Machine Learning

CMPT 726

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Linear Algebra and Calculus Review (cont’d)

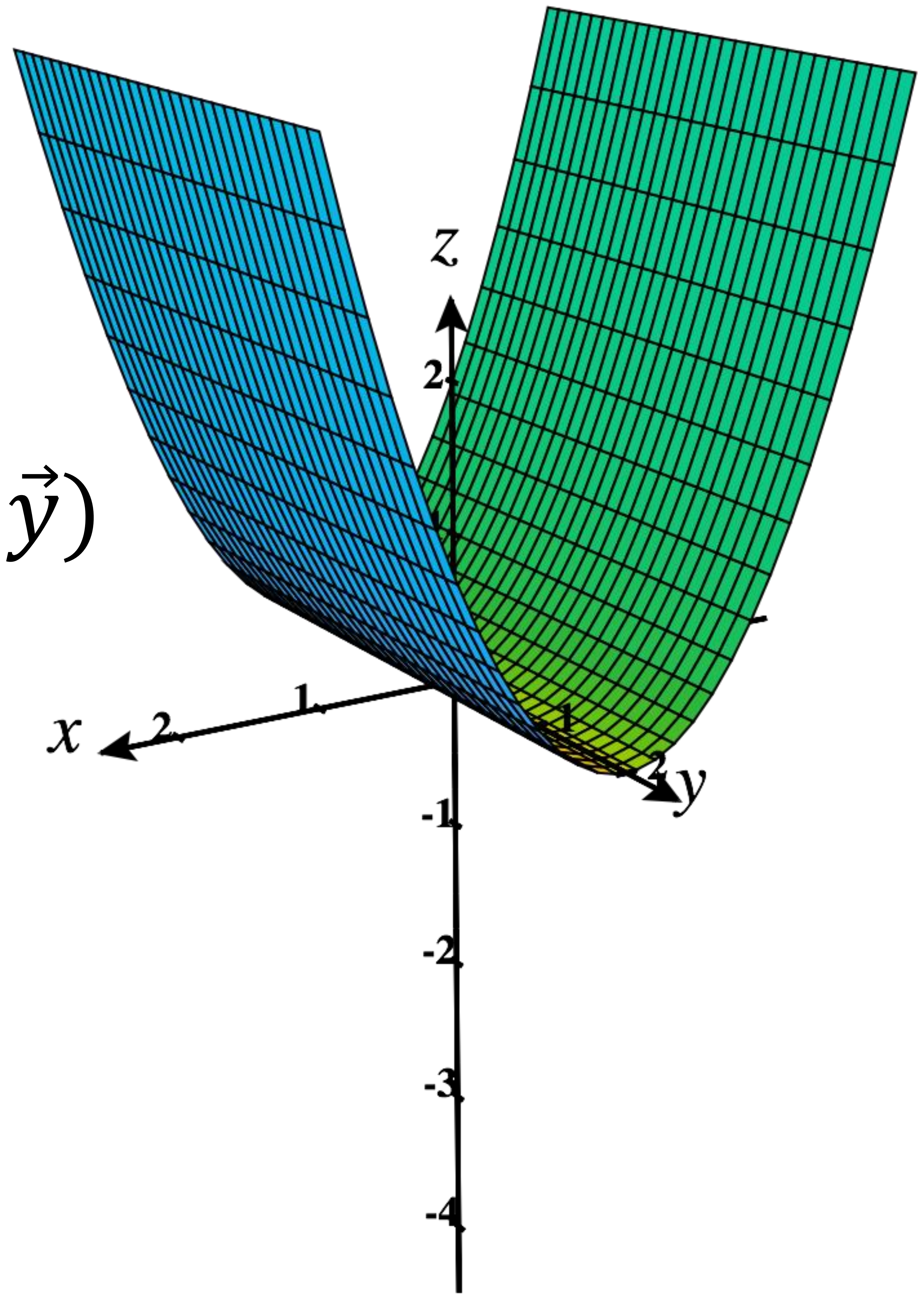
Convexity/Concavity

Convex function:

A line segment between **any** two points on the surface lies **on or above** the surface:

$$\alpha f(\vec{x}) + (1-\alpha) f(\vec{y}) \geq f(\alpha \vec{x} + (1-\alpha)\vec{y})$$

where $0 \leq \alpha \leq 1$



Or equivalently: Hessian of function is **positive semi-definite** everywhere, e.g.:

$$f(\vec{x}) = \vec{x}^{\top} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \vec{x} = x_1^2$$

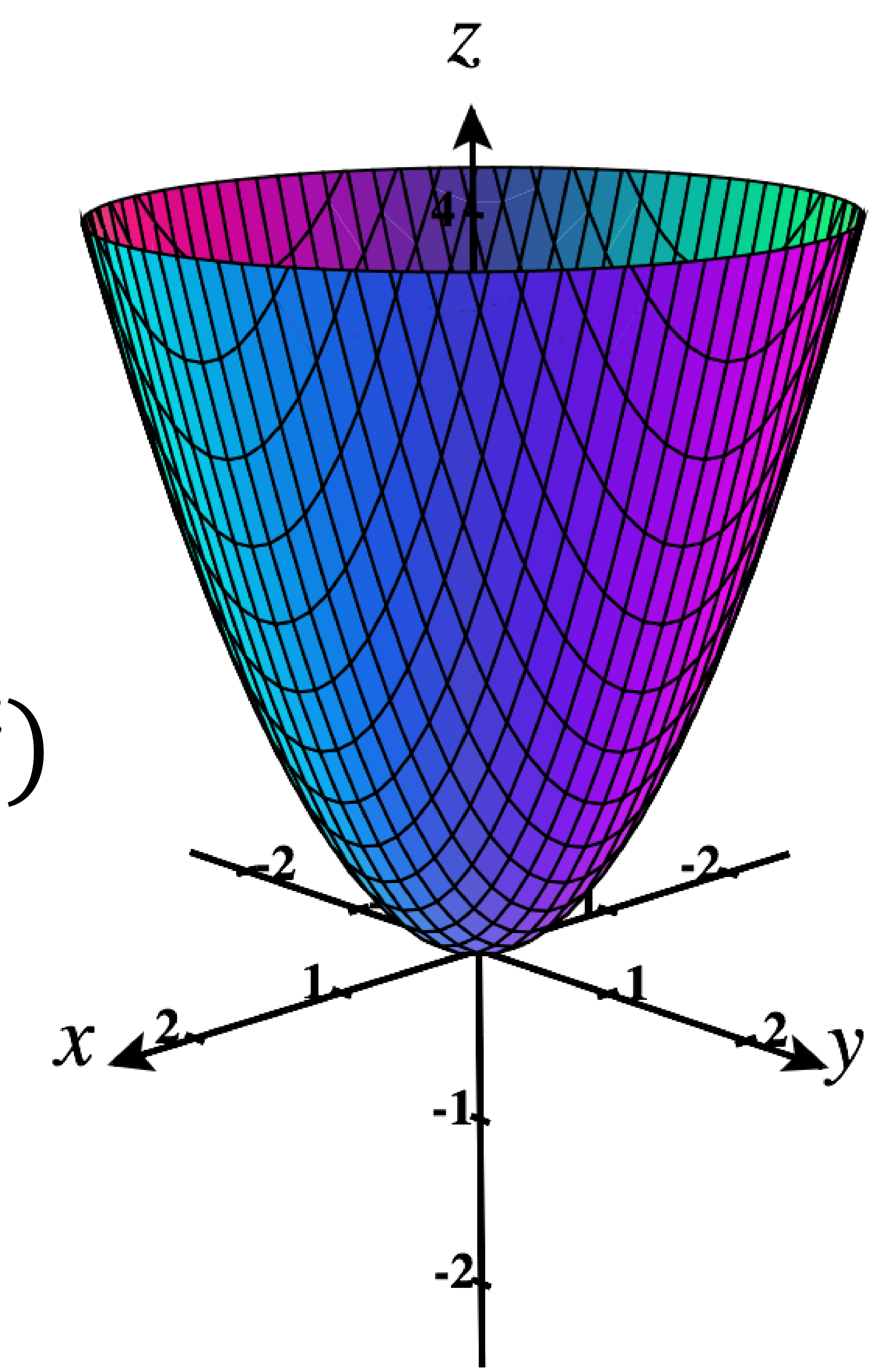
Convexity/Concavity

Strictly convex function:

A line segment between **any** two points on the surface lies **above** the surface:

$$\alpha f(\vec{x}) + (1-\alpha) f(\vec{y}) > f(\alpha \vec{x} + (1-\alpha)\vec{y})$$

where $0 \leq \alpha \leq 1$



Or equivalently: Hessian of function is **positive definite** everywhere, e.g.:

$$f(\vec{x}) = \vec{x}^\top \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \vec{x} = x_1^2 + x_2^2$$

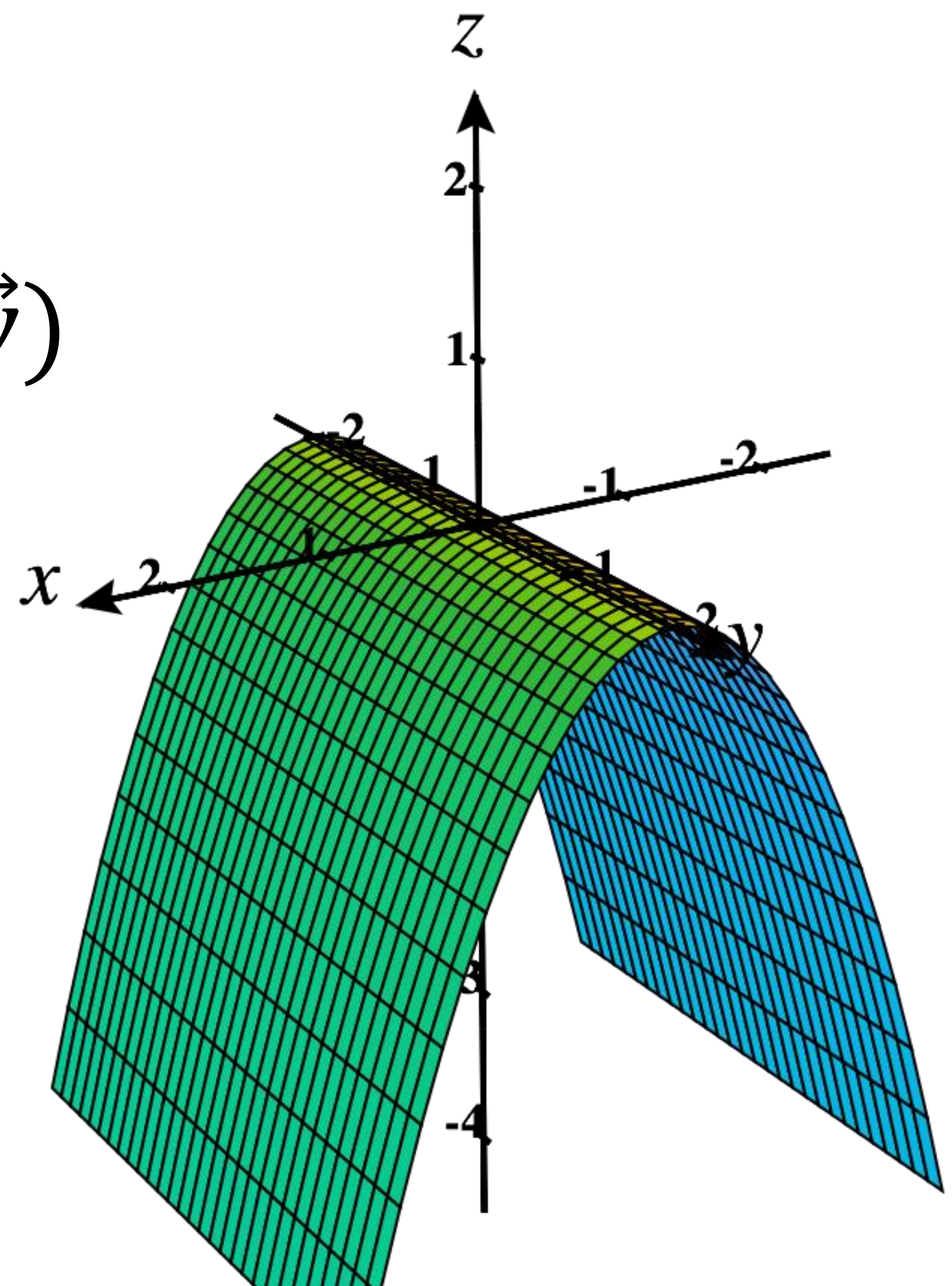
Convexity/Concavity

Concave function:

A line segment between **any** two points
on the surface lies **on or below** the surface:

$$\alpha f(\vec{x}) + (1-\alpha) f(\vec{y}) \leq f(\alpha\vec{x} + (1-\alpha)\vec{y})$$

where $0 \leq \alpha \leq 1$



Or equivalently: Hessian of function is
negative semi-definite everywhere, e.g.:

$$f(\vec{x}) = \vec{x}^{\top} \begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix} \vec{x} = -x_1^2$$

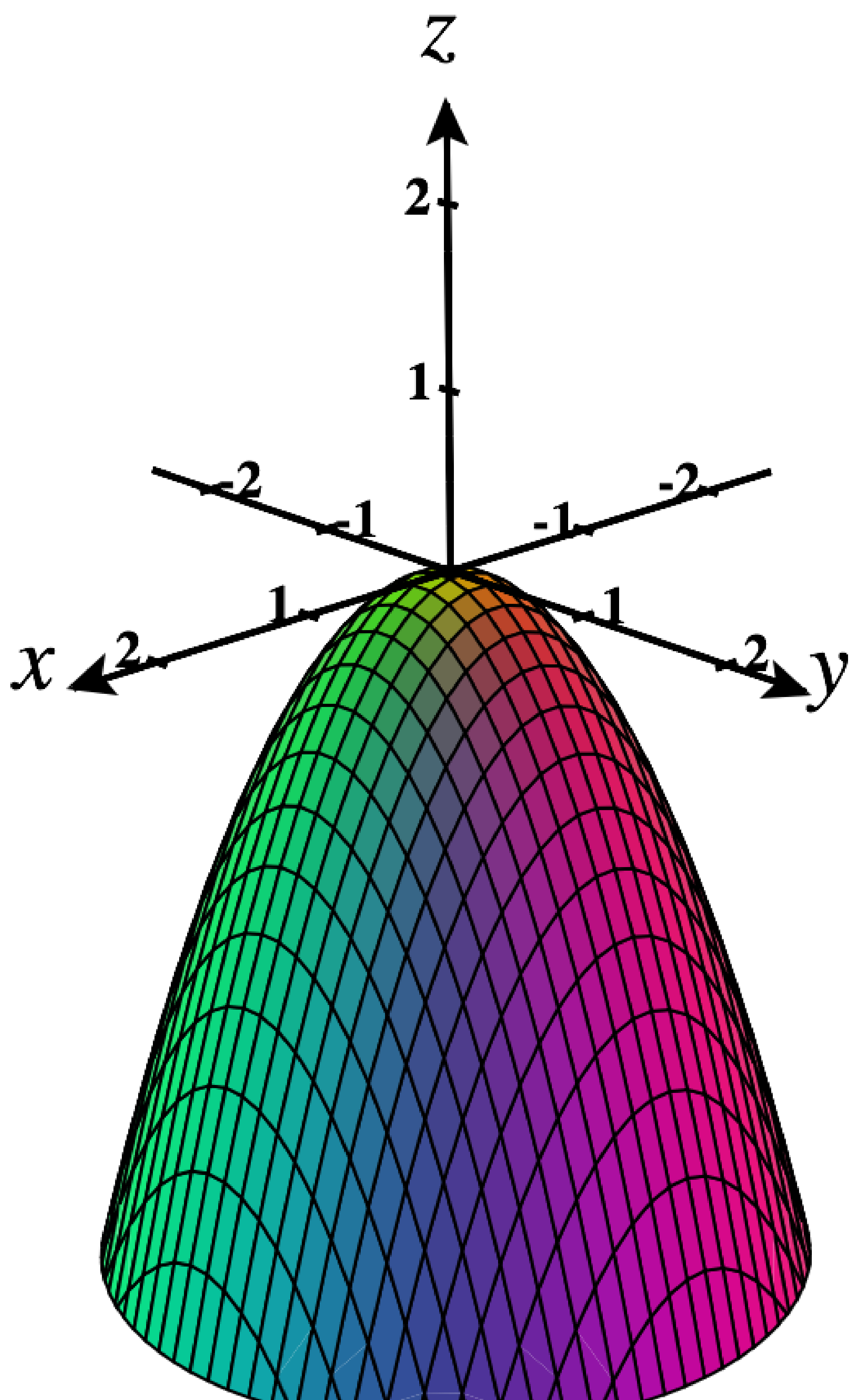
Convexity/Concavity

Strictly concave function:

A line segment between **any** two points on the surface lies **below** the surface:

$$\alpha f(\vec{x}) + (1-\alpha) f(\vec{y}) < f(\alpha\vec{x} + (1-\alpha)\vec{y})$$

where $0 \leq \alpha \leq 1$



Or equivalently: Hessian of function is **negative definite** everywhere, e.g.:

$$f(\vec{x}) = \vec{x}^\top \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \vec{x} = -x_1^2 - x_2^2$$