Some notes on homework 2

Question 1

The solution needs to characterize the menus of contracts such that

1. Agent θ chooses the contract $\Big(q(\theta), e(\theta), C \mapsto t(\theta, C)\Big)$ and willingly performs $e(\theta)$

2. Agent's expected payoff under $\Big(q(\theta), e(\theta), C \mapsto t(\theta, C)\Big)$ is more than his reservation value

Note that q is observed and it is not random, so the principal can just enforce whatever q she desires. C is observed but it is randrom, hence the contracted payment should depend on it to provide incentives. e is not contractable; the agent follows the recommendation of e if and only if e is incentive compatible.

As usual, the P/A problem should be tackled in two steps. We first look at agent's problem. For any menu of contracts $\{q(\theta'), e(\theta'), C \mapsto t(\theta', C)\}_{\theta' \in \Theta}$, the agent θ best-responds by choosing (θ^*, e^*),

$$(\theta^*, e^*) = \underset{(\theta', e')}{\arg\max}\ \mathbb{E}_C \left[U^A(\theta, q(\theta'), e', t(\theta', C)\right]. \tag{1}$$

Note that the choice of e' does not need to be $e(\theta')$ because effort is not observed. The randromness in C comes from ϵ. In any case, the offered contract is IC if $(\theta^*, e^*) = (\theta, e(\theta))$.

Assuming that (1) is solved, let the set of all incentive compatible contracts be denoted by $\mathcal{C}$. Assume the participation constraint binds, then usually, it can be eliminated via a substitution operation into the principal's objective function. Then, the principal solve

$$\max_{(q(\cdot), e(\cdot), t(\cdot)) \in \mathcal{C}} \mathbb{E}_{\theta, C} \left[U^P\Big(C, q(\theta), e(\theta), t(\theta, C)\Big)\right].$$

The exact resolution of the two optimization problems can be tedious to develop, but the general principles are simple. Details are in Laffont and Tirole's paper.

Question 2

Yes, and the proof is the same as in the case of dominant strategy, by just replacing the definition.

Question 3

We can look at a counter-example. Two players bidding for an item and the winner of the item gets private utility of $\theta_i(\theta_1+\theta_2)$, the loser gets 0. Without loss, let $\theta_1<\theta_2$. If they report truthfuly, then the VCG allocation is that the second player gets the item and the transfers are

$$\begin{cases} t^1 = \theta_2(\theta_1+\theta_2)+a^1(\theta_2) \\ t^2 = 0 + a^2(\theta_2), \end{cases}$$

for some functions of $a^i(\cdot)$. Then the agents' utilities are

$$\begin{cases} u^1 = \theta_2(\theta_1+\theta_2)+a^1(\theta_2) \\ u^2 = \theta_2(\theta_1+\theta_2)+a^2(\theta_1). \end{cases}$$

For a report of $\hat{\theta}_1<\theta_2$, the utilities are

$$\begin{cases} u^1 = \theta_2(\hat{\theta}_1+\theta_2)+a^1(\theta_2) \\ u^2 = \theta_2(\hat{\theta}_1+\theta_2)+a^2(\hat{\theta}_1). \end{cases}$$

Player 1 deviates.

When preferences are independent, i's VCG payment is not affected by i's reporting of θ_i, and can only change his utility by way of changing the outcome. By construction of the payment scheme, he would never want to change the outcome. When the preferences are inter-dependent, i can affect his own payment without changing the outcome, hence, the VCG scheme is not incentive compatible.

Question 4

The public good is provided at the level of $x = 1$ if and only if $K \leq \sum_i \theta^i$. If $K > \sum_i \theta^i$, no public good is provided and all payment is $a^i(\theta^{-i})$. If $K \leq \sum_i \theta^i$, then $x = 1$, and the payment is

$$t^i = K - \sum_{j \neq i} \theta_i + a^i(\theta^{-i}),$$

for some arbitrary function of $a^i(\cdot)$. To generate surplus, just let $a^i = C$ for some really large number. That is, everyone has to pay the government at least a trillion dollars to ensure a surplus.

Question 5

(a) Yes. For voter θ_i, he votes for the provision of the public good if and only if $\theta_i \geq K/N$ regardless of everyone's action.

(b) From the voters' point of view, if the VCG payment demands at least a trillion dollars from everyone, then clearly, they are better off in the voting scheme. Now consider a VCG payment such that there is a i^* with $a^{i^*} = C$, which is very large, and $a^i = -1$ for everyone else. Note that the surplus constraint is satisfied, and the choice of a does not affect incentives. In the case that no public good is provided, everyone except i^* strictly prefers the VCG payment to voting.

However, from the social planner's point of view, VCG allocation is more efficient than voting.