HW2 U67883657

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#1 Y - thenumber of ~~words~~ letters , E(Y) ?

① the possible values for Y : 1, 2, 3, 4, 6, 7, 8

The shortest distance between two points is a taxi

3 8 8 7 3 6 2 1 4

② the distribution of Y :

$P(Y=1) = \frac{1}{42}$, $P(Y=2) = \frac{2}{42} = \frac{1}{21}$, $P(Y=3) = \frac{6}{42} = \frac{1}{7}$

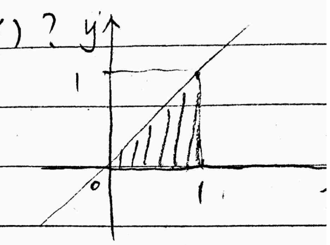
$P(Y=4) = \frac{4}{42} = \frac{2}{21}$, $P(Y=6) = \frac{6}{42} = \frac{1}{7}$, $P(Y=7) = \frac{7}{42} = \frac{1}{6}$

$P(Y=8) = \frac{2\times 8}{42} = \frac{8}{21}$

③ $E(Y) = \sum_k k \cdot P(Y=k) = 1\times\frac{1}{42} + 2\times\frac{1}{21} + 3\times\frac{1}{7} + 4\times\frac{2}{21} + 6\times\frac{1}{7}$

$+ 7\times\frac{1}{6} + 8\times\frac{8}{21} = 6$

#2 $f(x,y) = 12y^2$, $0 \le y \le x \le 1$, $E(XY)$?



$E(XY) = 12\int_0^1 x \int_0^x y^3\,dy\,dx = 12\int_0^1 x\cdot\frac{1}{4}[y^4]_0^x\,dx$

$= 3\int_0^1 x^5\,dx = 3\times\frac{1}{6}[x^6]_0^1 = \frac{1}{2}$

#3 $X_1, X_2, X_3 \sim U(0,1)$, $E[(X_1-2X_2+X_3)^2]$

① $f(X_k) = \begin{cases} 1 , & 0 < X_k < 1 \\ 0 , & \text{otherwise} \end{cases}$, $k = 1,2,3$

② $E(X_k^2) = \int_0^1 x^2\cdot 1\,dx = \frac{1}{3}[x^3]_0^1 = \frac{1}{3}$, $k = 1,2,3$

$f(X_1X_2) = f(X_1X_3) = f(X_2X_3) = \begin{cases} 1 , & 0 < X_k < 1 \quad (k=1,2,3) \\ 0 , & \text{otherwise} \end{cases}$

$E(X_1X_2) = E(X_1X_3) = E(X_2X_3) = \int_0^1 x_2 \int_0^1 x_3\,dx_3\,dx_2 = \frac{1}{4}$

③ $E[(X_1-2X_2+X_3)^2] = 6E(X_k^2) - 6E(X_1X_2) = 6\times(\frac{1}{3}-\frac{1}{4}) = \frac{1}{2}$

#4 $f(x) = e^{-x}$, $x > 0$, $Y = e^{\frac{3}{4}X}$, $E(Y)$?

$E(Y) = \int_0^\infty e^{\frac{3}{4}x}\cdot e^{-x}\,dx = \int_0^\infty e^{-\frac{1}{4}x}\,dx = -4[e^{-\frac{1}{4}x}]_0^\infty = 4$

#5 $Y = g(x) = 2X^2+1$, $E(Y)$? X : die

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X	P	Y
1	1/6	3
2	1/6	9
3	1/6	19
4	1/6	33
5	1/6	51
6	1/6	73

$\therefore E(Y) = \Sigma g(x)\cdot p(x)$

$= \frac{1}{6} \times (3+9+19+33+51+73)$

$= \frac{94}{3}$

#6 $f(x) = 2(1-x)$, $0<x<1$, $Y = 2X+1$, $E(Y^2)$?

$E(Y^2) = E[(2X+1)^2] = E(4X^2+4X+1]$

$\because E(X^2) = \int_0^1 x^2 \cdot 2(1-x)\,dx = 2\cdot\left[\frac{1}{3}x^3 - \frac{1}{4}x^4\right]_0^1 = \frac{1}{6}$

$E(X) = \int_0^1 x\cdot 2(1-x)\,dx = 2\cdot\left[\frac{1}{2}x^2 - \frac{1}{3}x^3\right]_0^1 = \frac{1}{3}$

$\therefore E(Y^2) = 4\times\frac{1}{6} + 4\times\frac{1}{3} + 1 = 3$

#7 Show: $E[(ax+b)^n] = \sum_{i=0}^{n} C_n^i a^{n-i} b^i E(X^{n-i})$

$E[(ax+b)^n] = E\left[\sum_{i=0}^{n} C_n^i (ax)^{n-i} b^i\right] = \sum_{i=0}^{n} C_n^i b^i E[a^{n-i}\cdot x^{n-i}]$

$= \sum_{i=0}^{n} C_n^i a^{n-i} b^i E(X^{n-i})$

#8 (1) defective: p X: defective Y: good , $E(X-Y)$?

$\because X \sim b(n,p)$ $Y \sim b(n, 1-p)$

$E(X) = x\cdot C_n^x \cdot p^x (1-p)^{n-x} = np$ $\because n = x+y$

$\therefore E(Y) = y\cdot C_n^y \cdot (1-p)^y \cdot (p)^{n-y} = (n-x)\, C_n^{n-x} \cdot (1-p)^{n-x} \cdot p^x = n(1-p)$

$\therefore E(X-Y) = E(X) - E(Y) = np - n + np = 2np - n$

(2) $n = 20$, $p = 0.05$, $E(X-Y)$?

$E(X-Y) = 2\times 20\times 0.05 - 20 = -18$

the expected value of $(X-Y)$ is -18