Week 10: Daily Morning Challenge

Day 1: Tuesday 10th March 2020

Question 1: Describe the properties of the following probability functions

Cumulative probability function: The cumulative distribution function (CDF) of a random variable X is denoted by F(x), and is defined as F(x) = Pr(X ≤ x).

Using our identity for the probability of disjoint events, if X is a discrete random variable, we can write

$$F(x) = \sum_{k=1}^{n} \Pr(X = x_k)$$

where x_n is the largest possible value of X that is less than or equal to x.
n other words, the cumulative distribution function for a random variable at x gives the probability that the random variable X is less than or equal to that number x.
Note that in the formula for CDFs of discrete random variables, we always have $n \leq N$, where N is the number of possible outcomes of X.

Probability mass function: A probability mass function (PMF) — also called a frequency function — gives you probabilities for discrete random variables. “Random variables” are variables from experiments like dice rolls, choosing a number out of a hat, or getting a high score on a test. The “discrete” part means that there’s a set number of outcomes. For example, you can only roll a 1,2,3,4,5, or 6 on a die.

A PMF equation looks like this:
P(X = x).
That just means “the probability that X takes on some value x”.

Probability density function: When we use a probability function to describe a continuous probability distribution we call it a probability density function. Sometimes we are concerned with the probabilities of random variables that have continuous outcomes. Examples include the height of an adult picked at random from a population or the amount of time that a taxi driver has to wait before their next job. For these examples, the random variable is better described by a continuous probability distribution. The probability density function for the normal distribution is defined as

$$f(x; \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

Where the parameters represent the mean, μ, and the standard deviation, σ, of the population.

Expectation: Mathematical expectation, also known as the expected value, is the summation or integration of possible values from a random variable. It is also known as the product of the probability of an event occurring, denoted P(x), and the value corresponding with the actual observed occurrence of the event. The expected value is a useful property of any random variable. Usually notated as E(X), the expect value can be computed by the summation overall the distinct values that the random variable can take. The mathematical expectation will be given by the mathematical formula as, $E(X)= \Sigma (x_1p_1, x_2p_2, ..., x_np_n)$, where x is a random variable with the probability function, f(x), p is the probability of the occurrence, and n is the number of all possible values In the case
The mathematical expectation of an indicator variable can be zero if there is no occurrence of an event A, and the mathematical expectation of an indicator variable can be one if there is an occurrence of an event A. Thus, it is a useful tool to find the probability of event A.

Variance: Variance is the expected value of the squared variation of a random variable from its mean value, in probability and statistics. In an informal way, it estimates how far a set of numbers (random) are spread out from their mean value.

In statistics, the variance is equal to the square of standard deviation, which is another central tool and is represented by σ^2, s^2, or Var(X).

Variance meaning – It is a measure of how data points differ from the mean. According to layman's terms, it is a measure of how far a set of data(numbers) are spread out from their mean (average) value.

For the purpose of solving questions, it is,

$Var(X) = E[(X - \mu))^2]$
The variance, var(X) of a random variable X has the following properties.

1. $Var(X + C) = Var(X)$, where C is a constant.

2. $Var(CX) = C^2.Var(X)$, where C is a constant.
3. $Var(aX + b) = a^2.Var(X)$, where a and b are constants.
4. If $X_1, X_2,......., X_n$ are n independent random variables, then

$Var(X_1 + X_2 +......+ X_n) = Var(X_1) + Var(X_2) +........+Var(X_n)$.

Question 2: Briefly illustrate with functions the definition of the following random variables

Discreet random variables

Bernoulli: A Bernouilli distribution is a discrete probability distribution for a Bernouilli trial — a random experiment that has only two outcomes (usually called a "Success" or a "Failure"). For example, the probability of getting a heads (a "success") while flipping a coin is 0.5. The probability of "failure" is 1 – P (1 minus the probability of success, which also equals 0.5 for a coin toss). It is a special case of the binomial distribution for n = 1. In other words, it is a binomial distribution with a single trial (e.g. a single coin toss).
The probability of a failure is labeled on the x-axis as 0 and success is labeled as 1.

The probability density function (pdf) for this distribution is $p^x (1 - p)^{1-x}$, which can also be written as:

$$P(n) = \begin{cases} 1 - p & \text{for } n = 0 \\ p & \text{for } n = 1 \end{cases}$$

The expected value for a random variable, X, from a Bernoulli distribution is:
$E[X] = p$.

Binomial: A binomial distribution is a specific probability distribution. It is used to model the probability of obtaining one of two outcomes, a certain number of times (k), out of fixed number of trials (N) of a discrete random event.

A binomial distribution has only two outcomes: the expected outcome is called a success and any other outcome is a failure. The probability of a successful outcome is p and the probability of a failure is 1 - p.

A successful outcome doesn't mean that it's a favorable outcome, but just the outcome being counted.

The binomial distribution is used to model the probabilities of occurrences when specific rules are met.

- Rule #1: There are only two mutually exclusive outcomes for a discrete random variable (i.e., success or failure).
- Rule #2: There is a fixed number of repeated trials (i.e., successive tests with no outcome excluded).
- Rule #3: Each trial is an independent event (meaning the result of one trial doesn't affect the results of subsequent trials).
- Rule #4: The probability of success for each trial is fixed (i.e., the probability of obtaining a successful outcome is the same for all trials).

Geometric: The geometric distribution represents the number of failures before you get a success in a series of Bernoulli trials. This discrete probability distribution is represented by the probability density function:
$f(x) = (1 - p)^{x-1}p$
For example, you ask people outside a polling station who they voted for until you find someone that voted for the independent candidate in a local election. The geometric distribution would represent the number of people who you had to poll before you found someone who voted independent. You would need to get a certain number of failures before you got your first success.

If you had to ask 3 people, then X=3; if you had to ask 4 people, then X=4 and so on. In other words, there would be X-1 failures before you get your success.

The three assumptions of Geometric distribution are:

- There are two possible outcomes for each trial (success or failure).
- The trials are independent.
- The probability of success is the same for each trial.

Poisson: A Poisson distribution is a tool that helps to predict the probability of certain events from happening when you know how often the event has occurred. It gives us the probability of a given number of events happening in a fixed interval of time.

The Poisson Distribution pmf is: $P(x; \mu) = (e^{-\mu} * \mu^{x}) / x!$

Where:

- The symbol "!" is a factorial.
- μ (the expected number of occurrences) is sometimes written as λ. Sometimes called the event rate or rate parameter.

Continuous random variables

Uniform: In statistics, a type of probability distribution in which all outcomes are equally likely; each variable has the same probability that it will be the outcome.
A deck of cards has within it uniform distributions because the likelihood of drawing a heart, a club, a diamond or a spade is equally likely. A coin also has a uniform distribution because the probability of getting either heads or tails in a coin toss is the same. There are two types of uniform distributions: discrete and continuous. In the former type of distribution, each outcome is discrete. In a continuous distribution, outcomes are continuous and infinite.

Exponential: In probability theory and statistics, the exponential distribution is the probability distribution of the time between events in a Poisson point process, i.e., a process in which events occur continuously and independently at a constant average rate. It is a particular case of the gamma distribution. It is the continuous analogue of the geometric distribution,

The probability density function (pdf) of an exponential distribution is

The most common form of the pdf is:
$F(x;\lambda) = e^{-\lambda x}$ $x > 0$.
Where:
- e = the natural number e,
- λ = mean time between events,
- x = a random variable.

For x less than 0, $F(x;\lambda) = 0$

Here $\lambda > 0$ is the parameter of the distribution, often called the rate parameter. The distribution is supported on the interval $[0, \infty)$. If a random variable X has this distribution, we write $X \sim \text{Exp}(\lambda)$.

Normal: The normal distribution is the most important probability distribution in statistics because it fits many natural phenomena. For example, heights, blood pressure, measurement error, and IQ scores follow the normal distribution. It is also known as the Gaussian distribution and the bell curve. The normal distribution is a probability function that describes how the values of a variable are distributed. It is a symmetric distribution where most of the observations cluster around the central peak and the probabilities for values further away from the mean taper off equally in both directions.

As with any probability distribution, the parameters for the normal distribution define its shape and probabilities entirely. The normal distribution has two parameters, the mean and standard deviation. The normal distribution does not have just one form. Instead, the shape changes based on the parameter values