Neural Networks for Learning Counterfactual G-Invariances from Single Environments

"Fixing the Image Rotation Problem"

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MACHINE LEARNING @ PURDUE

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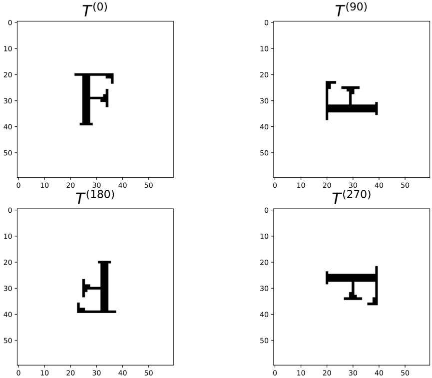
A$_3$: Data Augmentation (boring), **G-Invariant Transformations** (fun)!

Images as Transformations

We can visualize the image rotations as affine matrix transformations:

$$G_{rot} \equiv \{ T^{0^\circ}, T^{90^\circ}, T^{180^\circ}, T^{270^\circ} \} \tag{1}$$

$$x_{new} = Tx_{orig}; T \in G_{rot} \tag{2}$$



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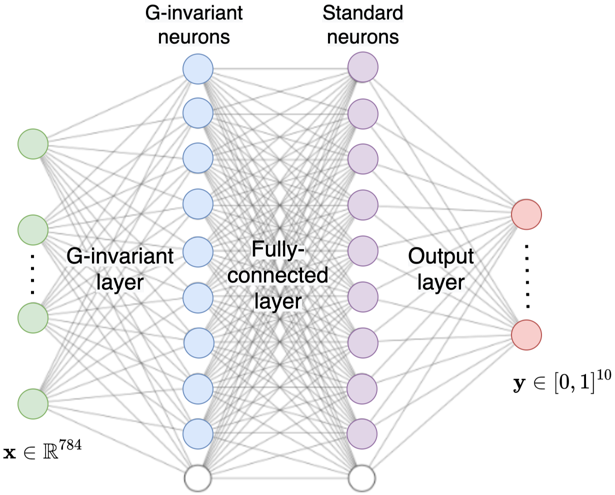
Finally, we construct our group invariant layer:

$$h_{inv} = \sigma(w^T \bar{T} x + b) \tag{6}$$

Let's Demonstrate!

Here's what the final architecture looks like:



Thank you!

Hopefully, this was cool!

Paper: `https://arxiv.org/abs/2104.10105/`

Slides: `https://cs.purdue.edu/homes/jsetpal/slides/gti.pdf`

Notebook: `https://cs.purdue.edu/homes/jsetpal/nb/gti.ipynb`

Presentation: `https://www.youtube.com/watch?v=znJsaCGiu10`