Assignment07

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[Apply K-means algorithm to both image value and its spatial domain]

For a given input image (either gray or color), apply a K-means algorithm that is designed to take into consideration of both the image intensity and its spatial domain with varying parameters: the number of clusters and the trade-off between the intensity energy and the spatial energy.

The objective function is given by:

$$\sum_k \sum_{\{} x \in I(k)\}[\|f(x) - m_k\|^2 + a * \|x - c_k\|^2]$$

where I(k) denotes the index set of x that belongs to cluster k, m_k denotes the centroid of image intensity for cluster k, c_k denotes the centroid of spatial location for cluster k, and a determines the importance between the image intensity and the spatial relation.

- Visualize the clustering results with varying k and a using the centroid color m_k for each cluster k.
- Visualize the energy curve for both the intensity energy and the spatial energy.

1 Initial array

```
In [1]: import matplotlib.pyplot as plt
        import numpy as np
        import random
        import cv2
        import math

        image = cv2.imread("keyring.jpg")
        im_color = cv2.cvtColor(image, cv2.COLOR_BGR2RGB)
        height, width = im_color.shape[:2]
        pix_num = height * width

        x = np.zeros((height, width), dtype = float)
        y = np.zeros((height, width), dtype = float)

        r = im_color[:,:,0]
        g = im_color[:,:,1]
        b = im_color[:,:,2]
```

```
g = np.zeros((height, width), dtype = float)
b = np.zeros((height, width), dtype = float)

for i in range(height):
    for j in range(width):
        x[i][j] = i
        y[i][j] = j

im_label = np.zeros((height, width), dtype = int)

E1 = []
E2 = []
E3 = []
```

2 Initial Image

In [2]:
```
plt.title('Original Image')
plt.imshow(im_color)
plt.axis('off')
plt.show()
```

Original Image

3 Nomalization

```
In [3]: def normalize(x,y):

            dx = np.zeros((height, width), dtype=float)
            dy = np.zeros((height, width), dtype=float)

            for i in range(height):
                for j in range(width):
                    dx[i,j] = i/height
                    dy[i,j] = j/width

            return dx, dy
```

4 Whitening

$g(x) = \frac{f(x)-\mu}{\sigma}, \mu = \frac{\sum f(x_i)}{|\Omega|}, \sigma^2 = \frac{\sum(f(x_i)-\mu)^2}{\Omega}$

```
In [4]: def whitening(data):
            mu = [0,0,0]
            sigma = [0,0,0]
            data_whitening = im_color.copy()

            r_mu = np.sum(data[:,:,0]) / pix_num
            g_mu = np.sum(data[:,:,1]) / pix_num
            b_mu = np.sum(data[:,:,2]) / pix_num
            mu = [r_mu, g_mu, b_mu]

            r_std = math.sqrt((np.sum((data[:,:,0] - r_mu)**2)) / pix_num)
            g_std = math.sqrt((np.sum((data[:,:,1] - g_mu)**2)) / pix_num)
            b_std = math.sqrt((np.sum((data[:,:,2] - b_mu)**2)) / pix_num)
            sigma = [r_std, g_std, b_std]

            data_whitening[:,:,0] = (data[:,:,0] - r_mu) / r_std
            data_whitening[:,:,1] = (data[:,:,1] - g_mu) / g_std
            data_whitening[:,:,2] = (data[:,:,2] - b_mu) / b_std

            return data_whitening, mu, sigma
```

5 Compute distance based on L2-norm square

$$||x-y||_2{}^2 = x^2 + y^2$$

```
In [5]: def distance(x,y):
            d = (x - y) ** 2
            s = np.sum(d)
```

```
        return s
```

6 Initialize Label

initialize all images with random label

```
In [6]: def initialLabel(cluster_num):
            for i in range(height):
                for j in range(width):
                    initial_num = random.randrange(0, cluster_num)
                    im_label[i][j] = initial_num

            return im_label
```

7 Initialize Centroid

```
In [7]: def initialCentroid(im_average_x, im_average_y, im_average,cluster_num, im_label, im_co
            for i in range(height):
                for j in range(width):
                    im_average_x[im_label[i,j]] += x[i][j]
                    im_average_y[im_label[i,j]] += y[i][j]
                    im_average[im_label[i,j]] += im_color[i,j]
                    im_count[im_label[i,j]] += 1


            for i in range(cluster_num):
                im_average_x[i] /= im_count[i]
                im_average_y[i] /= im_count[i]
                im_average[i] /= im_count[i]

            return im_average_x, im_average_y, im_average, im_count
```

8 Plot the Final Image

```
In [8]: def plot_average(im_average, cluster_num, mean, std):
            f1 = plt.figure(1)
            new_image = im_color.copy()

            for i in range(height) :
                for j in range(width):
                    new_image[i,j] = im_average[im_label[i,j]] * std + mean


            plt.title('Final Image')
            plt.imshow(new_image)
```

```
    plt.axis('off')
    plt.show()
```

9 Clustering

```
In [9]: def clustering(im_label, cluster_num, im_average_x, im_average_y, im_average, alpha):
            dist = 0
            label = 0

            for i in range(height):
                for j in range(width):
                    mins = 1000000000
                    for k in range(cluster_num):
                        nx = x[i][j]
                        ny = y[i][j]
                        ax = im_color[i,j]
                        ay = im_average[k]
                        cx = im_average_x[k]
                        cy = im_average_y[k]
                        dist = distance(nx, cx) + distance(ny, cy)+ alpha*distance(ax,ay)
                        if dist < mins:
                            mins = dist
                            label = k
                    im_label[i][j] = label

            return im_label
```

10 Compute Centroid

```
In [10]: def computeCentroid(cluster_num, im_label):
             num = np.zeros(cluster_num, dtype = int)
             c = np.zeros((cluster_num, 3), dtype = float)
             cx = np.zeros(cluster_num, dtype = float)
             cy = np.zeros(cluster_num, dtype = float)

             for i in range(height):
                 for j in range(width):
                     cx[im_label[i][j]] += x[i][j]
                     cy[im_label[i][j]] += y[i][j]
                     c[im_label[i][j]] += im_color[i,j]
                     num[im_label[i][j]] += 1

             for k in range(cluster_num):
                 if num[k] != 0:
                     cx[k] /= num[k]
                     cy[k] /= num[k]
                     c[k] /= num[k]
```

```
            return cx, cy, c
```

11 Compute Energy

$E = \frac{1}{n} \sum_{x \in \Omega} \| f(x) - m_c \|^2$,

where Ω denotes the image domain and the number of pixels $|\Omega|$ is n, and m_c denotes the centroid for cluster c that is the cluster label of $f(x)$.

```
In [11]: def computeEnergy(im_label, cluster_num, im_average_x, im_average_y,im_average):
             total_sum1 = 0
             total_sum2 = 0
             total_sum = 0
             num = 0
             for i in range(height):
                 for j in range(width):
                     total_sum1 += distance(x[i][j], im_average_x[im_label[i][j]])
                     total_sum1 += distance(y[i][j], im_average_y[im_label[i][j]])
                     total_sum2 += distance(im_color[i,j], im_average[im_label[i,j]])
                     num += 1

             total_sum1 /= num
             total_sum2 /= num
             total_sum = total_sum1+total_sum2

             return total_sum1, total_sum2, total_sum
```

12 Initialize Cluster

```
In [12]: def initialCluster(cluster_num):
             im_average_x = np.zeros(cluster_num, dtype=float)
             im_average_y = np.zeros(cluster_num, dtype=float)
             im_average = np.zeros((cluster_num,3), dtype=float)
             im_count = np.zeros(cluster_num, dtype=int)

             im_label = initialLabel(cluster_num)

             im_average_x, im_average_y, im_average, im_count = initialCentroid(im_average_x, i

             E1.clear()
             E2.clear()
             E3.clear()

             return im_average_x, im_average_y, im_average, im_count, im_label
```

13 Clustering until no change

no change means energy is maintained

In [13]:
```
def iteration(cluster_num, alpha, im_average_x, im_average_y, im_average, im_count, im
    iter_num = 0

    while True:
        im_label = clustering(im_label, cluster_num, im_average_x, im_average_y, im_av
        im_average_x, im_average_y,im_average  = computeCentroid(cluster_num,im_label

        a,b,c = computeEnergy(im_label, cluster_num, im_average_x, im_average_y, im_av
        E1.append(a)
        E2.append(b)
        E3.append(c)

        if iter_num > 0:
            if E3[iter_num-1] == E3[iter_num]:
                break
        iter_num += 1

    plot_average(im_average,cluster_num,mean, std)

    return iter_num
```

14 Energy graph per each iteration

In [14]:
```
def drawEnergy(iter_num, name, energy):
    plt.figure(3)
    x_range = np.arange(iter_num+1)

    plt.plot(x_range, energy, "g")
    plt.title(str(name) + " Energy")
    plt.grid(True)

    plt.show()
```

15 Normalize data

In [15]:
```
x,y  = normalize(x,y)
im_color, mean, std = whitening(im_color)
```

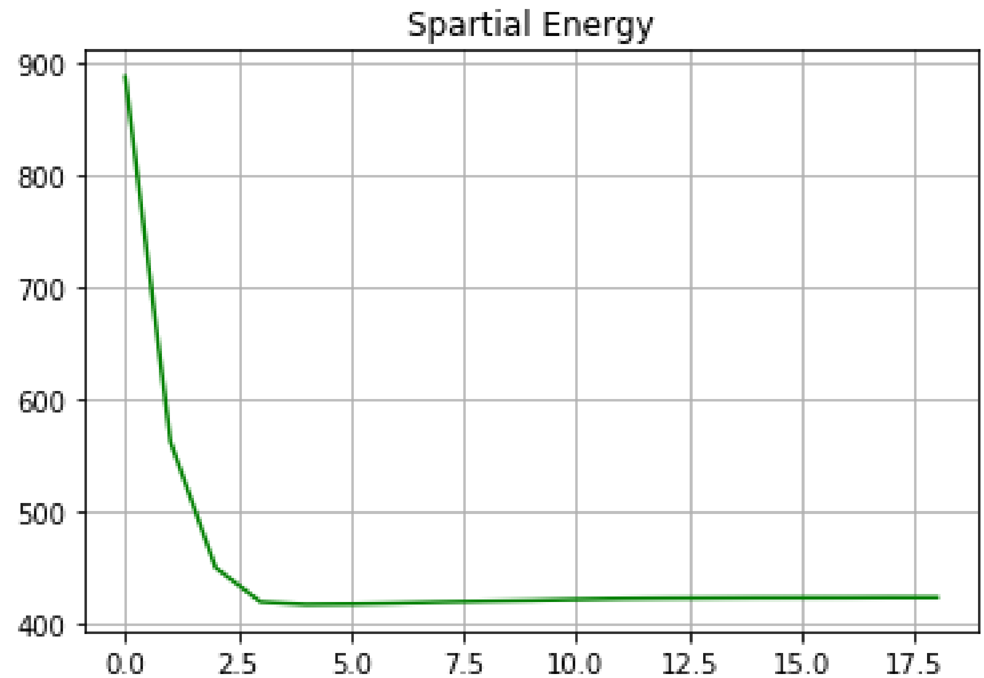
16 K = 7, a = 0.1

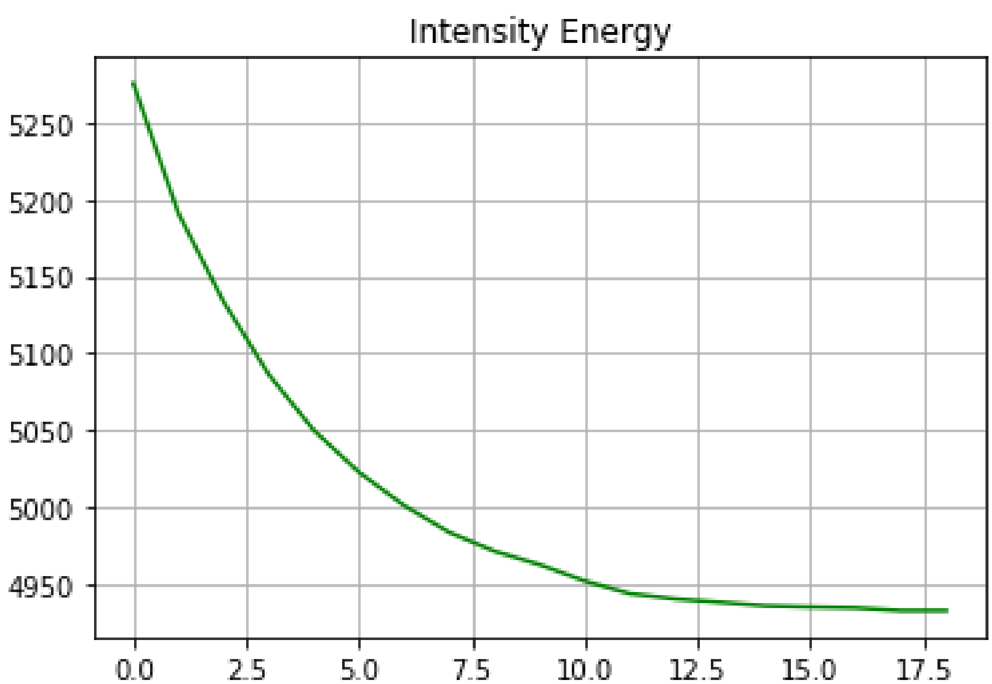
16.1 Final average image

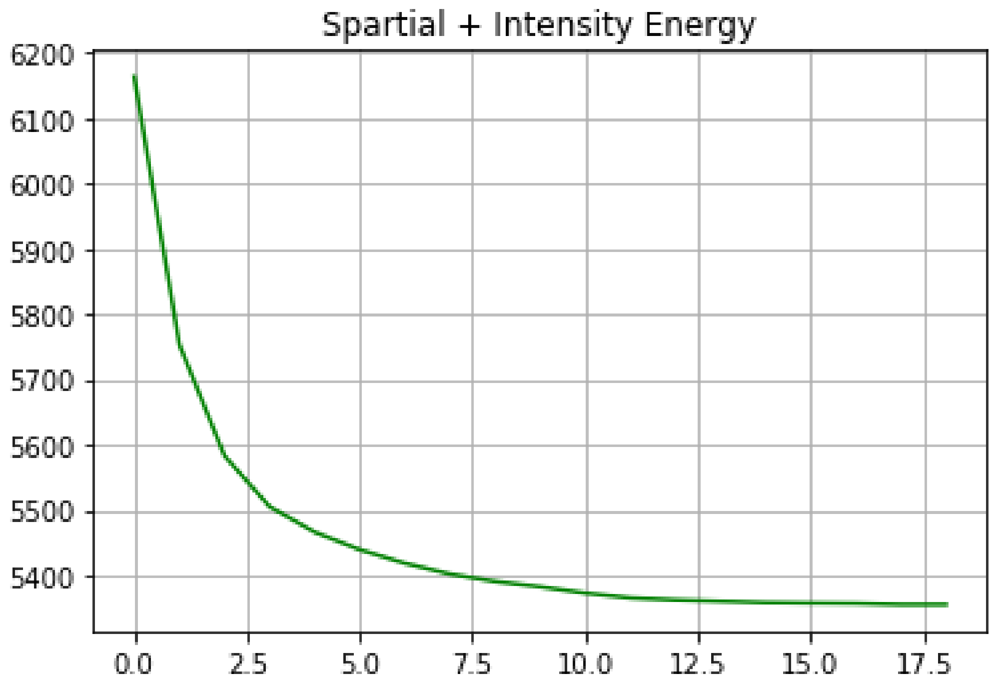
In [16]:
```
im_average_x, im_average_y,im_average, im_count,im_label = initialCluster(7)
iter_num = iteration(7,0.1,im_average_x, im_average_y, im_average, im_count, im_label
```

16.2 Energy graphs

In [17]:
```
drawEnergy(iter_num, "Spartial", E1)
drawEnergy(iter_num, "Intensity", E2)
drawEnergy(iter_num, "Spartial + Intensity", E3)
```


Spartial Energy
900
800
700
600
500
400
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5


Intensity Energy
5250
5200
5150
5100
5050
5000
4950
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5

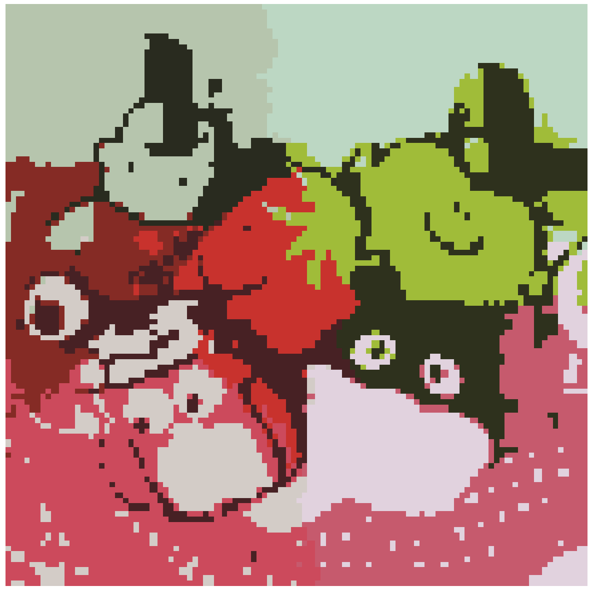


17 K = 12, a = 0.1

17.1 Final average Image

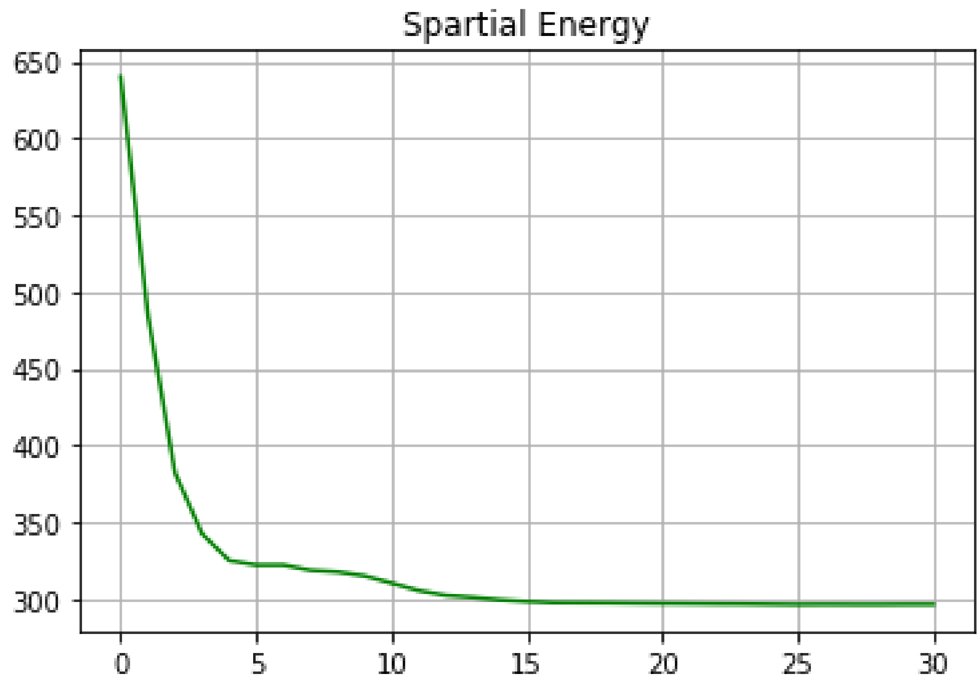
```
In [20]: im_average_x, im_average_y,im_average, im_count,im_label = initialCluster(12)
         iter_num = iteration(12,0.1,im_average_x, im_average_y, im_average, im_count, im_label
```

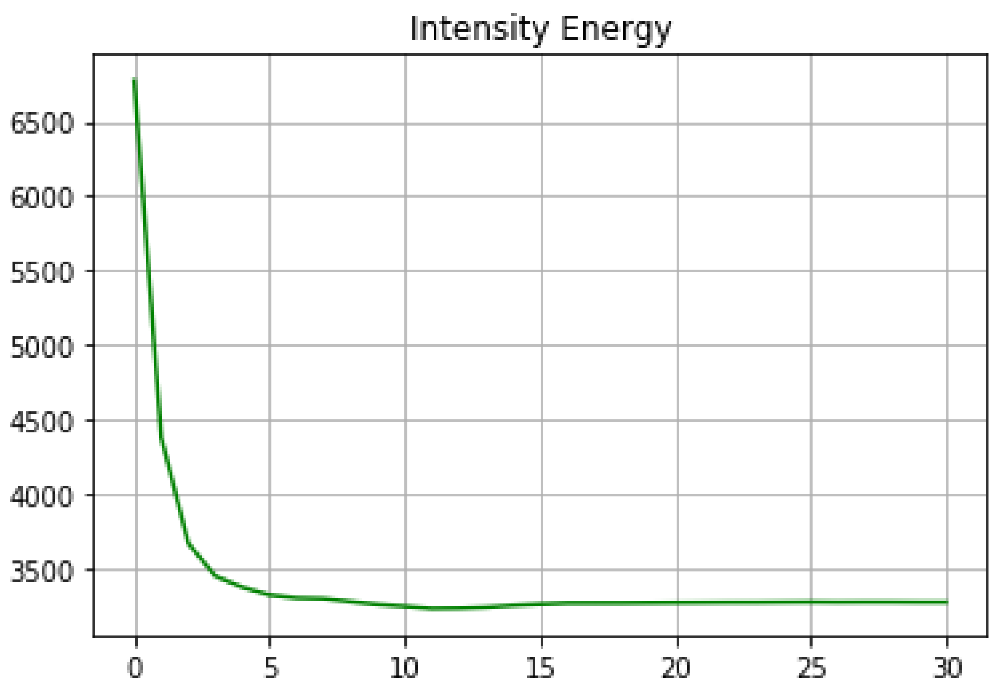
Final Image

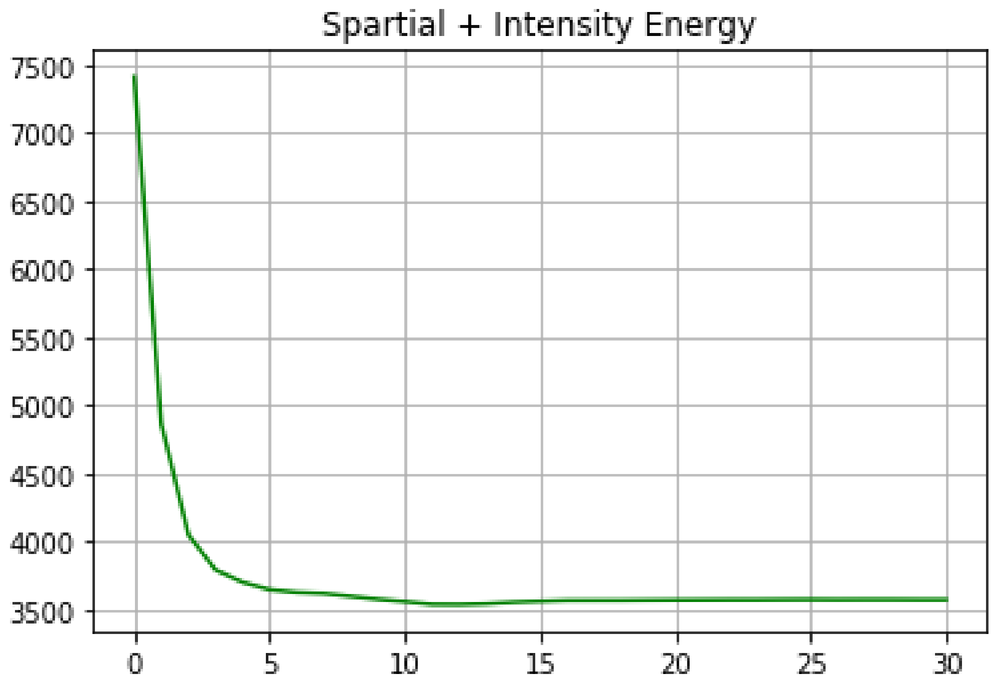


17.2 Energy graph

In [21]:
```
drawEnergy(iter_num, "Spartial", E1)
drawEnergy(iter_num, "Intensity", E2)
drawEnergy(iter_num, "Spartial + Intensity", E3)
```


Spartial Energy
650
600
550
500
450
400
350
300
0
5
10
15
20
25
30


Intensity Energy
6500
6000
5500
5000
4500
4000
3500
0
5
10
15
20
25
30



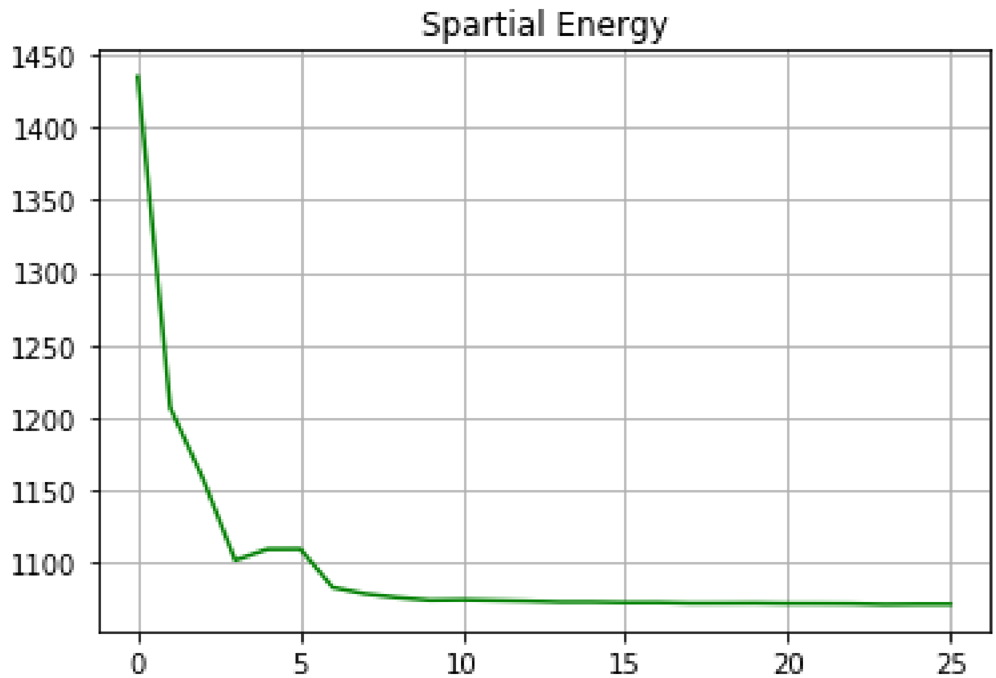
18 K = 7, a = 0.7

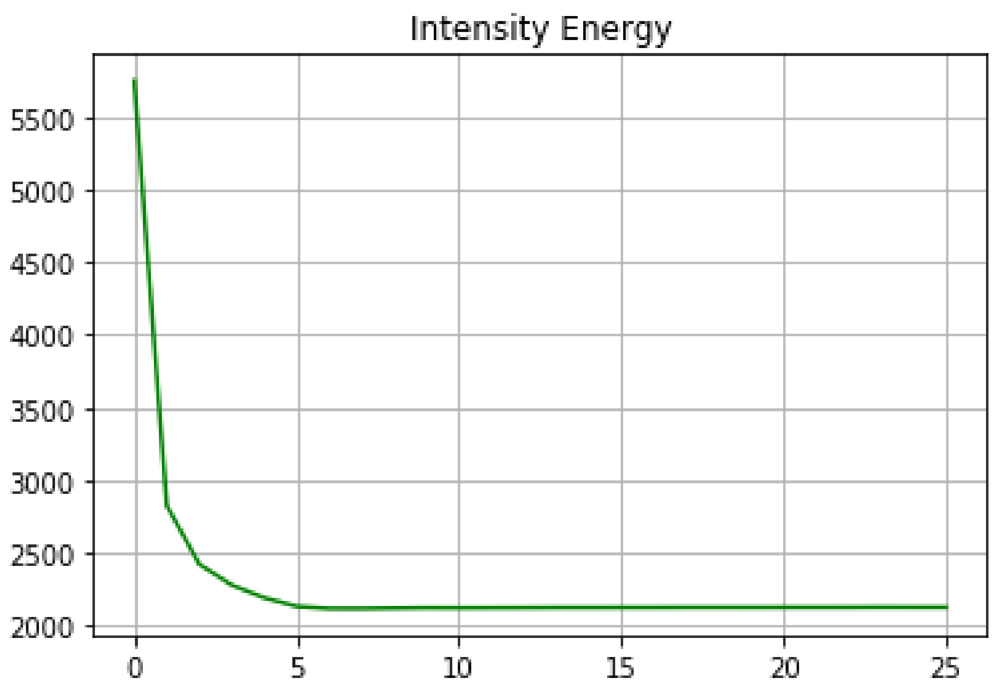
18.1 Final average image

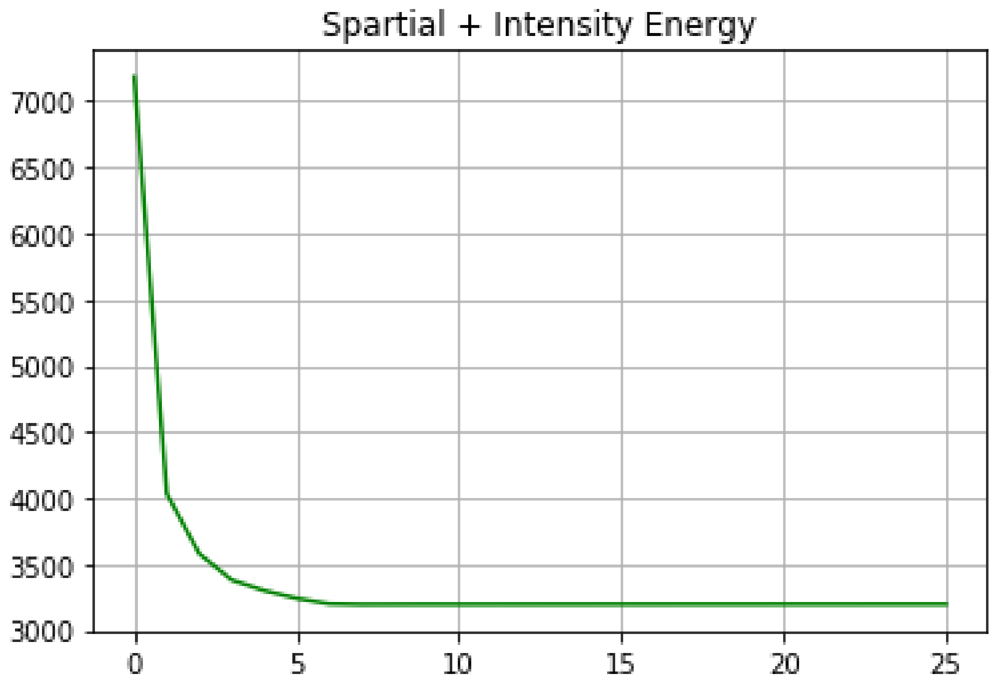
In [18]:
```
im_average_x, im_average_y,im_average, im_count,im_label = initialCluster(7)
iter_num = iteration(7,0.7,im_average_x, im_average_y, im_average, im_count, im_label
```

19 Energy graph

```
In [19]: drawEnergy(iter_num, "Spartial", E1)
         drawEnergy(iter_num, "Intensity", E2)
         drawEnergy(iter_num, "Spartial + Intensity", E3)
```


Spartial Energy
1450
1400
1350
1300
1250
1200
1150
1100
0
5
10
15
20
25


Intensity Energy
5500
5000
4500
4000
3500
3000
2500
2000
0
5
10
15
20
25



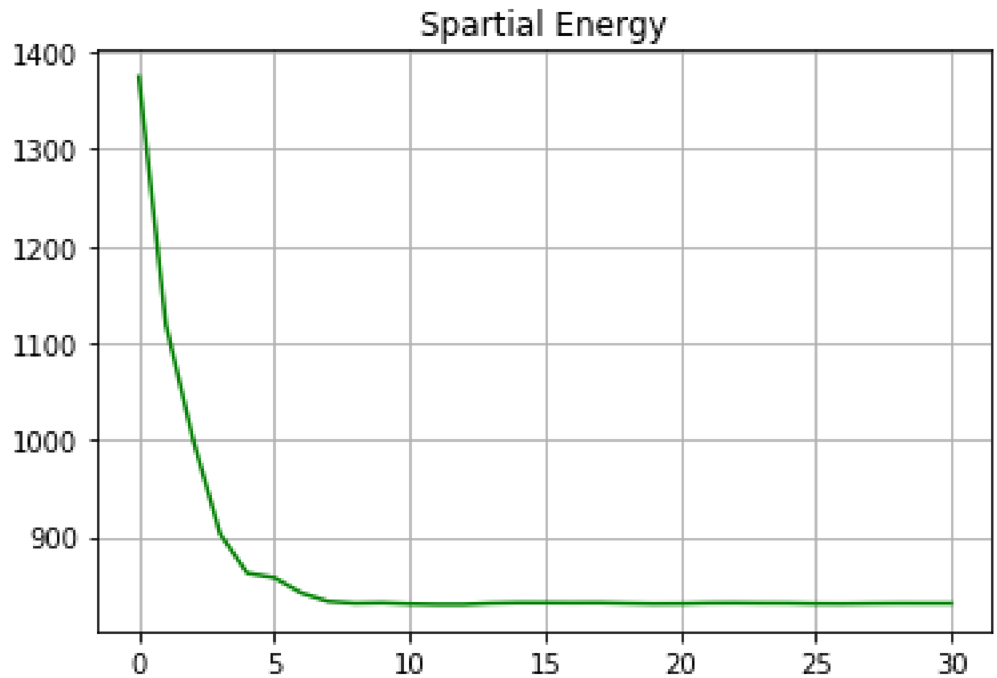
20 k=12, a = 0.7

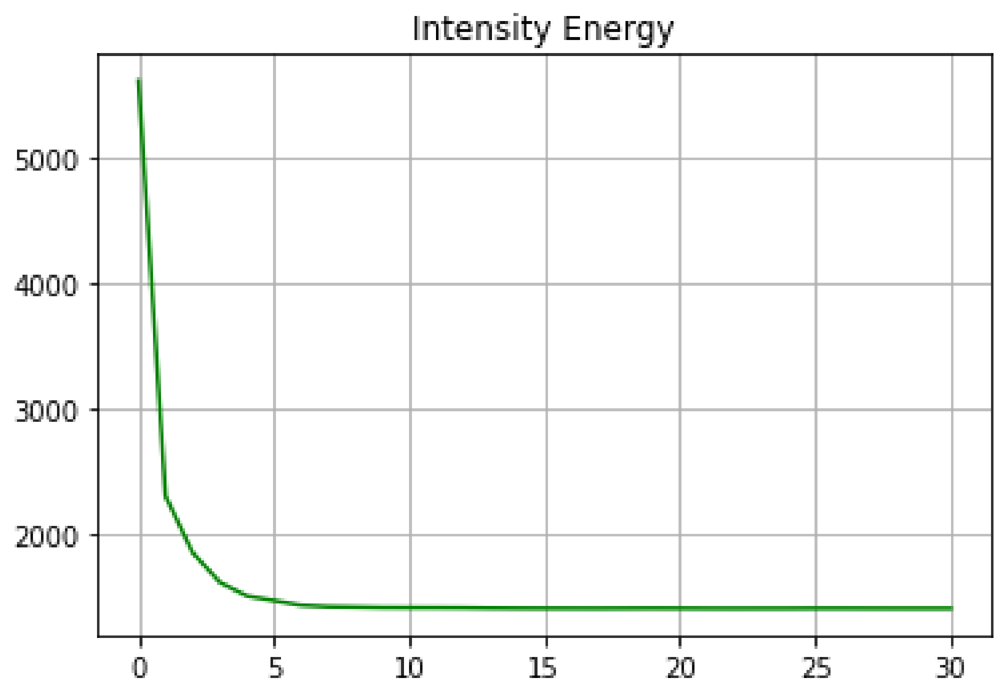
20.1 Final average Image

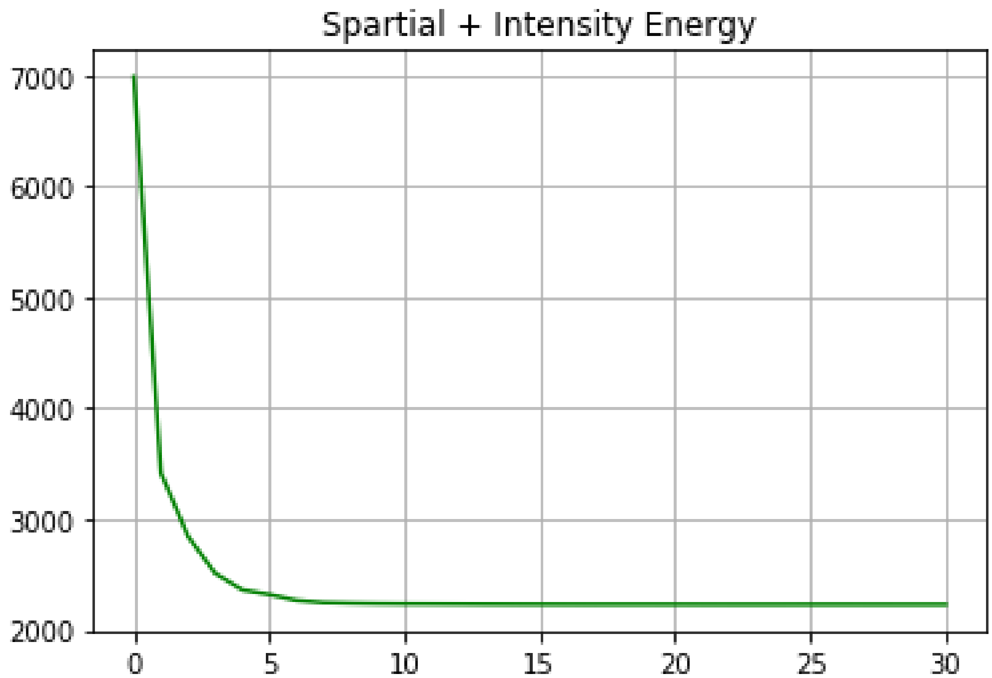
In [24]:
```
im_average_x, im_average_y,im_average, im_count,im_label = initialCluster(12)
iter_num = iteration(12,0.7,im_average_x, im_average_y, im_average, im_count, im_label
```

20.2 Energy graph

```
In [25]: drawEnergy(iter_num, "Spartial", E1)
         drawEnergy(iter_num, "Intensity", E2)
         drawEnergy(iter_num, "Spartial + Intensity", E3)
```


Spartial Energy
1400
1300
1200
1100
1000
900
0
5
10
15
20
25
30


Intensity Energy
5000
4000
3000
2000
0
5
10
15
20
25
30


Spartial + Intensity Energy
7000
6000
5000
4000
3000
2000
0
5
10
15
20
25
30