Machine Intelligence

Lecture 12: Multi-agent systems

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Tentative course overview

Topics:

- Introduction
- Search-based methods
- Constrained satisfaction problems
- Logic-based knowledge representation
- Representing domains endowed with uncertainty.
- Bayesian networks
- Inference in Bayesian networks
- Machine learning: classification
- Machine learning: clustering
- Planning
- **Multi-agent systems**

Multi-Agent Systems

So far ...

We have modeled an agent that decides/plans in a world with/without uncertainty.

New Dimension

Agent acts in an environment containing other agents. Other agents might have competing/conflicting objectives.

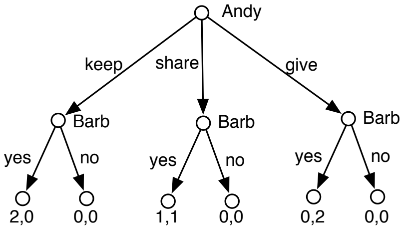
Using Uncertainty

The actions of other agents can partly be represented as uncertainty in effects of own actions (uncertainty of state transitions).

Better: take explictly into account

- Competing objectives of other agents
- Reasoning about what other agents will do (reduce uncertainty)
- Possibility to collaborate to achieve common objectives

The sharing “game”: Andy and Barb share two pieces of pie:

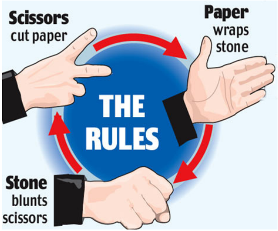


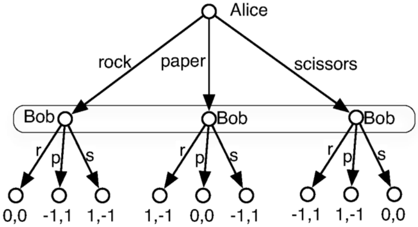
Extensive Form Representation

Representation by **game tree**:

- tree whose nodes are labeled with agents
- outgoing arcs labeled by actions of agent
- leaves labeled with one utility value for each agent
- (can also have *nature* nodes that represent uncertainty from random effects, e.g. dealing of cards, rolling of dice)

Representation of game with simultaneous moves:





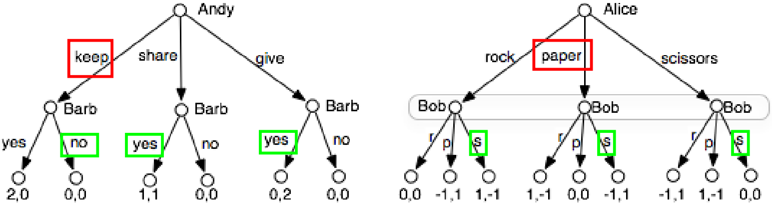
Collect in an **information set** the nodes that the agent (Bob) can not distinguish (at all nodes in an information set the same actions must be possible).

Other sources for imperfect information:

- Unobserved, random moves by nature (dealing of cards).
- Hidden moves by other agent

A **(pure) strategy** for one agent is a mapping from information sets to (possible) actions.

Example strategies for A and strategies for B:



(A strategy is essentially the same as a policy)

A **strategy profile** consists of a strategy for each agent.

Utility for each agent given a strategy profile:

- each node has the utilities that will be reached at a leaf by following the strategy profile
- the utilities at the node represent the outcome of the game (given the strategy profile)
- (utilities at a *nature* node are computed by taking the *expectation* over the utilities of its successors)

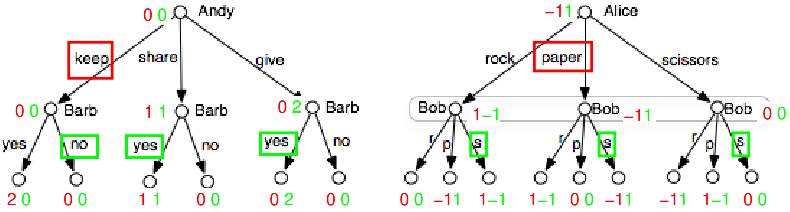


Figure shows the utilities for A and utilities for B at all nodes.

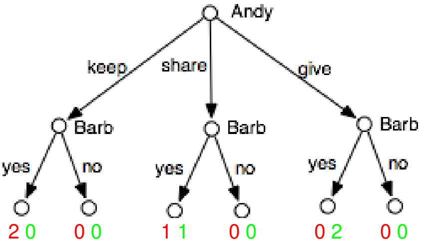
Solving Perfect Information Gain

If

- game is perfect information (no information sets with more than 1 node)
- both agents play rationally (optimize their own utility)

then the optimal strategies for both players are determined by

- bottom-up propagation of utilities under optimal strategies, where
- each player selects the action that leads to the child with the highest utility (for that player)



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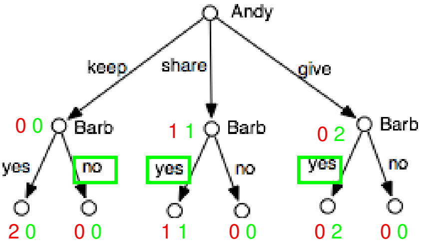

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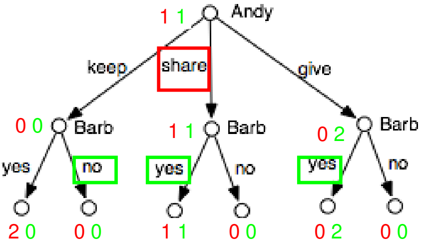


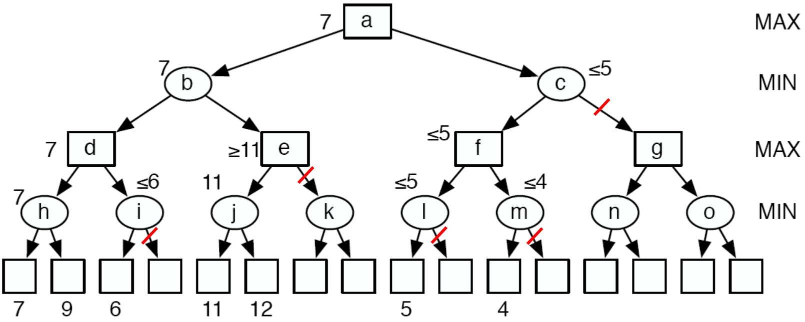
Zero Sum Game for two players:

$$\textit{utility of player 1} = -\textit{utility of player 2}$$

In this case:

- need only one utility value at the leaves
- one player (called *Max*) wants to reach leaf with maximal value, the other (*Min*) wants to reach leaf with minimal value.

In the bottom-up utility computation some sub-trees can then be **pruned** (α-β-pruning):



Solving Checkers

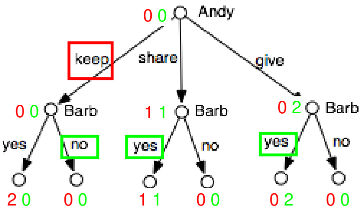
J.Schaeffer et al.: *Checkers Is Solved*. Science, July 2007

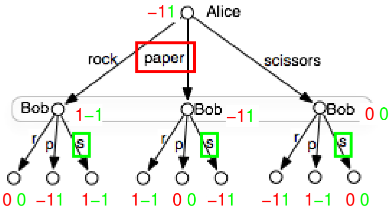
- Schaeffer et al. proved: there is no winning strategy for either player: perfect play by both players will always result in a draw
- checkers has approximately $5 \cdot 10^{20}$ different positions
- in the proof only about 10^{14} positions were explored
- reduction by several techniques, including α-β-pruning.

Imperfect Information

Normal Form

For each strategy profile, utilities of game are determined:





Share game:

- Strategy Andy: *keep*
- Strategy Barb: *no if keep*, *yes if share*, *yes if give*
- Utilities: 0 for Andy, 0 for Barb

Rock Paper Scissors:

- Strategy Alice: *paper*
- Strategy Bob: *scissors*
- Utilities: -1 for Alice, 1 for Bob

Can view game simply as consisting of

- Choice of action by A (possibly: action=strategy)
- Choice of action by B (possibly: action=strategy)
- Utilities determined by these choices

Share game

Barb	Andy: keep	share	give
$k \to y, s \to y, g \to y$	2 0	1 1	0 2
$k \to y, s \to y, g \to n$	2 0	1 1	0 0
$k \to y, s \to n, g \to y$	2 0	0 0	0 2
$k \to y, s \to n, g \to n$	2 0	0 0	0 0
$k \to n, s \to y, g \to y$	0 0	1 1	0 2
$k \to n, s \to y, g \to n$	0 0	1 1	0 0
$k \to n, s \to n, g \to y$	0 0	0 0	0 2
$k \to n, s \to n, g \to n$	0 0	0 0	0 0

Rock Paper Scissors

Bob	Alice: rock	paper	scissors
rock	0 0	1 -1	-1 1
paper	-1 1	0 0	1 -1
scissors	1 -1	-1 1	0 0

Difference between perfect and imperfect information not directly visible in normal form representation!

Consider optimal strategy profile for share game:

	Andy		
Barb	keep	share	give
$k \rightarrow y, s \rightarrow y, g \rightarrow y$	2 0	1 1	0 2
$k \rightarrow y, s \rightarrow y, g \rightarrow n$	2 0	1 1	0 0
$k \rightarrow y, s \rightarrow n, g \rightarrow y$	2 0	0 0	0 2
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The two strategies are in **Nash equilibrium**:

- no agent can improve utility by switching strategy while other agent keeps its strategy
- this also means: agent will stick to strategy when it knows the strategy of the other player

Alice and Bob are arrested for burglary. They are separately questioned by police. Alice and Bob are both given the offer to *testify*, in which case

- they will receive a sentence of 5 years each if both testify
- if only one testifies, that person will receive 1 year, and the other 10 years
- if neither testifies, both will get 2 years

	Alice	
Bob	*testify*	*not testify*
testify	-5 -5	-10 -1
not testify	-1 -10	-2 -2

- The only Nash equilibrium is Alice:*testify*, Bob:*testify*
- Nash equilibria do not represent cooperative behavior!

Mixed Strategies

No pure strategy Nash equilibrium in Rock Paper Scissors:

	Alice		
Bob	rock	paper	scissors
rock	0 0	1 -1	-1 1
paper	-1 1	0 0	1 -1
scissors	1 -1	-1 1	0 0

A **mixed strategy** is a probability distribution over actions.

Mixed Strategy for Alice: $r : 1/3 \; p : 1/3 \; s : 1/3$
Mixed Strategy for Bob: $r : 1/3 \; p : 1/3 \; s : 1/3$

Expected utility for Alice = expected utility for Bob =

$$1/9(0 + 1 - 1 - 1 + 0 + 1 + 1 - 1 + 0) = 0$$

Mixed Strategies

No pure strategy Nash equilibrium in Rock Paper Scissors:

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rock	0 0	1 -1	-1 1
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Mixed Strategy for Alice: $r : 1/3\ p : 1/3\ s : 1/3$
Mixed Strategy for Bob: $r : 1/3\ p : 1/3\ s : 1/3$

Expected utility for Alice = expected utility for Bob =

$$1/9(0 + 1 - 1 - 1 + 0 + 1 + 1 - 1 + 0) = 0$$

Suppose Alice plays some other strategy: $r : p_r\ p : p_p\ s : p_s$. Expected utility for Alice then:

$$1/3(p_r \cdot 0 + p_p \cdot 1 - p_s \cdot 1 - p_r \cdot 1 + p_p \cdot 0 + p_s \cdot 1 p_r \cdot +1 - p_p \cdot 1 + p_s \cdot 0) =$$
$$1/3(p_p + p_r + p_s - p_p - p_r - p_s) = 0$$

- If Bob plays $r : 1/3\ p : 1/3\ s : 1/3$, Alice can not do better than playing $r : 1/3\ p : 1/3\ s : 1/3$ also.
- Same for Bob
- Both playing $r : 1/3\ p : 1/3\ s : 1/3$ is a (the only) Nash equilibrium

Key Results

- Every (finite) game has a Nash equilibrium (using mixed strategies)
- There can be multiple Nash equilibria
- Playing a Nash equilibrium strategy profile does not necessarily lead to optimal utilities for the agents (prisoner's dilemma)

The Exam

The exam

Some practical issues

- January 21st, 2019.
- Written exam with internal censor.
- Graded exam.
- Answers should be written in English.
- A “question session” is scheduled for ??.

What the course has covered

The course has covered the following issues:

- Introduction
- Search-based methods
- Constrained satisfaction problems
- Logic-based knowledge representation
- Reasoning under uncertainty.
- Bayesian networks
- Inference in Bayesian networks
- Machine learning
- Planning
- Multi-agent systems

This corresponds to the following literature:

- David L. Poole and Alan K. Mackworth, Artificial Intelligence: Foundations of computational agents (Second edition): Preface, Ch. 1, 3-3.6, 3.7-3.7.1, 3.7.3, 3.8.2 - 3.8.3, 4-4.7.3, 5-5.2, 7-7.5 (except 7.4.2), 7.7, 8-8.4.1, 8.6-8.6.5, 9-9.4 (except 9.1.3), 10.1.2, 10.2, 11-11.4, A.3
- Finn V. Jensen and Thomas D. Nielsen, Bayesian networks and decision graphs: Sections 2-2.2, 3-3.1.
- The slides from the course.