Procurement auction for power reserves

a short review

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Market participants

- N generators with private information about its costs, (K_i, c_i). K_i is i's capacity cost, and c_i is its energy production unit cost.

- There is a system operator (SO) who performs the procurement, and needs to secure Q unit of reserved energy.

- Two kind of services provided (availability and energy), hence two-part bids. (R_i, P_i)

Reserve capacities and the real-time market

- A real-time market operates in parallel; price p.
- A reserve of capacity Q is secured by the SO the day ahead or the hour ahead.
- Each unit of energy in reserve is ranked in a merit order, i.e. $P_1 \leq P_2 \leq \cdots \leq P_Q$, and is called up in that order.
- The reserve is called when
  1. market shortages
  2. spot price p is too high
  3. high level of instability

Procurement process (1)

$t = 0$

- Agent i learns his own cost structure (K_i, c_i).

$t = 1$

- Agent i bids (R_i, P_i).
- SO decides who will be contributing to the reserve, picking a subset $S \subset N$ s.t. $|S| = Q$.

$t = 2$

- Calls up energy from the reserve as needed.

Procurement process (2)

The scoring rule $I(\cdot)$ and the settlement rule $R^*(\cdot), P^*(\cdot)$.

$t = 1$

- Each bid is evaluated as $I(R_i, P_i) = I_i$; a threshold level I^* is set.
- The bid is accepted iff $I_i \leq I^*$.
- If its bid is accepted, then i gets a capacity payment $R^*(R_i, P_i, R_{-i}, P_{-i})$

$t = 2$

- If i is called to supply its share, it gets a payment $P^*(R_i, P_i, R_{-i}, P_{-i})$.

A simple scoring and settlement rule

- $I_i = R_i + HP_i$, where H is a fixed fractional number.
- Capacity payment is R_i.
- Energy payment is hP_i at $t = 2$, where h is the actual fraction of hour that i supplies energy to the SO.

Incentive problem

The agent solves

$$\max_{R_i, P_i} \; (R_i - K_i) + h(P_i)[P_i - c_i]$$

FOC $\Rightarrow h'(P_i)(P_i - c_i) + h(P_i) = 0.$

Then, $P^*(c_i)$ is a nonlinear bidding function. In particular, $P_i^* \neq c_i$. It is not incentive compatible.

What are the magic scoring rule and settlement rule that work

$I(\cdot), R^*(\cdot), P^*(\cdot)$ that are ...

1. incentive compatible
2. optimal, i.e. costs the SO the least amount of money.
3. simple

First approach: a simultaneous auction

- θ is the r.v. representing the actual number of unit of energy that is called from the reserve at $t = 2$.
- q_i is i's maximal generating capacity.

With full information, the SO would like to clear the market by minimizing the producers' costs

$$\min_{\{g_j(\theta), x_j\}_{j,\theta}} \quad \sum_{j \in N} k_j x_j + \mathbb{E}^{\theta}[c_j g_j(\theta)]$$

$$\text{subject to: } \sum_j g_j(\theta) = \theta, \ \sum_j x_j = Q,$$

$$g_j(\theta) \leq x_j, \ x_j \leq q_j,$$

$$g_j(\theta) \geq 0, \ x_j \geq 0$$

The key results of a VCG payment scheme

1. The SO solicits a report of hidden costs $(\hat{K}, \hat{c}) = \{(\hat{K}_i, \hat{c}_i)\}_i$.

2. The allocation is specified by a contingency plan $g_i^*(\hat{K}, \hat{c})$ and units of commitment $x_i^*(\hat{K}, \hat{c})$.

3. The payment is calculated explicitly for each generator $V_i(\hat{K}_i, \hat{c}_i, \hat{K}_{-i}, \hat{c}_{-i})$.

4. The reported costs would have be truthful if the agents are optimizing, i.e. $(\hat{K}, \hat{c}) = (K, c)$.

Allocation and payment calculations (1)

Perform two optimizations:

First: $$\min_{\{g_j(\theta), x_j\}_{j,\theta}} \sum_{j \in N} k_j x_j + \mathbb{E}^\theta[c_j g_j(\theta)]$$

subject to some linear constraint

Let the optimizers be g^*, x^*.

Second: $$\min_{\{g_j(\theta), x_j\}_{j,\theta}} \sum_{j \neq i} k_j x_j + \mathbb{E}^\theta[c_j g_j(\theta)]$$

subject to some linear constraints

Let the optimizers be $\tilde{g}, \tilde{x}$.

Allocation and payment calculations (2)

1. The allocation is such that x_i^* unit of capacity is requested from i, and i delivers $g_i^*(\theta)$ unit of energy.

2. The payment is

$$V_i = \left[\sum_{j \neq i} k_i x_i^* + \mathbb{E}^\theta [c_i g_i^*(\theta)] \right] - \left[\sum_{j \neq i} k_i \tilde{x}_i + \mathbb{E}^\theta [c_i \tilde{g}_i(\theta)] \right]$$

3. Generators report K_i, c_i.

Properties of this scheme

1. $V_i \geq 0$
2. If we ignore payments, the SO minimizes costs.
3. However, this scheme is not guaranteed to be an optimal contract, i.e. the SO pays generators too much.
4. Also, it is possible that if implemented by the bidding (R_i, P_i), capacity bid is too large and P_i is small or negative.
5. If $P_i < c_i$, the SO should worry about energy withholding at $t = 2$.

To get an optimal, IC contract, we need to consider a more complicated problem

$$\min_{\{g_j(\theta), x_j\}_{j,\theta}, T(\cdot)} \sum_{j \in N} T(K_j, c_j, K_{-j}, c_{-j})$$

$$\text{subject to: } \begin{array}{l} \text{1. some linear constraints and} \\ \text{2. For each } j,\ K_j, c_j = \\ \displaystyle\arg\max_{\hat{K}_i, \hat{c}_i} T(\hat{K}_j, \hat{c}_j, K_{-j}, c_{-j}) - \hat{k}_j x_i - \mathbb{E}^{\theta}[\hat{c}_i g_i(\theta)] \end{array}$$

Issues with this formulation

1. A larger search space
2. Has embedded optimization problems
3. No general way to simplify the problem

Second approach: a sequential auction

The goal is to justify the following two-step scheme is incentive compatible.

$t = 1$

1. the scoring rule is $I_i = I(R_i, P_i) = R_i$
2. the first stage uniform payment is I^*, the lowest rejected I_i.

$t = 2$

3. Rearrange the P_i s.t. $P_1 \leq P_2 \leq \cdots \leq P_Q$. The demanded unit is θ, hence the unit payment is the lowest rejected $P_{\theta+1}$ to all $i \leq \theta$.

The main claim of the paper

The sequential auction as proposed is incentive compatible.

An example

Let $K_A = 1, K_B = 1, K_C = 1$, and $c_A = 1, c_B = 2, c_C = 3$.

Let's assume that the SO needs to secure 2 unit of reserve and without uncertainty, only one unit is called at $t = 2$.

Then,

1. Report $\{(R_i, P_i)\}_i = \{(0,1),(1,2),(1,3)\}$.
2. $I^* = 1$.
3. $P^* = 2$.
4. Both A and B are recruited. The net total profit: A gets 2 and B gets 0.

A backward induction argument (1)

- Assume a competitive market
- Let the spot price be a r.v. $p \sim dG(p)$.

At $t = 2$, the expected profit conditioning on being accepted into the pool is

$$\Pi(P_i, c_i) = [1 - G(P_i)]\mathbb{E}^p[p - c_i | p \geq P_i]$$

Rewriting

$$[1 - G(P_i)] \int_{P_i}^{\infty} \frac{(p - c_i)}{1 - G(P_i)} dG(p)$$

and taking the FOC yields that the optimizer is $P_i^*(c_i) = c_i$.

A backward induction argument (2)

- Assume the scoring rule takes the additive form, i.e. $I(R_i, P_i) = R_i + H(P_i)$.

At $t = 1$, the expected profit from an accepted bid is

$$R^*(P_i) - R(P_i, c_i) = [I^* - H(P_i)] - [K_i - \Pi(P_i, c_i)].$$

Taking the FOC with respect to P_i,

$$H'(P_i) - \Pi'(P_i,) = 0.$$

Applying $\Pi'(P_i, c_i)|_{P_i = c_i} = \Pi'(c_i, c_i) = 0$, we pin down H to be a constant function. Hence, $H = 0$ is good enough.

A backward induction argument (3)

Second stage IC forces the scoring rule to be simple, i.e. the initial procurement auction does not consider P_i and compare capacity bids R_i only.

Finally, using Vickrey auction in the first stage ensures first stage IC. $\square$

Properties of sequential auctions

1. Use VCG payment rule for both the capacity and energy price settlement, i.e. pay the first rejected bid.
2. The scoring rule is simple: only look at capacity bids.
3. Settlement rules are sequential.