Elo Paper

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November 2017

1 The Man Himself

Arpad Elo was born in Hungary in 1903. His family moved to Milwaukee, Wisconsin in 1913 when Elo was ten years old. He later became a physics professor at Marquette University in Milwaukee. He was also an amateur chess player and won the Wisconsin State Chess Championship eight times. In 1959 Elo was appointed chairman of the United States Chess Federation (USCF) where he took on the task of creating a new ranking system of chess players and scores. Elo's system replaced the existing USCF method in 1960 and in 1970 FIDE, the World Chess Federation, adopted Elo's system. Elo manually did the rating calculations for FIDE until the mid 1980s.

2 Basic Idea

The general idea behind Elo's rating system was that if you win your rating goes up and if you lose your rating goes down. However, the amount your score goes up or down depends on the strength of your opponent. Thus,draws against stronger opponents increase your rating while draws against weaker opponents decrease your rating. It is also noteworthy that the sum of all ratings remains constant.

3 Method

K is a variable that controls volatility/stagnation. The values of K come from FIDE's July 1, 2014 handbook.

For new players with less than 30 games K=40.

K also equals 40 for all players under 18 unless they are rated 2300+.

K=20 as long as a player's rating is under 2400.

K=10 if a player's published rating is above 2400.

A different variable, ξ, controls the spread of ratings. Currently FIDE uses 400.

Elo's original assumption was that an established player's performance level is normally distributed. From here he looked at data and revised his assumption, coming to the conclusion that a logistic function using the difference between two players' ratings as input accurately predicts each player's probability. ratings, which is true regardless of the actual ratings.

The logistic function for Elo's method is as follows:

$$L(x) = \frac{1}{1 + 10^{-x}}$$

Next we let r_i be player $i's$ rating, r_j be player $j's$ rating, $d_{ij} = r_i - r_j$ be the difference of i and $j's$ ratings, and μ_{ij} is the probability that i will defeat j:

$$\mu_{ij} = L(d_{ij}, \xi) = \frac{1}{1 + 10^{\frac{-d_{ij}}{\xi}}}$$

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Once a new game is played, a new rating is calculated.

$$r_{i(new)} = r_{i(old)} + K(S_{ij} - \mu_{ij})$$

where

$$f(x) = \begin{cases} 1 & \text{if } i \text{ beats } j \\ 0 & \text{if } j \text{ beats } i \\ \frac{1}{2} & \text{if } i \text{ and } j \text{ ties} \end{cases}$$

4 Example

For this example, five teams were used.

Teams	Team A	Team B	Team C	Team D	Team E
Team A	-	Win 16-12	Loss 41-57	Win 13-12	Win 27-21
Team B	Loss 12-16	-	Loss 30-38	Win 36-28	Loss 35-10
Team C	Win 57-41	Win 38-30	-	Loss 18-29	Loss 14-37
Team D	Loss 12-13	Loss 28-36	Win 29-18	-	Loss 0-39
Team E	Loss 21-27	Win 35-10	Win 37-14	Win 39-0	-

Every teams started out with a rating of 0. Since there are only 5 teams, we are just going to use 100 for our value of K and 100 for ξ. Once the Elo method was applied, the ratings and rankings were as follows:

Ranking	Team	Elo Rating
1	Team E	81.34
2	Team A	67.71
3	Team D	-24.83
4	Team B	-71.10
5	Team C	-74.21

5 Sources

"Arpad Elo and the Elo Rating System." *Chess News*, 16 Dec, 2007. https://en.chessbase.com/post/arpad-elo-and-the-elo-rating-system Professor Wessell's Applied Linear Algebra Class.