Assignment 2: Regression/Probability

Due Oct. 29 at 11:59pm

This assignment is to be done individually.

Important Note: The university policy on academic dishonesty (cheating) will be taken very seriously in this course. You may not provide or use any solution, in whole or in part, to or by another student.

You are encouraged to discuss the concepts involved in the questions with other students. If you are in doubt as to what constitutes acceptable discussion, please ask! Further, please take advantage of office hours offered by the instructor and the TA if you are having difficulties with this assignment.

DO NOT:

- Give/receive code or proofs to/from other students
- Use Google to find solutions for assignment

DO:

- Meet with other students to discuss assignment (it is best not to take any notes during such meetings, and to re-work assignment on your own)
- Use online resources (e.g. Wikipedia) to understand the concepts needed to solve the assignment

---

Submitting Your Assignment

The assignment must be submitted online on Canvas. In order to simplify grading, you must adhere to the following structure.

You must submit two files:

1. You must create an assignment report in **PDF format**, called `report.pdf`. This report must contain the solutions to questions 1, 3 as well as the figures / explanations requested for 2.(please take screenshots from your entire screen for the figures requested for question 2.)

2. You must submit a .zip file of all your code, called `code.zip`. This must contain a single directory called `code` (no sub-directories, no leading path names), in which all of your files must appear[1]. There must be the 3 scripts with the specific names referred to in Question 2, as well as a common codebase you create and name.

   As a check, if one runs

   ```
   unzip code.zip
   cd code
   ./polynomial_regression_1d.py
   ```

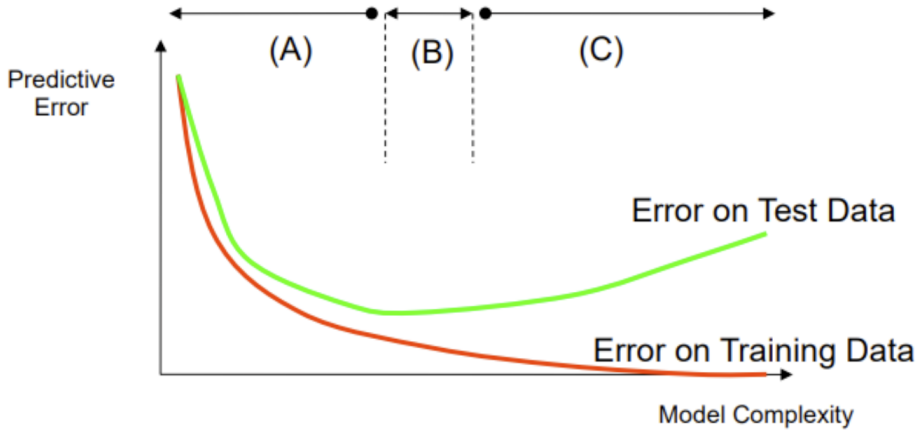
   the script produces the plots in your report from the relevant question.

[1] This includes the data files and others which are provided as part of the assignment.

1 Cross Validation

a) In order to get an unbiased estimate of how well our ML models and algorithms perform, it is typical to do a 60/20/20 split for our data: 60% train, 20% test and 20% validation. Several years ago this was widely considered best practice in machine learning. Do you still agree such ratios in the modern big data era? Why or why not? (Hint: consider different scenarios)

b) Consider the hypothetical graph below of predictive error (y-axis) vs. model complexity (x-axis), and how test/training error varies as model complexity increases.



i) Which part of the plot means that the model is Overfitting on the data? (Choose A, B, or C)

ii) Which part of the plot means that the model is Underfitting on the data? (Choose A, B, or C)

iii) Which part of the plot representing the ideal model complexity? (Choose A, B, or C)

c) For a decision tree model, whose train/test errors are in region A, which of the following is most likely to improve the model performance on real data? (Choose one)

i) Acquire more training data.

ii) Reduce the depth of the decision tree.

iii) Increase the depth of the decision tree.

2 Regression

In this question you will train models for regression and analyze a dataset. Start by downloading the code and dataset from the website.

The dataset is created from data provided by UNICEF's State of the World's Children 2013 report: `http://www.unicef.org/sowc2013/statistics.html`

Child mortality rates (number of children who die before age 5, per 1000 live births) for 195 countries, and a set of other indicators are included.

2.1 Getting started

Run the provided script `polynomial_regression.py` to load the dataset and names of countries / features.

Answer the following questions about the data. Include these answers in your report.

1. Which country had the highest child mortality rate in 1990? What was the rate?
2. Which country had the highest child mortality rate in 2011? What was the rate?
3. Some countries are missing some features (see original .xlsx/.csv spreadsheet). How is this handled in the function `assignment2.load_unicef_data()`?

For the rest of this question use the following data and splits for train/test and cross-validation.

- **Target value**: column 2 (Under-5 mortality rate (U5MR) 2011)[2].
- **Input features**: columns 8-40.
- **Training data**: countries 1-100 (Afghanistan to Luxembourg).
- **Testing data**: countries 101-195 (Madagascar to Zimbabwe).
- **Cross-validation**: subdivide training data into folds with countries 1-10 (Afghanistan to Austria), 11-20 (Azerbaijan to Bhutan), ... I.e. train on countries 11-100, validate on 1-10; train on 1-10 and 21-100, validate on 11-20, ...

2.2 Polynomial Regression

Implement linear basis function regression with polynomial basis functions. Use only monomials of a single variable (eg. x_1, x_1^2, x_2^2) and no cross-terms (eg. $x_1 x_2$).

Perform the following experiments:

a) Create a python script `polynomial_regression.py` for the following.

[2]Zero-indexing, hence `values[:,1]`.

Fit a polynomial basis function regression (unregularized) for degree 1 to degree 8 polynomials. Plot training error and test error (in RMS error) versus polynomial degree.

Put this plot in your report, along with a brief comment about what is "wrong" in your report.

Normalize the input features before using them (not the targets, just the inputs $\boldsymbol{x}$). Use `assignment2.normalize_data()`.

Run the code again, and put this new plot in your report.

b) Create a python script `polynomial_regression_1d.py` for the following.

Perform regression using just a single input feature.

Try features 8-15 (Total population - Low birthweight). For each (un-normalized) feature fit a degree 3 polynomial (unregularized).

Plot training error and test error (in RMS error) for each of the 8 features. This should be a bar chart (e.g. use `matplotlib.pyplot.bar()`).

Put this bar chart in your report.

The testing error for feature 11 (GNI per capita) is very high. To see what happened, produce plots of the training data points, learned polynomial, and test data points. The code `visualize_1d.py` may be useful.

In your report, include plots of the fits for degree 3 polynomials for features 11 (GNI), 12 (Life expectancy), 13 (literacy).

2.3 Regularized Polynomial Regression

Create a python script `polynomial_regression_reg.py` for the following.

Implement L_2-regularized regression. Fit a degree 2 polynomial using $\lambda = \{0, .01, .1, 1, 10, 10^2, 10^3, 10^4, 10^5\}$. Use normalized features as input. Use 10-fold cross-validation to decide on the best value for λ. Produce a plot of average validation set error versus λ. Use a `matplotlib.pyplot.semilogx` plot, putting λ on a log scale[3].

Put this plot in your report, and note which λ value you would choose from the cross-validation.

[3]The unregularized result will not appear on this scale. You can either add it as a separate horizontal line as a baseline, or report this number separately.

3 Probabilistic Modeling and Bayes' Rule

a) Assume the probability of being infected with Malaria disease is 0.01. The probability of test positive given that a person is infected with Malaria is 0.95 and the probability of test positive given the person is not infected with Malaria is 0.05.
(a) Calculate the probability of test positive.
(b) Use Bayes' Rule to calculate the probability of being infected with Malaria given that the test is positive.

b) Suppose P(rain today) = 0.30, P(rain tomorrow) = 0.60, P(rain today and tomorrow) = 0.25. Given that it rains today, what is the probability it will rain tomorrow?

c) A biased die has the following probabilities of landing on each face:

face	1	2	3	4	5	6
P(face)	0.2	0.1	0.1	0.2	0.1	0.3

i) I win if the die shows odd. What is the probability that I win? Compare this to a fair die (i.e., a die with equal probabilities for each face).

ii) What is the entropy of this die? Compare this to a fair die.