Dynamic Contract for Electricity Procurement

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The deregulated market for electricity services

Generators $\underset{\substack{\uparrow \\ \text{market}}}{-}$ Load Serving Entities $\underset{\substack{\uparrow \\ \text{market}}}{-}$ Customers

- Wholesale market
  - ▷ centralized spot market
  - ▷ one-on-one bilateral contract
  - ▷ financial contract

- Retail market
  - ▷ retailer competition
  - ▷ demand response

Why does the wholesale market need bilateral contracts

- Volatile prices
  - ▷ inelastic demand
  - ▷ physical constraints (generation and transmission)
  - ▷ no storage

- Provide stable revenue for new capacities, especially for renewable sources

- Allow large loads direct access to suppliers

The current model of bilateral contract uses a fixed unit price

- An existing contract pays the supplier $\int_0^T \bar{p} D_s ds$

- Perverse incentive
  - ▷ encourage excessive supply when the market price is low
  - ▷ mis-represent high averaged cost

- stochastic dynamics
  - ▷ supply uncertainties (e.g. renewable sources, equipment failures)
  - ▷ demand uncertainties
  - ▷ grid operations (e.g. transmission scheduling, frequency/voltage control, reactive power)

Desirable features of a dynamic contract

- Allow for stochastic fluctuations
- Account for price manipulation (moral hazard)
- Market responsiveness

Related literature: electricity market design

- The trouble with electricity markets: Borenstein (2002), Joskow (2008)
- Different components: reserve market (Chao et al. 2002), forward market (Murphy et al. 2010), bilateral contract for wind generators (Cai et al. 2011)
- Integrating market and engineering control: Cho et al. (2010), Wang et al. (2011), Mathieu et al. (2013)

Related literature: dynamic contract

- State process: Holmstrom et al. (1987), He (2009), Biais et al. (2010)
- Time horizon and payment: lump-sum (Cvitanic et al. 2009), continuous (Sannikov 2008)
- Information: moral hazard, adverse selection (Sung 2005, Cvitanic et al. 2012), learning (Prat et al. 2013)
- Modeling approach: weak or strong formulation
- Application: corporate finance (DeMarzo et. al 2006), surgeon qualities (Fong 2009)

Contracting environment

- The contract uses the spot price:

$$P_t = P_0 + \int_0^t \rho(A_s - P_s)ds + \int_0^t \sigma dM_s + \sum_{i=1}^{N_t} L_i$$

- Price is a mean reverting jump diffusion
- Agent (supplier) manipulates price by
  - ▷ withholding supplies
  - ▷ virtual bids
  - ▷ proxy
- Principal (e.g. LADWP) cannot contract on actions

Contract

- Principal pays a direct payment $\int_0^T P_s D_s ds$ and an adjustment fee C_T
- Agent's utility: $U(C_T) + \int_0^T \Big(u(P_s D_s) - h(A_s)\Big) ds$
- $U(\cdot)$ and $u(\cdot)$ are agent's utility functions, and $h(a)$ is his cost of performing action a.

- Principal offers a take-it-or-leave-it contract (C_T, A)
- The recommended action A_t could be a function of the history of prices up to time t, i.e. $\{P_s : 0 \leq s \leq t\}$
- C_T could depend on the entire path history of P

Notations and variables

- $D = \{D_s : 0 \leq s \leq T\}$ is the deterministic process that represents the amount of electricity demanded by the principal
- $M = \{M_s : 0 \leq s \leq T\}$ is a standard Brownian motion
- $N = \{N_s : 0 \leq s \leq T\}$ is a Poisson process with intensity λ
- L_i is uniformly distributed over $[-L, L]$
- J denotes the compound Poisson process

$$J_t \triangleq \sum_{i=1}^{N_t} L_i \quad \text{and} \quad \Delta J_t \triangleq J_t - J_{t-},$$

- Action A_t takes value on a compact subset $\mathbf{A} \subseteq \mathbb{R}$
- $\rho > 0$ is the rate of reversion to the mean
- $\sigma > 0$ is the scale of the volatility of M

Agent's continuation value

- Let $\mathcal{F}_t$ to represent the information at time t
- Fix a contract (C_T, A)
- The agent's continuation value

$$W_t^A \triangleq \mathbb{E}_t^A \left[U(C_T) + \int_t^T \Big(u(P_s D_s) - h(A_s) \Big) ds \right]$$

Lemma (Representation)

There exist unique, up to measure zero sets in $[0, T] \times \Omega$, predictable processes (Z^c, Z^d) such that

$$W_t^A = U(C_T) + \int_t^T \Big(u(P_s D_s) - h(A_s) \Big) ds - \int_t^T Z_s^c \sigma dM_s - \sum_{t < s \leq T} Z_s^d \Delta J_s.$$

The set of incentive compatible contracts

We say (C_T, A) is *incentive compatible* if A is an optimal response to C_T for the agent. Denote the set of all incentive compatible contracts by $\mathcal{C}$.

Proposition (Incentives)

The following two conditions are equivalent.
1. (C_T, A) *is incentive compatible.*
2. $A_t(\omega) \in \arg\max_a \{\rho Z_t^c(\omega)a - h(a)\}$, *for almost every* $(t, \omega) \in [0, T] \times \Omega$.

Resolution of the agent's problem

Let $\xi(a)$ (resp. $\eta(z)$) be the solution of $\rho z - h'(a) = 0$ in terms of z (resp. a).

1. Given C_T, the response $A_t = \eta(Z_t^c)$ is optimal, where (W, Z^c, Z^d) is the unique solution of the BSDE

$$\begin{aligned} W_t &= U(C_T) + \int_t^T \Big(u(P_s D_s) - h(\eta(Z_s^c)) + \rho Z_s^c \eta(Z_s^c) \Big) ds \\ &\quad - \int_t^T Z_s^c \sigma dB_s - \sum_{t<s\leq T} Z_s^d \Delta J_s. \end{aligned}$$

2. Given A, set the payment $C_T = I(\tilde{W}_T)$, where $I(\cdot)$ is the inverse of $U(\cdot)$, and

$$\tilde{W}_t = W_0 - \int_0^t \Big(u(P_s D_s) - h(A_s) \Big) ds + \int_0^t \xi(A_s) \sigma dM_s.$$

Principal's expected payment

- The principal's value function is

$$F(t,p,w) \triangleq \min_{A\in\mathcal{A}} \mathbb{E}\left[I(W_T^{t,p,w}) + \int_t^T P_s^{t,p} D_s ds\right]$$

$$\begin{cases} P_s^{t,p} \triangleq p + \int_t^s \rho(A_v - P_v)dv + \int_t^s \sigma dM_v + \sum_{t<v\le s} \Delta J_v \\ W_s^{t,p,w} \triangleq w - \int_t^s \left[u(P_v^{t,p} D_v) - h(A_v)\right] dv + \int_t^s \xi(A_v)\sigma dM_v. \end{cases}$$

derivation

- The incentive compatibility constraint is resolved by the choice of the dynamics of $W_s^{t,p,w}$

- Sovling for a stochastic control problem

Computing the optimal A

- The HJB equation

$$\begin{aligned} -\partial_t F &= \min_a \; \rho(a-p)\partial_p F + \frac{1}{2}\sigma^2 \partial_{pp} F + (h(a) - u(pD_t))\partial_w F \\ &+ \frac{1}{2}\xi(a)^2\sigma^2\partial_{ww} F + \; \xi(a)\sigma^2 \partial_{pw} F \\ &+ \int_{-L}^{L} \Big(F(t, p+dp', w) - F(t,p,w) \Big) \frac{\lambda}{2L} dp' + pD_t, \end{aligned}$$

  with terminal condition $F(T, p, w) = I(w)$, for all p and w.

- The optimal policy function $a^*(t, p, w)$ is the minimizer of

$$\rho a \partial_p F + h(a)\partial_w F + \frac{1}{2}\xi(a)^2\sigma^2\partial_{ww} F + \xi(a)\sigma^2\partial_{pw} F \qquad (1)$$

Properties of the optimal policy

- The state variables (p, w) summarize all the useful information; the agent's incentive is not affected by the path it takes to get to (p, w)

- In general, $a^* \neq min\{\mathbf{A}\}$;
  - ▷ with linear utilities, $a^* = min\{\mathbf{A}\}$
  - ▷ risk aversion and participation constraint
  - ▷ balancing present and future compensation

- The principal pays for the cost of $h(a)$ indirectly, through future price P_t and continuation value W_t

Optimal contract

Theorem (Optimal contract)

Let $a^*(t,p,w)$ be the minimizer in (1), and the agent is paid $C_T = I(W_T)$, where

$$\begin{cases} P_t = P_0 + \int_0^t \rho\Big(a^*(s,P_{s-},W_s) - P_s\Big)ds + \int_0^t \sigma dM_s + \sum_{0<s\le t}\Delta J_s \\ W_t = R + \int_0^t \Big(h(a^*(s,P_{s-},W_s)) - u(P_s D_s)\Big)ds \\ \qquad\qquad + \int_0^t \xi(a^*(s,P_{s-},W_s))\sigma dM_s. \end{cases} \tag{2}$$

Then, the contract $(C_T, A) = \Big(C_T, \{a^*(t,P_{t-},W_t) : 0 \le t \le T\}\Big)$ is incentive compatible for the agent, and optimal for the principal among all incentive compatible contracts that deliver an initial expected value of at least W_0 to the agent.

Verification step

Properties of the contract

- Keep track of P_t to find the minimizing balance between $P_t D_t$ and C_T; not necessary if there is no restriction on when payment is made

- Keep track of agent's continuation value W_t to provide incentive; not necessary in first-best contract or with linear utilities

- No full insurance: C_T needs to be random to provide incentives

- Income effect: $\partial_{ww} F > 0$, due to concavity of U and u

- Sensitivity to market fluctuation:
  - ▷ $\xi > 0$; in the same direction as the price movement
  - ▷ $\xi' > 0$; higher action $a \rightarrow$ higher volatility

An implementation

1. The agent is asked to perform A.

2. By time t, the agent is paid $\int_0^t P_s D_s ds$

3. By time t, the agent is also promised

$$R - \int_0^t \Big(u(P_s D_s) - h(A_s) \Big) ds + \int_0^t \xi(A_s) \sigma dM_s$$

   Note that P is observable, and M is the noise term that can be deduced from P. Hence, we can calculate the promised payment.

Qualitative lessons

- We do not know U, u, or h. We could assume that u and U are linear, but the incentive device ξ depends crucially on the form of h
- Contracted to pay either $\int \bar{p} D_s ds$ or $\int P_s D_s ds$ is not optimal
- The calculation of the adjustment fee must consider: averaged cost and market manipulation
- Adverse selection is not considered here
- The reward/punishment is balanced by two opposing effect:
    - ▷ consistently higher $P_t \to$ lower fee
    - ▷ market fluctuation trending upward $\to$ higher fee

Qualitative lessons, cont.

- Assume linear utilities
- R: Learn the ballpark figure of a fair value of the contract
- $\int_0^T h\big(\min\{\mathbf{A}\}\big)ds$: Estimate the cost of participating in the market, potentially from the information in the virtual bid markets, or let it be a small fraction of R, or set it to the level of expected subsidies
- ξ: Calibrate a fixed number ξ so that with respect to historical data, the realized contract value does not fluctuate too much

Conclusion

- Dynamic bilateral contract needs to account for stochastic fluctuations and be market responsive
- Market price aggregates market information and acts as a performance measure
- Payment should be contingent on market conditions (in particular, P)
- Subsidies should be sensitive on price volatilities.

Further research

- Numerical procedure and illustration
- Extension
  - ▷ control jumps
  - ▷ averaged cost is not known, i.e. the reservation value R is private information (adverse selection)
- A dynamic equilibrium pricing model to study the interaction between the centralized and bilateral market
- Retail market design

Reduction to a stochastic control problem

- Principal's constrained optimization problem is

$$\min_{(C_T,A)\in\mathcal{C}} \mathbb{E}\left[C_T + \int_0^T P_s D_s ds\right] \text{ subject to } W_0^A \geq R.$$

- Using the incentive proposition,

$$\min_{A,W_0\geq R} \mathbb{E}\left[I(W_T) + \int_0^T P_s D_s ds\right], \text{ and}$$

$$W_t = W_0 - \int_0^t \big(u(P_s D_s) - h(A_s)\big) ds + \int_0^t \xi(A_s)\sigma dM_s.$$

- We ignore the control W_0 because we can show that any incentive compatible contract with $W_0 > R$ is not optimal for the principal.

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Verification of the main theorem

Lemma (1)

The contract (C_T, A) is incentive compatible for the agent, and his initial expected utility is R.

Lemma (2)

Let the contract $(\bar{C}_T, \bar{A})$ be incentive compatible and delivers an initial expected utility W_0 to the agent. Then, the contract $(I(\tilde{W}_T), \bar{A})$ is also incentive compatible and delivers W_0, where

$$\tilde{W}_t = W_0 + \int_0^t \Big[h(\bar{A}_s) - u(\bar{P}_s D_s)\Big] ds + \int_0^t \xi(\bar{A}_s)\sigma dM_s.$$

Further, the principal does not strictly prefer $(\bar{C}_T, \bar{A})$ to $(I(\tilde{W}_T), \bar{A})$.

Verification of the main theorem, cont.

Lemma (3)

If a contract $(\bar{C}_T, \bar{A})$ is incentive compatible that delivers an initial expected utility R to the agent, then the principal's initial payment is at least $F(0, P_0, R)$, where $F(\cdot)$ is the solution to (1). That is,

$$\mathbb{E}\left[\bar{C}_T + \int_0^T \bar{P}_s D_s ds\right] \geq F(0, P_0, R),$$

where $\bar{P}_t = P_0 + \int_0^t \rho(\bar{A}_s - \bar{P}_s)ds + \int_0^t \sigma dM_s + \sum_{0<s\leq t} \Delta J_s$.

Verification of the main theorem, cont.

Lemma (4)

Under the contract (C_T, A), the principal's initial expected payment is $F(0, P_0, R)$.

Lemma (5)

Let $(\bar{C}_T, \bar{A})$ be an incentive compatible contract that deliver an initial expected value $\bar{W}_0$ to the agent. If $\bar{W}_0 > R$, then

$$\mathbb{E}\left[\bar{C}_T + \int_0^T \bar{P}_s D_s ds\right] > \mathbb{E}\left[C_T + \int_0^T P_s D_s ds\right].$$

In other words, the principal never wants to offer an incentive compatible contract that gives the agent an initial value higher than R.

Verification of the main theorem, cont.

- The contract is incentive compatible by Lemma 1.
- Set $Z^d = 0$ by Lemma 2.
- Principal starts $W_0 = R$ by Lemma 5.
- Lemma 3 and 4 say that among all incentive compatible contracts in which the agent gets an initial expected payoff R, (C_T, A) is at least as good to the principal as any others.

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