

Course No. MATH F424      Course title: Applied Stochastic Process

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- (1) The Canadian forest fire weather index is widely as means of to estimate the risk of wildfire. The Ontario Ministry of Natural Resources uses the index to classify each day's risk of forest fire as either nil, low, moderate, high or extreme. Transition probability matrix for the five state Markov chain for the daily changes in the index is given as:

$$P = \begin{matrix} & \begin{matrix} Nil & Low & Moderate & High & Extreme \end{matrix} \\ \begin{matrix} Nil \\ Low \\ Moderate \\ High \\ Extreme \end{matrix} & \begin{pmatrix} .575 & .118 & .172 & .109 & .026 \\ .453 & .243 & .148 & .123 & .033 \\ .104 & .343 & .367 & .167 & .019 \\ .015 & .066 & .318 & .505 & .096 \\ .000 & .060 & .149 & .567 & .224 \end{pmatrix} \end{matrix}$$

Using R find the long term likelihood of risk for a typical day in the early summer.

- (2) University administrators have developed a Markov model to simulate graduation rates at their school. Student might drop out, repeat a year or move on to the next year. Student have a 3% chance of repeating a year. First years and second years have a 6% of dropping out. For third years and fourth years the drop out rate is 4%. The transition matrix for the model is: Simulate the long term probability that a new student graduates.
- (3) After work, angel goes to the gym and either does aerobics, weights, yoga or goes for jogging. Each day Angle decides her workout routine based on what she did the previous day according to the Markov transition matrix:

$$P = \begin{matrix} & \begin{matrix} Aerobics & Jogging & Weights & Yoga \end{matrix} \\ \begin{matrix} Aerobics \\ Jogging \\ Weights \\ Yoga \end{matrix} & \begin{pmatrix} .1 & .2 & .4 & .3 \\ .4 & 0 & .4 & .2 \\ .3 & .3 & 0 & .4 \\ .2 & .1 & .4 & .3 \end{pmatrix} \end{matrix}$$

Simulate the long term probability that she goes for jogging. Compare this with stationary distribution.

- (4) Consider a Markov chain with transition probability matrix

$$P = \begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{pmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 1/3 & 1/3 & 1/3 \end{pmatrix} \end{matrix}$$

Simulate  $\mu_1$ .

- (5) A biased coin comes up head with probability  $2/3$  and tails with probability  $1/3$ . The coin is repeatedly flipped. Simulate to find the total numbers average number of flips needed, until the pattern HTHTH first appears. Compare it with theoretical result.

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