Attention Is All You Need:

Deriving the Seminal Transformer Architecture from First Principles

J. Setpal

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MACHINE LEARNING @ PURDUE

Glossary

Some context-relevant terms:

a. **Neuron:** The unit of a nueral network $y = \sigma(XW)$
b. **Logit:** Pre-activation scores for the *final layer*.
c. **Dot / Inner Product:** $\langle a, b\rangle = \sum_{i=1}^{N} a_i b_i$
d. **Matrix Multiplication:** For $X \in \mathbb{R}^{a\times b},\ Y \in \mathbb{R}^{b\times c}$, $Z = XY \in \mathbb{R}^{a\times c}$ $s.t.$ $z_{ij} = \sum_{k=1}^{b} a_{ik} b_{kj}$
e. **Activation:** Non-linear function over the output of matrix multiplication.
f. **Gradient:** $\nabla_w \mathcal{L}(w)$, derivative of $\mathcal{L} : \mathbb{R}^a \rightarrow \mathbb{R}$
g. **Latent Vector:** $h^{(\ell)}$, intermediary output from within a neural network.
h. **Embedding:** A look-up table that translates categorical values (words) to vectors.

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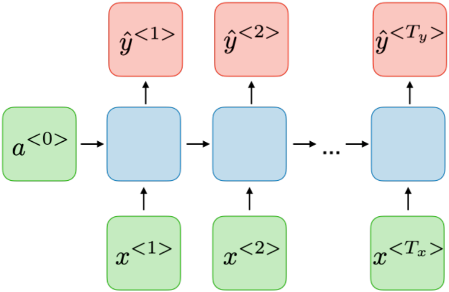

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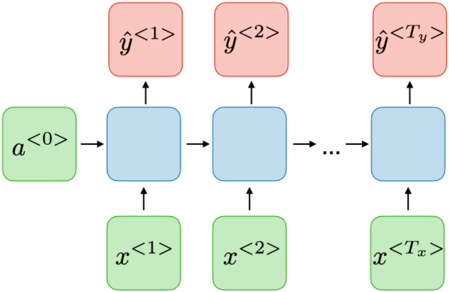

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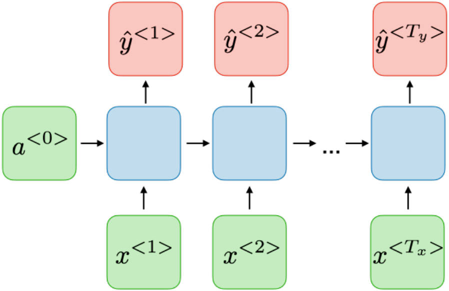

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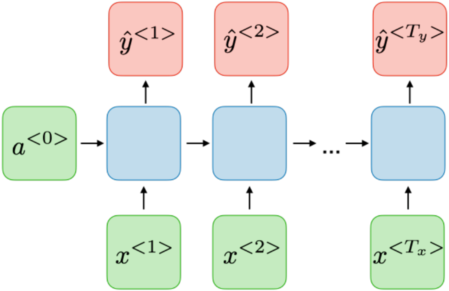

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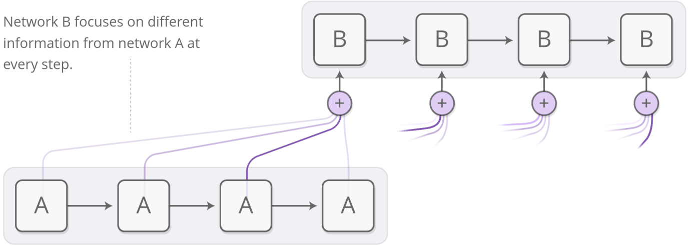

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Where q_i, k_i, v_i are each independently computed latent matrices.

$$\text{Self-Attention}(Q, K, V) = \left(\frac{QK^T}{\sqrt{d_{out}}}\right) V \tag{7}$$

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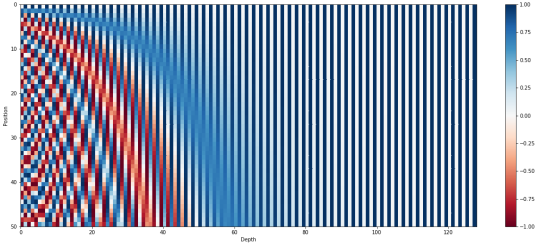

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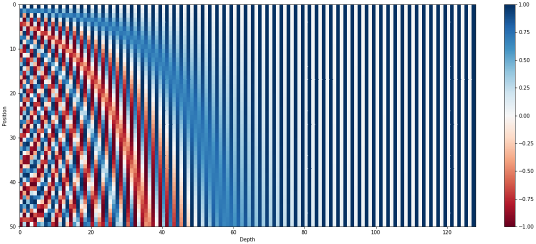

Modern-day references: **Rotary Positional Encodings (RoPE)**.

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Modern-day references: **Flash Attention**, **Multi-Head Latent Attention**.

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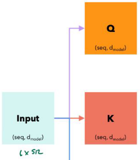

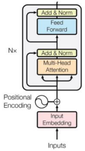

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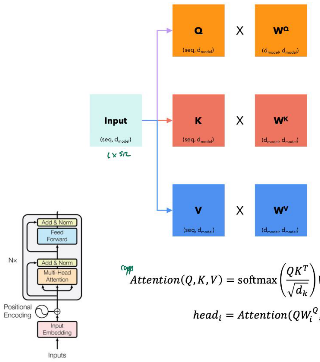

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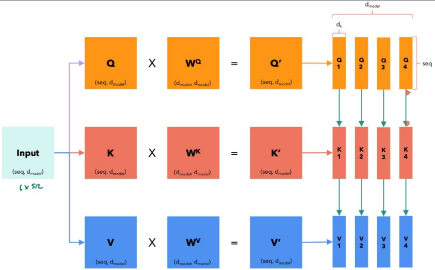

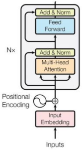

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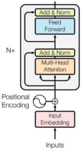

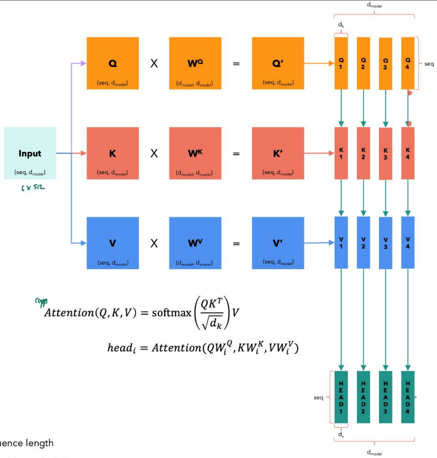

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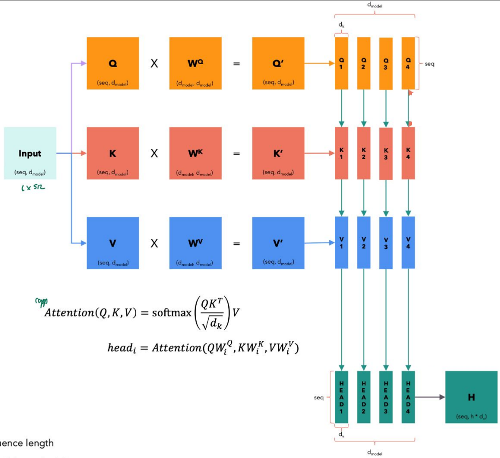

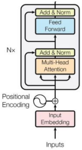

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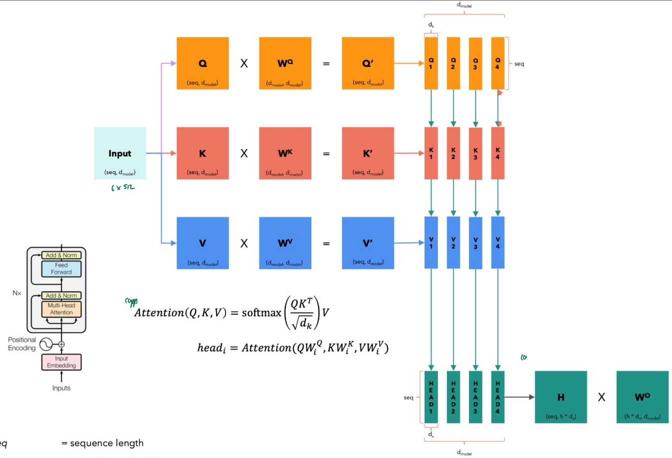

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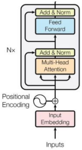

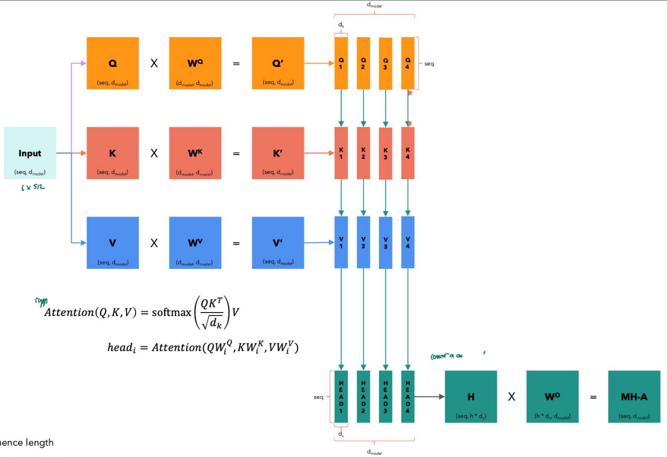

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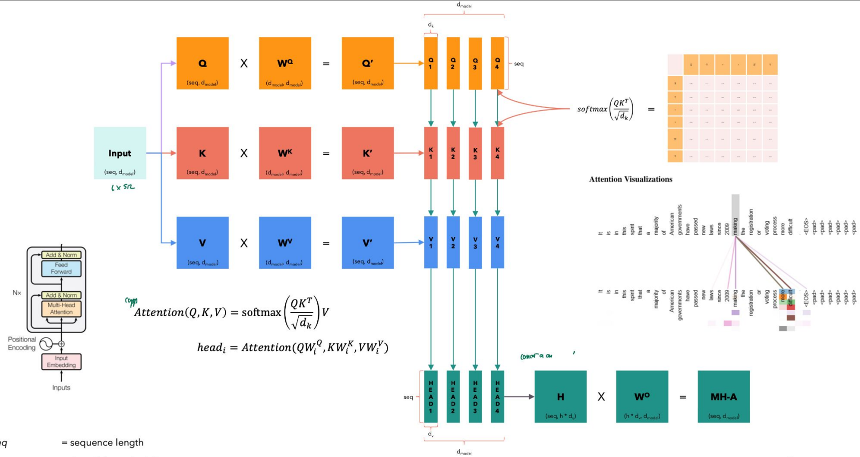

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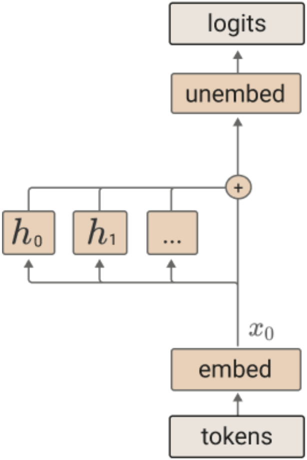
For this, a two-layer MLP is instantiated that expands and consequently contracts the input dimension.

$$FFN(x) = \sigma_{relu}(xW_1 + b_1)W_2 + b_2 \tag{12}$$

The original paper uses a factor of 4.

Scaling to Deeper Networks – A Better Interpretation



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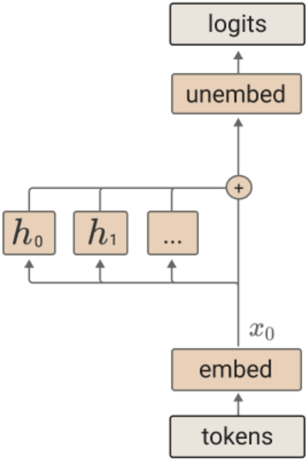

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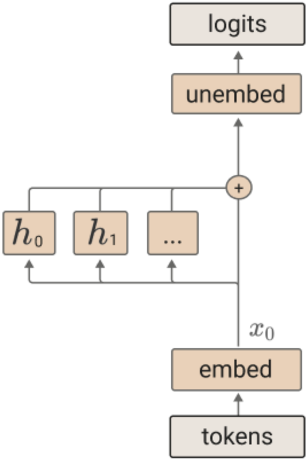

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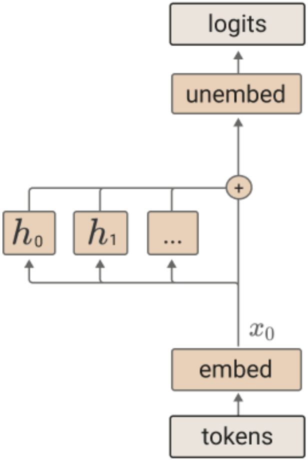

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In addition, we also perform layer normalization over the latent vectors before MLP & self-attention.

Inference & Training

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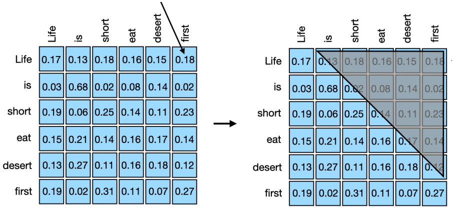
$$\mathcal{L}_{CE}(\{x\}_{t=1}^{T}, \theta) = \frac{1}{T-1} \sum_{t'=1}^{T-1} \mathcal{L}_{CE}(f_\theta(\{x_{t'+1} | x_t\}_{t=1}^{t'}), x_{t'+1}) \quad (13)$$

However, training these blocks means attention would be able to *look into the future*. We correct this by applying an upper-trianguler causal mask:



Reviewing NanoGPT

If you can view this screen, I am making a mistake.

Thank you!

Have an awesome rest of your day!

Slides: `https://cs.purdue.edu/homes/jsetpal/slides/transformer.pdf`