Level M Regression Models Examples

Writing linear model for Drug Carbinazole data

In a study of the drug Carbinazole in the treatment of thyrotoxicosis, there is interest in comparing patients who are responders (R) to the drug with those who are dormant (D), i.e. non-responders. The patients are compared using their Protein Bound Iodine (PBI) levels.

Data from 20 responders and 20 dormants are available. The model for these data can be written as:

Data: $y_{ij}; i = 1, 2; j = 1, ..., 20$
Model: $E(Y_{1j}) = \theta_R$, $E(Y_{2j}) = \theta_D$, $\quad Var(Y_{ij}) = \sigma^2$
i.e. $E(Y_{ij}) = x_{1j}\theta_R + x_{2j}\theta_D$ where

$$(x_{1j}, x_{2j}) = \begin{cases} (1,0), & \text{for } i = 1 \\ (0,1), & \text{for } i = 2 \end{cases}$$

In vector-matrix form, this model is represented as $E(\mathbf{Y}) = \mathbf{X}\boldsymbol{\beta}$. Using the general formula for $\hat{\boldsymbol{\beta}}$, in vector-matrix notation, find the least squares estimators of θ_R and θ_D. Use the summary statistics below to calculate the least squares estimates for θ_R and θ_D and the residual sum-of-squares.

$$\sum_{j=1}^{20} y_{1j} = 196.7, \sum_{j=1}^{20} y_{2j} = 271.9, \sum_{j=1}^{20} y_{ij}^2 = 5816.20$$

Writing linear model and centering

Consider the following model:

Data: $(y_i, x_i), \quad i = 1, \ldots, n$
Model: $E(Y_i) = \alpha + \beta x_i, Var(Y_i) = \sigma^2$
This model was suggested for the Crime data studied in the lectures.

a) Write this model in vector-matrix form, $E(\mathbf{Y}) = \mathbf{X}\boldsymbol{\beta}$, clearly identifying the elements of $\mathbf{Y}$, $\mathbf{X}$ and $\boldsymbol{\beta}$.

b) Identify formulae for the elements of $\mathbf{X}^T\mathbf{X}$ and $\mathbf{X}^T\mathbf{Y}$.

c) Now consider a re-expression of this model

$$E(Y_i) = \alpha' + \beta(x_i - \bar{x})$$

Derive $\mathbf{X}^T\mathbf{X}$ for this model and hence comment on any benefits for evaluation of $(\mathbf{X}^T\mathbf{X})^{-1}$.

d) Use the general formula for $\hat{\boldsymbol{\beta}}$, in vector matrix notation, to find the least squares estimators for α' and β.