Neural networks

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– Internal knowledge representation –

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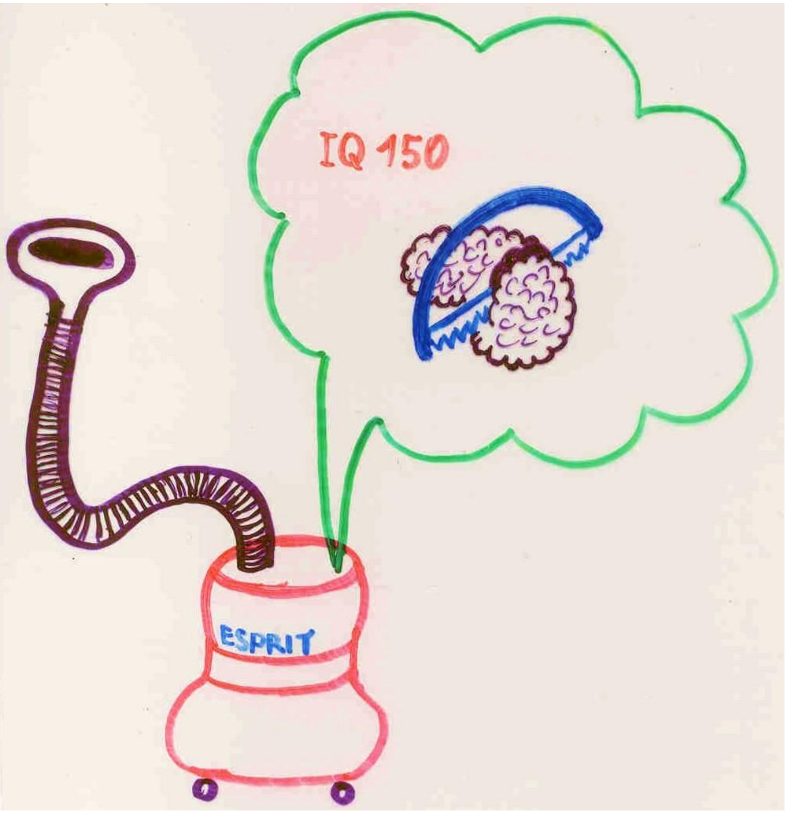
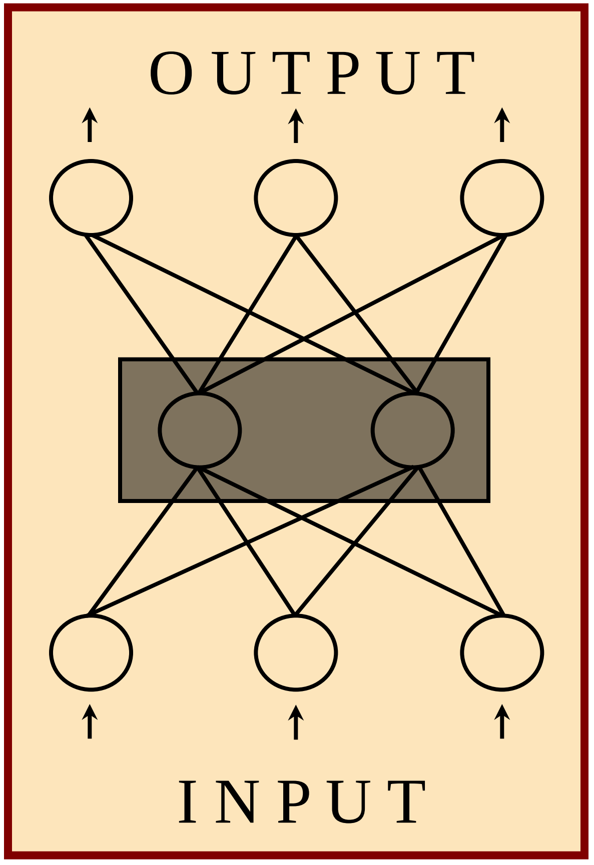
Internal knowledge representation



- **the number of neurons and generalization capa-bilities of the network**

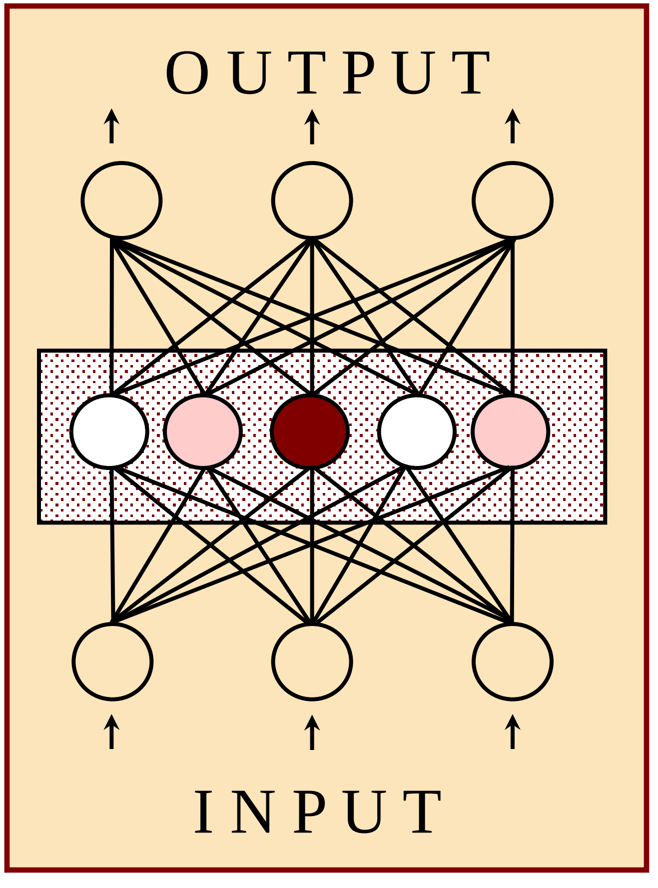
  → **pruning and retraining**

Internal knowledge representation



Excuse me, I always mix up ours with the enemy!

Condensed internal representation



- interpret the activity of hidden neurons:
  - 1 ⟷ active ⟷ YES
  - 0 ⟷ passive ⟷ NO
  - $\frac{1}{2}$ ⟷ silent ⟷
  - ⟷ „impossible to decide“
- transparent structure
- detection of redundant neurons and pruning
- **improved generalization**

Condensed internal representation

D: For a BP-network $\boldsymbol{B}$ procesing an input pattern $\vec{x}$:

- A hidden neuron with the weights $(w_1, \ldots, w_n)$, threshold ϑ, input pattern $\vec{z}$ and transfer function $f[\vec{w}, \vartheta](\vec{z})$ forms a **representation** $\boldsymbol{r}$:
  $\boldsymbol{r} = \boldsymbol{y} = f[\vec{w}, \vartheta](\vec{z})$
- The vector $\vec{r}$ of representations formed by a layer of hidden neurons is called an **internal representation** of $\vec{x}$

Condensed internal representation

D: For a BP-network $\boldsymbol{B}$:

- Internal representation $\vec{r} = (r_1, \ldots, r_m)$ is **binary**, if $r_i \in \{0,1\}; \quad 1 \le i \le m$
- Internal representation $\vec{r} = (r_1, \ldots, r_m)$ is **condensed**, if $r_i \in \{0, 0.5, 1\}; \quad 1 \le i \le m$

Requirements on forcing the condensed internal representation

- formulate the „desired properties“ in the form of an objective function:

standard error function

$$G = E + c_s F$$

representation error function

amount of influence of F on G

- local minima of the representation error function correspond to active, passive and silent states:

$$F = \sum_p \sum_h y_{h,p}^s \left(1 - y_{h,p}\right)^s \left(y_{h,p} - 0.5\right)^2$$

patterns

hidden neurons

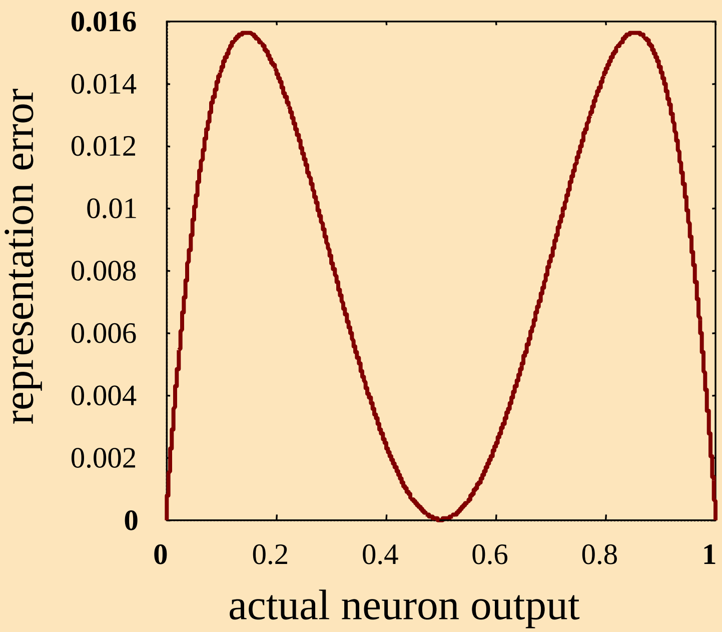
passive state

active state

the shape of F

silent state

The influence of the parameters on condensed internal representation



$$w_{ij}(t+1) = w_{ij}(t) + \alpha\delta_j y_i + \alpha_r \rho_j y_i + \\ + \alpha_m \left(w_{ij}(t) - w_{ij}(t-1)\right)$$

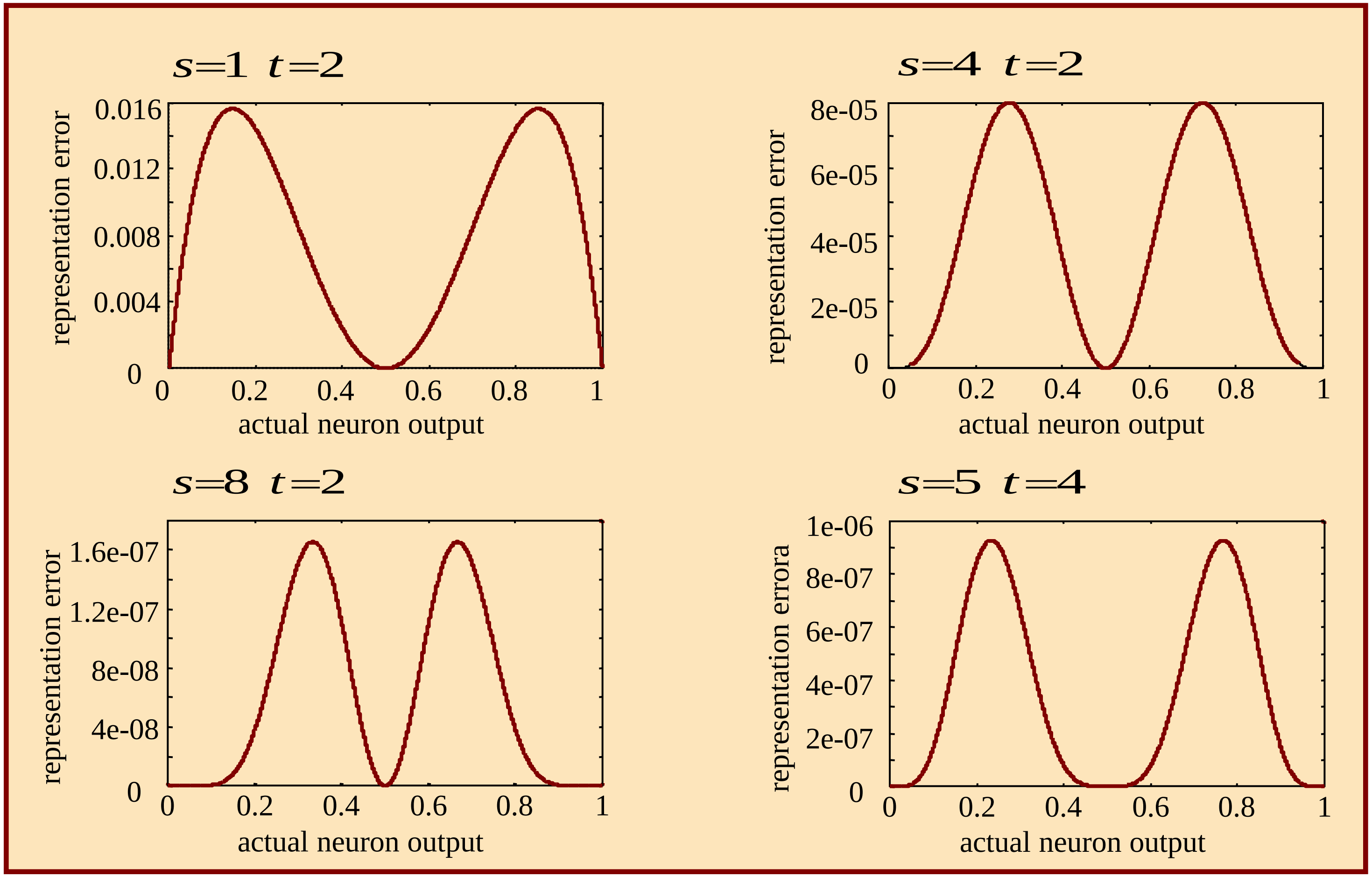
- slower enforcement of the internal representation and required network function
- stability of the formed internal representation and optimal network architecture
- form of the representation error function, speed and form of the enforced internsl representation
- time complexity of weight adjustment

Error term enforcing the condensed internal representation

Condensed internal representation ($\boldsymbol{y_j^s\,(1 - y_j)^s\,(y_j - 0.5)^2}$):

$$\rho_j = \begin{cases} 0 & \text{for the output neurons} \\[2ex] -\left[2(s+1)y_j\,(1-y_j) - \dfrac{s}{2}\right]\cdot y_j^s\,(1-y_j)^s\left(y_j - 0.5\right) & \\ & \text{for neurons from the last hidden layer} \\[2ex] \left(\displaystyle\sum_k \rho_k w_{jk}\right) y_j (1-y_j) & \\ & \text{for other hidden neurons} \end{cases}$$

The shape of the representation error function $F = y^s(1-y)^s(y-0.5)^t$



Unambiguous internal representation

- Varying outputs should be represented by varying internal representations
- Formulation of the requirements in the form of a modified objective function: $\boldsymbol{G = E + F + H}$
- The criterion for unambiguity of the IR:

$$H = -\frac{1}{2}\sum_{p}\sum_{q \neq p}\sum_{j}\sum_{o}\left(d_{o,p} - d_{o,q}\right)^2\left(y_{j,p} - y_{j,q}\right)^2$$

= const. for a fixed q

patterns

hidden neurons

output neurons

= const. for a fixed p

= const. for a fixed q

Pruning according to internal representation (1)

D: For a given BP-network ***B*** and a set ***S*** of input patterns yielding the input vectors: $\vec{z}$

- A hidden neuron with the weights ***(w*** $\boldsymbol{(w_1, \ldots, w_n)}$, the threshold $\boldsymbol{\vartheta}$ and a transfer function $f[\vec{w}, \vartheta](\vec{z})$ forms a **uniform representation** ***r***, if:

$$r = f[\vec{w}, \vartheta](\vec{z}) = const \qquad \text{for all input patterns} \quad \vec{x} \in S$$

Pruning according to internal representation (2)

D: For a given BP-network ***B*** and a set ***S*** of input patterns yielding the input vectors $\vec{z}$:

- A hidden neuron $i \in N$ with the weights $\boldsymbol{(w_{i1}, \dots, w_{in})}$, the threshold $\boldsymbol{\vartheta_i}$ and a transfer function $f_i[\vec{w}_i, \vartheta_i](\vec{z})$ forms a **representation $\boldsymbol{r_i}$** , **identical to the representation $\boldsymbol{r_j}$** formed by the hidden neuron $j \in N$ with the weights $\boldsymbol{(w_{j1}, \dots, w_{jn})}$, the threshold $\boldsymbol{\vartheta_j}$ and a transfer function $f_j[\vec{w}_j, \vartheta_j](\vec{z})$, if:

$$f_i[\vec{w}_i, \vartheta_i](\vec{z}) = f_j[\vec{w}_j, \vartheta_j](\vec{z}) \quad \text{for all input patterns } \vec{x} \in S$$

Pruning according to internal representation (3)

D: For a given BP-network $\boldsymbol{B}$ and a set $\boldsymbol{S}$ of input patterns yielding the input vectors $\vec{z}$:

- A hidden neuron $i \in N$ with the weights $(w_{i1}, \dots, w_{in})$, the threshold ϑ_i and a transfer function $f_i[\vec{w}_i, \vartheta_i](\vec{z})$ forms a **representation r_i , inverse to the representation r_j** formed by the hidden neuron $j \in N$ with the weights $(w_{j1}, \dots, w_{jn})$, the threshold ϑ_j and a transfer function $f_j[\vec{w}_j, \vartheta_j](\vec{z})$, if:
- $f_i[\vec{w}_i, \vartheta_i](\vec{z}) = 1 - f_j[\vec{w}_j, \vartheta_j](\vec{z})$ for all input patterns $\vec{x} \in S$

Pruning according to internal representation (4)

D: For a given BP-network ***B*** and a set ***S*** of input patterns:
a reduced layer is a layer, for which it holds that:

- no neuron forms a uniform representation,
- no neuron ***i*** forms a representation identical to the representation formed by another neuron ***j*** and
- no neuron ***i*** forms a representation inverse to the representation formed by another neuron ***j***.

Internal representation formed by a reduced layer is called **reduced**.

Pruning according to internal representation (5)

D: For a given set of input patterns $\boldsymbol{S}$:

- a BP-network $\boldsymbol{B}$ is **reduced**, if all its hidden layers are reduced.
- a BP-network $\boldsymbol{B}$ is **equivalent** to a BP-network $\boldsymbol{B'}$, if for any input pattern $\vec{x} \in S$, the actual output $\vec{y}_B$ of the network $\boldsymbol{B}$ is equal to the actual output $\vec{y}_{B'}$ of the network $\boldsymbol{B'}$: $\vec{y}_B = \vec{y}_{B'}$

Pruning according to internal representation (6)

T: To each BP-network $\boldsymbol{B}$ and a set of input patterns $\boldsymbol{S}$ there exists an equivalent reduced BP-network $\boldsymbol{B}'$.

Proof (idea):

Construction of a reduced BP-network $\boldsymbol{B}'$:

Let $\boldsymbol{B = (N, C, I, O, w, t)}$ is the original BP-network.

1. Sequential elimination of all those neurons $\boldsymbol{j}$, that form a uniform representation $\boldsymbol{r_j^k}$ and addition of the product $\boldsymbol{w_{ij}\, r_j^k}$ to all the thresholds $\boldsymbol{\vartheta_j}$ in the following layer.

Pruning according to internal representation (7)

Proof (continue):

2. Sequential elimination of all those neurons j, that form a representation r_j^{id} identical to the representation r_k formed by another neuron k and addition of the weights w_{ij} to all the weights w_{ik} , where i denotes the neurons in the following layer.
3. Sequential elimination of all those neurons j, that form a representation r_j^{in} inverse to the representation r_k formed by another neuron k and replacement of all the weights w_{ik} , where i denotes the neurons from the following layer, by the difference $w_{ik} - w_{ij}$ and addition of the weights w_{ij} to the threshold ϑ_i of each neuron i.

Pruning according to internal representation (8)

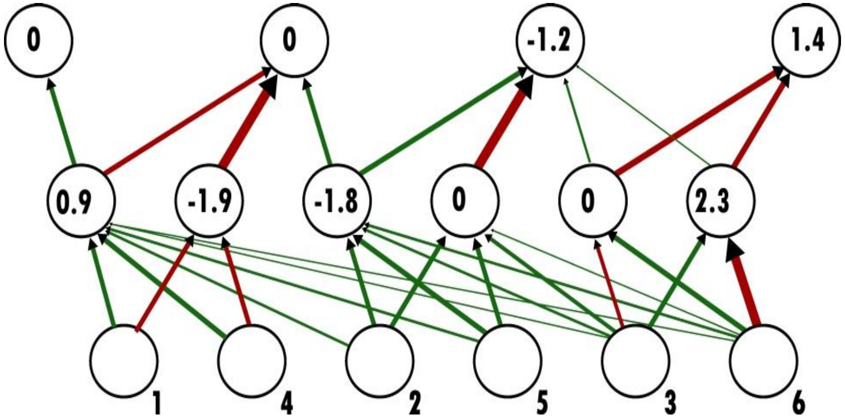
Proof (continue):

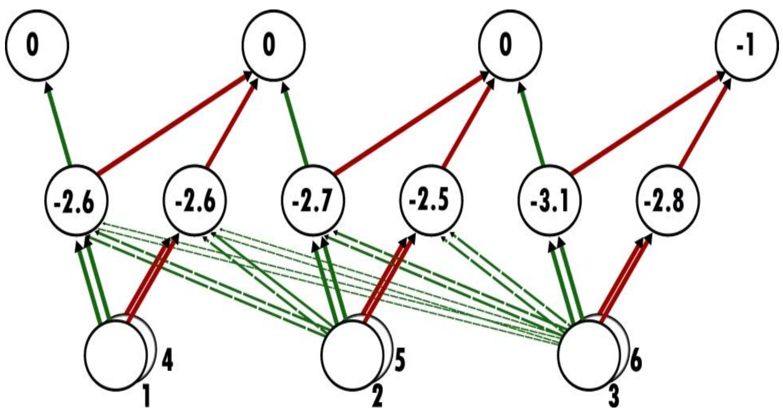
Then, the actual output $\vec{y}_{B'}$ of the BP-network ***B*´** will be equal to the actual output $\vec{y}_B$ of a BP-network ***B*** for any input pattern $\vec{x}$.

The BP-network ***B*´** constructed from the BP-network ***B*** in the above-discussed way is reduced and equivalent to ***B***.

QED

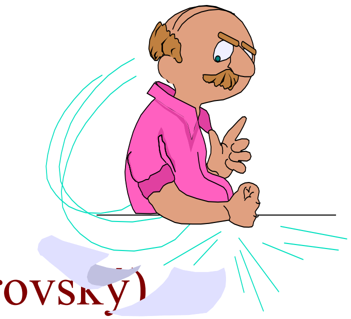
Experimental results: binary addition
[5(≈(1,-1,1)) + 3(≈(-1,1,1)) = 8(≈(1,-1,-1,-1))]



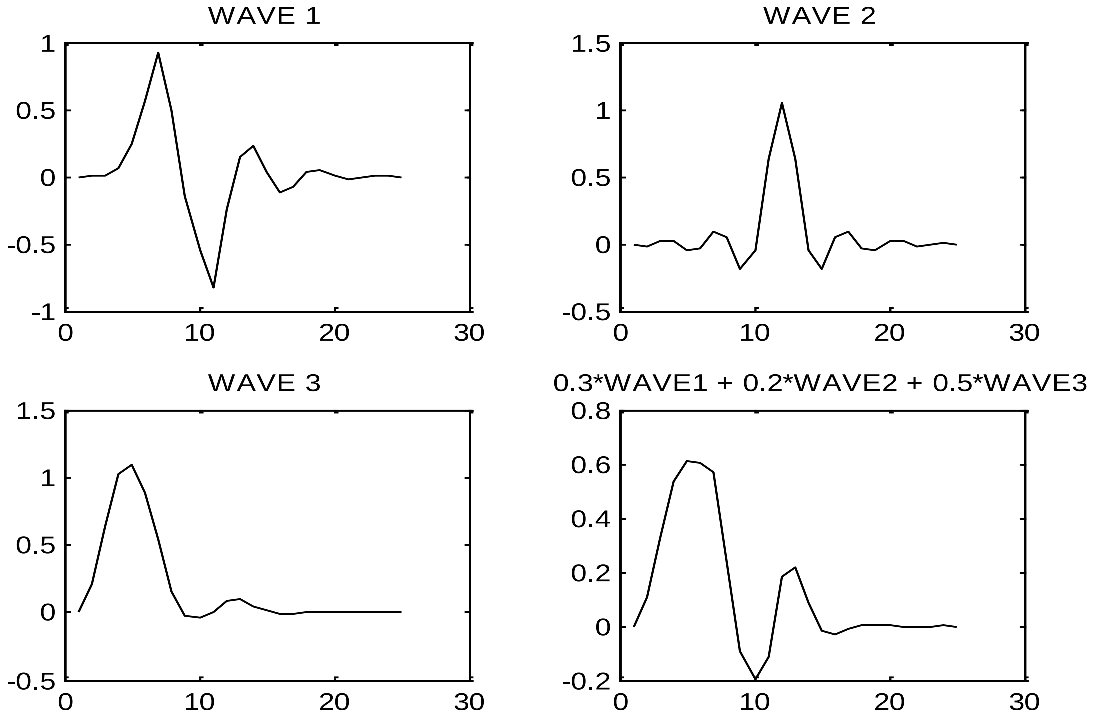


- SCG-with hints (carry to the 2nd output neuron)
  - 'carry' of the first and second output bit – hidden neurons 1 and 3
  - the function of other hidden neurons not clear
- SCGIR-with hints (carry to the 2nd output neuron)
  - 'carry' to higher output bits – hidden neurons 1, 3, 5
  - a similar function is apparent for the respective output neurons

Acoustic emission: simulation (with M. Chlada and Z. Převorovský)

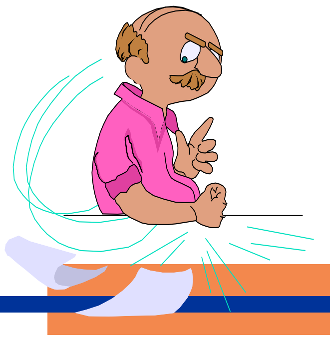


MODELED SIGNAL

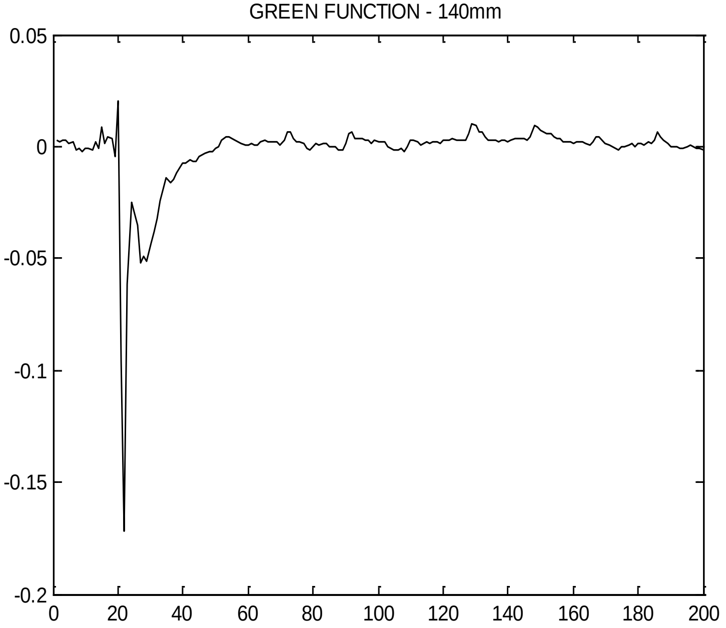


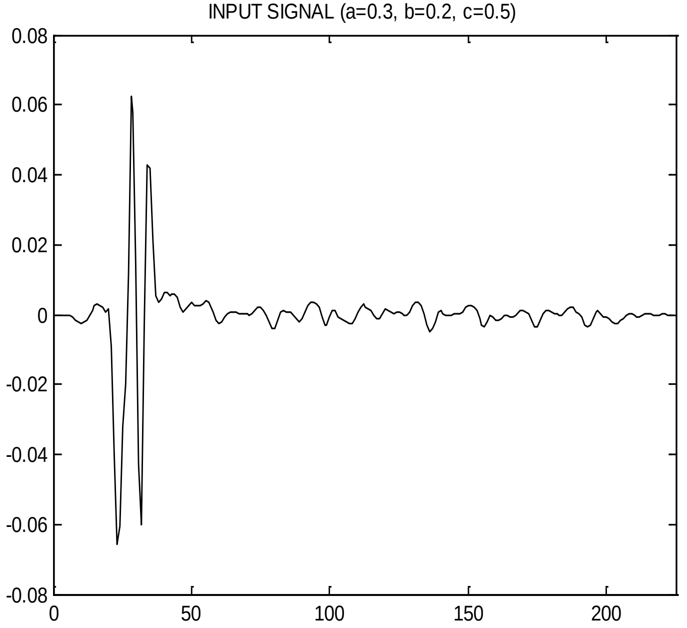
Simulated AE-data

(with M. Chlada and Z. Převorovský)



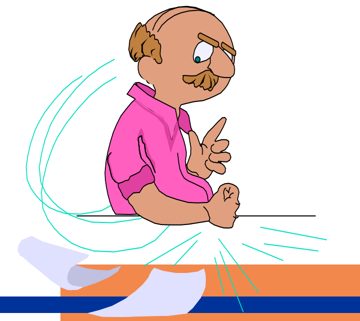
CONVOLUTION WITH THE GREEN FUNCTION





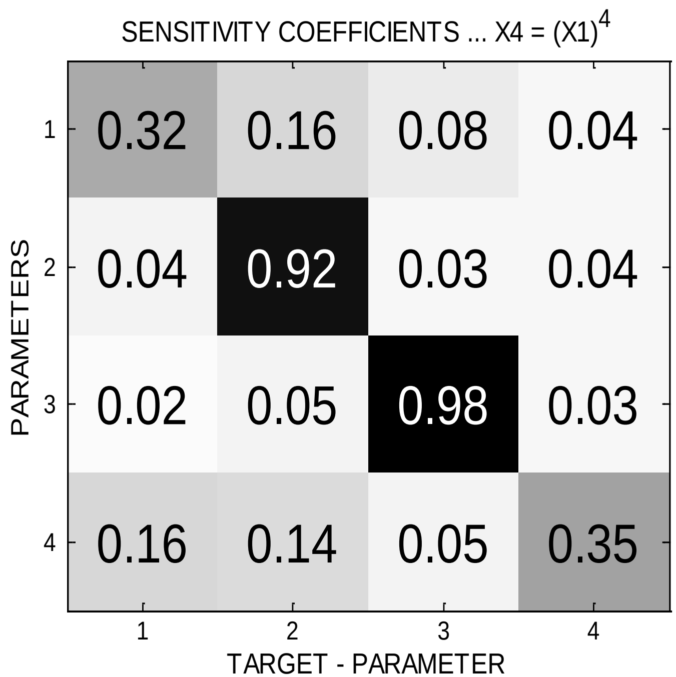
Dependency model

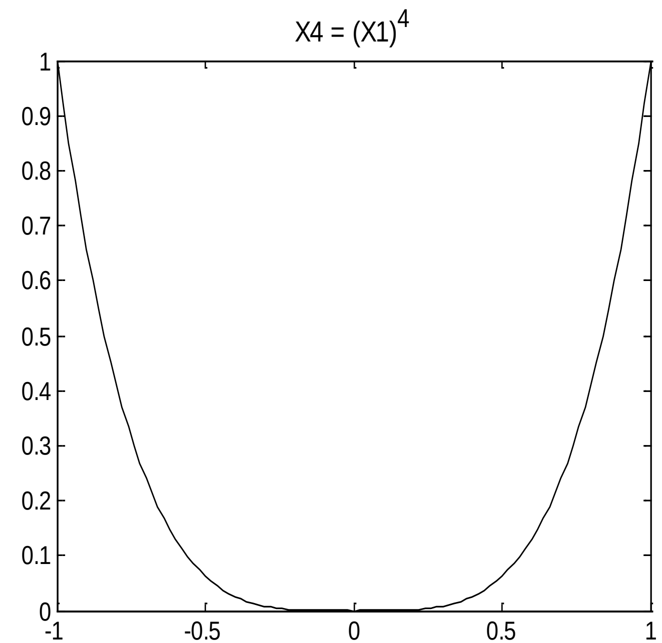
(with M. Chlada and Z. Převorovský)



Overall **network sensitivity** of the *s*-th output to the *r*-th input (over *Q* patterns) :

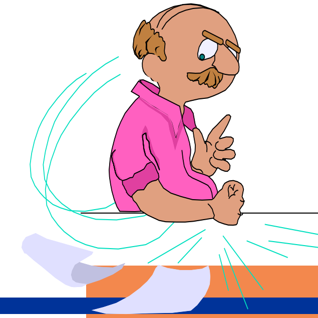
$$sens_r = 1/Q \sum_q \sum_s \left| \partial y_{q,s} / \partial y_{q,r} \right|$$

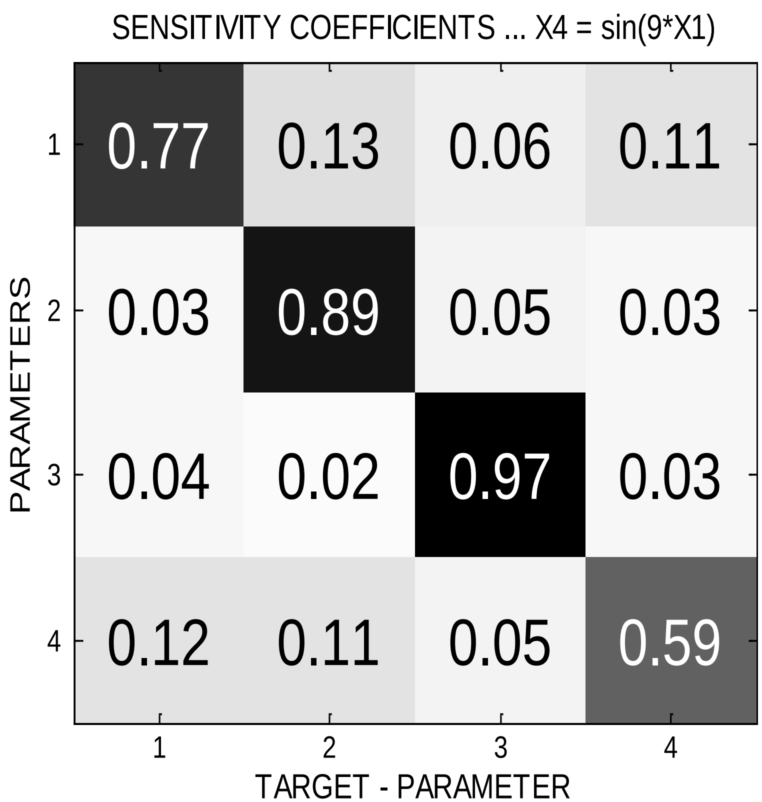


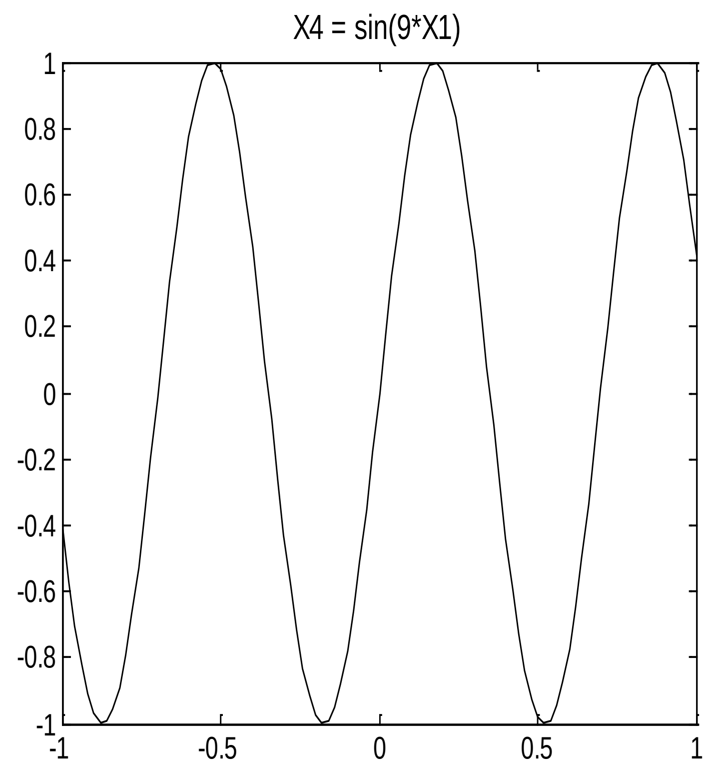


Dependency model

(with M. Chlada and Z. Převorovský)

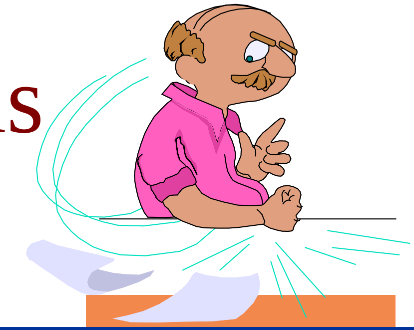






Factor vs. sensitivity analysis of input parameters

(with M. Chlada and Z. Převorovský)



input parametrs / selected factors

	1	2	3	4	5	6	7	8	9
1	0.04	0.91	0.16	0.10	0.01	0.07	0.06	0.01	0.23
2	0.09	0.02	0.01	0.19	0.03	0.03	0.95	0.03	0.16
3	0.10	0.96	0.15	0.00	0.03	0.00	0.02	0.05	0.07
4	0.13	0.91	0.02	0.03	0.05	0.06	0.04	0.06	0.20
5	0.30	0.05	0.04	0.41	0.06	0.49	0.17	0.07	0.66
6	0.26	0.00	0.04	0.03	0.08	0.93	0.02	0.10	0.20
7	0.29	0.06	0.03	0.27	0.03	0.17	0.16	0.04	0.86
8	0.12	0.06	0.01	0.88	0.02	0.02	0.24	0.03	0.36
9	0.90	0.15	0.09	0.08	0.15	0.17	0.06	0.21	0.18
10	0.93	0.14	0.06	0.08	0.10	0.17	0.06	0.09	0.19
11	0.25	0.10	0.12	0.03	0.25	0.11	0.03	0.90	0.05
12	0.20	0.07	0.14	0.02	0.93	0.09	0.03	0.23	0.04
13	0.04	0.09	0.97	0.00	0.05	0.00	0.00	0.06	0.02
14	0.08	0.18	0.95	0.02	0.09	0.04	0.01	0.06	0.02

selected factors

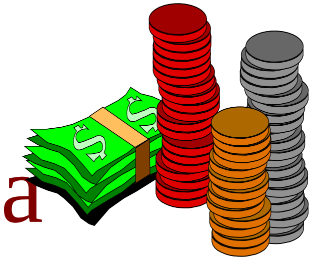
SENSITIVITY COEFFICIENTS

INPUTS \ OUTPUTS	1	2	3
1	0.173	0.266	0.149
2	0.093	0.068	0.047
3	0.320	0.193	0.184
4	0.301	0.178	0.196
5	0.564	0.250	0.206
6	0.196	0.322	0.158
7	0.099	0.063	0.043
8	0.065	0.015	0.030
9	0.022	0.014	0.016
10	0.053	0.020	0.012
11	0.035	0.012	0.032
12	0.039	0.050	0.022
13	0.081	0.134	0.082
14	0.260	0.172	0.109

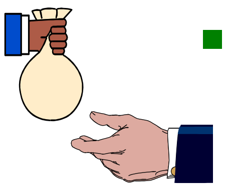
- **9 factors selected ("explain" 98.4% of variables)**
- **elimination of linearly dependent input parameters**

- **7 features selected**
- **detection of non-linear dependencies among input parameters (1, 3, 4, 5, 6, 13, 14)**

Analysis of the World Bank data



WDI-indicators (indicators of world development)



- published every year by the World Bank
  - support developing countries - loans / investments
  - assess the state of economies and their development
- data origin - incomplete and not accurate

- **used techniques**
  - regression analysis - linear dependencies
  - categorization of economies used in developed countries (G. Ip, Wall Street Journal)
  - categorization of economies according to GDP (World Bank)
  - Kohonen maps (T. Kohonen, S. Kaski, G. Deboeck)

Analysis of the World Bank data: used WDI-indicators

- GDP implicit deflator
- External debt (% GNP)
- Total debt service (% of export of goods and services)
- High-technology exports (% of manufactured exports)
- Military expenditures (% GNP)
- Expenditures for research and development (% GNP)
- Total expenditures on health (% GDP)
- Public expenditure on education (% GNP)
- Male life expectancy at birth
- Fertility rates
- GINI-index (the distribution of income / consumption)
- Internet hosts per 10000 people
- Mobile phones per 1000 people

- Purchasing power parity (PPP)

- GNP per capita (in USD)
- Average annual growth rate of GDP (% per capita)

Analysis of the World Bank data preprocessing

- 99 states with 16 WDI-indicators
- elementwise transformation of patterns to the interval (0,1) by means of:

$$x' = \frac{x - x_{min}}{x_{max} - x_{min}} \quad \text{and} \quad x'' = \frac{1}{1 + e^{-4(x' - 1/2)}}$$

minimum over all patterns

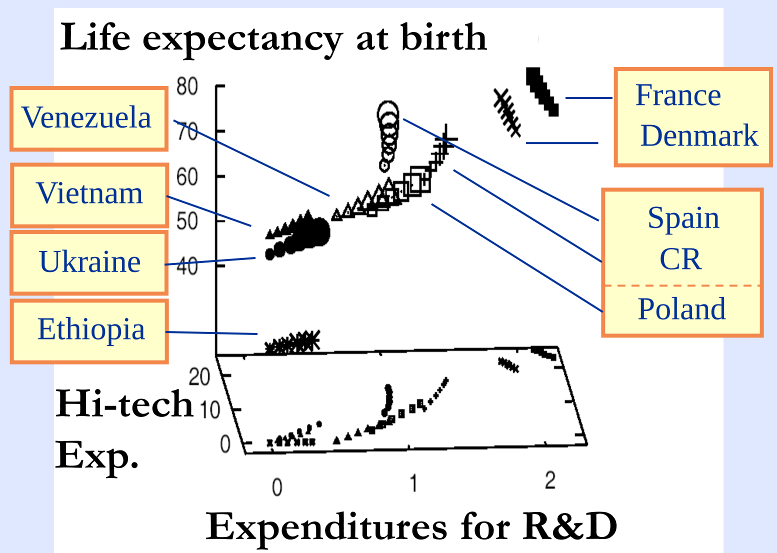
maximum over all patterns

- FCM-clustering: 7 clusters, $s = 1.4$
- controlled learning and iterative recall:
  - **99 (90+9)** states with **14 (13+1)** WDI-indicators
  - GREN-net **14-12-1**, BP-net **13-10-1**; **500-600** training cycles

Analysis of the World Bank data: impact of the indicators on the economy

Indicator	Net 1	Net 2
GDP defl.	0.0	0.0
External debt	5.6	10.9
Total debt service	5.5	8.1
High-tech export	12.2	6.6
Military expenditures	5.4	6.1
Expenditures fot R&D.	16.0	12.0
Internet users	11.1	12.4
Mobile phones	8.3	10.0
GINI-index	7.1	3.9
Life expectancy	12.3	7.6
Fertility	4.4	5.0
Expend. on health .	6.1	10.9
Expend. on education	6.1	6.1

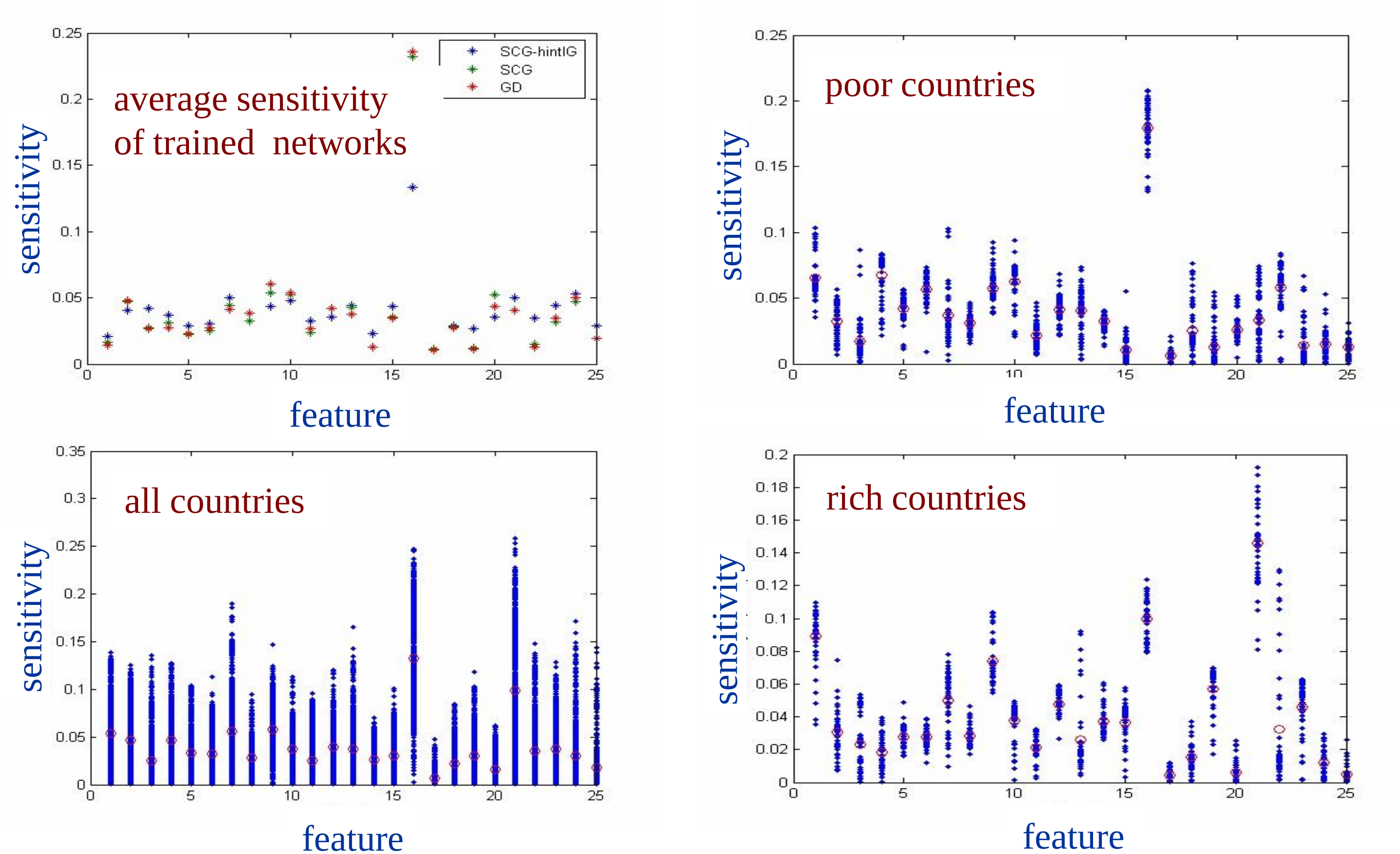
Sensitivity of GREN-networks



Iterative recognition – higher GNP / PPP (Net 1)

Sensitivity to input features

(with Z. Reitermanová)



Mutual dependency of parameters

(with Z. Reitermanová)