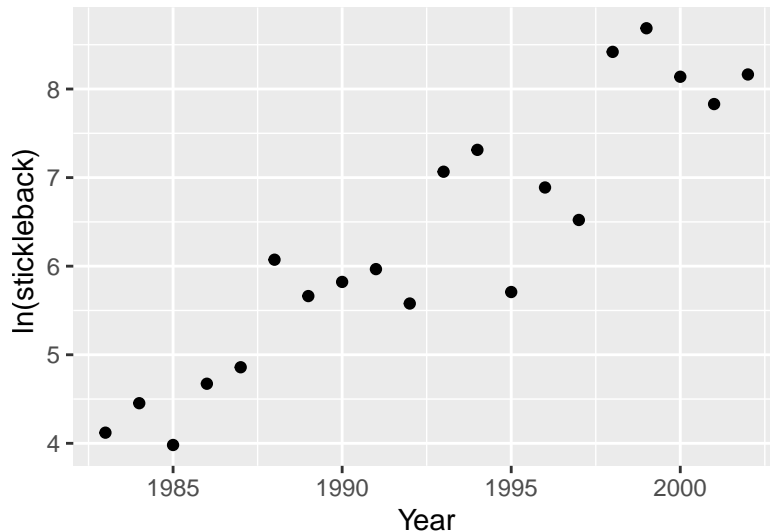


Level M Regression Models Examples

Modelling Stickleback catches



The number of stickleback (a species of fish) trapped in the filters at the pumping station at Ross Priory in Loch Lomond were recorded each year from 1983 to 2000. It was of interest to investigate the pattern over time for the number of stickleback. A natural log (ln) transformation has been applied to the stickleback data and it is plotted below for the years where data were available.

- Use the plot to comment on the relationship between year and ln(stickleback).
- Write down a possible statistical model to describe how ln(stickleback) depends on the year.

Identifying linear models

Which of the following models are linear models? Justify your answer.

- $Y_i = \alpha + \beta x_i + \gamma x_i^2 + \epsilon_i$
- $Y_i = \alpha + \gamma x_i^\beta + \epsilon_i$
- $E(Y_i) = e^{\beta x_i}$

Least Square Estimates

Consider the following model:

Data: (y_i, x_i) $i = 1, \dots, 24$

Model: $E(Y_i) = \beta x_i, \text{Var}(Y_i) = \sigma^2$

- a) Using the sum-of-squares function $S(\beta) = \sum_{i=1}^{24} (y_i - \beta x_i)^2$ derive, from first principles, the least-squares estimator for β .
- b) Use the summary statistics below to calculate the least-squares estimate for β and the residual sum-of-squares.

$$\sum_{i=1}^{24} x_i y_i = 12514, \sum_{i=1}^{24} x_i^2 = 29.518, \sum_{i=1}^{24} y_i^2 = 6511425$$