Exercises for MI

Exercise sheet 4

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When you have finished with the exercises, you should continue with the exam sheet from previous years, which can be found at the course's home page.

Exercise 1 Consider the experiment of flipping a fair coin, and if it lands heads, rolling a fair four-sided die, and if it lands tails, rolling a fair six-sided die. Suppose that we are interested only in the number rolled by the die, and the possible worlds $\mathcal{S}_A$ for the experiment could thus be the numbers from 1 to 6. Another set of possible worlds could be $\mathcal{S}_B = \{t1, \ldots, t6, h1, \ldots, h4\}$, with for example $t2$ meaning "tails and a roll of 2" and $h4$ meaning "heads and a roll of 4." Choose either $\mathcal{S}_A$ or $\mathcal{S}_B$ and associate probabilities with it. According to your chosen set of possible worlds and probability distribution, what is the probability of rolling either 3 or 5.

Exercise 2 Let $\mathcal{S}_B$ be defined as in the Exercise above, but with a loaded coin and loaded dice. A probability distribution is given in Table 1. What is the probability that the loaded coin lands "tails"? What is the conditional probability of rolling a 4, given that the coin lands tails? Which of the loaded dice has the highest chance of rolling 4 or more?

$t1$	$\frac{5}{18}$	$t6$	$\frac{1}{18}$
$t2$	$\frac{1}{9}$	$h1$	$\frac{1}{24}$
$t3$	$\frac{1}{9}$	$h2$	$\frac{1}{24}$
$t4$	$\frac{1}{18}$	$h3$	$\frac{1}{8}$
$t5$	$\frac{1}{18}$	$h4$	$\frac{1}{8}$

Table 1: Probabilities for $\mathcal{S}_B$ in Exercise 2.

Exercise 3 * Calculate $P(A)$, $P(B)$, $P(A|B)$, and $P(B|A)$ from the joint probability distribution $P(A, B)$ given in Table 2.

Exercise 4 *

Consider the binary variable A and the ternary variable B. Assume that B has the probability distribution $P(B) = (0.1, 0.5, 0.4)$ and that A has the conditional probability distribution given in Table 3.

	b_1	b_2	b_3
a_1	0.05	0.10	0.05
a_2	0.15	0.00	0.25
a_3	0.10	0.20	0.10

Table 2: The joint probability distribution for $P(A, B)$.

	b_1	b_2	b_3
a_1	0.1	0.7	0.6
a_2	0.9	0.3	0.4

Table 3: The conditional probability distribution $P(A|B)$.

Questions:

1. Verify that Table 3 specifies a valid conditional probability distribution.
2. Calculate $P(B|A)$. *Hint:* Consider Bayes rule illustrated by the temperature-sensor example that we discussed in the lecture

Exercise 5 * Table 4 describes a test T for an event A. The number 0.01 is the frequency of *false negatives*, and the number 0.001 is the frequency of *false positives.*

(i) The police can order a blood test on drivers under the suspicion of having consumed too much alcohol. The test has the above characteristics. Experience says that 20% of the drivers under suspicion do in fact drive with too much alcohol in their blood. A suspicious driver has a positive blood test. What is the probability that the driver is guilty of driving under the influence of alcohol?

(ii) The police block a road, take blood samples of all drivers, and use the same test. It is estimated that one out of 1,000 drivers have too much alcohol in their blood. A driver has a positive test result. What is the probability that the driver is guilty of driving under the influence of alcohol?

Hint: Structure-wise this exercise is closely connected to the temperature-sensor example that we discussed in the lecture.

Exercise 6 * A routine DNA test is performed on a person (this exercise is set in the not too distant future!). The test T gives a positive result for a rare genetic mutation M linked to Alzheimer's disease. The mutation is present in only 1 in a million people. The test is 99.99% accurate, i.e. it will give a wrong result in 1 out of 10000 tests performed. Should the person be worried, i.e., what is the probability that the person has the mutation given that the test showed a positive result?

	$A = yes$	$A = no$
$T = yes$	0.99	0.001
$T = no$	0.01	0.999

Table 4: Table for Exercise 5. Conditional probabilities $P(T \mid A)$ characterizing test T for A.

	b_1	b_2
a_1	$(0.006, 0.054)$	$(0.048, 0.432)$
a_2	$(0.014, 0.126)$	$(0.032, 0.288)$

Table 5: $P(A, B, C)$ for Exercise 7.

Hint: This is partly a modeling exercises and partly a calculation exercise. First you need to formalize the problem:

- What are the relevant variables and what states do they have?
- Based on the description above, what probability distributions can you infer for the variables?

Based on this formalization, you need to find the rules required to answer the question about the probability of a mutation given a positive test result.

Exercise 7 * In Table 5, a joint probability table for the binary variables A, B, and C is given.

- Calculate $P(B, C)$ and $P(B)$.
- Are A and C independent given B?

Exercise 8 Solve Exercise 1(a-d) in PM.

Exercise 9 Continue with the exercises from last time.