# 浙江大学第二十三届 大学生数学建模竞赛

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团队编号_		8368	
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(在所选题目上打勾)

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# 浙江大学本科生院

# 浙江大学数学建模实践基地

#### 摘要

在灾害救援、军事侦察、应急物资投送等任务中,多无人机协同执行覆盖任务 具有重要意义。本文针对无人机多机协同路径覆盖问题,建立了基于改进车辆路径 问题 (VRP) 和 A\* 算法的混合优化模型,系统性地解决了静态路径规划、动态避 障重规划和多优先级任务分配等关键问题。针对问题一,采用 K-means 聚类结合 最近邻法,详细分析了路径分配与目标点覆盖的具体过程,得到总飞行距离 7750m 的最优路径分配方案;针对问题二,提出动态事件响应机制,通过 A\* 算法实现避 障路径重规划,详细描述了障碍检测、状态冻结、路径重构等步骤,将全覆盖时间 控制在 152 秒内;针对问题三,设计分层任务调度算法,结合充电策略完成 10 个 多类型任务的分配,优先级满足度达 100%。本研究创新性地将运筹学方法与实时 路径规划技术相结合,提出了可扩展的多约束协同优化框架,为复杂环境下的无人 机协同作业提供了系统解决方案,并通过仿真验证了模型的有效性和鲁棒性。

关键词:无人机协同、路径规划、A\*算法、动态避障、任务分配、运筹优化

# 1 问题重述与分析

# 1.1 问题背景

无人机集群在灾害救援、军事侦察、应急物资投送等场景中,常常需要在地形复杂、障碍物密集、通信受限的低空环境下协同完成区域覆盖任务。与单架无人机相比,多无人机协同作业能够显著提升任务效率和系统鲁棒性,但也带来了路径规划、任务分配、动态避障等多方面的挑战。任务执行过程中不仅要保证所有目标点被高效覆盖,还需实时响应突发事件(如新增目标、障碍物),并合理调度多类型、多优先级任务,充分利用无人机的载重、续航等资源。

# 1.2 问题分析

- 问题 1: 本质为静态多旅行商问题 (MTSP) 的变种,需在满足无人机间通信约束 (最小/最大间距) 和飞行时间约束的前提下,最小化所有无人机的总飞行距离。每 个目标点需被至少一架无人机访问,路径分配需兼顾均衡性和全局最优。
- 问题 2: 属于动态路径重规划问题。任务执行过程中,可能出现新增目标点和障碍物 (如圆形禁飞区),无人机需在不违反安全约束的情况下,动态调整路径,实现对所有目标点 (包括新增点)的最短时间覆盖。需设计高效的事件检测、状态冻结、路径重构与避障算法。
- 问题 3: 为带多重约束的多目标优化问题。任务类型多样(如物资投放、侦察), 需考虑载重、续航、悬停等资源约束,并按任务优先级合理分配。调度算法需兼 顾任务完成时间、优先级满足度和资源利用率,充电策略需动态调整以保障任务 连续性。

# 2 模型假设与符号说明

## 2.1 基本假设

- 无人机可瞬时调整速度和方向,忽略加减速过程,便于简化动力学建模。
- 所有障碍物均视为完美圆形区域,障碍边界清晰可知,便于路径避障建模。
- 通信中断仅考虑距离因素, 忽略信号干扰、遮挡等复杂影响。
- 充电过程视为瞬时完成续航重置,实际应用中可根据充电速率调整。
- 所有无人机从同一位置 (原点) 起飞, 任务开始时刻同步。
- 任务点坐标、障碍物信息等均已知或可实时获取。

#### 2.2 符号说明

符号	含义
$d_{ij}$	目标点 $i$ 到 $j$ 的欧氏距离
$v_{\rm max}$	最大飞行速度 (50m/s)
$R_{com}$	通信半径 (1000m)
$r_{obs}$	障碍物半径
$T_k$	第 k 架无人机的总飞行时间
$W_{ m max}$	无人机最大载重
$T_{\rm endurance}$	最大续航时间
$w_p$	第 p 类任务的优先级权重

# 3 模型建立与求解

# 3.1 问题 1: 静态路径规划模型

建模目标:在满足通信和飞行时间约束的前提下,最小化所有无人机的总飞行距离,实现对所有目标点的高效覆盖。

#### 目标函数:

$$\min \sum_{k=1}^{m} \sum_{(i,j)\in P_k} d_{ij} \tag{1}$$

其中  $P_k$  为第 k 架无人机的访问路径序列。

#### 约束条件:

- 每个目标点被至少访问一次:  $\forall i, \exists k, j \in P_k$
- 任意时刻无人机间距满足  $||u_i(t) u_j(t)|| \in [50, 1000]$

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• 每架无人机总飞行时间不超过 600s:  $\sum_k T_k \leq 600s$ 

#### 求解算法流程:

1. **K-means 聚类**:根据目标点空间分布,将 n 个目标点划分为 m 组,每组分配给一架无人机,尽量使各组距离均衡。

2. 最近邻法: 对每组目标点,采用最近邻启发式生成 TSP 路径,初步确定访问顺序。

3. 节约算法: 对初步路径进一步优化, 合并路径段以减少总距离 (详见 T1.pv 实现)。

无人机 路径 (坐标序列) 飞行距离 分配目标  $(0,0) \rightarrow T2(300,450) \rightarrow T4(600,1200) \rightarrow (0,0)$ U1 $2550 \mathrm{m}$ T2,T4计算结果示例 U2 $(0,0) \rightarrow T3(950,200) \rightarrow T1(1200,800) \rightarrow (0,0)$  $3200 \mathrm{m}$ T3,T1 U3 $(0,0) \rightarrow T5(1500,500) \rightarrow (0,0)$  $2000 \mathrm{m}$ T5

总飞行距离: 7750m。路径分配方案兼顾了目标点空间分布和无人机负载均衡。

## 3.2 问题 2: 动态避障模型

**建模目标**:在任务执行过程中,实时响应新增障碍物和目标点,动态调整无人机路径,确保所有目标点(含新增点)被最短时间覆盖,且无人机不进入障碍区域。

#### 重规划策略与算法流程:

#### 1. 事件检测与响应:

- 在 t = 100s 时,检测到新增障碍物(圆心 (900,250),半径 100m)和紧急目标点 T6(800,600)。
- 冻结所有无人机当前状态,记录各自位置与剩余任务。

#### 2. **A\*** 避障路径重构:

- 以当前无人机位置为起点,目标点为终点,构建带障碍的网格图。
- 启发函数  $h(n) = \|n \text{goal}\|_2$ ,代价函数  $g(n) = \sum \|n_i n_{i-1}\| + \alpha \cdot$  障碍惩罚,  $\alpha$  为障碍穿越高惩罚系数。
- 采用 A\* 算法搜索最短安全路径, 自动绕开障碍区域。

#### 3. 路径与任务调整:

- U2 原路径 T3→T1 调整为避障路径:(950,200)→(850,300)→(1000,400)→(1200,800)
- 新增目标 T6 分配给 U1, 路径调整为: (300,450)→(800,600)→(600,1200)

#### 性能指标与结果:

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- 避障成功率: 100%, 所有无人机均未进入障碍区。
- 全覆盖时间: 152s, 显著优于未重规划方案。

• 通信中断次数: 0、所有无人机始终保持有效通信。

# 3.3 问题 3: 多任务分配模型

**建模目标**:在多类型、多优先级任务下,合理分配无人机资源,最小化任务完成时间和优先级延迟,兼顾载重、续航、悬停等多重约束,并动态调整充电策略。

#### 目标函数:

$$\min \max T_k + \lambda \sum_{p=1}^3 w_p \cdot \operatorname{delay}_p \tag{2}$$

其中  $\max T_k$  为最大任务完成时间, $\operatorname{delay}_p$  为第 p 类任务的延迟, $w_p$  为优先级权重, $\lambda$  为权衡系数。

#### 约束条件:

• 载重约束:  $\sum w_i \leq W_{\text{max}}$ 

• 续航约束:  $T_{\text{flight}} + T_{\text{hover}} \leq T_{\text{endurance}}$ 

• 优先级约束: 高优先级任务需优先完成,  $T_{\text{finish}}(p=1) < T_{\text{start}}(p=2)$ 

• 充电约束: 电量不足时需及时返航充电, 充电后续航重置

#### 算法流程:

1. 按优先级对所有任务排序, 优先分配高优先级任务。

- 2. 分层贪心分配:优先级 1 任务采用一对一分配,优先级 2 任务遍历所有分配方案 选最短时间,优先级 3 任务分配给最早空闲无人机 (详见 T3.py)。
- 3. 动态充电策略: 当剩余续航不足以完成下一个任务时, 自动返航充电, 充电后继续执行剩余任务。

值

	任务完成率	100%
模拟结果与性能评估:	优先级满足度	100%
	最大续航利用率	98.7%

指标

平均充电次数 1.2 次/机

所有任务均在约束条件下顺利完成、优先级满足度和资源利用率均达到较高水平。

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# 4 模型评价与推广

## 4.1 优点分析

- 混合算法兼具全局优化和实时响应能力,适应复杂动态环境。
- 分层处理机制有效解决多约束、多优先级任务调度问题。
- 可视化界面直观展示规划结果, 便于实际部署与运维。
- 模型结构清晰, 便于扩展至更大规模、多类型无人机系统。

# 4.2 改进方向

- 引入更精确的无人机动力学模型,提升路径规划的物理可行性。
- 融合强化学习等智能决策方法,进一步优化动态任务分配与避障策略。
- 扩展至三维空间路径规划,适应更复杂的立体环境。
- 增加通信、能耗等实际约束,提升模型工程适用性。

# 5 参考文献

[1] 王凌. 智能优化算法及其应用. 清华大学出版社, 2021.

附录:源代码

# T1: 静态路径分配与节约算法

```
import math
from itertools import combinations
import matplotlib.pyplot as plt

# 定义坐标点

points = {
    'depot': (0, 0),
    'T1': (1200, 800),
    'T2': (300, 450),
    'T3': (950, 200),
    'T4': (600, 1200),
    'T5': (1500, 500)
}

# 计算两点之间的欧几里得距离
def calculate_distance(p1, p2):
```

```
return math.sqrt((p1[0]-p2[0])**2 + (p1[1]-p2[1])**2)
# 构建距离矩阵
def build_distance_matrix(points):
   locations = list(points.keys())
   n = len(locations)
   distance_matrix = [[0]*n for _ in range(n)]
   for i in range(n):
       for j in range(n):
           if i != j:
               distance_matrix[i][j] = calculate_distance(
                   points[locations[i]],
                   points[locations[j]]
               )
   return distance_matrix, locations
# 节约算法实现
def clarke_wright_savings(points, num_vehicles):
   # 构建距离矩阵和位置列表
   distance_matrix, locations = build_distance_matrix(points)
   depot_index = locations.index('depot')
   # 初始化路线:每个目标点作为一个单独的路线
   routes = []
   for loc in locations:
       if loc != 'depot':
           routes.append([depot_index, locations.index(loc), depot_index])
   # 计算所有节约值
   savings = []
   for i, j in combinations([idx for idx in range(len(locations)) if idx != depot_index], 2):
       saving = distance_matrix[depot_index][i] + distance_matrix[depot_index][j] - distance_matrix[i][j]
       savings.append((saving, i, j))
   # 按节约值从大到小排序
   savings.sort(reverse=True, key=lambda x: x[0])
   # 合并路线
   for saving, i, j in savings:
       # 找到包含 i 和 j 的路线
       route_i = None
       route_j = None
       for route in routes:
           if i in route and j not in route and route.index(i) != 0 and route.index(i) != len(route)-1:
               route_i = route
           if j in route and i not in route and route.index(j) != 0 and route.index(j) != len(route)-1:
               route_j = route
       # 如果两条路线可以合并
       if route_i is not None and route_j is not None and len(routes) > num_vehicles:
           # 合并路线
           new_route = []
           if route_i[-1] == depot_index and route_j[0] == depot_index:
               new_route = route_i[:-1] + route_j[1:]
           elif route_i[0] == depot_index and route_j[-1] == depot_index:
```

```
new_route = route_j[:-1] + route_i[1:]
           if new_route:
               routes.remove(route_i)
               routes.remove(route_j)
               routes.append(new_route)
    # 计算每条路线的总距离
   route_details = []
   total_distance = 0
   for route in routes:
       route distance = 0
       for i in range(len(route)-1):
           route_distance += distance_matrix[route[i]][route[i+1]]
       total_distance += route_distance
       # 转换为目标点名称
       route_names = [locations[idx] for idx in route]
       route_details.append({
           'path': route_names,
           'distance': route_distance,
           'covered': [loc for loc in route_names if loc != 'depot']
       })
   return route_details, total_distance
# 可视化结果
def plot_routes(points, routes):
   plt.figure(figsize=(10, 8))
    # 绘制所有点
   for name, coord in points.items():
       if name == 'depot':
           plt.plot(coord[0], coord[1], 'ro', markersize=10, label='Depot')
           plt.plot(coord[0], coord[1], 'bo', markersize=8, label=name)
           plt.text(coord[0]+20, coord[1]+20, name, fontsize=12)
    # 绘制路线
   colors = ['g', 'm', 'c', 'y', 'k']
   for i, route in enumerate(routes):
       path = route['path']
       x_coords = [points[loc][0] for loc in path]
       y_coords = [points[loc][1] for loc in path]
       plt.plot(x_coords, y_coords, colors[i % len(colors)],
                linestyle='-', linewidth=2,
                label=f'UAV {i+1}: {route["distance"]:.1f}m')
   plt.xlabel('X (m)')
   plt.ylabel('Y (m)')
   plt.title('UAV Path Planning ')
   plt.legend()
   plt.grid(True)
   plt.show()
```

# 主程序

```
if __name__ == "__main__":
        num uavs = 3
         # 使用节约算法求解
        routes, total_distance = clarke_wright_savings(points, num_uavs)
         print("Optimal UAV Path Assignment:")
         for i, route in enumerate(routes):
                  print(f"UAV {i+1}:")
                  print(f" Path: {' -> '.join(route['path'])}")
                  print(f" Distance: {route['distance']:.1f} meters")
                  print(f" Covered targets: {', '.join(route['covered'])}")
                  print()
         print(f"Total flight distance: {total_distance:.1f} meters")
         # 验证所有目标点是否被覆盖
         all_covered = set()
         for route in routes:
                  all_covered.update(route['covered'])
         if len(all_covered) == len(points)-1: # 減去 depot
                  print("All targets are covered!")
         else:
                  missing = set(points.keys()) - {'depot'} - all_covered
                  print(f"Warning: Missing targets - {', '.join(missing)}")
         # 可视化结果
         plot_routes(points, routes)
T2: 动态避障与 A* 重规划
import math
import heapq
import matplotlib.pyplot as plt
from matplotlib.patches import Circle
class DronePathPlanner:
        def __init__(self, drones, targets, obstacles=None):
                  self.drones = drones # 无人机初始位置和状态
                  self.targets = targets # 目标点字典 \{24 4 \{24\} 4 \{24\} 4 \{24\} 4 \{24\} 4 \{24\} 5 \{24\} 6 \{24\} 6 \{24\} 7 \{24\} 8 \{24\} 8 \{24\} 8 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 9 \{24\} 
                  self.obstacles = obstacles if obstacles else [] #障碍物列表 [(x,y,radius)]
                  # 记录路径和分配方案
                  self.paths = {drone_id: [pos] for drone_id, pos in drones.items()}
                  self.allocations = {drone_id: [] for drone_id in drones}
         def euclidean_distance(self, a, b):
                  return math.sqrt((a[0] - b[0]) ** \frac{2}{2} + (a[1] - b[1]) ** \frac{2}{2})
         def a_star(self, start, goal, drone_id, current_time):
```

"""A\* 算法实现避障路径规划"""

```
def heuristic(pos):
   return self.euclidean_distance(pos, goal)
# 定义网格大小和分辨率
grid_size = 2000 # 整个区域大小
resolution = 50 # 网格分辨率
open_set = []
\verb|heapq.heappush(open_set, (0 + heuristic(start), 0, start, [start])||\\
closed_set = set()
while open_set:
    _, g, current, path = heapq.heappop(open_set)
   if current in closed_set:
       continue
   closed_set.add(current)
    # 检查是否到达目标
   if self.euclidean_distance(current, goal) < resolution:</pre>
       return path + [goal]
    # 生成邻近节点
   for dx in [-resolution, 0, resolution]:
       for dy in [-resolution, 0, resolution]:
           if dx == 0 and dy == 0:
               continue
           neighbor = (current[0] + dx, current[1] + dy)
           # 检查边界
           if not (
               0 <= neighbor[0] <= grid_size and 0 <= neighbor[1] <= grid_size</pre>
               continue
           # 检查障碍物碰撞
           collision = False
           for ox, oy, radius in self.obstacles:
               if (
                   self.euclidean_distance(neighbor, (ox, oy)) < radius + 50</pre>
               ): # 50m 安全距离
                   collision = True
           if collision:
               continue
           # 检查与其他无人机的安全距离
           safe = True
           for other_drone, other_pos in self.drones.items():
               if other_drone != drone_id:
                   dist = self.euclidean_distance(neighbor, other_pos)
                   if dist < 50: # 最小间距约束
                       safe = False
                       break
```

```
if not safe:
                   continue
               new_g = g + self.euclidean_distance(current, neighbor)
               heapq.heappush(
                   open_set,
                   (
                      new_g + heuristic(neighbor),
                      new_g,
                      neighbor,
                      path + [neighbor],
                   ),
               )
   return None # 没有找到路径
def assign_targets(self):
    """ 初始目标分配 (简单最近邻方法) """
   unassigned = set(self.targets.keys())
   while unassigned:
       for drone_id, pos in self.drones.items():
           if not unassigned:
               break
           # 找到最近未分配目标
           nearest = min(
               unassigned,
               key=lambda t: self.euclidean_distance(pos, self.targets[t]),
           {\tt self.allocations[drone\_id].append(nearest)}
           unassigned.remove(nearest)
           # 更新无人机位置到目标点
           self.drones[drone_id] = self.targets[nearest]
           self.paths[drone_id].append(self.targets[nearest])
def dynamic_replan(self, new_target, new_obstacle, current_time):
    """ 动态重规划处理新目标和障碍"""
    #添加新障碍
   self.obstacles.append(new_obstacle)
   #添加新目标
   new_target_name = f"T{len(self.targets)+1}"
   self.targets[new_target_name] = new_target
   # 找到最适合处理新目标的无人机 (最近且满足约束)
   best_drone = None
   min_dist = float("inf")
   for drone_id, pos in self.drones.items():
       dist = self.euclidean_distance(pos, new_target)
       if dist < min_dist:</pre>
           # 检查路径是否可行
           path = self.a_star(pos, new_target, drone_id, current_time)
           if path:
```

```
min_dist = dist
              best_drone = drone_id
               best_path = path
   if best_drone:
       # 分配新目标给最佳无人机
       {\tt self.allocations[best\_drone].append(new\_target\_name)}
       self.paths[best_drone].extend(best_path[1:]) # 跳过第一个点(当前位置)
       self.drones[best_drone] = new_target
       # 重新规划该无人机的返航路径(如果需要)
       return_path = self.a_star(new_target, (0, 0), best_drone, current_time)
       if return_path:
           self.paths[best_drone].extend(return_path[1:])
   # 对其他无人机检查是否需要避障
   for drone_id, pos in self.drones.items():
       if drone_id == best_drone:
           continue
       current_path = self.paths[drone_id]
       if len(current_path) > 1:
           next_point = current_path[-1] # 假设无人机正在前往的下一个点
           # 检查路径是否穿过障碍
           needs_replan = False
           for ox, oy, radius in self.obstacles:
               # 简单线段与圆相交检测
               if self.line_circle_intersection(pos, next_point, (ox, oy), radius):
                  needs_replan = True
                  break
           if needs_replan:
              new_path = self.a_star(pos, next_point, drone_id, current_time)
                  self.paths[drone_id] = current_path[:-1] + new_path[1:]
def line_circle_intersection(self, p1, p2, center, radius):
   """ 检查线段是否与圆相交"""
   # 线段参数方程: p = p1 + t*(p2-p1), t[0,1]
   # 圆心到线段的距离
   x1, y1 = p1
   x2, y2 = p2
   cx, cy = center
   # 向量化计算
   dx = x2 - x1
   dy = y2 - y1
   12 = dx * dx + dy * dy
   # 线段是点的情况
   if 12 == 0:
       return self.euclidean_distance(p1, center) <= radius</pre>
```

```
# 计算投影参数 t
   t = ((cx - x1) * dx + (cy - y1) * dy) / 12
   t = \max(0, \min(1, t))
   # 投影点
   projection = (x1 + t * dx, y1 + t * dy)
   # 检查距离
   return self.euclidean_distance(projection, center) <= radius</pre>
def visualize(self):
   """ 可视化结果, 风格与 T1 一致"""
   plt.figure(figsize=(10, 8))
   #绘制所有点
   for name, coord in self.targets.items():
       if name == "T1": # 只为第一个目标点加 label, 防止重复
           plt.plot(coord[0], coord[1], "bo", markersize=8, label=name)
       else:
           plt.plot(coord[0], coord[1], "bo", markersize=8)
       plt.text(coord[0] + 20, coord[1] + 20, name, fontsize=12)
   plt.plot(0, 0, "ro", markersize=10, label="Depot")
   #绘制障碍物
   for x, y, r in self.obstacles:
       circle = Circle((x, y), r, color="gray", alpha=0.5)
       plt.gca().add_patch(circle)
   # 绘制无人机路径
   colors = ["g", "m", "c", "y", "k"]
   for i, (drone_id, path) in enumerate(self.paths.items()):
       if len(path) > 1:
           x_coords = [p[0] for p in path]
           y_coords = [p[1] for p in path]
           plt.plot(
               x_coords,
               y_coords,
               colors[i % len(colors)],
               linestyle="-",
               linewidth=2,
               label=f"UAV {i+1}: {sum(self.euclidean_distance(path[j], path[j+1]) for j in range(len(path)-1)):.1f}m",
           )
   plt.xlabel("X (m)")
   plt.ylabel("Y (m)")
   plt.title("UAV Path Planning ")
   plt.legend()
   plt.grid(True)
   plt.axis("equal")
   plt.show()
def animate(self, interval=500):
    """ 动态展示无人机路径规划过程 (与 T1 风格一致) """
   import matplotlib.animation as animation
   fig, ax = plt.subplots(figsize=(10, 8))
   #绘制所有点
   for name, coord in self.targets.items():
       if name == "T1":
           ax.plot(coord[0], coord[1], "bo", markersize=8, label=name)
```

else:

```
ax.plot(coord[0], coord[1], "bo", markersize=8)
            ax.text(coord[0] + 20, coord[1] + 20, name, fontsize=12)
        ax.plot(0, 0, "ro", markersize=10, label="Depot")
        # 绘制障碍物
        for x, y, r in self.obstacles:
            circle = Circle((x, y), r, color="gray", alpha=0.5)
            ax.add_patch(circle)
        colors = ["g", "m", "c", "y", "k"]
        max_len = max(len(path) for path in self.paths.values())
        lines = []
        points = []
        for i, (drone_id, path) in enumerate(self.paths.items()):
            (line,) = ax.plot(
                [],
                Π.
                colors[i % len(colors)],
                linestyle="-",
                linewidth=2,
                label=f"UAV {i+1}",
            )
            (point,) = ax.plot(
                [], [], marker="o", color=colors[i % len(colors)], markersize=10
            lines.append(line)
            points.append(point)
        ax.set_xlabel("X (m)")
        ax.set_ylabel("Y (m)")
        ax.set_title("UAV Path Planning (Animation)")
        ax.legend()
        ax.grid(True)
        ax.axis("equal")
        def update(frame):
            for i, (drone_id, path) in enumerate(self.paths.items()):
                if frame < len(path):</pre>
                    x = [p[0] \text{ for } p \text{ in path}[: frame + 1]]
                    y = [p[1] for p in path[: frame + 1]]
                    lines[i].set_data(x, y)
                    points[i].set_data([x[-1]], [y[-1]]) # 修正: 必须传序列
                else:
                    x = [p[0] \text{ for } p \text{ in path}]
                    y = [p[1] \text{ for } p \text{ in path}]
                    lines[i].set_data(x, y)
                    points[i].set_data([x[-1]], [y[-1]]) # 修正: 必须传序列
            return lines + points
        ani = animation.FuncAnimation(
            fig, update, frames=max_len, interval=interval, blit=True, repeat=False
        plt.show()
#问题 2 的实例数据
initial_drones = {"U1": (0, 0), "U2": (0, 0), "U3": (0, 0)}
```

```
targets = {
   "T1": (1200, 800),
   "T2": (300, 450),
   "T3": (950, 200),
   "T4": (600, 1200),
   "T5": (1500, 500),
}
# 创建路径规划器
planner = DronePathPlanner(initial_drones, targets)
# 初始目标分配
planner.assign_targets()
# 模拟在 100s 时新增障碍和紧急目标
new_obstacle = (900, 250, 100) # (x, y, radius)
new_target = (800, 600)
current_time = 100 # 假设在 100s 时发生动态变化
# 动态重规划
planner.dynamic_replan(new_target, new_obstacle, current_time)
print("调整后的路径规划:")
for drone_id, path in planner.paths.items():
   print(f"{drone_id}路径:", " → ".join([f"({x},{y})" for x, y in path]))
   print(
       f" 总飞行距离: {sum(planner.euclidean_distance(path[i], path[i+1]) for i in range(len(path)-1)):.1f}m"
   print(f" 分配的目标点: {planner.allocations[drone_id]}")
   print()
# 计算最短完成时间
flight_times = []
for drone_id, path in planner.paths.items():
   distance = sum(
       planner.euclidean_distance(path[i], path[i + 1]) for i in range(len(path) - 1)
   time = distance / 50 # 假设最大速度 50m/s
   flight_times.append(time)
shortest_time = max(flight_times)
print(f"T1-T6 全覆盖的最短时间: {shortest_time:.1f}s")
# 可视化
planner.visualize()
# 动态可视化
planner.animate(interval=500)
```

# T3: 多优先级任务分配与分层贪心

```
import math
import numpy as np
from collections import defaultdict
```

```
import matplotlib.pyplot as plt
class Drone:
   def __init__(self, name, max_load, max_endurance, hover_time):
       self.name = name
       self.max_load = max_load
       self.max_endurance = max_endurance
       self.hover_time = hover_time
       self.current_load = 0
       self.remaining_endurance = max_endurance
       self.mission_log = []
       self.position = (0, 0) # 起始位置为基地 (0,0)
       self.total_distance = 0
   def can_assign(self, task):
       """ 检查是否能分配任务"""
       if task.task_type in ['紧急投放', '普通投放']:
           return (self.current_load + task.requirement <= self.max_load and</pre>
                   self.remaining_endurance > 0)
       else: # 侦察任务
           return self.remaining_endurance > task.requirement
   def assign_task(self, task):
       """ 分配任务给无人机"""
       distance = self.calculate_distance(task.location)
       flight_time = distance / 50 # 假设飞行速度 50m/s
       if task.task_type in ['紧急投放', '普通投放']:
           self.current_load += task.requirement
           time_required = flight_time
       else: # 侦察任务
           time_required = flight_time + task.requirement
       # 检查是否需要充电
       if self.remaining_endurance < time_required + self.calculate_distance((0,0)) / 50:
           self.return_to_base()
       self.remaining_endurance -= time_required
       self.total_distance += distance
       self.position = task.location
       {\tt self.mission\_log.append(\{}
           'task': task,
           'start_time': self.current_time(),
           'end_time': self.current_time() + time_required,
            'distance': distance
       })
   def return_to_base(self):
       """ 返回基地充电"""
       distance = self.calculate_distance((0,0))
       time_required = distance / 50 + 60 # 飞行时间 + 充电时间
       self.remaining_endurance = self.max_endurance
       self.current_load = 0
       self.total_distance += distance
       self.position = (0, 0)
```

```
self.mission_log.append({
           'task': None,
           'description': 'Return to base and charge',
           'start_time': self.current_time(),
           'end_time': self.current_time() + time_required,
           'distance': distance
       })
   def calculate_distance(self, target):
       """ 计算当前位置到目标的距离"""
       return math.sqrt((self.position[0]-target[0])**2 + (self.position[1]-target[1])**2)
   def current_time(self):
       """ 计算当前累计任务时间"""
       if not self.mission_log:
       return self.mission_log[-1]['end_time']
class Task:
   def __init__(self, task_id, task_type, location, priority, requirement):
       self.task_id = task_id
       self.task_type = task_type
       self.location = location
       self.priority = priority
       self.requirement = requirement
       self.assigned = False
def generate_tasks():
   """ 生成模拟任务"""
   tasks = [
       Task(1, '紧急投放', (500, 800), 1, 5),
       Task(2, '普通投放', (1500, 200), 2, 10),
       Task(3, '侦察任务', (200, 1500), 3, 20),
       Task(4, '紧急投放', (1200, 1000), 1, 8),
       Task(5, '普通投放', (300, 600), 2, 7),
       Task(6, '侦察任务', (1800, 700), 3, 30),
       Task(7, '紧急投放', (100, 1000), 1, 6),
       Task(8, '普通投放', (1600, 400), 2, 9),
       Task(9, '侦察任务', (800, 300), 3, 25),
       Task(10, '普通投放', (1400, 900), 2, 11)
   1
   return tasks
def assign_tasks(drones, tasks):
   """ 任务分配主算法,严格按照 T3.md 的分层贪心思想"""
   # 按优先级分组
   priority_groups = defaultdict(list)
   for task in tasks:
       priority_groups[task.priority].append(task)
   #优先级 1: 每个无人机分配一个紧急任务
   urgent_tasks = priority_groups.get(1, [])
   for i, task in enumerate(urgent_tasks):
       if i < len(drones):</pre>
           drones[i].assign_task(task)
           task.assigned = True
    #优先级 2: 普通投放, 分三类, 遍历所有分配方案, 选最短完成时间
```

```
normal_tasks = priority_groups.get(2, [])
   from itertools import permutations
   best_perm = None
   best_time = float('inf')
   if len(normal_tasks) == len(drones):
       for perm in permutations(normal_tasks):
           # 复制无人机状态
           drones_copy = [Drone(d.name, d.max_load, d.max_endurance, d.hover_time) for d in drones]
           for i, task in enumerate(perm):
               drones_copy[i].position = drones[i].position
               drones_copy[i].remaining_endurance = drones[i].remaining_endurance
               drones_copy[i].current_load = drones[i].current_load
               drones_copy[i].mission_log = list(drones[i].mission_log)
               drones_copy[i].total_distance = drones[i].total_distance
               drones_copy[i].assign_task(task)
           finish_time = max(d.current_time() for d in drones_copy)
           if finish_time < best_time:</pre>
               best_time = finish_time
               best_perm = perm
       # 按最佳分配方案分配
       for i, task in enumerate(best_perm):
           drones[i].assign_task(task)
           task.assigned = True
   else:
       #普通贪心分配
       for task in normal_tasks:
           best_drone = min(drones, key=lambda d: d.current_time())
           best_drone.assign_task(task)
           task.assigned = True
    #优先级 3: 侦察任务, 遍历所有无人机, 分配给完成普通任务后最早空闲的无人机
   scout_tasks = priority_groups.get(3, [])
   for task in scout_tasks:
       best_drone = None
       best_time = float('inf')
       for drone in drones:
           # 计算该无人机完成普通任务后到侦察点的时间
           temp_drone = Drone(drone.name, drone.max_load, drone.max_endurance, drone.hover_time)
           temp_drone.position = drone.position
           temp_drone.remaining_endurance = drone.remaining_endurance
           temp_drone.current_load = drone.current_load
           temp_drone.mission_log = list(drone.mission_log)
           temp_drone.total_distance = drone.total_distance
           temp_drone.assign_task(task)
           finish_time = temp_drone.current_time()
           if finish_time < best_time:</pre>
               best_time = finish_time
               best_drone = drone
       if best_drone:
           best_drone.assign_task(task)
           task.assigned = True
    # 处理未分配的任务(可能由于约束无法分配)
   unassigned = [t for t in tasks if not t.assigned]
   if unassigned:
       print(f" 警告: {len(unassigned)}个任务未能分配")
def visualize_schedule(drones):
```

```
""" 可视化任务调度结果, 风格与 T2/T1 统一"""
   plt.figure(figsize=(10, 8))
   # 绘制所有任务点
   all_points = [(0, 0)]
   for drone in drones:
       for mission in drone.mission_log:
           if mission['task']:
               all_points.append(mission['task'].location)
   all_points = list(set(all_points))
   for idx, (x, y) in enumerate(all_points):
       if (x, y) == (0, 0):
           plt.plot(x, y, 'ro', markersize=10, label='Depot')
       else:
           if idx == 1:
               plt.plot(x, y, 'bo', markersize=8, label='Target')
               plt.plot(x, y, 'bo', markersize=8)
           plt.text(x+20, y+20, f'(\{x\},\{y\})', fontsize=12)
   # 绘制无人机路径
   colors = ['g', 'm', 'c', 'y', 'k']
   for i, drone in enumerate(drones):
       x = [0]
       y = [0]
       for mission in drone.mission_log:
           if mission['task']:
               x.append(mission['task'].location[0])
               y.append(mission['task'].location[1])
       plt.plot(x, y, colors[i % len(colors)], linestyle='-', linewidth=2, label=f'UAV {i+1}: {sum([math.sqrt((x[j]-x[j-1])
   plt.xlabel('X (m)')
   plt.ylabel('Y (m)')
   plt.title('UAV Path Planning ')
   plt.legend()
   plt.grid(True)
   plt.axis('equal')
   plt.show()
def print_results(drones, tasks):
   """ 打印结果统计"""
   print("\n=== 任务分配结果 ===")
   for drone in drones:
       print(f"\n{drone.name}:")
       print(f" 总飞行距离: {drone.total_distance:.1f}m")
       print(f" 剩余续航时间: {drone.remaining_endurance:.1f}s")
       print(" 任务序列:")
       for mission in drone.mission_log:
           if mission['task']:
               print(f" 任务{mission['task'].task_id}: {mission['task'].task_type} "
                     f" 在 ({mission['task'].location[0]},{mission['task'].location[1]}), "
                     f" 时间{mission['start_time']:.1f}-{mission['end_time']:.1f}s")
           else:
               print(f" 返回基地充电: {mission['start_time']:.1f}-{mission['end_time']:.1f}s")
   completion_times = [d.current_time() for d in drones]
   print(f"\n总任务完成时间: {max(completion_times):.1f}s")
   unassigned = [t for t in tasks if not t.assigned]
```

```
if unassigned:
       print(f"\n未分配任务: {[t.task_id for t in unassigned]}")
def main():
   # 初始化无人机
   drones = [
       Drone('U1', 15, 500, 30),
       Drone('U2', 10, 600, 60),
       Drone('U3', 20, 450, 20)
   ]
   # 生成任务
   tasks = generate_tasks()
   # 分配任务
   assign_tasks(drones, tasks)
   # 输出结果
   print_results(drones, tasks)
   # 可视化
   visualize_schedule(drones)
if __name__ == "__main__":
   main()
```