homework 07

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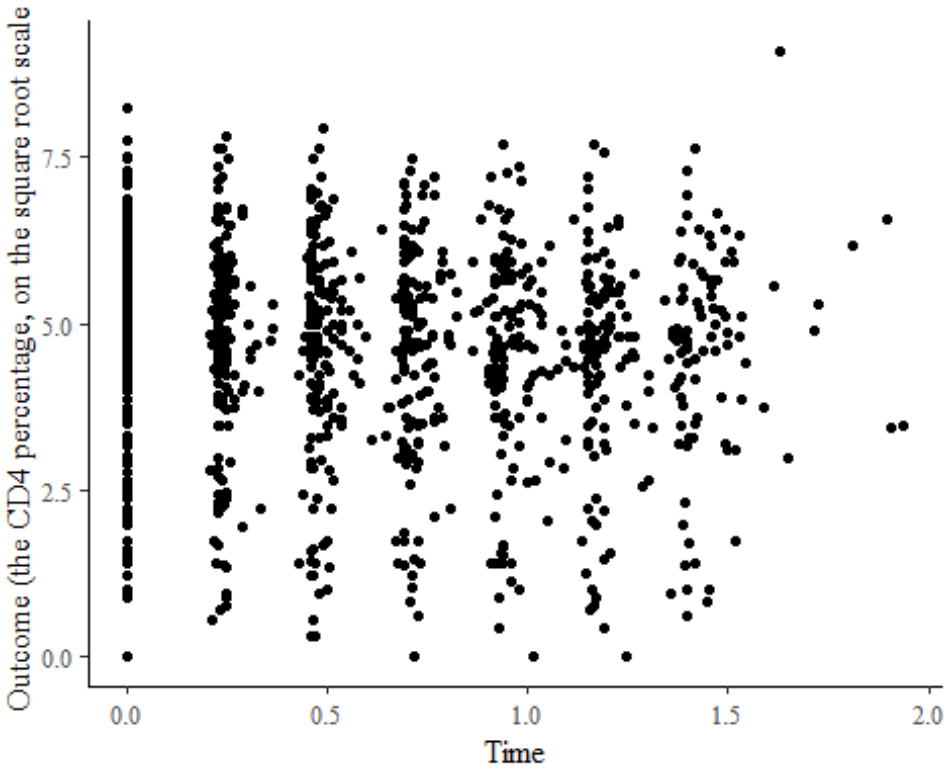
Data analysis

CD4 percentages for HIV infected kids

The folder `cd4` has CD4 percentages for a set of young children with HIV who were measured several times over a period of two years. The dataset also includes the ages of the children at each measurement.

1. Graph the outcome (the CD4 percentage, on the square root scale) for each child as a function of time.

```
ggplot(hiv.data)+geom_point(aes(x=time,y=y))+
  xlab("Time")+ ylab("Outcome (the CD4 percentage, on the square root s
cale)")
```



2. Each child's data has a time course that can be summarized by a linear fit. Estimate these lines and plot them for all the children.

```r
# No pooling
r_np <- lm(y~time+factor(newpid)-1, data = hiv.data)
summary(r_np)
```

```
##
## Call:
## lm(formula = y ~ time + factor(newpid) - 1, data = hiv.data)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -3.6595 -0.3293  0.0000  0.3347  4.0036
##
## Coefficients:
##                    Estimate Std. Error t value Pr(>|t|)
## time               -0.38629    0.05455  -7.081 3.07e-12 ***
## factor(newpid)1     4.56368    0.34896  13.078  < 2e-16 ***
## factor(newpid)2     0.81507    0.54578   1.493 0.135716
## factor(newpid)3     5.95004    0.29534  20.146  < 2e-16 ***
## factor(newpid)4     5.61374    0.31677  17.722  < 2e-16 ***
## factor(newpid)5     4.00000    0.77180   5.183 2.76e-07 ***
## factor(newpid)6     5.36947    0.31738  16.918  < 2e-16 ***
## factor(newpid)7     5.61896    0.29436  19.088  < 2e-16 ***
## factor(newpid)8     5.14703    0.38791  13.268  < 2e-16 ***
## factor(newpid)9     6.21645    0.34732  17.898  < 2e-16 ***
## factor(newpid)10    5.71848    0.31739  18.017  < 2e-16 ***
## factor(newpid)11    2.44507    0.29417   8.312 3.89e-16 ***
## factor(newpid)12    4.36330    0.31699  13.765  < 2e-16 ***
## factor(newpid)13    5.33903    0.44635  11.962  < 2e-16 ***
## factor(newpid)14    3.00000    0.77180   3.887 0.000110 ***
## factor(newpid)15    5.24008    0.31759  16.499  < 2e-16 ***
## factor(newpid)16    2.39908    0.38705   6.198 9.03e-10 ***
## factor(newpid)17    6.10066    0.31839  19.161  < 2e-16 ***
## factor(newpid)18    6.02588    0.34608  17.412  < 2e-16 ***
## factor(newpid)19    4.10797    0.38783  10.592  < 2e-16 ***
## factor(newpid)20    5.00962    0.44580  11.237  < 2e-16 ***
## factor(newpid)21    5.00000    0.77180   6.478 1.60e-10 ***
## factor(newpid)22    6.16441    0.77180   7.987 4.66e-15 ***
## factor(newpid)23    1.59920    0.34723   4.606 4.76e-06 ***
## factor(newpid)24    4.81823    0.44728  10.772  < 2e-16 ***
## factor(newpid)25    4.76132    0.31717  15.012  < 2e-16 ***
## factor(newpid)26    4.63303    0.31656  14.636  < 2e-16 ***
## factor(newpid)27    4.38498    0.31672  13.845  < 2e-16 ***
## factor(newpid)28    5.65959    0.54590  10.367  < 2e-16 ***
## factor(newpid)29    4.52845    0.38717  11.696  < 2e-16 ***
## factor(newpid)30    1.00000    0.77180   1.296 0.195454
## factor(newpid)31    4.45824    0.54608   8.164 1.22e-15 ***
## factor(newpid)32    4.64821    0.34892  13.322  < 2e-16 ***
## factor(newpid)33    5.03494    0.29431  17.108  < 2e-16 ***
## factor(newpid)34    6.49167    0.54579  11.894  < 2e-16 ***
## factor(newpid)35    4.93661    0.38757  12.737  < 2e-16 ***
```

```
## factor(newpid)37   3.98526    0.54579   7.302 6.72e-13 ***
## factor(newpid)38   6.15939    0.44617  13.805  < 2e-16 ***
## factor(newpid)39   4.84721    0.34613  14.004  < 2e-16 ***
## factor(newpid)40   3.60555    0.77180   4.672 3.49e-06 ***
## factor(newpid)41   5.00000    0.77180   6.478 1.60e-10 ***
## factor(newpid)42   3.26132    0.29446  11.076  < 2e-16 ***
## factor(newpid)43   4.93493    0.29446  16.759  < 2e-16 ***
## factor(newpid)44   2.49104    0.44579   5.588 3.13e-08 ***
## factor(newpid)45   5.16288    0.31782  16.245  < 2e-16 ***
## factor(newpid)46   3.50085    0.31798  11.010  < 2e-16 ***
## factor(newpid)47   4.85968    0.31796  15.284  < 2e-16 ***
## factor(newpid)48   4.45407    0.38739  11.498  < 2e-16 ***
## factor(newpid)49   5.39827    0.29437  18.339  < 2e-16 ***
## factor(newpid)50   4.32745    0.29426  14.706  < 2e-16 ***
## factor(newpid)51   3.94551    0.34618  11.397  < 2e-16 ***
## factor(newpid)52   1.79719    0.29417   6.109 1.54e-09 ***
## factor(newpid)53   4.81554    0.29411  16.373  < 2e-16 ***
## factor(newpid)54   4.46903    0.29419  15.191  < 2e-16 ***
## factor(newpid)55   2.37752    0.29410   8.084 2.24e-15 ***
## factor(newpid)56   2.79201    0.54578   5.116 3.90e-07 ***
## factor(newpid)57   2.14991    0.31692   6.784 2.24e-11 ***
## factor(newpid)58   2.01600    0.31692   6.361 3.32e-10 ***
## factor(newpid)59   5.12724    0.29440  17.416  < 2e-16 ***
## factor(newpid)60   2.04462    0.54578   3.746 0.000192 ***
## factor(newpid)61   5.23903    0.31671  16.542  < 2e-16 ***
## factor(newpid)62   5.65826    0.29448  19.215  < 2e-16 ***
## factor(newpid)63   1.92512    0.29426   6.542 1.07e-10 ***
## factor(newpid)64   5.42219    0.29418  18.431  < 2e-16 ***
## factor(newpid)65   1.42126    0.34611   4.106 4.42e-05 ***
## factor(newpid)66   6.46556    0.44592  14.499  < 2e-16 ***
## factor(newpid)67   2.50677    0.54579   4.593 5.06e-06 ***
## factor(newpid)68   5.87367    0.77180   7.610 7.50e-14 ***
## factor(newpid)69   5.37708    0.39062  13.766  < 2e-16 ***
## factor(newpid)70   5.04789    0.38676  13.052  < 2e-16 ***
## factor(newpid)71   2.64575    0.77180   3.428 0.000638 ***
## factor(newpid)72   3.79504    0.38672   9.813  < 2e-16 ***
## factor(newpid)73   6.85565    0.77180   8.883  < 2e-16 ***
## factor(newpid)74   5.15287    0.29412  17.519  < 2e-16 ***
## factor(newpid)75   5.83766    0.29416  19.845  < 2e-16 ***
## factor(newpid)76   4.92242    0.34748  14.166  < 2e-16 ***
## factor(newpid)77   4.01660    0.38672  10.386  < 2e-16 ***
## factor(newpid)78   5.99278    0.29415  20.373  < 2e-16 ***
## factor(newpid)79   4.90326    0.44575  11.000  < 2e-16 ***
## factor(newpid)81   0.97153    0.54589   1.780 0.075492 .
## factor(newpid)82   3.25905    0.34636   9.409  < 2e-16 ***
## factor(newpid)83   0.94868    0.77180   1.229 0.219356
## factor(newpid)84   2.25870    0.34701   6.509 1.32e-10 ***
## factor(newpid)85   1.58969    0.34705   4.581 5.36e-06 ***
## factor(newpid)86   6.44121    0.34644  18.593  < 2e-16 ***
## factor(newpid)87   6.09731    0.29421  20.724  < 2e-16 ***
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## factor(newpid)88   4.83296    0.54579   8.855  < 2e-16 ***
## factor(newpid)89   5.02052    0.34621  14.501  < 2e-16 ***
## factor(newpid)90   5.84808    0.77180   7.577 9.53e-14 ***
## factor(newpid)91   2.54897    0.38706   6.586 8.09e-11 ***
## factor(newpid)92   2.68623    0.54579   4.922 1.04e-06 ***
## factor(newpid)93   1.52443    0.38637   3.945 8.64e-05 ***
## factor(newpid)94   4.94328    0.44775  11.040  < 2e-16 ***
## factor(newpid)95   2.78151    0.54578   5.096 4.30e-07 ***
## factor(newpid)96   4.89898    0.77180   6.347 3.62e-10 ***
## factor(newpid)97   7.70878    0.44671  17.257  < 2e-16 ***
## factor(newpid)98   4.79583    0.77180   6.214 8.22e-10 ***
## factor(newpid)99   6.58753    0.38674  17.033  < 2e-16 ***
## factor(newpid)100  6.54584    0.34609  18.914  < 2e-16 ***
## factor(newpid)101  5.65685    0.77180   7.329 5.54e-13 ***
## factor(newpid)103  6.11117    0.29512  20.708  < 2e-16 ***
## factor(newpid)104  3.55877    0.31688  11.230  < 2e-16 ***
## factor(newpid)105  4.66845    0.29461  15.846  < 2e-16 ***
## factor(newpid)106  3.79964    0.38686   9.822  < 2e-16 ***
## factor(newpid)107  5.79041    0.38686  14.968  < 2e-16 ***
## factor(newpid)108  1.17737    0.38739   3.039 0.002447 **
## factor(newpid)109  4.04447    0.54579   7.410 3.13e-13 ***
## factor(newpid)110  5.32304    0.29448  18.076  < 2e-16 ***
## factor(newpid)111  2.13749    0.54580   3.916 9.74e-05 ***
## factor(newpid)112  4.04681    0.29465  13.734  < 2e-16 ***
## factor(newpid)113  6.34488    0.31739  19.991  < 2e-16 ***
## factor(newpid)114  4.95064    0.29459  16.805  < 2e-16 ***
## factor(newpid)115  5.62952    0.29454  19.113  < 2e-16 ***
## factor(newpid)116  4.25683    0.54612   7.795 1.95e-14 ***
## factor(newpid)117  4.41240    0.34852  12.660  < 2e-16 ***
## factor(newpid)118  5.31355    0.34636  15.341  < 2e-16 ***
## factor(newpid)119  1.92914    0.54582   3.534 0.000432 ***
## factor(newpid)120  6.83535    0.31712  21.555  < 2e-16 ***
## factor(newpid)121  6.12904    0.44703  13.711  < 2e-16 ***
## factor(newpid)122  5.43379    0.44651  12.169  < 2e-16 ***
## factor(newpid)123  2.96695    0.54578   5.436 7.18e-08 ***
## factor(newpid)124  3.16228    0.77180   4.097 4.60e-05 ***
## factor(newpid)126  4.48243    0.38753  11.567  < 2e-16 ***
## factor(newpid)127  5.25547    0.34628  15.177  < 2e-16 ***
## factor(newpid)128  4.75350    0.54668   8.695  < 2e-16 ***
## factor(newpid)129  0.97864    0.34636   2.825 0.004836 **
## factor(newpid)130  3.70472    0.38672   9.580  < 2e-16 ***
## factor(newpid)131  4.25708    0.38711  10.997  < 2e-16 ***
## factor(newpid)132  4.73853    0.38778  12.220  < 2e-16 ***
## factor(newpid)133  3.77490    0.31673  11.918  < 2e-16 ***
## factor(newpid)134  6.72519    0.29422  22.858  < 2e-16 ***
## factor(newpid)135  5.60776    0.29440  19.048  < 2e-16 ***
## factor(newpid)136  6.64977    0.29433  22.593  < 2e-16 ***
## factor(newpid)137  5.67273    0.29452  19.261  < 2e-16 ***
## factor(newpid)138  7.48331    0.77180   9.696  < 2e-16 ***
## factor(newpid)139  4.85189    0.29479  16.459  < 2e-16 ***
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```
## factor(newpid)140  5.47249    0.29452  18.581  < 2e-16 ***
## factor(newpid)141  7.16773    0.29440  24.347  < 2e-16 ***
## factor(newpid)142  2.82420    0.31707   8.907  < 2e-16 ***
## factor(newpid)143  2.88106    0.29437   9.787  < 2e-16 ***
## factor(newpid)144  6.04833    0.29423  20.556  < 2e-16 ***
## factor(newpid)145  5.55106    0.31688  17.518  < 2e-16 ***
## factor(newpid)146  5.46320    0.31677  17.246  < 2e-16 ***
## factor(newpid)147  6.18166    0.34655  17.838  < 2e-16 ***
## factor(newpid)148  5.34407    0.44578  11.988  < 2e-16 ***
## factor(newpid)149  5.67007    0.34615  16.381  < 2e-16 ***
## factor(newpid)150  4.39422    0.38642  11.372  < 2e-16 ***
## factor(newpid)151  5.68779    0.38640  14.720  < 2e-16 ***
## factor(newpid)152  4.61519    0.77180   5.980 3.33e-09 ***
## factor(newpid)153  7.21403    0.44577  16.183  < 2e-16 ***
## factor(newpid)154  5.71394    0.44580  12.817  < 2e-16 ***
## factor(newpid)155  6.27073    0.44579  14.067  < 2e-16 ***
## factor(newpid)156  6.34439    0.54578  11.624  < 2e-16 ***
## factor(newpid)157  6.41098    0.44609  14.371  < 2e-16 ***
## factor(newpid)158  6.08632    0.34692  17.544  < 2e-16 ***
## factor(newpid)159  5.29916    0.54594   9.706  < 2e-16 ***
## factor(newpid)160  5.04712    0.54579   9.247  < 2e-16 ***
## factor(newpid)161  5.14072    0.38657  13.298  < 2e-16 ***
## factor(newpid)162  4.69277    0.44588  10.525  < 2e-16 ***
## factor(newpid)163  7.42011    0.38647  19.200  < 2e-16 ***
## factor(newpid)164  7.07418    0.34873  20.286  < 2e-16 ***
## factor(newpid)165  4.40042    0.34744  12.665  < 2e-16 ***
## factor(newpid)166  5.63845    0.54812  10.287  < 2e-16 ***
## factor(newpid)167  4.93276    0.38713  12.742  < 2e-16 ***
## factor(newpid)168  5.79989    0.29425  19.711  < 2e-16 ***
## factor(newpid)169  2.83271    0.54605   5.188 2.69e-07 ***
## factor(newpid)170  4.52041    0.34670  13.039  < 2e-16 ***
## factor(newpid)171  6.70820    0.77180   8.692  < 2e-16 ***
## factor(newpid)172  5.26891    0.34643  15.209  < 2e-16 ***
## factor(newpid)173  1.59625    0.54592   2.924 0.003551 **
## factor(newpid)174  3.80765    0.34709  10.970  < 2e-16 ***
## factor(newpid)175  5.86770    0.34640  16.939  < 2e-16 ***
## factor(newpid)176  5.71388    0.44591  12.814  < 2e-16 ***
## factor(newpid)177  4.65448    0.38715  12.022  < 2e-16 ***
## factor(newpid)178  6.64100    0.34712  19.132  < 2e-16 ***
## factor(newpid)179  5.42868    0.44577  12.178  < 2e-16 ***
## factor(newpid)180  5.38254    0.29417  18.297  < 2e-16 ***
## factor(newpid)181  7.58231    0.31737  23.891  < 2e-16 ***
## factor(newpid)182  6.87445    0.44674  15.388  < 2e-16 ***
## factor(newpid)183  4.73226    0.54591   8.669  < 2e-16 ***
## factor(newpid)184  4.69042    0.77180   6.077 1.87e-09 ***
## factor(newpid)185  5.32106    0.31790  16.738  < 2e-16 ***
## factor(newpid)186  2.26637    0.34754   6.521 1.22e-10 ***
## factor(newpid)187  5.96108    0.31804  18.743  < 2e-16 ***
## factor(newpid)188  5.64729    0.34676  16.286  < 2e-16 ***
## factor(newpid)189  0.89556    0.54589   1.641 0.101277
```

```
## factor(newpid)190  3.93221    0.54593   7.203 1.34e-12 ***
## factor(newpid)191  4.73072    0.44582  10.611  < 2e-16 ***
## factor(newpid)192  4.63493    0.29415  15.757  < 2e-16 ***
## factor(newpid)193  3.51569    0.29414  11.952  < 2e-16 ***
## factor(newpid)194  1.67399    0.31665   5.286 1.60e-07 ***
## factor(newpid)195  6.57259    0.44708  14.701  < 2e-16 ***
## factor(newpid)196  4.28686    0.38778  11.055  < 2e-16 ***
## factor(newpid)197  4.52015    0.38659  11.692  < 2e-16 ***
## factor(newpid)198  6.11686    0.34677  17.640  < 2e-16 ***
## factor(newpid)199  3.58154    0.38734   9.247  < 2e-16 ***
## factor(newpid)200  6.33062    0.31871  19.863  < 2e-16 ***
## factor(newpid)201  4.88817    0.38837  12.586  < 2e-16 ***
## factor(newpid)202  6.08433    0.54598  11.144  < 2e-16 ***
## factor(newpid)203  6.31594    0.38792  16.282  < 2e-16 ***
## factor(newpid)204  5.44066    0.38672  14.069  < 2e-16 ***
## factor(newpid)205  3.66210    0.34771  10.532  < 2e-16 ***
## factor(newpid)206  5.98915    0.29415  20.361  < 2e-16 ***
## factor(newpid)207  6.08204    0.31761  19.149  < 2e-16 ***
## factor(newpid)208  4.17020    0.34723  12.010  < 2e-16 ***
## factor(newpid)209  6.43027    0.31684  20.295  < 2e-16 ***
## factor(newpid)210  5.21148    0.29412  17.719  < 2e-16 ***
## factor(newpid)211  5.34459    0.29419  18.167  < 2e-16 ***
## factor(newpid)212  5.21535    0.31670  16.468  < 2e-16 ***
## factor(newpid)213  4.67607    0.44578  10.490  < 2e-16 ***
## factor(newpid)214  6.54179    0.29428  22.230  < 2e-16 ***
## factor(newpid)215  5.04463    0.31666  15.931  < 2e-16 ***
## factor(newpid)216  3.74901    0.34628  10.827  < 2e-16 ***
## factor(newpid)217  3.09943    0.54578   5.679 1.88e-08 ***
## factor(newpid)218  4.76821    0.29420  16.207  < 2e-16 ***
## factor(newpid)219  5.47723    0.77180   7.097 2.76e-12 ***
## factor(newpid)220  6.34478    0.29424  21.564  < 2e-16 ***
## factor(newpid)221  5.78464    0.31662  18.270  < 2e-16 ***
## factor(newpid)222  5.27235    0.31785  16.587  < 2e-16 ***
## factor(newpid)223  5.34864    0.31661  16.894  < 2e-16 ***
## factor(newpid)224  3.80821    0.54578   6.978 6.19e-12 ***
## factor(newpid)225  6.47400    0.29413  22.010  < 2e-16 ***
## factor(newpid)226  6.85178    0.34695  19.748  < 2e-16 ***
## factor(newpid)227  6.21616    0.31664  19.631  < 2e-16 ***
## factor(newpid)228  4.67312    0.31665  14.758  < 2e-16 ***
## factor(newpid)229  5.25787    0.34628  15.184  < 2e-16 ***
## factor(newpid)230  5.96217    0.34628  17.218  < 2e-16 ***
## factor(newpid)231  5.95432    0.38653  15.405  < 2e-16 ***
## factor(newpid)232  6.17519    0.44620  13.840  < 2e-16 ***
## factor(newpid)233  4.36377    0.38636  11.295  < 2e-16 ***
## factor(newpid)234  6.22240    0.54578  11.401  < 2e-16 ***
## factor(newpid)235  3.21066    0.44635   7.193 1.43e-12 ***
## factor(newpid)236  2.83698    0.34674   8.182 1.06e-15 ***
## factor(newpid)237  5.43365    0.31707  17.137  < 2e-16 ***
## factor(newpid)238  5.05647    0.38660  13.079  < 2e-16 ***
## factor(newpid)239  5.54035    0.44593  12.424  < 2e-16 ***
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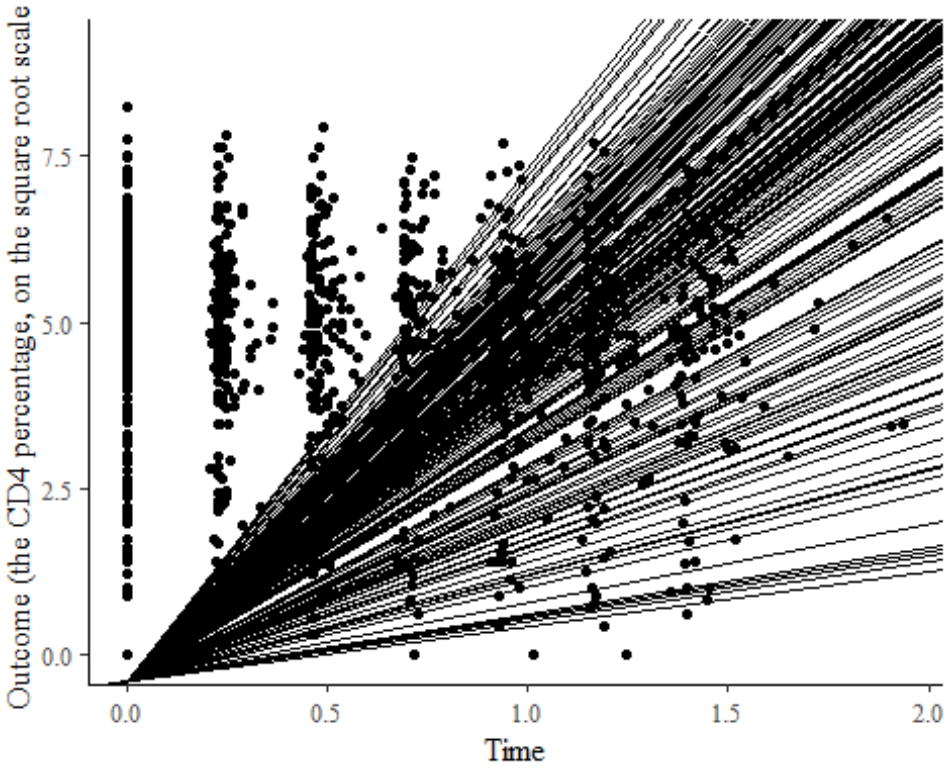
## factor(newpid)240  3.51138    0.34603  10.148  < 2e-16 ***
## factor(newpid)241  6.11555    0.77180   7.924 7.49e-15 ***
## factor(newpid)242  5.16910    0.44592  11.592  < 2e-16 ***
## factor(newpid)243  5.89800    0.44636  13.213  < 2e-16 ***
## factor(newpid)244  5.94175    0.54578  10.887  < 2e-16 ***
## factor(newpid)245  4.92484    0.38641  12.745  < 2e-16 ***
## factor(newpid)246  5.05558    0.54579   9.263  < 2e-16 ***
## factor(newpid)247  4.78539    0.77180   6.200 8.92e-10 ***
## factor(newpid)248  5.64132    0.54579  10.336  < 2e-16 ***
## factor(newpid)249  5.59464    0.77180   7.249 9.71e-13 ***
## factor(newpid)250  5.83524    0.54579  10.691  < 2e-16 ***
## factor(newpid)251  3.74166    0.77180   4.848 1.49e-06 ***
## factor(newpid)252  4.51291    0.54582   8.268 5.45e-16 ***
## factor(newpid)253  3.60555    0.77180   4.672 3.49e-06 ***
## factor(newpid)254  3.75520    0.54598   6.878 1.20e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.7718 on 821 degrees of freedom
## Multiple R-squared:  0.9809, Adjusted R-squared:  0.9751
## F-statistic: 168.1 on 251 and 821 DF,  p-value: < 2.2e-16

# plot fitted line
ggplot(aes(x=time, y=y), data = hiv.data)+
  geom_point()+
  geom_abline(intercept = coef(r_np)[1], slope=coef(r_np)[2:length(coef
(r_np))]) +
  xlab("Time")+ ylab("Outcome (the CD4 percentage, on the square root s
cale)")

## Warning in data.frame(intercept = intercept, slope = slope): row nam
es were
## found from a short variable and have been discarded
```



3. Set up a model for the children's slopes and intercepts as a function of the treatment and age at baseline. Estimate this model using the two-step procedure–first estimate the intercept and slope separately for each child, then fit the between-child models using the point estimates from the first step.

```
# Step1: individual level predictors: time, newpid indicators
r1 <- lm(y~time+factor(newpid)-1, data = hiv.data)
summary(r1)

##
## Call:
## lm(formula = y ~ time + factor(newpid) - 1, data = hiv.data)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -3.6595 -0.3293  0.0000  0.3347  4.0036
##
## Coefficients:
##                   Estimate Std. Error t value Pr(>|t|)
## time               -0.38629    0.05455  -7.081 3.07e-12 ***
## factor(newpid)1     4.56368    0.34896  13.078  < 2e-16 ***
## factor(newpid)2     0.81507    0.54578   1.493 0.135716
## factor(newpid)3     5.95004    0.29534  20.146  < 2e-16 ***
## factor(newpid)4     5.61374    0.31677  17.722  < 2e-16 ***
## factor(newpid)5     4.00000    0.77180   5.183 2.76e-07 ***
## factor(newpid)6     5.36947    0.31738  16.918  < 2e-16 ***
## factor(newpid)7     5.61896    0.29436  19.088  < 2e-16 ***
```

```
## factor(newpid)8    5.14703    0.38791  13.268  < 2e-16 ***
## factor(newpid)9    6.21645    0.34732  17.898  < 2e-16 ***
## factor(newpid)10   5.71848    0.31739  18.017  < 2e-16 ***
## factor(newpid)11   2.44507    0.29417   8.312 3.89e-16 ***
## factor(newpid)12   4.36330    0.31699  13.765  < 2e-16 ***
## factor(newpid)13   5.33903    0.44635  11.962  < 2e-16 ***
## factor(newpid)14   3.00000    0.77180   3.887 0.000110 ***
## factor(newpid)15   5.24008    0.31759  16.499  < 2e-16 ***
## factor(newpid)16   2.39908    0.38705   6.198 9.03e-10 ***
## factor(newpid)17   6.10066    0.31839  19.161  < 2e-16 ***
## factor(newpid)18   6.02588    0.34608  17.412  < 2e-16 ***
## factor(newpid)19   4.10797    0.38783  10.592  < 2e-16 ***
## factor(newpid)20   5.00962    0.44580  11.237  < 2e-16 ***
## factor(newpid)21   5.00000    0.77180   6.478 1.60e-10 ***
## factor(newpid)22   6.16441    0.77180   7.987 4.66e-15 ***
## factor(newpid)23   1.59920    0.34723   4.606 4.76e-06 ***
## factor(newpid)24   4.81823    0.44728  10.772  < 2e-16 ***
## factor(newpid)25   4.76132    0.31717  15.012  < 2e-16 ***
## factor(newpid)26   4.63303    0.31656  14.636  < 2e-16 ***
## factor(newpid)27   4.38498    0.31672  13.845  < 2e-16 ***
## factor(newpid)28   5.65959    0.54590  10.367  < 2e-16 ***
## factor(newpid)29   4.52845    0.38717  11.696  < 2e-16 ***
## factor(newpid)30   1.00000    0.77180   1.296 0.195454
## factor(newpid)31   4.45824    0.54608   8.164 1.22e-15 ***
## factor(newpid)32   4.64821    0.34892  13.322  < 2e-16 ***
## factor(newpid)33   5.03494    0.29431  17.108  < 2e-16 ***
## factor(newpid)34   6.49167    0.54579  11.894  < 2e-16 ***
## factor(newpid)35   4.93661    0.38757  12.737  < 2e-16 ***
## factor(newpid)37   3.98526    0.54579   7.302 6.72e-13 ***
## factor(newpid)38   6.15939    0.44617  13.805  < 2e-16 ***
## factor(newpid)39   4.84721    0.34613  14.004  < 2e-16 ***
## factor(newpid)40   3.60555    0.77180   4.672 3.49e-06 ***
## factor(newpid)41   5.00000    0.77180   6.478 1.60e-10 ***
## factor(newpid)42   3.26132    0.29446  11.076  < 2e-16 ***
## factor(newpid)43   4.93493    0.29446  16.759  < 2e-16 ***
## factor(newpid)44   2.49104    0.44579   5.588 3.13e-08 ***
## factor(newpid)45   5.16288    0.31782  16.245  < 2e-16 ***
## factor(newpid)46   3.50085    0.31798  11.010  < 2e-16 ***
## factor(newpid)47   4.85968    0.31796  15.284  < 2e-16 ***
## factor(newpid)48   4.45407    0.38739  11.498  < 2e-16 ***
## factor(newpid)49   5.39827    0.29437  18.339  < 2e-16 ***
## factor(newpid)50   4.32745    0.29426  14.706  < 2e-16 ***
## factor(newpid)51   3.94551    0.34618  11.397  < 2e-16 ***
## factor(newpid)52   1.79719    0.29417   6.109 1.54e-09 ***
## factor(newpid)53   4.81554    0.29411  16.373  < 2e-16 ***
## factor(newpid)54   4.46903    0.29419  15.191  < 2e-16 ***
## factor(newpid)55   2.37752    0.29410   8.084 2.24e-15 ***
## factor(newpid)56   2.79201    0.54578   5.116 3.90e-07 ***
## factor(newpid)57   2.14991    0.31692   6.784 2.24e-11 ***
## factor(newpid)58   2.01600    0.31692   6.361 3.32e-10 ***
```

```
## factor(newpid)59   5.12724    0.29440  17.416  < 2e-16 ***
## factor(newpid)60   2.04462    0.54578   3.746 0.000192 ***
## factor(newpid)61   5.23903    0.31671  16.542  < 2e-16 ***
## factor(newpid)62   5.65826    0.29448  19.215  < 2e-16 ***
## factor(newpid)63   1.92512    0.29426   6.542 1.07e-10 ***
## factor(newpid)64   5.42219    0.29418  18.431  < 2e-16 ***
## factor(newpid)65   1.42126    0.34611   4.106 4.42e-05 ***
## factor(newpid)66   6.46556    0.44592  14.499  < 2e-16 ***
## factor(newpid)67   2.50677    0.54579   4.593 5.06e-06 ***
## factor(newpid)68   5.87367    0.77180   7.610 7.50e-14 ***
## factor(newpid)69   5.37708    0.39062  13.766  < 2e-16 ***
## factor(newpid)70   5.04789    0.38676  13.052  < 2e-16 ***
## factor(newpid)71   2.64575    0.77180   3.428 0.000638 ***
## factor(newpid)72   3.79504    0.38672   9.813  < 2e-16 ***
## factor(newpid)73   6.85565    0.77180   8.883  < 2e-16 ***
## factor(newpid)74   5.15287    0.29412  17.519  < 2e-16 ***
## factor(newpid)75   5.83766    0.29416  19.845  < 2e-16 ***
## factor(newpid)76   4.92242    0.34748  14.166  < 2e-16 ***
## factor(newpid)77   4.01660    0.38672  10.386  < 2e-16 ***
## factor(newpid)78   5.99278    0.29415  20.373  < 2e-16 ***
## factor(newpid)79   4.90326    0.44575  11.000  < 2e-16 ***
## factor(newpid)81   0.97153    0.54589   1.780 0.075492 .
## factor(newpid)82   3.25905    0.34636   9.409  < 2e-16 ***
## factor(newpid)83   0.94868    0.77180   1.229 0.219356
## factor(newpid)84   2.25870    0.34701   6.509 1.32e-10 ***
## factor(newpid)85   1.58969    0.34705   4.581 5.36e-06 ***
## factor(newpid)86   6.44121    0.34644  18.593  < 2e-16 ***
## factor(newpid)87   6.09731    0.29421  20.724  < 2e-16 ***
## factor(newpid)88   4.83296    0.54579   8.855  < 2e-16 ***
## factor(newpid)89   5.02052    0.34621  14.501  < 2e-16 ***
## factor(newpid)90   5.84808    0.77180   7.577 9.53e-14 ***
## factor(newpid)91   2.54897    0.38706   6.586 8.09e-11 ***
## factor(newpid)92   2.68623    0.54579   4.922 1.04e-06 ***
## factor(newpid)93   1.52443    0.38637   3.945 8.64e-05 ***
## factor(newpid)94   4.94328    0.44775  11.040  < 2e-16 ***
## factor(newpid)95   2.78151    0.54578   5.096 4.30e-07 ***
## factor(newpid)96   4.89898    0.77180   6.347 3.62e-10 ***
## factor(newpid)97   7.70878    0.44671  17.257  < 2e-16 ***
## factor(newpid)98   4.79583    0.77180   6.214 8.22e-10 ***
## factor(newpid)99   6.58753    0.38674  17.033  < 2e-16 ***
## factor(newpid)100  6.54584    0.34609  18.914  < 2e-16 ***
## factor(newpid)101  5.65685    0.77180   7.329 5.54e-13 ***
## factor(newpid)103  6.11117    0.29512  20.708  < 2e-16 ***
## factor(newpid)104  3.55877    0.31688  11.230  < 2e-16 ***
## factor(newpid)105  4.66845    0.29461  15.846  < 2e-16 ***
## factor(newpid)106  3.79964    0.38686   9.822  < 2e-16 ***
## factor(newpid)107  5.79041    0.38686  14.968  < 2e-16 ***
## factor(newpid)108  1.17737    0.38739   3.039 0.002447 **
## factor(newpid)109  4.04447    0.54579   7.410 3.13e-13 ***
## factor(newpid)110  5.32304    0.29448  18.076  < 2e-16 ***
```

```
## factor(newpid)111  2.13749    0.54580   3.916 9.74e-05 ***
## factor(newpid)112  4.04681    0.29465  13.734  < 2e-16 ***
## factor(newpid)113  6.34488    0.31739  19.991  < 2e-16 ***
## factor(newpid)114  4.95064    0.29459  16.805  < 2e-16 ***
## factor(newpid)115  5.62952    0.29454  19.113  < 2e-16 ***
## factor(newpid)116  4.25683    0.54612   7.795 1.95e-14 ***
## factor(newpid)117  4.41240    0.34852  12.660  < 2e-16 ***
## factor(newpid)118  5.31355    0.34636  15.341  < 2e-16 ***
## factor(newpid)119  1.92914    0.54582   3.534 0.000432 ***
## factor(newpid)120  6.83535    0.31712  21.555  < 2e-16 ***
## factor(newpid)121  6.12904    0.44703  13.711  < 2e-16 ***
## factor(newpid)122  5.43379    0.44651  12.169  < 2e-16 ***
## factor(newpid)123  2.96695    0.54578   5.436 7.18e-08 ***
## factor(newpid)124  3.16228    0.77180   4.097 4.60e-05 ***
## factor(newpid)126  4.48243    0.38753  11.567  < 2e-16 ***
## factor(newpid)127  5.25547    0.34628  15.177  < 2e-16 ***
## factor(newpid)128  4.75350    0.54668   8.695  < 2e-16 ***
## factor(newpid)129  0.97864    0.34636   2.825 0.004836 **
## factor(newpid)130  3.70472    0.38672   9.580  < 2e-16 ***
## factor(newpid)131  4.25708    0.38711  10.997  < 2e-16 ***
## factor(newpid)132  4.73853    0.38778  12.220  < 2e-16 ***
## factor(newpid)133  3.77490    0.31673  11.918  < 2e-16 ***
## factor(newpid)134  6.72519    0.29422  22.858  < 2e-16 ***
## factor(newpid)135  5.60776    0.29440  19.048  < 2e-16 ***
## factor(newpid)136  6.64977    0.29433  22.593  < 2e-16 ***
## factor(newpid)137  5.67273    0.29452  19.261  < 2e-16 ***
## factor(newpid)138  7.48331    0.77180   9.696  < 2e-16 ***
## factor(newpid)139  4.85189    0.29479  16.459  < 2e-16 ***
## factor(newpid)140  5.47249    0.29452  18.581  < 2e-16 ***
## factor(newpid)141  7.16773    0.29440  24.347  < 2e-16 ***
## factor(newpid)142  2.82420    0.31707   8.907  < 2e-16 ***
## factor(newpid)143  2.88106    0.29437   9.787  < 2e-16 ***
## factor(newpid)144  6.04833    0.29423  20.556  < 2e-16 ***
## factor(newpid)145  5.55106    0.31688  17.518  < 2e-16 ***
## factor(newpid)146  5.46320    0.31677  17.246  < 2e-16 ***
## factor(newpid)147  6.18166    0.34655  17.838  < 2e-16 ***
## factor(newpid)148  5.34407    0.44578  11.988  < 2e-16 ***
## factor(newpid)149  5.67007    0.34615  16.381  < 2e-16 ***
## factor(newpid)150  4.39422    0.38642  11.372  < 2e-16 ***
## factor(newpid)151  5.68779    0.38640  14.720  < 2e-16 ***
## factor(newpid)152  4.61519    0.77180   5.980 3.33e-09 ***
## factor(newpid)153  7.21403    0.44577  16.183  < 2e-16 ***
## factor(newpid)154  5.71394    0.44580  12.817  < 2e-16 ***
## factor(newpid)155  6.27073    0.44579  14.067  < 2e-16 ***
## factor(newpid)156  6.34439    0.54578  11.624  < 2e-16 ***
## factor(newpid)157  6.41098    0.44609  14.371  < 2e-16 ***
## factor(newpid)158  6.08632    0.34692  17.544  < 2e-16 ***
## factor(newpid)159  5.29916    0.54594   9.706  < 2e-16 ***
## factor(newpid)160  5.04712    0.54579   9.247  < 2e-16 ***
## factor(newpid)161  5.14072    0.38657  13.298  < 2e-16 ***
```

```
## factor(newpid)162  4.69277    0.44588  10.525  < 2e-16 ***
## factor(newpid)163  7.42011    0.38647  19.200  < 2e-16 ***
## factor(newpid)164  7.07418    0.34873  20.286  < 2e-16 ***
## factor(newpid)165  4.40042    0.34744  12.665  < 2e-16 ***
## factor(newpid)166  5.63845    0.54812  10.287  < 2e-16 ***
## factor(newpid)167  4.93276    0.38713  12.742  < 2e-16 ***
## factor(newpid)168  5.79989    0.29425  19.711  < 2e-16 ***
## factor(newpid)169  2.83271    0.54605   5.188 2.69e-07 ***
## factor(newpid)170  4.52041    0.34670  13.039  < 2e-16 ***
## factor(newpid)171  6.70820    0.77180   8.692  < 2e-16 ***
## factor(newpid)172  5.26891    0.34643  15.209  < 2e-16 ***
## factor(newpid)173  1.59625    0.54592   2.924 0.003551 **
## factor(newpid)174  3.80765    0.34709  10.970  < 2e-16 ***
## factor(newpid)175  5.86770    0.34640  16.939  < 2e-16 ***
## factor(newpid)176  5.71388    0.44591  12.814  < 2e-16 ***
## factor(newpid)177  4.65448    0.38715  12.022  < 2e-16 ***
## factor(newpid)178  6.64100    0.34712  19.132  < 2e-16 ***
## factor(newpid)179  5.42868    0.44577  12.178  < 2e-16 ***
## factor(newpid)180  5.38254    0.29417  18.297  < 2e-16 ***
## factor(newpid)181  7.58231    0.31737  23.891  < 2e-16 ***
## factor(newpid)182  6.87445    0.44674  15.388  < 2e-16 ***
## factor(newpid)183  4.73226    0.54591   8.669  < 2e-16 ***
## factor(newpid)184  4.69042    0.77180   6.077 1.87e-09 ***
## factor(newpid)185  5.32106    0.31790  16.738  < 2e-16 ***
## factor(newpid)186  2.26637    0.34754   6.521 1.22e-10 ***
## factor(newpid)187  5.96108    0.31804  18.743  < 2e-16 ***
## factor(newpid)188  5.64729    0.34676  16.286  < 2e-16 ***
## factor(newpid)189  0.89556    0.54589   1.641 0.101277
## factor(newpid)190  3.93221    0.54593   7.203 1.34e-12 ***
## factor(newpid)191  4.73072    0.44582  10.611  < 2e-16 ***
## factor(newpid)192  4.63493    0.29415  15.757  < 2e-16 ***
## factor(newpid)193  3.51569    0.29414  11.952  < 2e-16 ***
## factor(newpid)194  1.67399    0.31665   5.286 1.60e-07 ***
## factor(newpid)195  6.57259    0.44708  14.701  < 2e-16 ***
## factor(newpid)196  4.28686    0.38778  11.055  < 2e-16 ***
## factor(newpid)197  4.52015    0.38659  11.692  < 2e-16 ***
## factor(newpid)198  6.11686    0.34677  17.640  < 2e-16 ***
## factor(newpid)199  3.58154    0.38734   9.247  < 2e-16 ***
## factor(newpid)200  6.33062    0.31871  19.863  < 2e-16 ***
## factor(newpid)201  4.88817    0.38837  12.586  < 2e-16 ***
## factor(newpid)202  6.08433    0.54598  11.144  < 2e-16 ***
## factor(newpid)203  6.31594    0.38792  16.282  < 2e-16 ***
## factor(newpid)204  5.44066    0.38672  14.069  < 2e-16 ***
## factor(newpid)205  3.66210    0.34771  10.532  < 2e-16 ***
## factor(newpid)206  5.98915    0.29415  20.361  < 2e-16 ***
## factor(newpid)207  6.08204    0.31761  19.149  < 2e-16 ***
## factor(newpid)208  4.17020    0.34723  12.010  < 2e-16 ***
## factor(newpid)209  6.43027    0.31684  20.295  < 2e-16 ***
## factor(newpid)210  5.21148    0.29412  17.719  < 2e-16 ***
## factor(newpid)211  5.34459    0.29419  18.167  < 2e-16 ***
```

```
## factor(newpid)212  5.21535    0.31670  16.468  < 2e-16 ***
## factor(newpid)213  4.67607    0.44578  10.490  < 2e-16 ***
## factor(newpid)214  6.54179    0.29428  22.230  < 2e-16 ***
## factor(newpid)215  5.04463    0.31666  15.931  < 2e-16 ***
## factor(newpid)216  3.74901    0.34628  10.827  < 2e-16 ***
## factor(newpid)217  3.09943    0.54578   5.679 1.88e-08 ***
## factor(newpid)218  4.76821    0.29420  16.207  < 2e-16 ***
## factor(newpid)219  5.47723    0.77180   7.097 2.76e-12 ***
## factor(newpid)220  6.34478    0.29424  21.564  < 2e-16 ***
## factor(newpid)221  5.78464    0.31662  18.270  < 2e-16 ***
## factor(newpid)222  5.27235    0.31785  16.587  < 2e-16 ***
## factor(newpid)223  5.34864    0.31661  16.894  < 2e-16 ***
## factor(newpid)224  3.80821    0.54578   6.978 6.19e-12 ***
## factor(newpid)225  6.47400    0.29413  22.010  < 2e-16 ***
## factor(newpid)226  6.85178    0.34695  19.748  < 2e-16 ***
## factor(newpid)227  6.21616    0.31664  19.631  < 2e-16 ***
## factor(newpid)228  4.67312    0.31665  14.758  < 2e-16 ***
## factor(newpid)229  5.25787    0.34628  15.184  < 2e-16 ***
## factor(newpid)230  5.96217    0.34628  17.218  < 2e-16 ***
## factor(newpid)231  5.95432    0.38653  15.405  < 2e-16 ***
## factor(newpid)232  6.17519    0.44620  13.840  < 2e-16 ***
## factor(newpid)233  4.36377    0.38636  11.295  < 2e-16 ***
## factor(newpid)234  6.22240    0.54578  11.401  < 2e-16 ***
## factor(newpid)235  3.21066    0.44635   7.193 1.43e-12 ***
## factor(newpid)236  2.83698    0.34674   8.182 1.06e-15 ***
## factor(newpid)237  5.43365    0.31707  17.137  < 2e-16 ***
## factor(newpid)238  5.05647    0.38660  13.079  < 2e-16 ***
## factor(newpid)239  5.54035    0.44593  12.424  < 2e-16 ***
## factor(newpid)240  3.51138    0.34603  10.148  < 2e-16 ***
## factor(newpid)241  6.11555    0.77180   7.924 7.49e-15 ***
## factor(newpid)242  5.16910    0.44592  11.592  < 2e-16 ***
## factor(newpid)243  5.89800    0.44636  13.213  < 2e-16 ***
## factor(newpid)244  5.94175    0.54578  10.887  < 2e-16 ***
## factor(newpid)245  4.92484    0.38641  12.745  < 2e-16 ***
## factor(newpid)246  5.05558    0.54579   9.263  < 2e-16 ***
## factor(newpid)247  4.78539    0.77180   6.200 8.92e-10 ***
## factor(newpid)248  5.64132    0.54579  10.336  < 2e-16 ***
## factor(newpid)249  5.59464    0.77180   7.249 9.71e-13 ***
## factor(newpid)250  5.83524    0.54579  10.691  < 2e-16 ***
## factor(newpid)251  3.74166    0.77180   4.848 1.49e-06 ***
## factor(newpid)252  4.51291    0.54582   8.268 5.45e-16 ***
## factor(newpid)253  3.60555    0.77180   4.672 3.49e-06 ***
## factor(newpid)254  3.75520    0.54598   6.878 1.20e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.7718 on 821 degrees of freedom
## Multiple R-squared:  0.9809, Adjusted R-squared:  0.9751
## F-statistic: 168.1 on 251 and 821 DF,  p-value: < 2.2e-16
```

```r
# Step2: child level predictors: age.baseline, treatment
library(tidyverse)

## -- Attaching packages ---------------------------------------------
------------------------- tidyverse 1.2.1 --

## √ tibble  1.4.2     √ purrr   0.2.5
## √ tidyr   0.8.1     √ dplyr   0.7.7
## √ readr   1.1.1     √ forcats 0.3.0

## -- Conflicts ------------------------------------------------------
------------------- tidyverse_conflicts() --
## x dplyr::between()   masks data.table::between()
## x dplyr::combine()   masks gridExtra::combine()
## x tidyr::expand()    masks Matrix::expand()
## x tidyr::extract()   masks rstan::extract()
## x dplyr::filter()    masks stats::filter()
## x dplyr::first()     masks data.table::first()
## x dplyr::lag()       masks stats::lag()
## x dplyr::last()      masks data.table::last()
## x dplyr::recode()    masks car::recode()
## x dplyr::select()    masks MASS::select()
## x purrr::some()      masks car::some()
## x purrr::transpose() masks data.table::transpose()

child <- hiv.data %>%
  select(newpid, age.baseline, treatment)
child <- unique(child)
r1.coef <- data.frame(child, r1$coefficients[2:length(r1$coefficients)])
colnames(r1.coef) <- c("newpid", "age.baseline", "treatment","coef.id")
rownames(r1.coef) <- 1:250

r1_coef.id <- lm(coef.id ~ age.baseline + factor(treatment),data = r1.c
oef)
summary(r1_coef.id)

##
## Call:
## lm(formula = coef.id ~ age.baseline + factor(treatment), data = r1.c
oef)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -4.1594 -0.7039  0.2265  1.1215  2.7256
##
## Coefficients:
##                     Estimate Std. Error t value Pr(>|t|)
## (Intercept)          5.10627    0.18728  27.265  < 2e-16 ***
## age.baseline        -0.12088    0.04023  -3.005  0.00293 **
## factor(treatment)2   0.14558    0.18421   0.790  0.43012
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.455 on 247 degrees of freedom
## Multiple R-squared:  0.03753,    Adjusted R-squared:  0.02974
## F-statistic: 4.816 on 2 and 247 DF,  p-value: 0.008875
```

4. Write a model predicting CD4 percentage as a function of time with varying intercepts across children. Fit using `lmer()` and interpret the coefficient for time.

```
# varying intercepts across children
M0 <- lmer (y ~ time + (1 | newpid), data = hiv.data)
display(M0)

## lmer(formula = y ~ time + (1 | newpid), data = hiv.data)
##             coef.est coef.se
## (Intercept)  4.76     0.10
## time        -0.37     0.05
##
## Error terms:
##  Groups   Name        Std.Dev.
##  newpid   (Intercept) 1.40
##  Residual             0.77
## ---
## number of obs: 1072, groups: newpid, 250
## AIC = 3148.8, DIC = 3126.9
## deviance = 3133.9

M0.coef <- data.frame(unique(hiv.data$newpid),coef(M0)$newpid)
colnames(M0.coef) <- c("newpid","intercept","time")
head(coef(M0)$newpid)

##   (Intercept)       time
## 1    4.557250 -0.3660932
## 2    1.335566 -0.3660932
## 3    5.884129 -0.3660932
## 4    5.561130 -0.3660932
## 5    4.178397 -0.3660932
## 6    5.326751 -0.3660932
```

We can see the coefficient for time is -0.3660932, which is constant across the children. It means, whichever a child is, if the time increases by 1 unit, then the CD4 percentage on the square root scale will decrease by 0.3660932 units.

5. Extend the model in (4) to include child-level predictors (that is, group-level predictors) for treatment and age at baseline. Fit using `lmer()` and interpret the coefficients on time, treatment, and age at baseline.

```
M1 <- lmer (y ~ time + factor(treatment) + age.baseline +
              (1 | newpid), data = hiv.data)
display(M1)
```

```
## lmer(formula = y ~ time + factor(treatment) + age.baseline +
##     (1 | newpid), data = hiv.data)
##                    coef.est coef.se
## (Intercept)         5.09     0.19
## time               -0.36     0.05
## factor(treatment)2  0.18     0.18
## age.baseline       -0.12     0.04
##
## Error terms:
##  Groups   Name        Std.Dev.
##  newpid   (Intercept) 1.37
##  Residual             0.77
## ---
## number of obs: 1072, groups: newpid, 250
## AIC = 3149.2, DIC = 3110.9
## deviance = 3124.1
```

```
head(coef(M1)$newpid)
```

```
##   (Intercept)       time factor(treatment)2 age.baseline
## 1    5.012677 -0.3621573          0.1800822   -0.1194538
## 2    1.607624 -0.3621573          0.1800822   -0.1194538
## 3    6.593175 -0.3621573          0.1800822   -0.1194538
## 4    5.834945 -0.3621573          0.1800822   -0.1194538
## 5    4.320103 -0.3621573          0.1800822   -0.1194538
## 6    5.499405 -0.3621573          0.1800822   -0.1194538
```

We can see the coefficients for time, treatment and age.baseline are all constant across the children.

The coefficient for time is -0.3621573, which means whichever a child is, if the time increases by 1 unit, then the CD4 percentage on the square root scale will decrease by 0.3621573 units, with the same other factors.

The coefficient for treatment is 0.1800822, which means whichever a child is, the CD4 percentage on the square root scale for children who are under treatment 2 is 0.1800822 more than that for children who are under treatment 1, with the same other factors.

**The coefficient for time is -0.1194538, which means whichever a child is, if the age.baseline increases by 1 unit, then the CD4 percentage on the square root scale will decrease by 0.1194538 units, with the same other factors.*

6. Investigate the change in partial pooling from (4) to (5) both graphically and numerically.

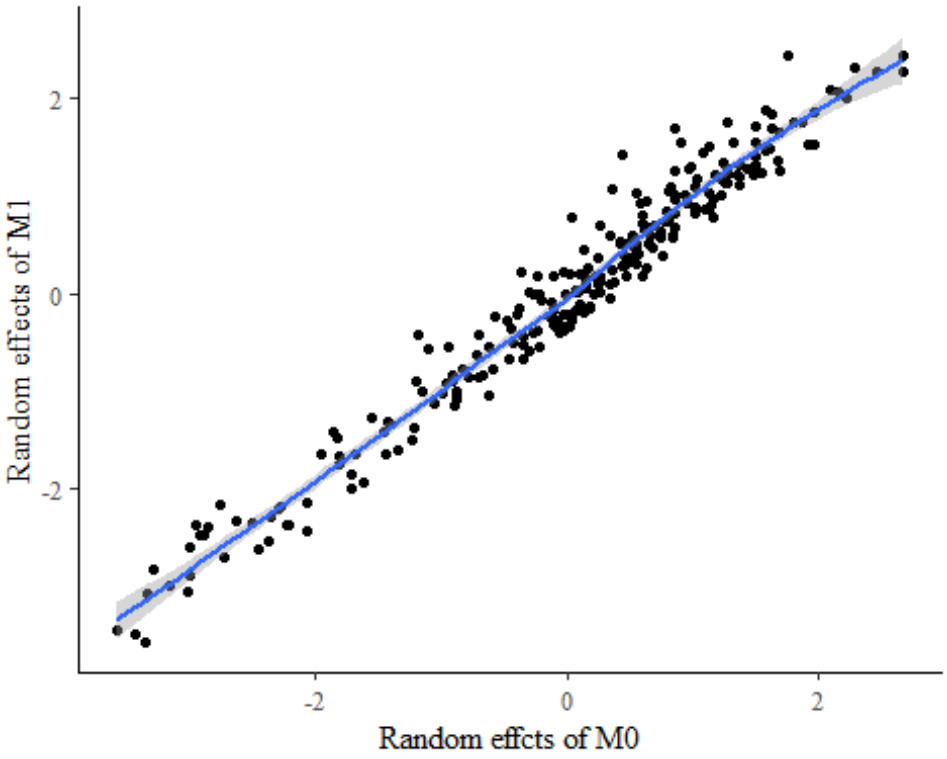
```
# Graphically

#ggplot(data = hiv.data) + geom_point(aes(x=time, y=y)) +
#  geom_abline(intercept = M0.coef$intercept,
#              slope=M0.coef$time, color=M0.coef$newpid) +
```

```
# xlab("Time")+ ylab("Outcome (the CD4 percentage, on the square root s
cale)")

data_plot <- as.data.frame(cbind(unlist(ranef(M0)),unlist(ranef(M1))))
colnames(data_plot) <- c("M0","M1")

ggplot(data=data_plot,aes(x=M0,y=M1))+geom_point()+geom_smooth()+
  xlab("Random effcts of M0")+
  ylab("Random effects of M1")
```




```
# Numerically
display(M0)

## lmer(formula = y ~ time + (1 | newpid), data = hiv.data)
##             coef.est coef.se
## (Intercept)  4.76     0.10
## time        -0.37     0.05
##
## Error terms:
##  Groups   Name        Std.Dev.
##  newpid   (Intercept) 1.40
##  Residual             0.77
## ---
## number of obs: 1072, groups: newpid, 250
## AIC = 3148.8, DIC = 3126.9
## deviance = 3133.9
```

```
display(M1)

## lmer(formula = y ~ time + factor(treatment) + age.baseline +
##     (1 | newpid), data = hiv.data)
##                     coef.est coef.se
## (Intercept)          5.09     0.19
## time                -0.36     0.05
## factor(treatment)2   0.18     0.18
## age.baseline        -0.12     0.04
##
## Error terms:
##  Groups   Name        Std.Dev.
##  newpid   (Intercept) 1.37
##  Residual             0.77
## ---
## number of obs: 1072, groups: newpid, 250
## AIC = 3149.2, DIC = 3110.9
## deviance = 3124.1
```

From the two display results, we can see the group-level standard deviation in (4) is 1.40 while in (5) is 1.37; the deviance in (4) is 3133.9 while in (5) is 3110.9. Both the group-level standard deviation and deviance in (5) are lower than those in (4).

The group-level predictors play a special role in multilevel modeling by reducing the unexplained group-level variation and thus reducing the group-level standard deviation.

7. Use the model fit from (5) to generate simulation of predicted CD4 percentages for each child in the dataset at a hypothetical next time point.

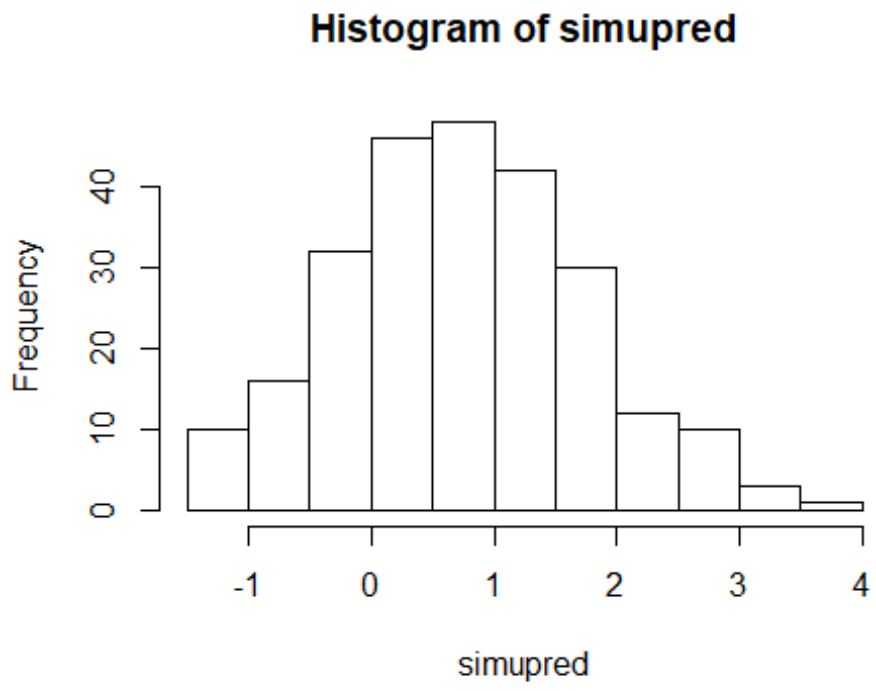
```
library(dplyr)
predict_data <- hiv.data %>%
  filter(is.na(hiv.data$treatment)==FALSE) %>%
  filter(is.na(age.baseline)==FALSE) %>%
  select(time,treatment,age.baseline,newpid,y)
predict_new <- predict(M1,newdata=predict_data)
predict_cmb <- cbind(predict_data, predict_new)
colnames(predict_cmb)[1] <- c("prediction")
```

8. Use the same model fit to generate simulations of CD4 percentages at each of the time periods for a new child who was 4 years old at baseline.

```
predict_data2 <- hiv.data %>%
  filter(is.na(hiv.data$treatment)==FALSE) %>%
  filter(is.na(age.baseline)==FALSE) %>%
  select(time,treatment,age.baseline,newpid,y) %>%
  filter(round(age.baseline)==4)
predict_new2 <- predict(M1,newdata=predict_data2)
predict_cmb2 <- cbind(predict_data2, predict_new2)
colnames(predict_cmb2)[1] <- c("prediction")
```

9. Posterior predictive checking: continuing the previous exercise, use the fitted model from (5) to simulate a new dataset of CD4 percentages (with the same sample size and ages of the original dataset) for the final time point of the study, and record the average CD4 percentage in this sample. Repeat this process 1000 times and compare the simulated distribution to the observed CD4 percentage at the final time point for the actual data.

```
pred<-hiv.data[,list(time=max(time),age.baseline=unique(age.baseline),
                        treatment=unique(treatment)),by =newpid]
cm<-coef(M1)$newpid
sigy<-sigma.hat(M1)$sigma$data
predy<-cm[,1]+cm[,2]*pred$time+cm[,3]*pred$age.baseline+cm[,4]*(pred$tr
eatment-1)
avg.pred.CD4PCT<-NULL
simupred<-matrix(NA,nrow(pred),1000)
for (i in 1:1000){
  ytilde<-rnorm(predy,sigy)
  simupred[,1]<-ytilde
}
hist(simupred)
```



10. Extend the model to allow for varying slopes for the time predictor.

```
M2<-lmer(hiv.data$y~hiv.data$time+(1+hiv.data$time|hiv.data$newpid))
```

11. Next fit a model that does not allow for varying slopes but does allow for different coefficients for each time point (rather than fitting the linear trend).

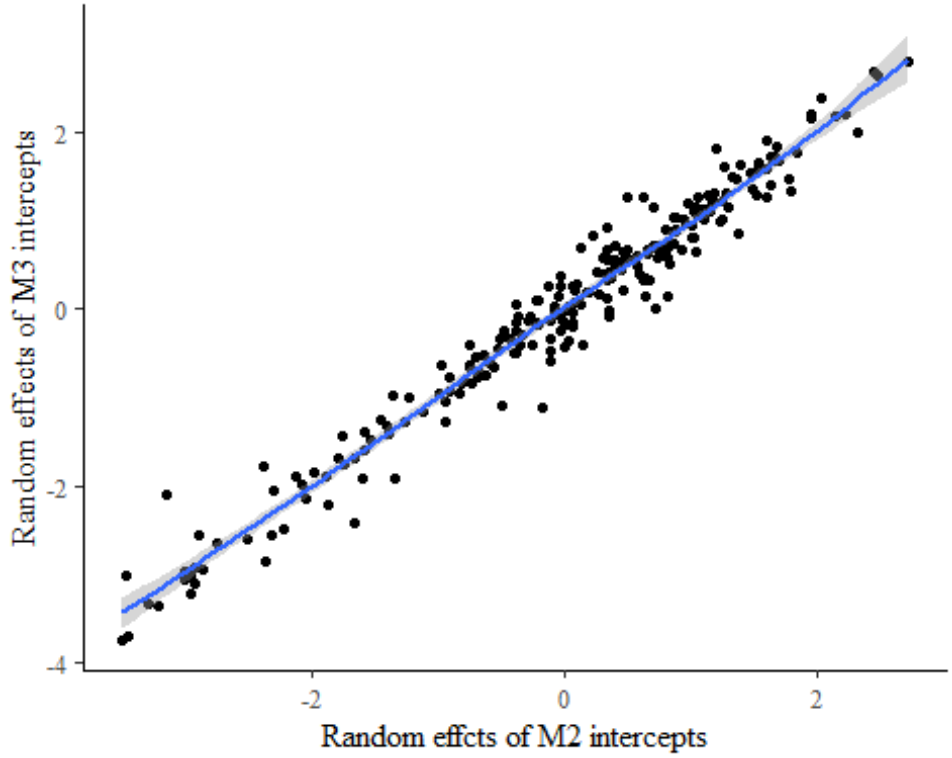
```
M3<-lmer(hiv.data$y ~ factor(hiv.data$time) + (1 | hiv.data$newpid))
```

12. Compare the results of these models both numerically and graphically.

```
# Graphically
data_plot2_inter <- as.data.frame(cbind(unlist(ranef(M2))[1:250],unlist
(ranef(M3))[1:250]))
colnames(data_plot2_inter) <- c("M2","M3")

ggplot(data=data_plot2_inter,aes(x=M2,y=M3))+geom_point()+geom_smooth()
+
  xlab("Random effcts of M2 intercepts")+
  ylab("Random effects of M3 intercepts")

## `geom_smooth()` using method = 'loess' and formula 'y ~ x'
```




```
# Numerically
display(M2)

## lmer(formula = hiv.data$y ~ hiv.data$time + (1 + hiv.data$time |
##     hiv.data$newpid))
##                coef.est coef.se
## (Intercept)     4.76     0.09
## hiv.data$time -0.36     0.07
##
## Error terms:
##  Groups           Name           Std.Dev. Corr
##  hiv.data$newpid (Intercept)    1.39
##                  hiv.data$time 0.58     -0.05
##  Residual                       0.72
```

```
## ---
## number of obs: 1072, groups: hiv.data$newpid, 250
## AIC = 3123.2, DIC = 3098.2
## deviance = 3104.7

display(M3)

## lmer(formula = hiv.data$y ~ factor(hiv.data$time) + (1 | hiv.data$ne
wpid))
##                                          coef.est coef.se
## (Intercept)                               4.77     0.10
## factor(hiv.data$time)0.205               -1.23     0.67
## factor(hiv.data$time)0.209999999999999    0.21     0.89
## factor(hiv.data$time)0.213333333333333    0.16     0.94
## factor(hiv.data$time)0.213333333333334   -1.20     0.94
## factor(hiv.data$time)0.215833333333332    1.47     0.90
## factor(hiv.data$time)0.215833333333334   -0.25     0.84
## factor(hiv.data$time)0.216666666666667   -0.35     0.80
## factor(hiv.data$time)0.218333333333334    0.07     0.90
## factor(hiv.data$time)0.219166666666667   -0.48     0.85
## factor(hiv.data$time)0.221666666666667    0.19     0.94
## factor(hiv.data$time)0.224166666666666    1.65     0.86
## factor(hiv.data$time)0.224166666666667   -1.53     0.63
## factor(hiv.data$time)0.226666666666667    1.42     0.59
## factor(hiv.data$time)0.227499999999999   -1.56     0.89
## factor(hiv.data$time)0.2275               0.07     0.46
## factor(hiv.data$time)0.229999999999999   -0.36     0.59
## factor(hiv.data$time)0.23                -0.11     0.12
## factor(hiv.data$time)0.2325              -0.59     0.40
## factor(hiv.data$time)0.233333333333333    0.02     0.84
## factor(hiv.data$time)0.235000000000001   -1.96     0.80
## factor(hiv.data$time)0.235833333333333    0.04     0.29
## factor(hiv.data$time)0.235833333333334    0.18     0.62
## factor(hiv.data$time)0.2375               1.44     0.89
## factor(hiv.data$time)0.238333333333333   -0.27     0.49
## factor(hiv.data$time)0.238333333333334    0.85     0.82
## factor(hiv.data$time)0.240833333333333   -0.21     0.78
## factor(hiv.data$time)0.240833333333334    0.34     0.59
## factor(hiv.data$time)0.243333333333333   -0.51     0.89
## factor(hiv.data$time)0.244166666666667    0.09     0.48
## factor(hiv.data$time)0.245833333333333    0.09     0.43
## factor(hiv.data$time)0.245833333333334   -0.25     0.60
## factor(hiv.data$time)0.246666666666666   -0.51     0.65
## factor(hiv.data$time)0.246666666666667    0.38     0.85
## factor(hiv.data$time)0.249166666666666    0.15     0.39
## factor(hiv.data$time)0.249166666666667   -0.48     0.19
## factor(hiv.data$time)0.251666666666667    0.25     0.43
## factor(hiv.data$time)0.251666666666668    0.31     0.80
## factor(hiv.data$time)0.2525              -0.05     0.94
## factor(hiv.data$time)0.254166666666667   -0.63     0.84
```

```
## factor(hiv.data$time)0.255               0.33     0.80
## factor(hiv.data$time)0.256666666666667   0.29     0.63
## factor(hiv.data$time)0.257499999999999   0.09     0.84
## factor(hiv.data$time)0.2575              0.45     0.63
## factor(hiv.data$time)0.2625             -0.04     0.84
## factor(hiv.data$time)0.265              -0.16     0.85
## factor(hiv.data$time)0.265833333333333 -0.36     0.84
## factor(hiv.data$time)0.268333333333333 -0.34     0.59
## factor(hiv.data$time)0.268333333333334  0.07     0.49
## factor(hiv.data$time)0.2875              0.50     0.53
## factor(hiv.data$time)0.289999999999999 -0.77     0.94
## factor(hiv.data$time)0.293333333333333 -0.28     0.94
## factor(hiv.data$time)0.304166666666667  0.31     0.89
## factor(hiv.data$time)0.306666666666666 -0.28     0.89
## factor(hiv.data$time)0.306666666666667  0.22     0.59
## factor(hiv.data$time)0.325833333333334 -0.16     0.89
## factor(hiv.data$time)0.328333333333333 -0.87     0.94
## factor(hiv.data$time)0.331666666666667  0.05     0.94
## factor(hiv.data$time)0.358333333333333  0.57     0.86
## factor(hiv.data$time)0.364166666666667 -0.07     0.61
## factor(hiv.data$time)0.429166666666666 -0.29     0.94
## factor(hiv.data$time)0.429166666666667 -0.44     0.90
## factor(hiv.data$time)0.438333333333333 -0.85     0.90
## factor(hiv.data$time)0.440833333333333 -0.10     0.77
## factor(hiv.data$time)0.443333333333332  0.15     0.85
## factor(hiv.data$time)0.449166666666667 -0.08     0.79
## factor(hiv.data$time)0.454166666666666  0.27     0.90
## factor(hiv.data$time)0.454166666666667  0.09     0.86
## factor(hiv.data$time)0.455              -1.45     0.89
## factor(hiv.data$time)0.456666666666667  0.24     0.94
## factor(hiv.data$time)0.4575             -0.14     0.49
## factor(hiv.data$time)0.459166666666667  0.27     0.62
## factor(hiv.data$time)0.459999999999999 -0.19     0.46
## factor(hiv.data$time)0.46               -0.27     0.17
## factor(hiv.data$time)0.460000000000001 -0.27     0.31
## factor(hiv.data$time)0.462499999999999 -0.77     0.48
## factor(hiv.data$time)0.4625              0.43     0.41
## factor(hiv.data$time)0.463333333333333 -0.70     0.81
## factor(hiv.data$time)0.465              -0.88     0.48
## factor(hiv.data$time)0.465833333333333  0.32     0.59
## factor(hiv.data$time)0.465833333333334 -0.60     0.83
## factor(hiv.data$time)0.4675              1.61     0.61
## factor(hiv.data$time)0.468333333333335 -0.80     0.85
## factor(hiv.data$time)0.470833333333333 -0.36     0.46
## factor(hiv.data$time)0.470833333333334 -0.46     0.61
## factor(hiv.data$time)0.473333333333333 -0.17     0.62
## factor(hiv.data$time)0.473333333333334  0.45     0.79
## factor(hiv.data$time)0.474166666666666 -0.67     0.82
## factor(hiv.data$time)0.474166666666667 -0.81     0.80
## factor(hiv.data$time)0.475833333333333 -0.46     0.80
```

```
## factor(hiv.data$time)0.475833333333334 -1.41     0.58
## factor(hiv.data$time)0.476666666666667  0.61     0.40
## factor(hiv.data$time)0.479166666666666 -0.24     0.58
## factor(hiv.data$time)0.479166666666667 -0.03     0.25
## factor(hiv.data$time)0.481666666666666  0.11     0.59
## factor(hiv.data$time)0.481666666666667  0.22     0.43
## factor(hiv.data$time)0.484166666666667 -2.71     0.81
## factor(hiv.data$time)0.485              0.90     0.62
## factor(hiv.data$time)0.487499999999999 -0.23     0.77
## factor(hiv.data$time)0.4875             1.80     0.82
## factor(hiv.data$time)0.487500000000001  1.76     0.80
## factor(hiv.data$time)0.489999999999999  0.09     0.85
## factor(hiv.data$time)0.495             -0.14     0.60
## factor(hiv.data$time)0.495833333333333 -0.47     0.86
## factor(hiv.data$time)0.495833333333334 -0.12     0.94
## factor(hiv.data$time)0.498333333333333 -0.59     0.39
## factor(hiv.data$time)0.498333333333334 -0.58     0.38
## factor(hiv.data$time)0.500833333333333 -0.26     0.79
## factor(hiv.data$time)0.500833333333334  0.05     0.81
## factor(hiv.data$time)0.501666666666667 -1.18     0.61
## factor(hiv.data$time)0.503333333333334  0.59     0.84
## factor(hiv.data$time)0.504166666666666  0.24     0.52
## factor(hiv.data$time)0.505833333333333 -1.53     0.82
## factor(hiv.data$time)0.509166666666666 -0.42     0.90
## factor(hiv.data$time)0.511666666666667 -1.10     0.87
## factor(hiv.data$time)0.514166666666666  0.12     0.62
## factor(hiv.data$time)0.515              0.16     0.94
## factor(hiv.data$time)0.5175            -0.42     0.38
## factor(hiv.data$time)0.517500000000001 -1.13     0.82
## factor(hiv.data$time)0.533333333333333 -0.27     0.64
## factor(hiv.data$time)0.533333333333334 -1.07     0.87
## factor(hiv.data$time)0.534166666666667  0.22     0.62
## factor(hiv.data$time)0.536666666666667 -0.48     0.44
## factor(hiv.data$time)0.555833333333333 -0.22     0.62
## factor(hiv.data$time)0.558333333333333  1.74     0.94
## factor(hiv.data$time)0.564166666666667 -0.77     0.89
## factor(hiv.data$time)0.575             -0.35     0.63
## factor(hiv.data$time)0.580833333333333 -0.28     0.94
## factor(hiv.data$time)0.5825             0.64     0.94
## factor(hiv.data$time)0.594166666666667 -0.48     0.89
## factor(hiv.data$time)0.610833333333333 -1.09     0.94
## factor(hiv.data$time)0.6375             1.80     0.90
## factor(hiv.data$time)0.648333333333333 -2.11     0.89
## factor(hiv.data$time)0.651666666666666 -1.28     0.87
## factor(hiv.data$time)0.6575            -1.01     0.94
## factor(hiv.data$time)0.67              -0.43     0.85
## factor(hiv.data$time)0.670833333333333 -0.99     0.49
## factor(hiv.data$time)0.673333333333333  0.05     0.94
## factor(hiv.data$time)0.675833333333333 -0.15     0.67
## factor(hiv.data$time)0.684166666666667  0.53     0.63
```

```
## factor(hiv.data$time)0.685               0.61     0.84
## factor(hiv.data$time)0.6875             -1.59     0.61
## factor(hiv.data$time)0.689166666666666 -0.09     0.89
## factor(hiv.data$time)0.689166666666667  0.16     0.81
## factor(hiv.data$time)0.69               -0.18     0.16
## factor(hiv.data$time)0.6925              0.52     0.55
## factor(hiv.data$time)0.692500000000001  0.90     0.84
## factor(hiv.data$time)0.693333333333334 -0.48     0.85
## factor(hiv.data$time)0.695               0.37     0.84
## factor(hiv.data$time)0.695833333333333 -0.32     0.86
## factor(hiv.data$time)0.695833333333334 -1.68     0.83
## factor(hiv.data$time)0.695833333333335  0.82     0.59
## factor(hiv.data$time)0.697500000000001 -1.49     0.79
## factor(hiv.data$time)0.698333333333332  0.05     0.84
## factor(hiv.data$time)0.698333333333333 -0.61     0.59
## factor(hiv.data$time)0.700833333333333  1.56     0.49
## factor(hiv.data$time)0.703333333333333 -0.89     0.58
## factor(hiv.data$time)0.703333333333334 -1.07     0.82
## factor(hiv.data$time)0.704166666666667 -4.77     0.80
## factor(hiv.data$time)0.705833333333333 -0.48     0.49
## factor(hiv.data$time)0.705833333333334 -0.45     0.77
## factor(hiv.data$time)0.706666666666667  2.36     0.80
## factor(hiv.data$time)0.709166666666666 -0.11     0.80
## factor(hiv.data$time)0.709166666666667  0.23     0.28
## factor(hiv.data$time)0.711666666666666  0.10     0.84
## factor(hiv.data$time)0.711666666666667 -0.47     0.81
## factor(hiv.data$time)0.711666666666668 -0.88     0.57
## factor(hiv.data$time)0.714166666666667 -1.43     0.58
## factor(hiv.data$time)0.714999999999999 -0.72     0.79
## factor(hiv.data$time)0.715000000000001 -0.62     0.86
## factor(hiv.data$time)0.7175             -1.01     0.81
## factor(hiv.data$time)0.72               -4.32     0.78
## factor(hiv.data$time)0.725              -0.65     0.60
## factor(hiv.data$time)0.725833333333332  0.35     0.84
## factor(hiv.data$time)0.725833333333333  0.72     0.83
## factor(hiv.data$time)0.725833333333334  0.33     0.94
## factor(hiv.data$time)0.728333333333333 -0.26     0.29
## factor(hiv.data$time)0.730833333333333  0.08     0.51
## factor(hiv.data$time)0.733333333333333 -0.74     0.84
## factor(hiv.data$time)0.734166666666666 -2.81     0.81
## factor(hiv.data$time)0.735833333333333 -0.93     0.67
## factor(hiv.data$time)0.736666666666666  0.05     0.85
## factor(hiv.data$time)0.736666666666667  1.51     0.84
## factor(hiv.data$time)0.7425             -0.48     0.94
## factor(hiv.data$time)0.744166666666667  0.15     0.57
## factor(hiv.data$time)0.745              -0.39     0.82
## factor(hiv.data$time)0.7475             -0.36     0.37
## factor(hiv.data$time)0.752500000000001 -1.65     0.84
## factor(hiv.data$time)0.758333333333334  0.22     0.86
## factor(hiv.data$time)0.761666666666667 -0.59     0.81
```

```
## factor(hiv.data$time)0.763333333333333 -0.13     0.81
## factor(hiv.data$time)0.763333333333335 -0.02     0.87
## factor(hiv.data$time)0.764166666666666 -0.45     0.86
## factor(hiv.data$time)0.765833333333333 -1.22     0.87
## factor(hiv.data$time)0.766666666666667 -0.44     0.39
## factor(hiv.data$time)0.775             -1.57     0.81
## factor(hiv.data$time)0.78               0.89     1.58
## factor(hiv.data$time)0.783333333333333  1.05     0.89
## factor(hiv.data$time)0.785             -0.23     0.83
## factor(hiv.data$time)0.785833333333333  0.44     0.80
## factor(hiv.data$time)0.788333333333333 -0.74     0.94
## factor(hiv.data$time)0.794166666666667 -0.66     0.82
## factor(hiv.data$time)0.8025            -0.56     0.58
## factor(hiv.data$time)0.805             -1.58     0.82
## factor(hiv.data$time)0.805000000000001  0.08     0.79
## factor(hiv.data$time)0.807500000000001 -0.32     0.89
## factor(hiv.data$time)0.824166666666667 -0.01     0.63
## factor(hiv.data$time)0.8625            -0.45     0.61
## factor(hiv.data$time)0.8675             0.56     0.90
## factor(hiv.data$time)0.878333333333334 -0.23     0.89
## factor(hiv.data$time)0.881666666666666  0.94     0.94
## factor(hiv.data$time)0.895833333333333 -0.73     0.89
## factor(hiv.data$time)0.900833333333333 -0.49     0.85
## factor(hiv.data$time)0.900833333333334  0.30     0.58
## factor(hiv.data$time)0.903333333333333 -1.03     0.76
## factor(hiv.data$time)0.903333333333334  2.25     0.94
## factor(hiv.data$time)0.905833333333334  1.26     0.90
## factor(hiv.data$time)0.908333333333334 -0.44     0.90
## factor(hiv.data$time)0.909166666666666  2.46     0.94
## factor(hiv.data$time)0.909166666666667 -0.30     0.81
## factor(hiv.data$time)0.911666666666667 -0.26     0.79
## factor(hiv.data$time)0.914166666666667  0.58     0.59
## factor(hiv.data$time)0.9175            -0.49     0.84
## factor(hiv.data$time)0.919166666666667  0.05     0.48
## factor(hiv.data$time)0.919999999999998 -0.94     0.79
## factor(hiv.data$time)0.92              -0.95     0.27
## factor(hiv.data$time)0.920000000000001 -0.15     0.46
## factor(hiv.data$time)0.9225            -0.53     0.59
## factor(hiv.data$time)0.925833333333333 -1.74     0.59
## factor(hiv.data$time)0.925833333333334  0.34     0.84
## factor(hiv.data$time)0.928333333333333  0.04     0.79
## factor(hiv.data$time)0.928333333333334 -0.94     0.83
## factor(hiv.data$time)0.930833333333333 -0.72     0.56
## factor(hiv.data$time)0.930833333333334 -1.97     0.81
## factor(hiv.data$time)0.933333333333332  0.43     0.82
## factor(hiv.data$time)0.933333333333333  0.01     0.59
## factor(hiv.data$time)0.934166666666664 -0.82     0.84
## factor(hiv.data$time)0.934166666666667 -1.03     0.80
## factor(hiv.data$time)0.935833333333333 -0.52     0.59
## factor(hiv.data$time)0.935833333333334 -0.73     0.83
```

```
## factor(hiv.data$time)0.936666666666667  0.14     0.79
## factor(hiv.data$time)0.938333333333334  0.03     0.80
## factor(hiv.data$time)0.939166666666666 -0.19     0.33
## factor(hiv.data$time)0.939166666666667  0.04     0.33
## factor(hiv.data$time)0.939166666666668 -1.72     0.82
## factor(hiv.data$time)0.941666666666666 -0.30     0.41
## factor(hiv.data$time)0.941666666666667 -0.49     0.81
## factor(hiv.data$time)0.944166666666667  0.12     0.59
## factor(hiv.data$time)0.9475             0.66     0.59
## factor(hiv.data$time)0.952500000000001 -0.45     0.79
## factor(hiv.data$time)0.955              1.56     0.84
## factor(hiv.data$time)0.955000000000001 -0.47     0.81
## factor(hiv.data$time)0.955833333333333 -1.06     0.85
## factor(hiv.data$time)0.9575             0.39     0.79
## factor(hiv.data$time)0.958333333333333  0.11     0.36
## factor(hiv.data$time)0.958333333333334 -0.14     0.46
## factor(hiv.data$time)0.960833333333333 -1.47     0.82
## factor(hiv.data$time)0.964166666666666 -0.01     0.84
## factor(hiv.data$time)0.964166666666667  0.50     0.83
## factor(hiv.data$time)0.965833333333333  0.05     0.85
## factor(hiv.data$time)0.976666666666667 -0.51     0.84
## factor(hiv.data$time)0.977499999999999 -3.22     0.81
## factor(hiv.data$time)0.9775            -0.39     0.36
## factor(hiv.data$time)0.977500000000001  0.58     0.86
## factor(hiv.data$time)0.982499999999999 -0.31     0.82
## factor(hiv.data$time)0.9825            -0.08     0.84
## factor(hiv.data$time)0.983333333333333 -0.02     0.88
## factor(hiv.data$time)0.985833333333333 -0.41     0.94
## factor(hiv.data$time)0.996666666666666 -0.61     0.57
## factor(hiv.data$time)0.996666666666667 -0.95     0.42
## factor(hiv.data$time)0.999166666666667 -0.98     0.78
## factor(hiv.data$time)1                 -0.99     0.84
## factor(hiv.data$time)1.00166666666667  -0.36     0.84
## factor(hiv.data$time)1.0025            -1.30     0.84
## factor(hiv.data$time)1.01083333333333  -0.32     0.81
## factor(hiv.data$time)1.0125            -0.75     0.81
## factor(hiv.data$time)1.01583333333333  -0.96     0.61
## factor(hiv.data$time)1.02083333333333  -1.29     0.85
## factor(hiv.data$time)1.02333333333333  -0.57     0.90
## factor(hiv.data$time)1.0325             0.79     0.89
## factor(hiv.data$time)1.035             -0.37     0.39
## factor(hiv.data$time)1.04833333333333  -1.29     0.87
## factor(hiv.data$time)1.05333333333333  -0.13     0.83
## factor(hiv.data$time)1.05416666666667  -0.54     0.58
## factor(hiv.data$time)1.07583333333333  -1.00     0.85
## factor(hiv.data$time)1.08666666666667   0.13     1.58
## factor(hiv.data$time)1.09              -1.53     0.94
## factor(hiv.data$time)1.0925            -0.63     0.50
## factor(hiv.data$time)1.11416666666667   0.10     0.85
## factor(hiv.data$time)1.11666666666667   0.94     0.94
```

```
## factor(hiv.data$time)1.13083333333333   0.28     0.85
## factor(hiv.data$time)1.13583333333333  -2.80     0.94
## factor(hiv.data$time)1.13916666666667   0.11     0.79
## factor(hiv.data$time)1.14166666666667  -0.48     0.57
## factor(hiv.data$time)1.14416666666667   1.06     0.86
## factor(hiv.data$time)1.145             -0.42     0.89
## factor(hiv.data$time)1.1475            -1.35     0.84
## factor(hiv.data$time)1.14916666666667  -0.67     0.57
## factor(hiv.data$time)1.15              -0.41     0.22
## factor(hiv.data$time)1.1525            -0.92     0.46
## factor(hiv.data$time)1.15583333333333  -1.17     0.49
## factor(hiv.data$time)1.1575            -0.14     0.46
## factor(hiv.data$time)1.15833333333333  -1.63     0.59
## factor(hiv.data$time)1.16083333333333  -1.02     0.41
## factor(hiv.data$time)1.16333333333333   0.86     0.59
## factor(hiv.data$time)1.16416666666667   0.31     0.84
## factor(hiv.data$time)1.16583333333333  -1.58     0.83
## factor(hiv.data$time)1.16833333333333  -0.32     0.80
## factor(hiv.data$time)1.16916666666667  -0.52     0.31
## factor(hiv.data$time)1.17166666666667  -1.39     0.57
## factor(hiv.data$time)1.17666666666667  -1.31     0.83
## factor(hiv.data$time)1.1775            -0.10     0.80
## factor(hiv.data$time)1.18              -0.45     0.80
## factor(hiv.data$time)1.18833333333333  -0.10     0.22
## factor(hiv.data$time)1.19666666666667   0.32     0.63
## factor(hiv.data$time)1.20166666666667   0.09     0.48
## factor(hiv.data$time)1.20416666666667  -0.62     0.59
## factor(hiv.data$time)1.20666666666667   0.30     0.79
## factor(hiv.data$time)1.2075            -1.07     0.47
## factor(hiv.data$time)1.2125             0.10     0.84
## factor(hiv.data$time)1.22416666666667  -0.31     0.86
## factor(hiv.data$time)1.22583333333333   0.02     0.86
## factor(hiv.data$time)1.22666666666667  -0.02     0.59
## factor(hiv.data$time)1.22916666666667  -0.12     0.59
## factor(hiv.data$time)1.23166666666667  -1.42     0.80
## factor(hiv.data$time)1.2325            -0.12     0.80
## factor(hiv.data$time)1.24583333333333  -1.10     0.48
## factor(hiv.data$time)1.24833333333333  -1.65     0.84
## factor(hiv.data$time)1.25333333333333  -0.99     0.90
## factor(hiv.data$time)1.26166666666667  -0.42     0.81
## factor(hiv.data$time)1.265             -0.19     0.47
## factor(hiv.data$time)1.2675            -0.26     0.84
## factor(hiv.data$time)1.28416666666667  -2.30     0.86
## factor(hiv.data$time)1.3025            -0.69     0.87
## factor(hiv.data$time)1.30333333333333  -0.34     0.57
## factor(hiv.data$time)1.31166666666667  -1.86     0.81
## factor(hiv.data$time)1.34166666666667  -1.11     0.79
## factor(hiv.data$time)1.35              -0.55     0.87
## factor(hiv.data$time)1.35833333333333  -1.66     0.84
## factor(hiv.data$time)1.36              -0.09     0.89
```

```
## factor(hiv.data$time)1.36083333333333  -0.05     0.89
## factor(hiv.data$time)1.36583333333333  -0.75     0.79
## factor(hiv.data$time)1.37166666666667  -0.35     0.57
## factor(hiv.data$time)1.37416666666667  -0.69     0.79
## factor(hiv.data$time)1.375             -0.83     0.79
## factor(hiv.data$time)1.37666666666667  -3.03     0.76
## factor(hiv.data$time)1.37916666666667   0.03     0.78
## factor(hiv.data$time)1.38              -0.42     0.32
## factor(hiv.data$time)1.3825            -1.52     0.55
## factor(hiv.data$time)1.38583333333333  -0.78     0.59
## factor(hiv.data$time)1.38583333333334   0.17     0.83
## factor(hiv.data$time)1.3875            -0.22     0.57
## factor(hiv.data$time)1.38833333333333  -1.47     0.80
## factor(hiv.data$time)1.39083333333333  -0.58     0.57
## factor(hiv.data$time)1.39583333333333  -0.57     0.60
## factor(hiv.data$time)1.39666666666667   0.32     0.77
## factor(hiv.data$time)1.39833333333333  -0.33     0.82
## factor(hiv.data$time)1.39916666666667  -0.47     0.31
## factor(hiv.data$time)1.40166666666667  -2.16     0.60
## factor(hiv.data$time)1.41              -1.28     0.55
## factor(hiv.data$time)1.4125            -0.69     0.83
## factor(hiv.data$time)1.415             -0.18     0.57
## factor(hiv.data$time)1.41833333333333  -0.14     0.46
## factor(hiv.data$time)1.42083333333333  -0.74     0.94
## factor(hiv.data$time)1.42416666666667   0.39     0.79
## factor(hiv.data$time)1.42583333333333  -0.19     0.81
## factor(hiv.data$time)1.42916666666667   0.67     0.87
## factor(hiv.data$time)1.43166666666667  -0.13     0.58
## factor(hiv.data$time)1.43666666666667   0.45     0.81
## factor(hiv.data$time)1.4375            -0.88     0.79
## factor(hiv.data$time)1.44583333333333  -3.39     0.81
## factor(hiv.data$time)1.45333333333333  -0.85     0.90
## factor(hiv.data$time)1.45416666666667  -0.44     0.86
## factor(hiv.data$time)1.45583333333333  -0.56     0.79
## factor(hiv.data$time)1.45666666666667  -0.19     0.41
## factor(hiv.data$time)1.4625             0.31     0.80
## factor(hiv.data$time)1.47               0.18     0.58
## factor(hiv.data$time)1.4725             0.13     0.82
## factor(hiv.data$time)1.475              0.27     0.84
## factor(hiv.data$time)1.48166666666667  -1.52     0.84
## factor(hiv.data$time)1.48333333333333  -0.60     0.90
## factor(hiv.data$time)1.4925            -2.59     0.80
## factor(hiv.data$time)1.495              0.20     0.48
## factor(hiv.data$time)1.4975             0.43     0.63
## factor(hiv.data$time)1.5               -0.30     0.81
## factor(hiv.data$time)1.50583333333333  -0.83     0.89
## factor(hiv.data$time)1.51416666666667   0.25     0.79
## factor(hiv.data$time)1.51666666666667  -1.94     0.84
## factor(hiv.data$time)1.51916666666667  -3.13     0.86
## factor(hiv.data$time)1.53               0.51     0.85
```

```
## factor(hiv.data$time)1.53083333333333   0.11     0.89
## factor(hiv.data$time)1.53333333333333  -0.13     0.58
## factor(hiv.data$time)1.54166666666667  -0.91     0.81
## factor(hiv.data$time)1.59083333333333  -1.28     0.87
## factor(hiv.data$time)1.615             -0.73     0.83
## factor(hiv.data$time)1.62916666666667   3.59     0.80
## factor(hiv.data$time)1.64833333333333  -1.52     0.85
## factor(hiv.data$time)1.71666666666667   0.00     0.84
## factor(hiv.data$time)1.725             -0.38     0.79
## factor(hiv.data$time)1.81166666666667   0.35     0.85
## factor(hiv.data$time)1.89666666666667  -0.41     0.82
## factor(hiv.data$time)1.90833333333333  -0.73     0.86
## factor(hiv.data$time)1.93833333333333  -0.88     0.94
##
## Error terms:
##  Groups          Name        Std.Dev.
##  hiv.data$newpid (Intercept) 1.41
##  Residual                    0.70
## ---
## number of obs: 1072, groups: hiv.data$newpid, 250
## AIC = 2980.5, DIC = 2698.6
## deviance = 2434.5
```

Figure skate in the 1932 Winter Olympics

The folder olympics has seven judges' ratings of seven figure skaters (on two criteria: "technical merit" and "artistic impression") from the 1932 Winter Olympics. Take a look at http://www.stat.columbia.edu/~gelman/arm/examples/olympics/olympics1932.txt

1. Construct a $7 \times 7 \times 2$ array of the data (ordered by skater, judge, and judging criterion).

```
#install.packages("reshape")
library(reshape)
arr_olym<-melt(data = olympics1932,id.vars=c("pair","criterion"),
               measure.vars=c(colnames(olympics1932)[3:9]))
arr_olym
```

```
##    pair   criterion variable value
## 1     1     Program  judge_1   5.6
## 2     1 Performance  judge_1   5.6
## 3     2     Program  judge_1   5.5
## 4     2 Performance  judge_1   5.5
## 5     3     Program  judge_1   6.0
## 6     3 Performance  judge_1   6.0
## 7     4     Program  judge_1   5.6
## 8     4 Performance  judge_1   5.6
## 9     5     Program  judge_1   5.4
## 10    5 Performance  judge_1   4.8
```

```
## 11    6     Program  judge_1   5.2
## 12    6 Performance  judge_1   4.8
## 13    7     Program  judge_1   4.8
## 14    7 Performance  judge_1   4.3
## 15    1     Program  judge_2   5.5
## 16    1 Performance  judge_2   5.5
## 17    2     Program  judge_2   5.2
## 18    2 Performance  judge_2   5.7
## 19    3     Program  judge_2   5.3
## 20    3 Performance  judge_2   5.5
## 21    4     Program  judge_2   5.3
## 22    4 Performance  judge_2   5.3
## 23    5     Program  judge_2   4.5
## 24    5 Performance  judge_2   4.8
## 25    6     Program  judge_2   5.1
## 26    6 Performance  judge_2   5.6
## 27    7     Program  judge_2   4.0
## 28    7 Performance  judge_2   4.6
## 29    1     Program  judge_3   5.8
## 30    1 Performance  judge_3   5.8
## 31    2     Program  judge_3   5.8
## 32    2 Performance  judge_3   5.6
## 33    3     Program  judge_3   5.8
## 34    3 Performance  judge_3   5.7
## 35    4     Program  judge_3   5.8
## 36    4 Performance  judge_3   5.8
## 37    5     Program  judge_3   5.8
## 38    5 Performance  judge_3   5.5
## 39    6     Program  judge_3   5.3
## 40    6 Performance  judge_3   5.0
## 41    7     Program  judge_3   4.7
## 42    7 Performance  judge_3   4.5
## 43    1     Program  judge_4   5.3
## 44    1 Performance  judge_4   4.7
## 45    2     Program  judge_4   5.8
## 46    2 Performance  judge_4   5.4
## 47    3     Program  judge_4   5.0
## 48    3 Performance  judge_4   4.9
## 49    4     Program  judge_4   4.4
## 50    4 Performance  judge_4   4.8
## 51    5     Program  judge_4   4.0
## 52    5 Performance  judge_4   4.4
## 53    6     Program  judge_4   5.4
## 54    6 Performance  judge_4   4.7
## 55    7     Program  judge_4   4.0
## 56    7 Performance  judge_4   4.0
## 57    1     Program  judge_5   5.6
## 58    1 Performance  judge_5   5.7
## 59    2     Program  judge_5   5.6
## 60    2 Performance  judge_5   5.5
```

```
## 61    3     Program  judge_5   5.4
## 62    3 Performance  judge_5   5.5
## 63    4     Program  judge_5   4.5
## 64    4 Performance  judge_5   4.5
## 65    5     Program  judge_5   5.5
## 66    5 Performance  judge_5   4.6
## 67    6     Program  judge_5   4.5
## 68    6 Performance  judge_5   4.0
## 69    7     Program  judge_5   3.7
## 70    7 Performance  judge_5   3.6
## 71    1     Program  judge_6   5.2
## 72    1 Performance  judge_6   5.3
## 73    2     Program  judge_6   5.1
## 74    2 Performance  judge_6   5.3
## 75    3     Program  judge_6   5.1
## 76    3 Performance  judge_6   5.2
## 77    4     Program  judge_6   5.0
## 78    4 Performance  judge_6   5.0
## 79    5     Program  judge_6   4.8
## 80    5 Performance  judge_6   4.8
## 81    6     Program  judge_6   4.5
## 82    6 Performance  judge_6   4.6
## 83    7     Program  judge_6   4.0
## 84    7 Performance  judge_6   4.0
## 85    1     Program  judge_7   5.7
## 86    1 Performance  judge_7   5.4
## 87    2     Program  judge_7   5.8
## 88    2 Performance  judge_7   5.7
## 89    3     Program  judge_7   5.3
## 90    3 Performance  judge_7   5.7
## 91    4     Program  judge_7   5.1
## 92    4 Performance  judge_7   5.5
## 93    5     Program  judge_7   5.5
## 94    5 Performance  judge_7   5.2
## 95    6     Program  judge_7   5.0
## 96    6 Performance  judge_7   5.2
## 97    7     Program  judge_7   4.8
## 98    7 Performance  judge_7   4.8
```

2. Reformulate the data as a 98 × 4 array (similar to the top table in Figure 11.7), where the first two columns are the technical merit and artistic impression scores, the third column is a skater ID, and the fourth column is a judge ID.

```
olym_984 <- rename(arr_olym, c("pair"="skater_ID", "variable"="judge_ID
"))
olym_984 <- olym_984[order(olym_984$judge_ID),]
olym_984 <- olym_984[c("criterion", "value", "skater_ID", "judge_ID")]
```

3. Add another column to this matrix representing an indicator variable that equals 1 if the skater and judge are from the same country, or 0 otherwise.

```
olym_984$SameCountry <-ifelse(olym_984[,3] == " 1"&olym_984[,4] == "jud
ge_5",1,
  ifelse(olym_984[,3] == " 2"&olym_984[,4] == "judge_7",1,
  ifelse(olym_984[,3] == " 3"&olym_984[,4] == "judge_1",1,
  ifelse(olym_984[,3] == " 4"&olym_984[,4] == "judge_1",1,
  ifelse(olym_984[,3] == " 7"&olym_984[,4] == "judge_7",1,0
  )))))
```

4. Write the notation for a non-nested multilevel model (varying across skaters and judges) for the technical merit ratings and fit using lmer().

```
#Divide the data into technical rating and artistic rating
data_tech <- olym_984 %>%
  filter(criterion=="Program")
data_art <- olym_984 %>%
  filter(criterion=="Performance")

reg_tech <- lmer(value ~ 1 + (1|skater_ID) + (1|judge_ID),data=data_tec
h)
summary(reg_tech)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: value ~ 1 + (1 | skater_ID) + (1 | judge_ID)
##    Data: data_tech
##
## REML criterion at convergence: 60
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -2.51025 -0.45646 -0.05459  0.63866  1.89709
##
## Random effects:
##  Groups    Name        Variance Std.Dev.
##  skater_ID (Intercept) 0.17488  0.4182
##  judge_ID  (Intercept) 0.07664  0.2768
##  Residual              0.11057  0.3325
## Number of obs: 49, groups:  skater_ID, 7; judge_ID, 7
##
## Fixed effects:
##             Estimate Std. Error t value
## (Intercept)   5.1347     0.1954   26.28
```

5. Fit the model in (4) using the artistic impression ratings.

```
reg_art <- lmer(value ~ 1 + (1|skater_ID) + (1|judge_ID),data=data_art)
summary(reg_tech)
```

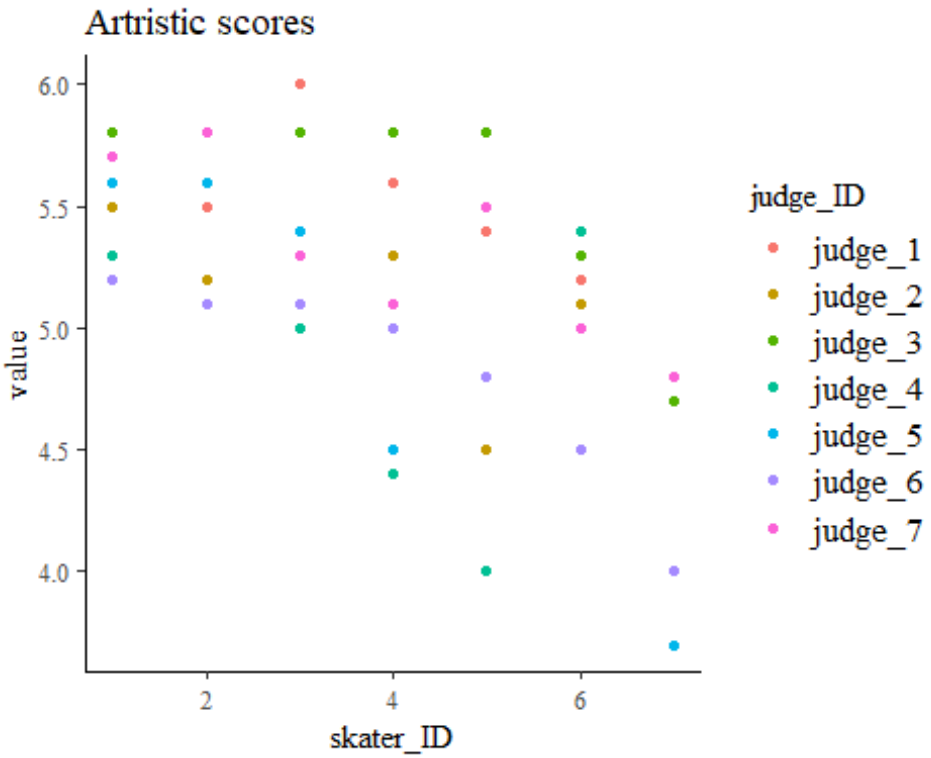
```
## Linear mixed model fit by REML ['lmerMod']
## Formula: value ~ 1 + (1 | skater_ID) + (1 | judge_ID)
##    Data: data_tech
##
## REML criterion at convergence: 60
```

```
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -2.51025 -0.45646 -0.05459  0.63866  1.89709
##
## Random effects:
##  Groups    Name        Variance Std.Dev.
##  skater_ID (Intercept) 0.17488  0.4182
##  judge_ID  (Intercept) 0.07664  0.2768
##  Residual              0.11057  0.3325
## Number of obs: 49, groups:  skater_ID, 7; judge_ID, 7
##
## Fixed effects:
##             Estimate Std. Error t value
## (Intercept)   5.1347     0.1954   26.28
```

6. Display your results for both outcomes graphically.

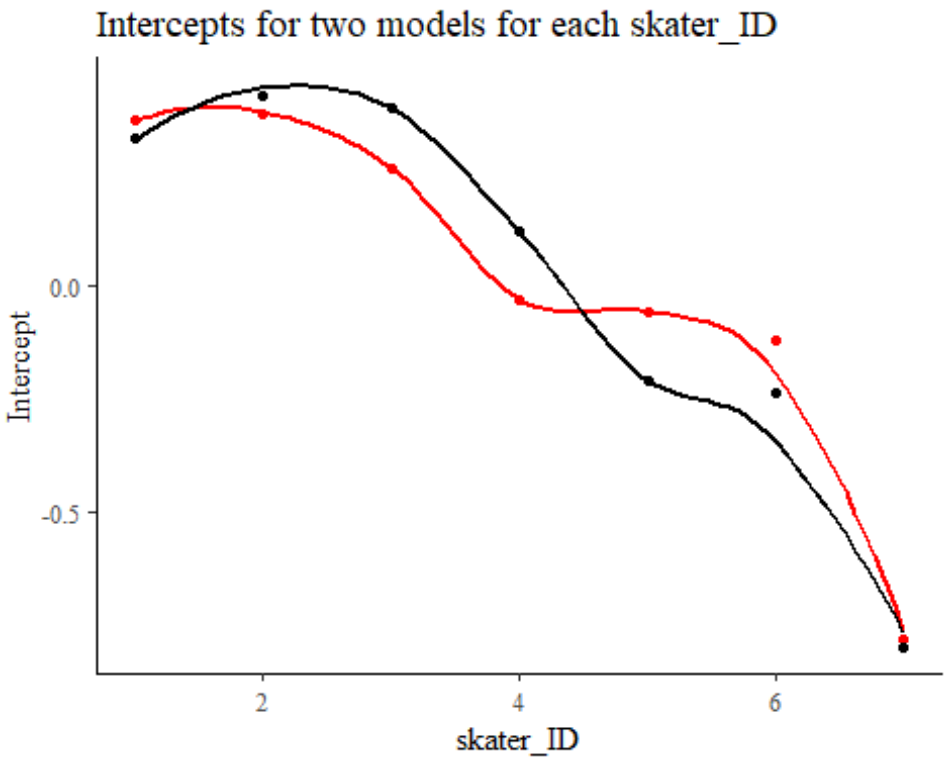
```
ggplot(data_tech,aes(x=skater_ID,y=value,color=judge_ID))+geom_point()+
  ggtitle("Technical scores")
```


```
ggplot(data_tech,aes(x=skater_ID,y=value,color=judge_ID))+geom_point()+
  ggtitle("Artristic scores")
```



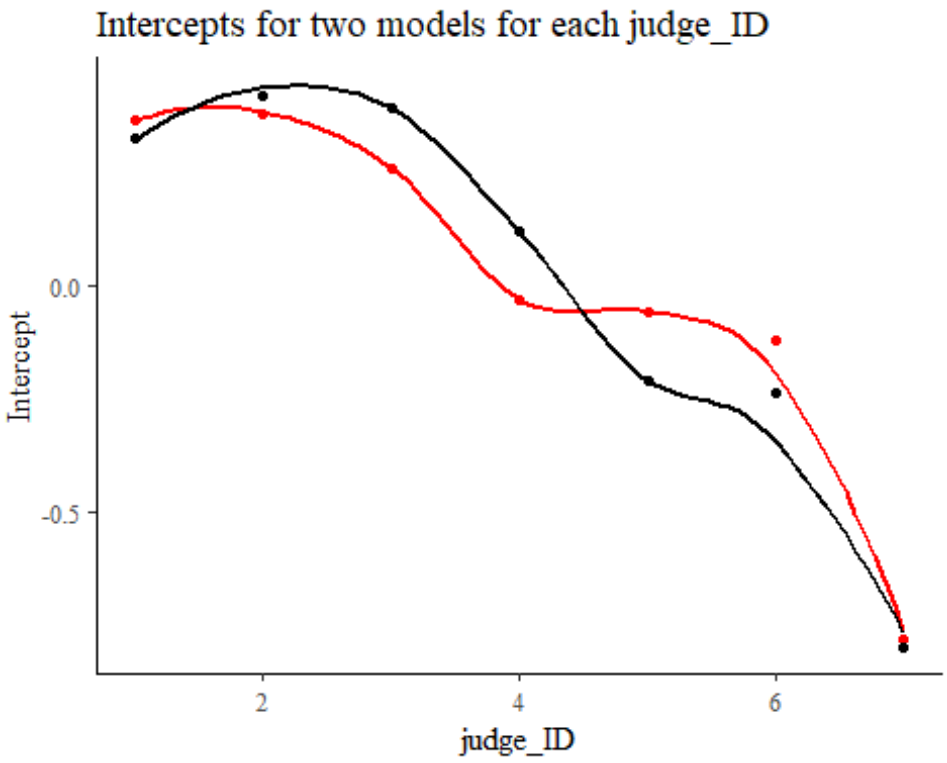

```
#A plot displaying Intercepts for two models for each skater_ID
inter_skate <- as.data.frame(cbind(unlist(ranef(reg_tech))[1:7],unlist
(ranef(reg_art))[1:7]))
inter_skate$skater_ID <-c(1:7)
ggplot(data=inter_skate)+
  geom_point(col="red",aes(x=skater_ID,y=V1))+geom_smooth(col="red",aes
(x=skater_ID,y=V1),se=FALSE)+
  geom_point(col="black",aes(x=skater_ID,y=V2))+geom_smooth(col="black",
aes(x=skater_ID,y=V2),se=FALSE)+
  ggtitle("Intercepts for two models for each skater_ID")+
  ylab("Intercept")

## `geom_smooth()` using method = 'loess' and formula 'y ~ x'
## `geom_smooth()` using method = 'loess' and formula 'y ~ x'
```




```
##A plot displaying Intercepts for two models for each judge_ID
inter_judge <- as.data.frame(cbind(unlist(ranef(reg_tech))[1:7],unlist
(ranef(reg_art))[1:7]))
inter_judge$judge_ID <-c(1:7)
ggplot(data=inter_judge)+
  geom_point(col="red",aes(x=judge_ID,y=V1))+geom_smooth(col="red",aes
(x=judge_ID,y=V1),se=FALSE)+
  geom_point(col="black",aes(x=judge_ID,y=V2))+geom_smooth(col="black",
aes(x=judge_ID,y=V2),se=FALSE)+
  ggtitle("Intercepts for two models for each judge_ID")+
  ylab("Intercept")

## `geom_smooth()` using method = 'loess' and formula 'y ~ x'
## `geom_smooth()` using method = 'loess' and formula 'y ~ x'
```



7. (optional) Use posterior predictive checks to investigate model fit in (4) and (5).

Different ways to write the model:

Using any data that are appropriate for a multilevel model, write the model in the five ways discussed in Section 12.5 of Gelman and Hill.

```
lmer(formula=hiv.data$y~hiv.data$time+hiv.data$age.baseline+hiv.data$tr
eatment+(1|hiv.data$newpid))

## Linear mixed model fit by REML ['lmerMod']
## Formula:
## hiv.data$y ~ hiv.data$time + hiv.data$age.baseline + hiv.data$treatm
ent +
##     (1 | hiv.data$newpid)
## REML criterion at convergence: 3137.209
## Random effects:
##  Groups          Name        Std.Dev.
##  hiv.data$newpid (Intercept) 1.3747
##  Residual                    0.7726
## Number of obs: 1072, groups:  hiv.data$newpid, 250
## Fixed Effects:
##           (Intercept)          hiv.data$time  hiv.data$age.baseline

##                4.9061               -0.3622                -0.1195
```

```
##    hiv.data$treatment
##                0.1801
```

Original formula of the multilevel model:

$$y = \beta_{0[j]i} + X_{i1} * \beta_{1[j]i} + X_{i2} * \beta_{2[j]i} + X_{i3} * \beta_{3[j]i} + \epsilon_i$$

$$y = \alpha_{j[i]} + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i3} + \epsilon_i$$

$$\alpha_j \sim N(\mu_i, \sigma_i^2)$$

$$X1 = time,\ X2 = age.baseline,\ X3 = treatment$$

.

Method1: Allowing regression coefficeints to vary accross groups

$$y = 4.91 + X_{i1} * (-0.36) + X_{i2} * (-0.12) + X_{i3} * 0.18 + 0.77\ ,\ for\ i = 1, \ldots, n_{250}$$

$$\alpha_j \sim \mathrm{N}(0, 1.37^2)$$

Method2: Combining separate local regressions

$$y \sim N(4.91 + X_{i1} * (-0.36) + X_{i2} * (-0.12) + X_{i3} * 0.18,\ 0.77^2)\ ,\ for\ i = 1, \ldots, n_{250}$$

$$\alpha_j \sim \mathrm{N}(random\ intercept, 1.37^2)$$

Method3: Modeling the coefficients of a large regression model

$$y_i \sim N(4.91 + X_{i1} * (-0.36) + X_{i2} * (-0.12) + X_{i3} * 0.18,\ 0.77^2)$$

$$\beta_j \sim N(0, 1.37^2)$$

Method4: Regression with multiple error terms

$$y_i \sim N(4.91 + X_{i1} * (-0.36) + X_{i2} * (-0.12) + X_{i3} * 0.18 + 1.37^2,\ 0.77^2)$$

Method5: Large regression with correlated errors

$$y_i \sim N(4.91 + X_{i1} * (-0.36) + X_{i2} * (-0.12) + X_{i3} * 0.18,\ 1.37^2 + 0.77^2)$$

Models for adjusting individual ratings:

A committee of 10 persons is evaluating 100 job applications. Each person on the committee reads 30 applications (structured so that each application is read by three people) and gives each a numerical rating between 1 and 10.

1. It would be natural to rate the applications based on their combined scores; however, there is a worry that different raters use different standards, and we would like to correct for this. Set up a model for the ratings (with parameters for the applicants and the raters).

lmer(rating_scores~applicants_ID+raters_ID+(1|raters_ID))

2. It is possible that some persons on the committee show more variation than others in their ratings. Expand your model to allow for this.

lmer(rating_scores~applicants_ID+raters_ID+(1+raters_ID|raters_ID))