Exercises for MI

Exercise sheet 5

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When you have finished with the exercises, you should continue with the exam sheet from the previous years, which can be found at the course's home page.

Exercise 1* In the graphs in Figures 1 and 2, determine which variables are d-separated from A. Note that it is sufficient to find a single open path along which evidence can be transmitted; if such a path exists then the variables are *not* d-separated (instead they are said to be d-connected).

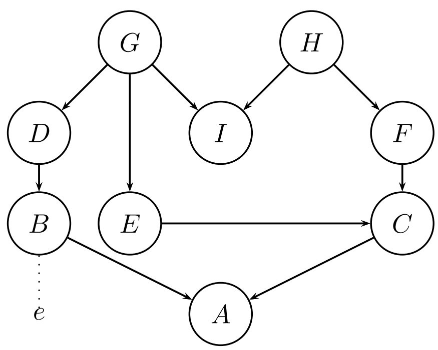


Figure 1: Figure for Exercise 1.

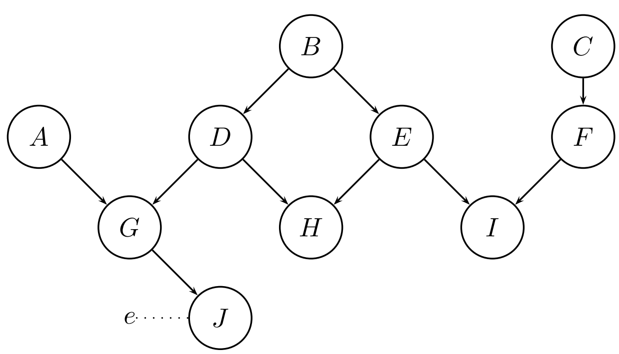


Figure 2: Figure for Exercise 1.

Exercise 2* Consider the network in Figure 3.

- What are the minimal set(s) of variables that we should have evidence on in order to d-separate C and E (that is, sets of variables for which no proper subset d-separates C and E)?
- What are the minimal set(s) of variables we should have evidence on in order to d-separate A and B?
- What are the maximal set(s) of variables that we can have evidence on and still d-separate C and E (that is, sets of variables for which no proper superset d-separates C and E)?
- What are the maximal set(s) of variables that we can have evidence on and still d-separate A and B?

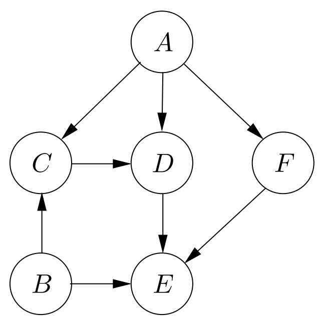


Figure 3: A causal network for Exercise 3.

Exercise 3 Consider the network in Figure 3. Which conditional probability tables must be specified to turn the graph into a Bayesian network?

Exercise 4 Construct a Bayesian network for Exercise 5 in the last exercise sheet.

Exercise 5* Peter is currently taking three courses on the topics of probability theory, linguistics, and algorithmics. At the end of the term he has to take an exam in two of the courses, but he has yet to be told which ones. Previously he has passed a mathematics and an English course, with good grades in the mathematics course and outstanding grades in the English course. At the moment, the workload from all three courses combined is getting too big, so Peter is considering dropping one of the courses, but he is unsure how this will affect his chances of getting good grades in the remaining ones. What are reasonable variables of interest in assessing Peter's situation? How do they group into information, hypothesis, and mediating variables?

Hint: The grades that Peter has already received constitute evidence about certain variables (i.e., variables for which we observe the states they are in).

Exercise 6* Construct a Bayesian network (ignoring the probabilities) and follow the reasoning in the following story based on how information is transmitted in the networks (according the d-separation rules). Mr. Holmes is working in his office when he receives a phone call from his neighbor, who tells him that Holmes' burglar alarm has gone off. Convinced that a burglar has broken into his house, Holmes rushes to his car and heads for home. On his way, he listens to the radio, and in the news it is reported that there has been a small earthquake in the area. Knowing that earthquakes have a tendency to turn on burglar alarms, he returns to work.

Exercise 7 We want to construct a Bayesian network for the following random variables (all with domain *true,false*):

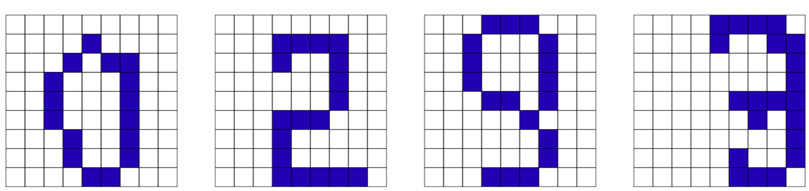
Mexico	Person X has recently travelled to Mexico
Svine_flu_infection	Person X has been infected with the svine flu virus
Vaccination	Person X has been vaccinated against svine flu
Svine_flu_sick	Person X is sick from svine flu
Fever	Person X has fever

a. Use the above ordering of the variables to determine a Baysian network structure based on the chain rule and conditional independence relations.

b. Repeat the construction with the alternative variable ordering:

Vaccination,Mexico,Svine_flu_sick,Fever,Svine_flu_infection

Exercise 8* Design a Bayesian network that can be used to recognize handwritten digits 0,1,2,. . . ,9 from scanned, pixelated images like these:



- What are hypothesis and information variables?
- Could there be any useful mediating variables (consider e.g. the last image above)?
- How could you design a network structure
  - so that the conditional independencies are (approximately) reasonable

  - so that specification and inference complexity remain feasible

- How do you fill in the conditional probability tables?

Exercise 9 You are confronted with three doors, A, B, and C. Behind exactly one of the doors there is $10,000. When you have pointed at a door, an official will open another door with nothing behind it. After he has done so, you are allowed to alter your choice. Should you do that (i.e., will altering your choice improve your chances of winning the prize)?

Exercise 10 *This exercise is intended to be solved using Hugin. If you would like more experience doing probability updating manually, you can also solve the first part of the exercise by hand. In that case, you should take into account the evidence given in the description when constructing the tables so that you have fewer numbers to deal with, i.e., throw away the parts of the tables inconsistent with the evidence.*

Consider the insemination example from Section 3.1.13 in BNDG (pdf available form the lecture sheet). Let the probabilities be as in Table 1 ($Ho = y$ means that hormonal changes have taken place) $P(Pr) = (0.87, 0.13)$.

	$Pr = y$	$Pr = n$
$Ho = y$	0.9	0.01
$Ho = n$	0.1	0.99

	$Ho = y$	$Ho = n$
$BT = y$	0.7	0.1
$BT = n$	0.3	0.9

	$Ho = y$	$Ho = n$
$UT = y$	0.8	0.1
$UT = n$	0.2	0.9

Table 1: Tables for Exercise 10.

(i) What is $P(Pr \mid BT = n, UT = n)$?

(ii) Construct a naive Bayes model. Determine the conditional probabilities for the model by making inference queries in the model above using Hugin. What is $P(Pr \mid BT = n, UT = n)$ in this model and how does it compare to the result you got above? Try to (qualitatively) account for any differences.

Exercise 11 *Use Hugin to solve this exercise* The following relations hold for the Boolean variables
A, B, C, D, E, and F:

$$(A \vee \neg B \vee C) \wedge (B \vee C \vee \neg D) \wedge (\neg C \vee E \vee \neg F) \wedge (\neg A \vee D \vee F) \wedge$$
$$(A \vee B \vee \neg C) \wedge (\neg B \vee \neg C \vee D) \wedge (C \vee \neg E \vee \neg F) \wedge (A \vee \neg D \vee F).$$

(i) Is there a truth value assignment to the variables making the expression true? (Hint: Represent the expression as a Bayesian network.)

(ii) We receive the evidence that A is false and B is true. Is there a truth value assignment to the other variables making the expression true?

Exercise 12 (* only part of it)

For 10000 emails in your inbox you determine the values of the following three boolean variables:

Spam	the email is spam
Caps	the subject line is in all capital letters
Pills	Body of the message contains the word "pills"

You obtain the following counts:

	Caps			
	yes		*no*	
	Pills		*Pills*	
Spam	*yes*	*no*	*yes*	*no*
yes	150	850	600	3400
no	1	99	49	4851

Are

- *Spam* and *Caps* independent?
- *Pills* and *Caps* independent?
- *Pills* and *Caps* independent given *Spam*?
- *Spam* and *Caps* independent given *Pills*?

Exercise 13 Continue with the exercises from last time.