Exercises for MI

Exercise sheet 3

Thomas Dyhre Nielsen

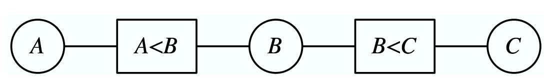
I have marked the possible exam questions with an *. The questions are, however, of varying difficulty (with the latter group being the more difficult), and that would also be reflected in the 'number of points' each exercise would give at the exam.

When you have finished with the exercises, you should continue with the exam sheet from the previous years, which can be found at the course's home page.

Exercise 1 * Consider the same situation as in Exercise 8 (from the last exercise sheet), but now you only want to know whether there is a menu that costs no more than 150. Express this problem as a constraint satisfaction problem:

- what are suitable *variables* that describe the possible worlds?
- what are the constraints? For the specification it is sufficient to give them in intensional form.
- draw (a part of) the state space graph that you would use if you wanted to solve this problem using search.

Exercise 2 *



$D_A = \{1, 2, 3\}$ $D_B = \{2, 3, 4\}$ $D_C = \{3, 4\}$

- Which arcs in the above constraint network are arc consistent?
- How can the whole network be made arc consistent, i.e., which changes should be made to the domains of the variables making the network arc-consistent without eliminating potential solutions?
- Check your result by implementing the model in the CSP-implementation found at http://aispace.org/constraint/.

Exercise 3

Consider the following mini-Sudoku:

		4	
2			
		1	
3			

The empty fields have to be filled with numbers 1,2,3, or 4, such that each row, each column, and each of the 2×2 sub-squares contain each of these numbers exactly once.

a. * Formalize this problem as a constraint satisfaction problem: define an appropriate set of variables, and a set of constraints that define the solutions of the sudoku (hint: instead of using constraints as on the lecture slide, define a smaller set of constraints, each constraint expressing the full condition for one row, one column, or one sub-square. We have not discussed off-the-shelf formal languages for expressing such constraints, so you should try to express them as formally as possible).

b. * Draw the constraint network for this problem. If this gets too large, draw only the part of the network that is sufficient to answer the next question.

c. Apply the generalized arc consistency algorithm to show that the top left square must contain a 1. Does the operation of the algorithm resemble how you would come to that conclusion yourself?

Exercise 4 * Solve exercise 4.12 in PM.

Exercise 5 *

Use Variable Elimination to solve the following CSP given by extensional constraints on Boolean variables A, B, C:

A	B
t	f
t	t
f	t

A	C
t	f
f	t

B	C
t	f
f	t

Exercise 6 Solve exercise 4.3 (except d) in PM and using only the network displayed in Figure 4.15(b). For exercise (a), (b), (c) you may use the CSP-implementation found at http://aispace.org/constraint/; observe that the

CSP-implementation has a special constraint for the word relations appearing in a cross-word.

Exercise 7 Solve exercise 4.5 in PM.

Exercise 8 *

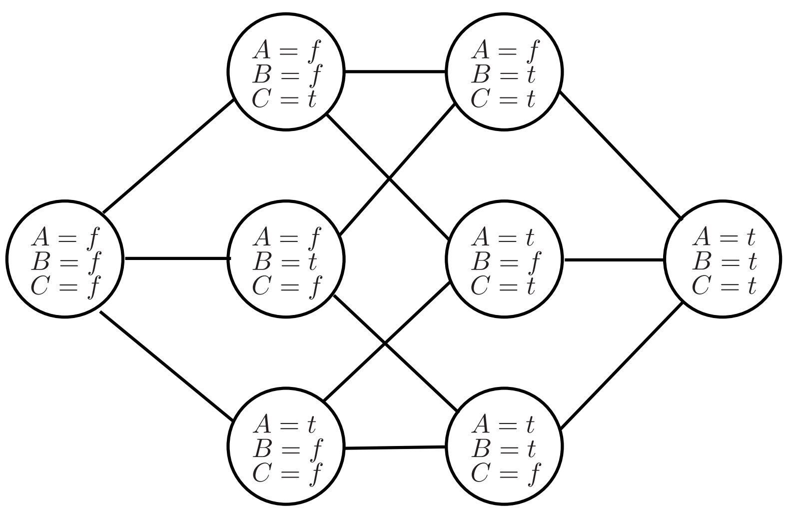
Consider the following soft constraints on three binary variables A, B, C:

A	B	Cost
t	t	2
t	f	5
f	t	1
f	f	5

A	C	Cost
t	t	8
t	f	0
f	t	1
f	f	8

B	Cost
t	4
f	0

- Label the nodes in the state space graph below with the resulting (additive) cost function.
- What will be the solution found if local hill climbing search is started at the possible world $A = f, B = t, C = f$?
- Where would you have to start local hill climbing search in order to find the globally optimal solution?



Exercise 9 Consider the cryptarithmetic puzzle

	T	W	O
+	T	W	O
F	O	U	R

Each letter in a cryptarithmetic problem represents a digit; note that F cannot be 0.

- Construct a constraint network representation for the puzzle. *Hint:* For each of the four columns in the table representation above we have a constraint. E.g. for the first column we have the constraint

$$O + O = R + 10 \cdot C_1,$$

  where C_1 is an auxiliary variable representing what is carried over in the 10 column. Use similar auxiliary variables, say C_2 and C_3, for encoding what is carried over to the 100 and 1000 column, respectively.

- Perform a forward-backward search to find a solution to the puzzle. *Hint:* The ordering of the variables has a big impact on the solution size, so a good strategy would be to chose the variable with the smallest state space. E.g. consider starting with $C_3 = 1$.

Exercise 10

a. Let π be the interpretation that assigns the following truth values:

$$\pi(a) = \mathit{true}, \pi(b) = \mathit{false}, \pi(c) = \mathit{false}, \pi(d) = \mathit{true}$$

Determine the truth values for the following propositions:

$$\begin{array}{l} \neg a \to b \\ (\neg b \vee c) \wedge (d \to a) \\ (a \to c) \to c \end{array}$$

b. For the following propositions, find an interpretation in which they are true:

$$\begin{array}{l} (a \vee (a \to c)) \to b \\ (a \wedge (\neg b \vee c)) \wedge (a \to (c \to b)) \end{array}$$

Exercise 11

Show that if a knowledge base KB contains the two propositions a and $a \to b$, then

$$KB \models b$$

(This means that the Modus Ponens inference rules is *sound*).