Exercises for MI

Exercise sheet 11

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When you have finished with the exercises, you should continue with the exam sheet from the previous years, which can be found at the course's home page.

Exercise 1*

A contestant on the show "who wants to be a millionaire" might be faced with the following situation: she is asked a question, which she can choose to answer or not to answer. If she chooses not to answer, then the game is over and she wins 10.000. If she answers, and the answer is incorrect, then she wins 5.000. If she answers and the answer is correct she wins 20.000. Assume the contestant is about 60% sure she knows the answer, and decides not to answer.

- Represent the scenario as a choice between two lotteries.
- Given the decision of the contestant, what can you say about the utility of money function for this contestant? (draw a partial graph of this function, indicating the relative position of some points on this graph).
- How would the utility of money function look for a person who decides to try to answer the question even if she had only a 40% confidence in knowing the right answer?

Exercise 2* In your computer science studies you attend two courses, *Graph Algorithms* and *Machine Intelligence.* In the middle of the term, you realize that you cannot keep pace. You can either reduce your effort in both courses slightly or you can decide to attend one of the courses superficially. What is the best decision?

You have three possible actions:

Gm: Keep pace in Graph Algorithms and follow Machine Intelligence superficially.

SB: Slow down in both courses.

Mg: Keep pace in Machine Intelligence and follow Graph Algorithms superficially.

The results of the actions are your final marks for the courses (excluding -03). You have certain expectations for the marks given your effort in the rest of the term. They are shown in Table 1.

	kp	*sd*	*fs*
0	0	0	0.1
2	0.1	0.2	0.1
4	0.1	0.1	0.4
7	0.2	0.4	0.2
10	0.4	0.2	0.2
12	0.2	0.1	0

$P(GA \mid \mathit{effort})$

	kp	*sd*	*fs*
0	0	0	0.1
2	0	0.1	0.2
4	0.1	0.2	0.2
7	0.2	0.2	0.3
10	0.4	0.4	0.2
12	0.3	0.1	0

$P(MI \mid \mathit{effort})$

Table 1: The conditional probabilities of the final marks in Graph Algorithms (*GA*) and Machine Intelligence (*MI*) given the efforts *keep pace* (*kp*), *slow down* (*sd*), and *follow superficially* (*fs*).

- Assuming that you wish to maximize the sum of the expected marks, what is your best course of action?
- Specify a reasonable utility function for the marks and determine your best course of action according to the utility function.

Exercise 3* Solve Exercise 9.6 in PM.

Exercise 4*

(a)

Construct a decision network for the following version of the exam preparation problem: you have to decide whether you *prepare some* or *prepare all* of the questions. At the exam, you get one of 0,7,10 as a grade (to simplify matters, we consider only three grades). You take into consideration whether this is your 1., 2. or 3. attempt at this exam (include in your network a chance node *Attempt* which you observe before you make your decision).

(b) Make a table containing all possible worlds with *Attempt*=1. For each possible world ω in the table compute $P(\omega \mid \mathit{Prepare} = p) \cdot U(\omega)$, where p is the value of *Prepare* in ω. From the table, determine the optimal decision for the case *Attempt*=1.

(c)

Solve the decision network to obtain the optimal decision rule for the *Prepare* decision node (i.e. the optimal decision for all possible states of *Attempt*).

For finding an optimal decision rule for Prepare we calculate

$$\delta(\mathit{Attempt}) = \arg\max_{\mathit{Prepare}} \sum_{\mathit{Grade}} P(\mathit{Grade}|\mathit{Prepare})U(\mathit{Prepare}, \mathit{Attempt}),$$

which prescribes a decision for each state of Attempt.

The actual calculations requested in the exercise will depend on the numbers that you have chosen to quantify the decision network.

Exercise 5

Use Hugin to construct a sequential decision model for the preparation of (at most) two exams: the model should contain the two decision nodes *Prepare_exam*, *Prepare_reexam*, both with the two possible values *prepare some* and *prepare all*. Consider only 0,7, and 10 as possible outcomes of the exam. Note that there is a somewhat subtle peculiarity with the problem: if you pass the course the first time, then you are *not* allowed to take the exam again.

Model the problem as a decision network and solve it using Hugin. What are the optimal decision functions?

Exercise 6 Consider again the Monty hall problem from one of the previous exercise sessions: You are confronted with three doors, A, B, and C. Behind exactly one of the doors there is $10,000. When you have pointed at a door, an official will open another door with nothing behind it. After he has done so, you are allowed to alter your choice. Should you do that?

Model the problem as a decision network and solve it using Hugin.

Exercise 7 An oil wildcatter must decide whether to drill or not to drill. The cost of drilling is $70,000. If he decides to drill, the hole may be soaking (with a return of $270,000), wet (with a return of $120,000), or dry (with a return of $0). The prior probabilities for soaking, wet, and dry are (0.2, 0.3, 0.5). At the cost of $10,000, the oil wildcatter could decide to take seismic soundings of the geological structure at the site. The specifics of the test are given in Table 2.

$T \setminus S$	dr	wt	so
n	0.6	0.3	0.1
o	0.3	0.4	0.4
c	0.1	0.3	0.5

$P(Test \mid Structure)$

Table 2: Table for Exercise 7. The states n, o, and c are the outcomes of the test.

- Construct a decision network for the problem above.*
- Solve the problem using Hugin.

Exercise 8 One morning, a farmer goes out into his field to inspect the quality of his crops. At this time of the year, the farmer estimates that the prior probability for the crops being in a good condition is 0.9. To get more information the farmer takes a sample of his crops to determine its quality (which can either be good or bad). The farmer expects that if the general quality of the crops is good, then the sample will also have a good quality with probability 0.95 and if the quality of the crops is bad, then the sample will also be bad with probability 0.90. Based on the quality of the sample, the farmer then decides whether to apply fertilization. He knows that if the quality of the crops is bad, then a week after applying fertilization the quality of the crops will become good (with probability 0.8). If the quality is already good, then (with probability 1) the quality will still be good no matter whether fertilization is applied. On the other hand, if the quality is bad, then (without applying fertilization) the quality of the crops may improve to good with probability 0.3. The cost of applying fertilization is $20,000$ Dkr.

A week after having applied the fertilization, the farmer should then decide whether to harvest now or wait two more weeks. The value of this decision is determined by the quality of the crops: if the quality is good, then the farmer expects to earn $100,000$ Dkr if he harvests now, but only $80,000$ if he waits. On the other hand, if the quality is bad, then harvesting now will only give him $20,000$ Dkr, whereas he expects that by waiting the quality of the crops will improve so much that he can get $40,000$ Dkr from harvesting.

- Construct an influence diagram for the farmer from the description above.*
- Solve the influence diagram using Hugin.

Exercise 9 Solve Exercise 9.7 in PM.

Exercise 10* A farmer inseminated a cow five weeks ago, and he should now decide whether to repeat the insemination or wait an additional five weeks before doing the insemination. The probability that the cow is pregnant is 0.8, and the cost of repeating the insemination is 1000 Dkr regardless of whether the cow is pregnant. On the other hand, waiting with the insemination will incur an additional loss of 500 Dkr if the cow is not pregnant (giving a total of 1500 Dkr).

Before making this decision the farmer can decide to perform a scanning test of the cow at the cost of 100 Dkr. The scanning test's frequency of false positives and false negatives is 0.3 and 0.05, respectively.

- Perform a value of information analysis of the decision problem above.

Exercise 11 Using Hugin solve the decision problem in Exercise 7 as a value of information problem.

Exercise 12 Complete the exercises from last time.