Omnipredictors[1]: One Predictor to Rule Them All

Heavily adapted from P. Gopalan's Talk at IAS

J. Setpal

April 18, 2024

[1] Gopalan, Kalai, Reingold, Sharan, Wieder

Supervised Learning Synopsis

We'll start with an *overview* of supervised learning paradigm:

1. Dataset $\mathcal{D} := \{(x_i, y_i)\}_{i=1}^{N}$; $N \ll \infty$; $\mathcal{D} \sim$ "Real World"

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L is minimized over $\mathcal{D}$, not over the real world. This is **empirical risk**:

$$\min_{\boldsymbol{\theta}} \frac{1}{N} \sum_{i=1}^{N} L(f_{\boldsymbol{\theta}}(x_i), y_i) \tag{1}$$

Generalization Error

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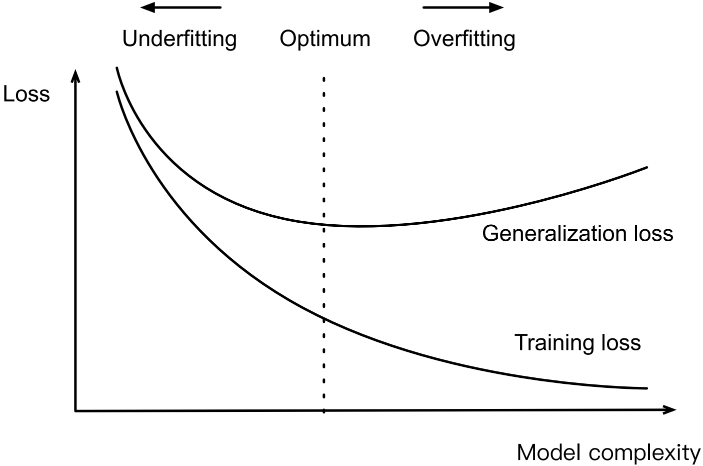
Usually $L_{valid} \not\approx L_{train}$ after training. That's our generalization gap.



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Let's evaluate this empirically on ℓ_1 and ℓ_2 losses, which optimize for median and mean respectively:

$$\ell_1 = |y - \hat{y}|,\ \ell_2 = (y - \hat{y})^2 \tag{2}$$

$$x \sim f(\epsilon \sim \mathcal{U}[0,1]) := \begin{cases} 0 & \epsilon \leq 0.4 \\ \mathcal{U}[0.8,1] & \text{otherwise} \end{cases} \tag{3}$$

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Omnipredictors provides a framework for rigorous guarantees, deriving $\tilde{p} \approx p^*$: a predictor that is able to *simultaneously minimize* a family of convex loss functions.

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Multigroup Fairness

We can split $\mathcal{D}$ into various *subgroups* based on **shared characteristics**. These can be explicit or implicit (i.e. subgroups we don't know of):

	Group-1	**Group-2**	**Group-3**	**Group-4**
Accuracy	0.9593	0.6249	0.3157	0.2664
Loss	0.0021	0.4102	1.3457	1.7664
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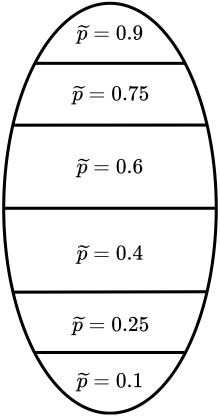
One notion of fairness stipulates equal risk for every subgroup. However, finding subgroups is hard for high-dimensional data.

Multiaccuracy & Multigroup Fairness



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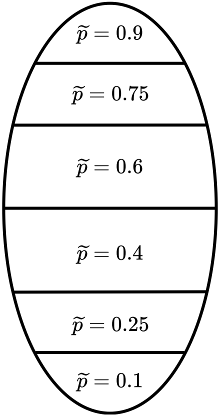

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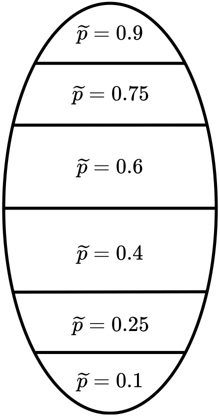

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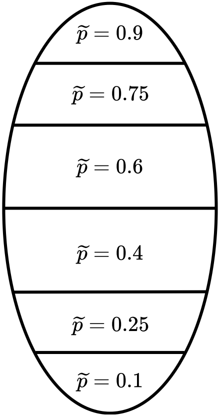

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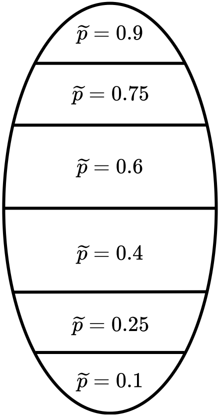

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If we can find correlation with the error, there's some advantage to be gained. We minimize this to train a **weak agnostic learner**.

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Multicalibration implies omniprediction for all convex loss functions.

Training Agnostic Predictors

We can use this framework to train a predictor s.t. a new model trained just on <u>one loss function</u> performs equivalently to the omnipredictor.

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We use this to compute an α-multicalibrated partition by a layered branching program that runs in $\mathcal{O}(\frac{l}{w(\alpha/2)})^{\mathcal{O}(l)}$.

Thank you!

Have an awesome rest of your day!

Slides:

`https://cs.purdue.edu/homes/jsetpal/slides/omnipredictors.pdf`