

**Birla Institute of Technology and Science, Pilani K K Birla Goa Campus**  
**First Semester 2016-2017**  
**Test-1 (Closed Book)**

Course No: MATH F424  
Date: 22/09/2016

**Applied Stochastic Processes**

Max. Mark: 60  
Time: 1 Hour

---

**Instructions:**

1. Start a new question on a fresh page.
  2. Write all steps clearly and give explanation for complete credit.
- 

Q1. Let  $X_1, X_2, \dots, X_n$  be iid and follows uniform  $(0,1)$ . If  $Y_n = \max_{1 \leq i \leq n} X_i$ , then show that  $Y_n \xrightarrow{P} 1$ . (15)

Q2. Consider a stochastic process  $\{Z_n\}_{n \geq 1}$  such that  $Z_n$  are iid standard normal random variables. Check if the process  $\{X_n\}_{n \geq 1}$  defined as:

$$X_n = \begin{cases} Z_n, & \text{if } n \text{ is even} \\ \frac{1}{\sqrt{2}}(Z_{n-1}^2 - 1), & \text{if } n \text{ is odd} \end{cases}$$

is weakly stationary process. Also check if  $\{X_n\}_{n \geq 1}$  is strongly stationary process. (15)

Q3. One step transition probability matrix for a Markov chain defined over a state space  $S = \{1, 2, 3\}$  is give as

$$P = \begin{bmatrix} \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 1 & 0 & 0 \end{bmatrix}.$$

Check if chain is irreducible, aperiodic and positive recurrent. Also find the stationary distribution if exists. (15)

Q4. An unbiased coin is flipped repeatedly until three heads in a row appears. Use absorbing Markov chain model to calculate expected number of flips needed. (15)