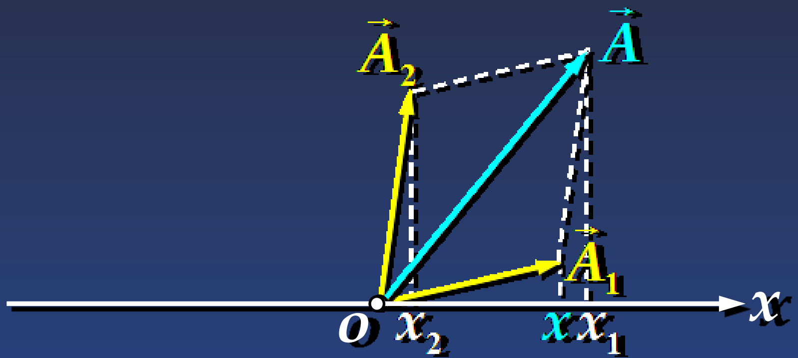
§ 9.4 简谐振动的合成

一、同方向同频率的谐振动合成

$$\begin{cases} x_1 = A_1 \cos(\omega t + \varphi_1) \\ x_2 = A_2 \cos(\omega t + \varphi_2) \end{cases}$$



合振动:

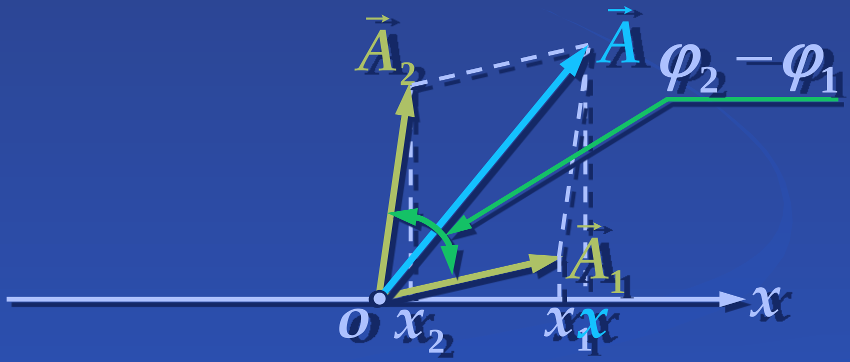
$$x = x_1 + x_2 = A\cos(\omega t + \varphi)$$

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\varphi_2 - \varphi_1)}$$

$$tg\varphi = \frac{A_1\sin\varphi_1 + A_2\sin\varphi_2}{A_1\cos\varphi_1 + A_2\cos\varphi_2}$$

明确几点

☻**同频率同方向的简谐振动的合成仍然为简谐振动。**



合振动:

$$x=x_1+x_2=A\cos(\omega t+\varphi)$$

$$A=\sqrt{A_1^2+A_2^2+2A_1A_2\cos(\varphi_2-\varphi_1)}$$

$$tg\varphi=\frac{A_1\sin\varphi_1+A_2\sin\varphi_2}{A_1\cos\varphi_1+A_2\cos\varphi_2}$$

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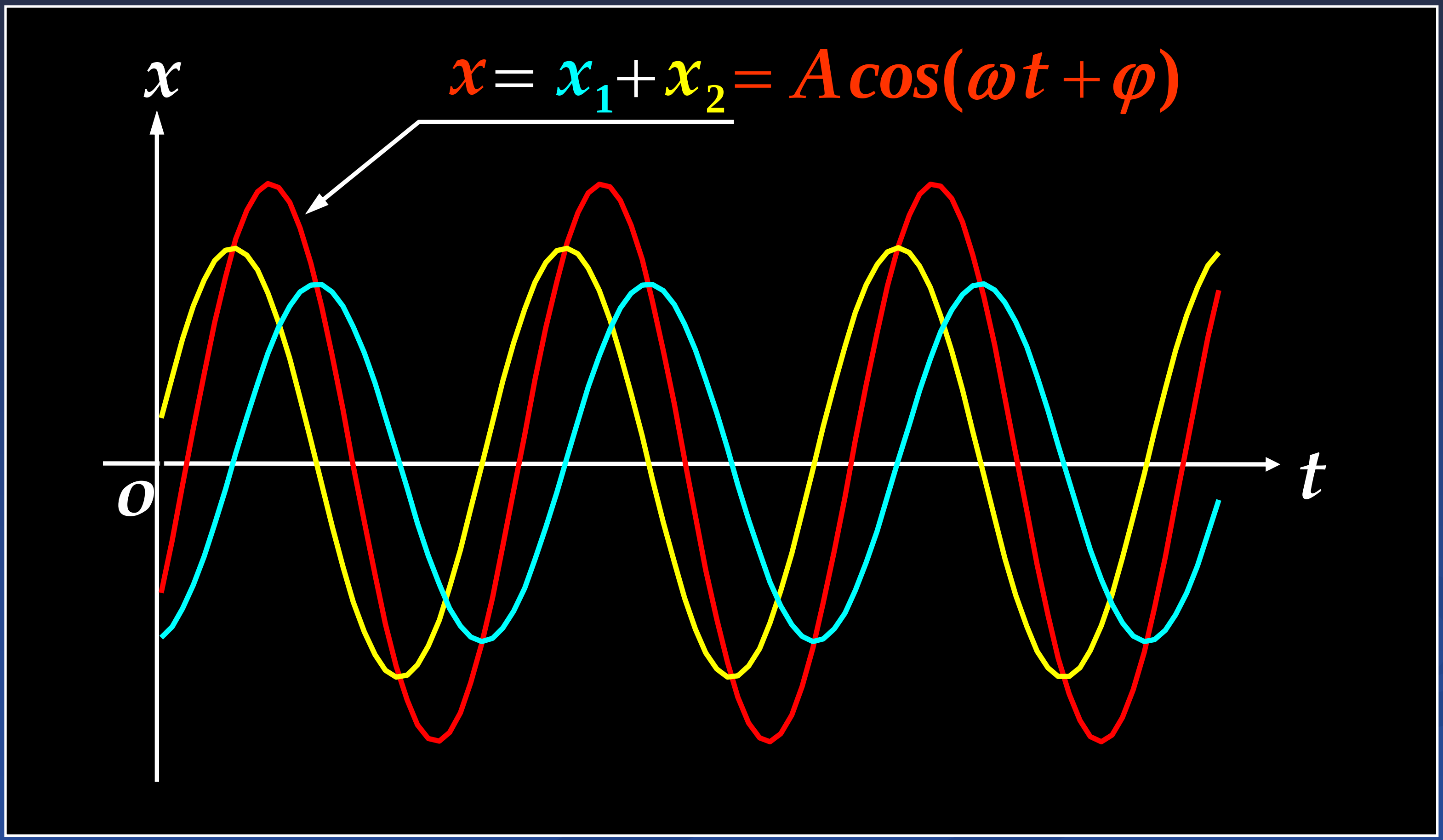


☻ $$\begin{cases} A=\sqrt{A_1^2+A_2^2+2A_1A_2\cos(\varphi_2-\varphi_1)}=A(\Delta\varphi) \\ \Delta\varphi=\varphi_2-\varphi_1 \end{cases}$$

当 $\Delta\varphi=\pm 2k\pi$ 时，$A=A_1+A_2=A_{max}$

合振动加强；

当 $\Delta\varphi=\pm(2k+1)\pi$ 时，$A=\left|A_1-A_2\right|=A_{min}$

合振动减弱；

($k=0,\ 1,\ 2,\ \cdots$)

一般情况下，相位差可取任意值，因此：|A1-A2| <A< A1+A2

例　一质点同时参与两个同方向的简谐运动，其运动方程分别为：

$$x_1 = 5\times10^{-2}\cos(4t + \frac{1}{3}\pi)\mathrm{m}$$

$$x_2 = 3\times10^{-2}\sin(4t - \frac{1}{6}\pi)\mathrm{m}$$

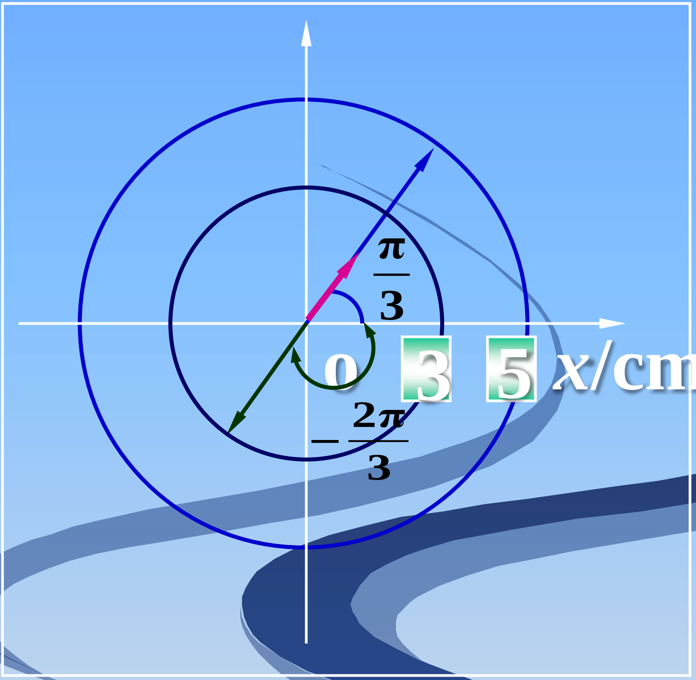
画出两运动的旋转矢量图，并求合运动的运动方程.

解 $x_1 = 5\times10^{-2}\cos(4t+\frac{1}{3}\pi)$

$$x_2 = 3\times10^{-2}\sin(4t-\frac{1}{6}\pi)$$

$$= 3\times10^{-2}\cos(4t-\frac{1}{6}\pi-\frac{1}{2}\pi)$$

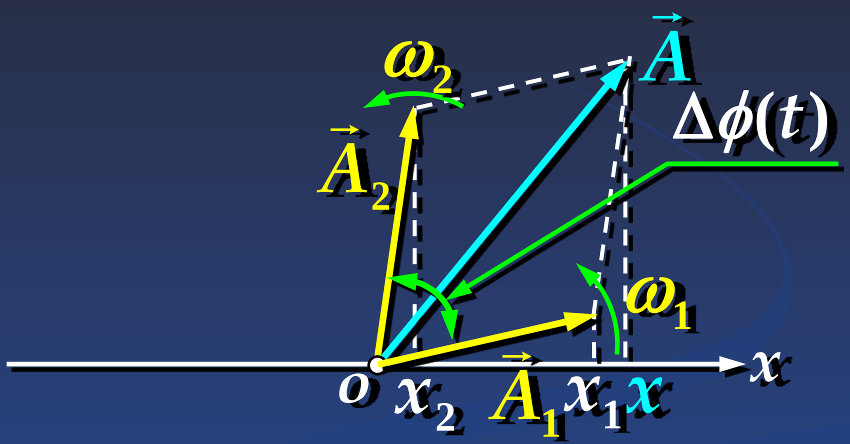
$$= 3\times10^{-2}\cos(4t-\frac{2}{3}\pi)$$



$$x = x_1 + x_2 = 2\times10^{-2}\cos(4t+\frac{1}{3}\pi)\mathrm{m}$$

二、同方向不同频率的谐振动合成

$$\begin{cases} x_1 = A_1 \cos(\omega_1 t + \varphi_1) \\ x_2 = A_2 \cos(\omega_2 t + \varphi_2) \end{cases}$$



合振动 $x = x_1 + x_2$ 振幅:

当 $\Delta\varphi = \pm(2k+1)\pi$ 时，$A = |A_1 - A_2| = A_{min}$

合振动减弱;

$(\ k = 0, 1, 2, \cdots\ )$

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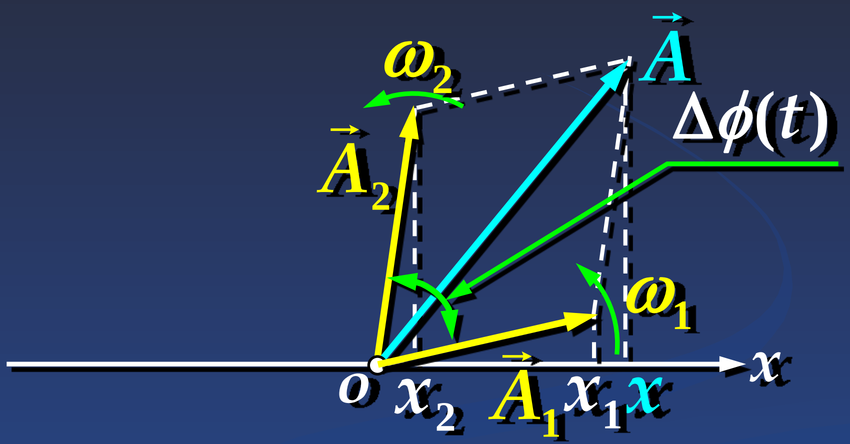


合振动 $x = x_1 + x_2$ 振幅:

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos\Delta\phi(t)}$$

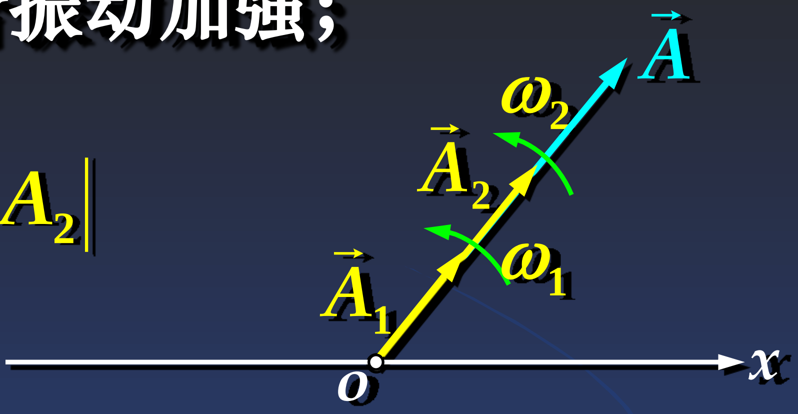
$$\Delta\phi(t) = (\omega_2 t + \varphi_2) - (\omega_1 t + \varphi_1)$$

$$= (\omega_2 - \omega_1)t + (\varphi_2 - \varphi_1)$$

$\Delta\phi(t)=\pm 2k\pi$： $A=A_1+A_2$ 合振动加强；

$\Delta\phi(t)=\pm(2k+1)\pi$： $A=\left|A_1-A_2\right|$

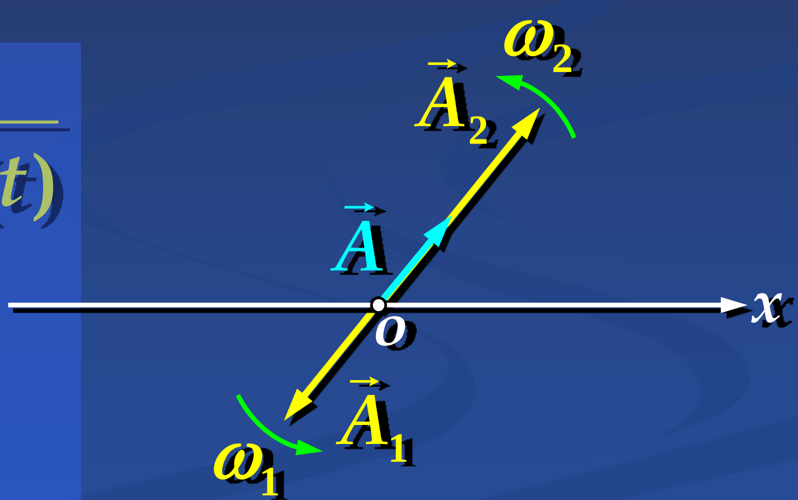
合振动减弱；



$$A=\sqrt{A_1^2+A_2^2+2A_1A_2\cos\Delta\phi(t)}$$

$$\Delta\phi(t)=(\omega_2 t+\varphi_2)-(\omega_1 t+\varphi_1)$$

$$=(\omega_2-\omega_1)t+(\varphi_2-\varphi_1)$$



频率较大而频率之差很小的两个同方向简谐运动的合成，其合振动的振幅时而加强时而减弱的现象叫拍.

$$\begin{cases} x_1 = A_1\cos\omega_1 t = A_1\cos 2\pi\nu_1 t \\ x_2 = A_2\cos\omega_2 t = A_2\cos 2\pi\nu_2 t \end{cases}$$

$$x = x_1 + x_2$$

讨论 $A_1 = A_2$, $|\nu_2 - \nu_1| << \nu_1 + \nu_2$ **的情况**

$$x = x_1 + x_2 = A_1 \cos 2\pi\nu_1 t + A_2 \cos 2\pi\nu_2 t$$

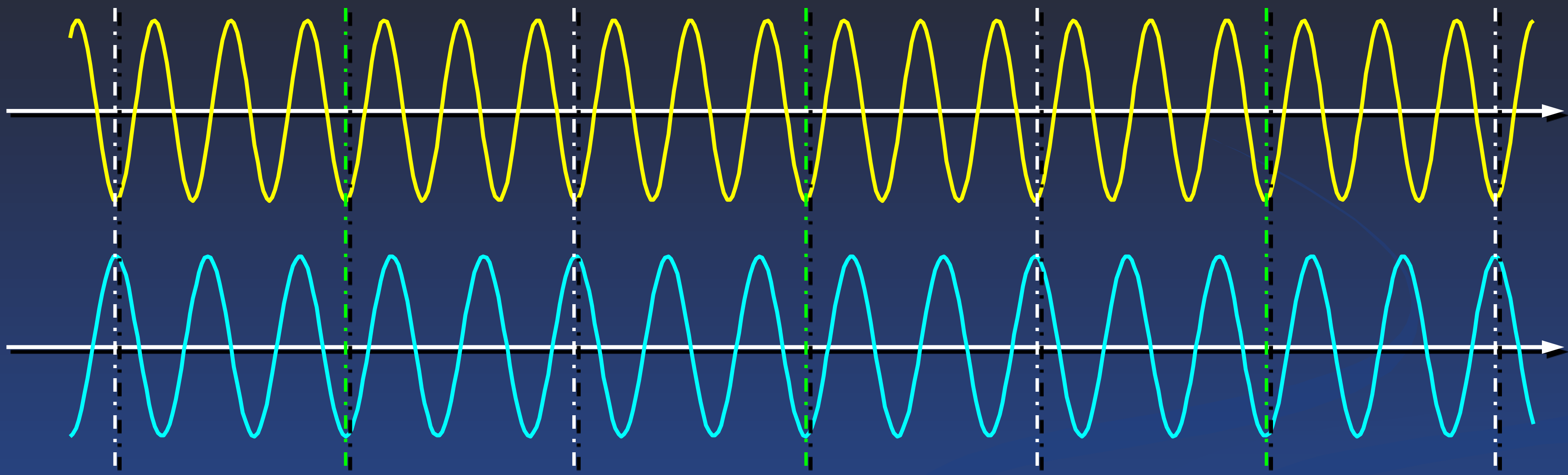
$$x = \underbrace{\left(2A_1 \cos 2\pi \frac{\nu_2 - \nu_1}{2} t\right)}_{\text{振幅部分}} \cos 2\pi \underbrace{\frac{\nu_2 + \nu_1}{2} t}_{\text{合振动频率}}$$

振动频率 $\nu = (\nu_1 + \nu_2)/2$

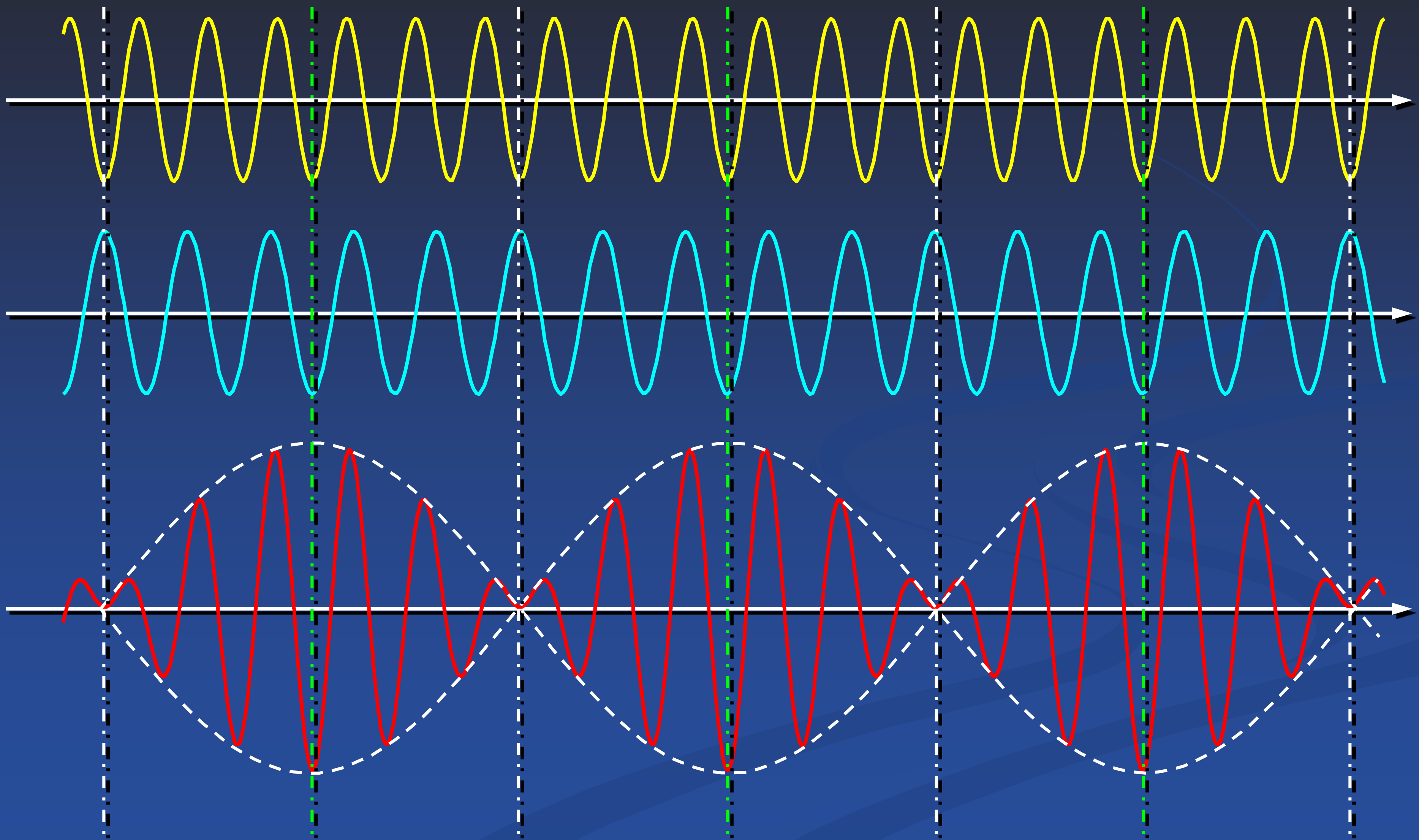
振幅 $A = \left| 2A_1 \cos 2\pi \frac{\nu_2 - \nu_1}{2} t \right|$

$$\begin{cases} A_{max} = 2A_1 \\ A_{min} = 0 \end{cases}$$

拍：



拍：



$$x = (2A_1 \cos 2\pi \frac{\nu_2 - \nu_1}{2} t) \cos 2\pi \frac{\nu_2 + \nu_1}{2} t$$

$$A = \left| 2A_1 \cos 2\pi \frac{\nu_2 - \nu_1}{2} t \right|$$

余弦函数的绝对值以π为周期

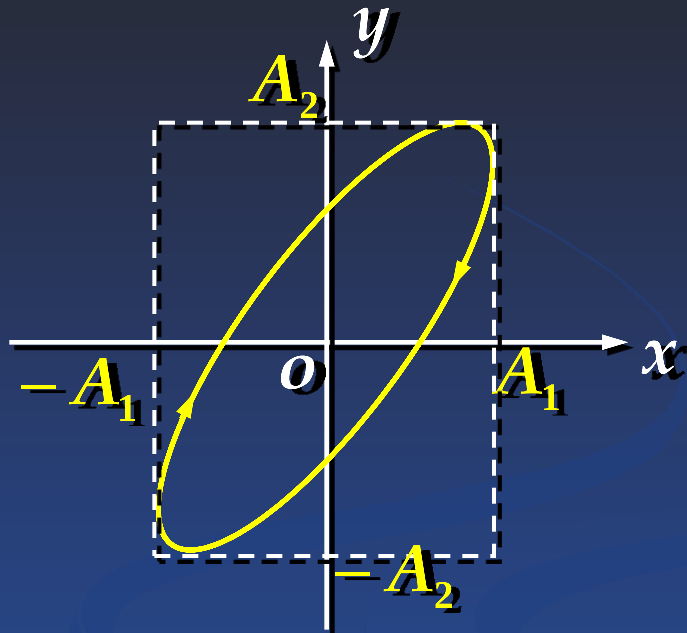
$$2\pi \frac{\nu_2 - \nu_1}{2} T = \pi \quad \Rightarrow \quad T = \frac{1}{\nu_2 - \nu_1}$$

$$\boxed{\nu = \nu_2 - \nu_1}$$

拍频（振幅变化的频率）

三、同频率相互垂直方向的谐振动合成

$$\begin{cases} x = A_1 \cos(\omega t + \varphi_1) \\ y = A_2 \cos(\omega t + \varphi_2) \end{cases}$$



轨迹方程:

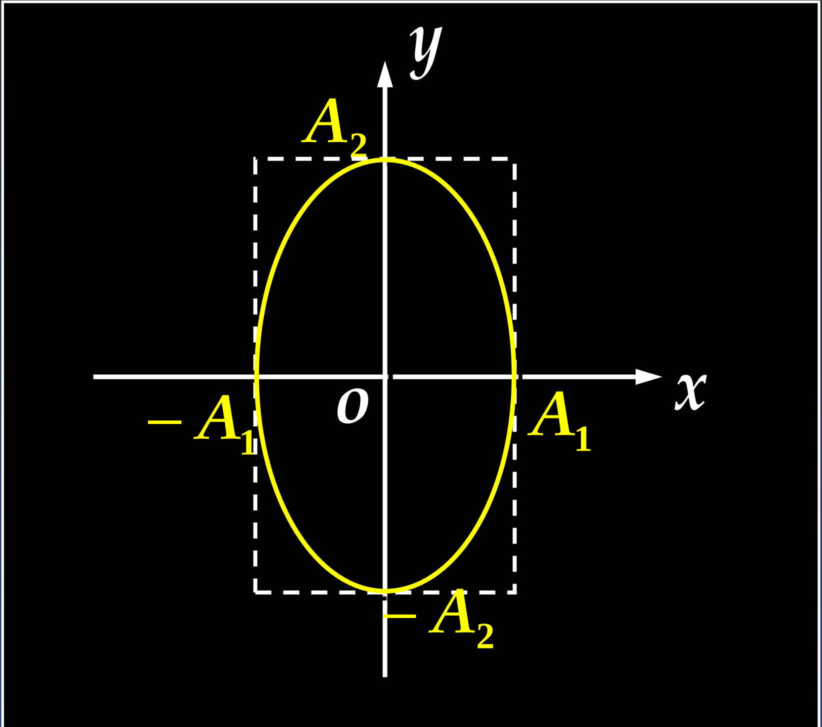
$$\frac{x^2}{A_1^2} + \frac{y^2}{A_2^2} - 2\frac{xy}{A_1 A_2}\cos(\varphi_2 - \varphi_1) = \sin^2(\varphi_2 - \varphi_1)$$

轨迹为斜椭圆。运动方向与 $\varphi_2 - \varphi_1$ 有关。

(1) 当 $\varphi_2-\varphi_1=\pm\dfrac{\pi}{2}$ 时，

$$\frac{x^2}{A_1^2}+\frac{y^2}{A_2^2}=1$$

若 $\varphi_2-\varphi_1=\dfrac{\pi}{2}$ ，则



$$\frac{x^2}{A_1^2}+\frac{y^2}{A_2^2}-2\frac{xy}{A_1A_2}\cos(\varphi_2-\varphi_1)=\sin^2(\varphi_2-\varphi_1)$$

轨迹为**斜椭圆**。运动方向与 $\varphi_2-\varphi_1$ 有关。

(1) 当 $\varphi_2-\varphi_1=\pm\frac{\pi}{2}$ 时，

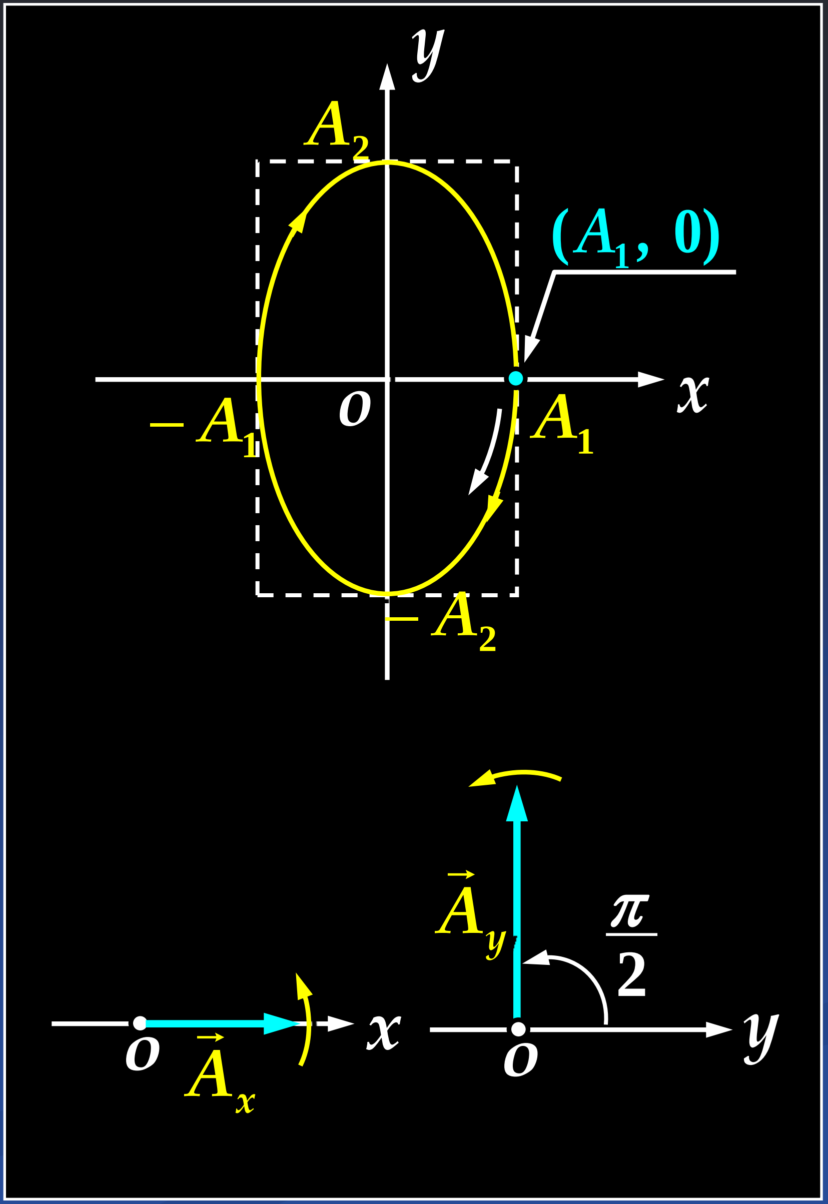
$$\frac{x^2}{A_1^2}+\frac{y^2}{A_2^2}=1$$

若 $\varphi_2-\varphi_1=\frac{\pi}{2}$ ，则图中质点运动方向：

$(A_1,\ 0)\longrightarrow(x>0,\ y<0)$

顺时针运动！

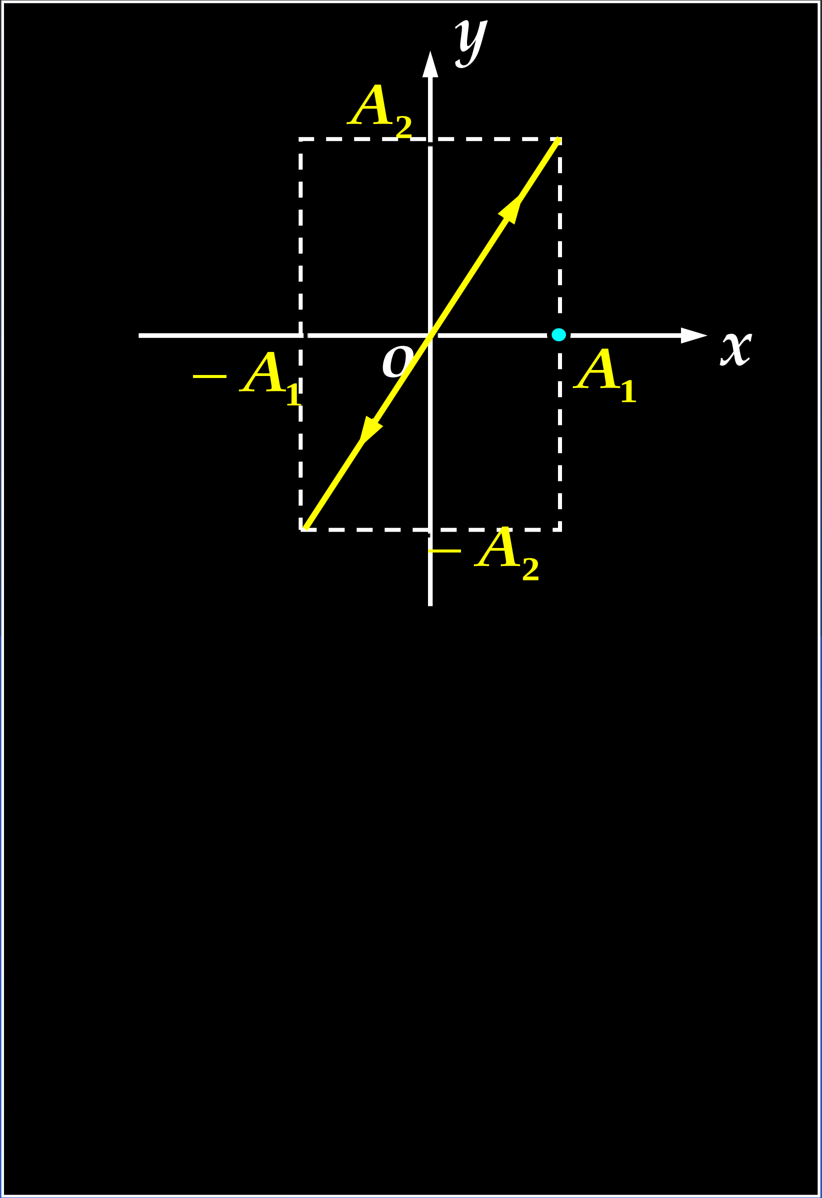
$\varphi_2-\varphi_1=-\frac{\pi}{2}$：逆时针运动！



(2) 当 $\varphi_2-\varphi_1=0$ 时,

$$y=\frac{A_2}{A_1}x$$

合振动退化为简谐振动:



$(A_1,\ 0) \longrightarrow (x>0,\ y<0)$

顺时针运动!

$\varphi_2-\varphi_1=-\frac{\pi}{2}$: 逆时针运动!

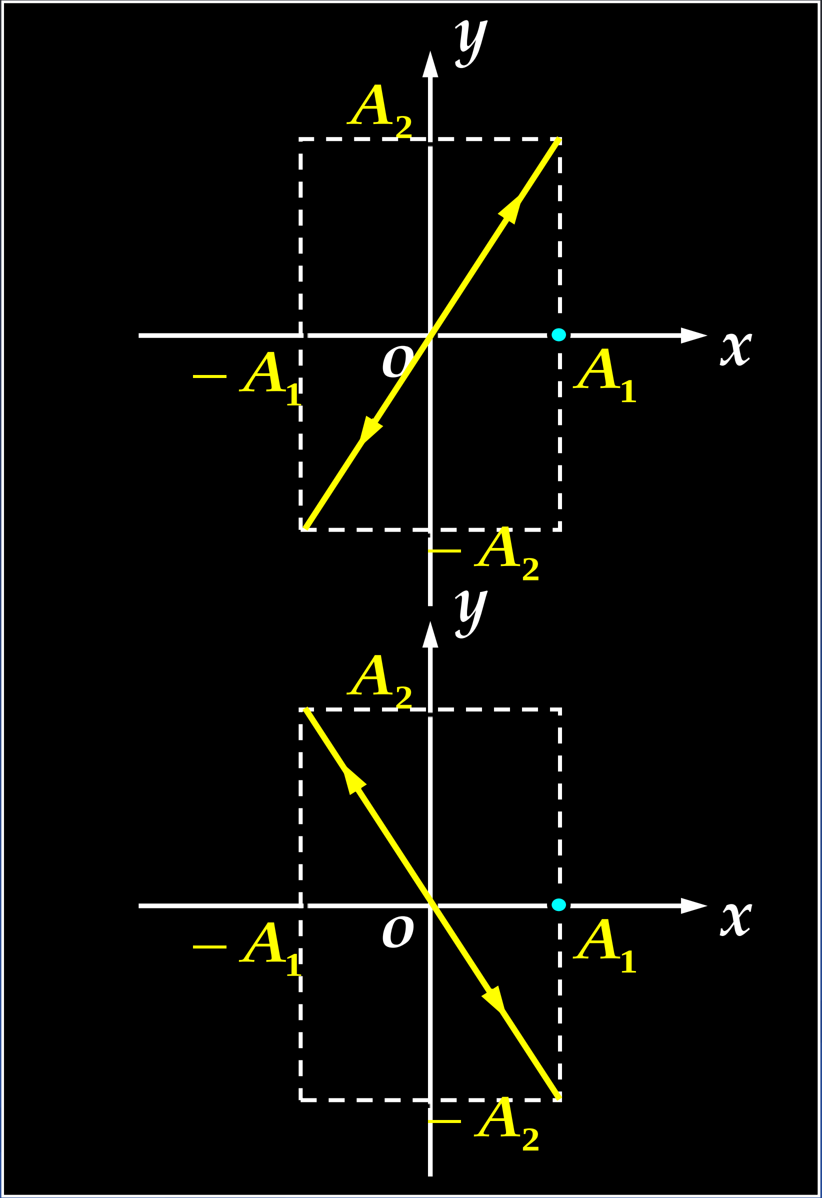
(2) 当 $\varphi_2-\varphi_1=0$ 时，

$$y=\frac{A_2}{A_1}x$$

合振动退化为简谐振动：

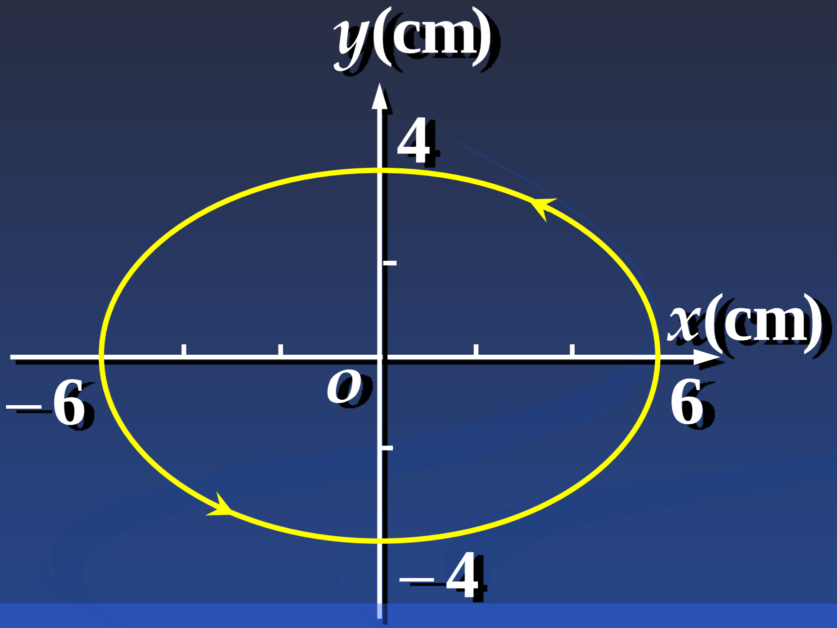
(3) 当 $\varphi_2-\varphi_1=\pi$ 时，

$$y=-\frac{A_2}{A_1}x$$



例 如图，质点按椭圆轨迹作逆时针运动，已知坐标 x：

$x = 6\cos(\pi t)$ (cm)，求 $y(t)$ 。



解

(3) 当 $\varphi_2-\varphi_1=\pi$ 时，

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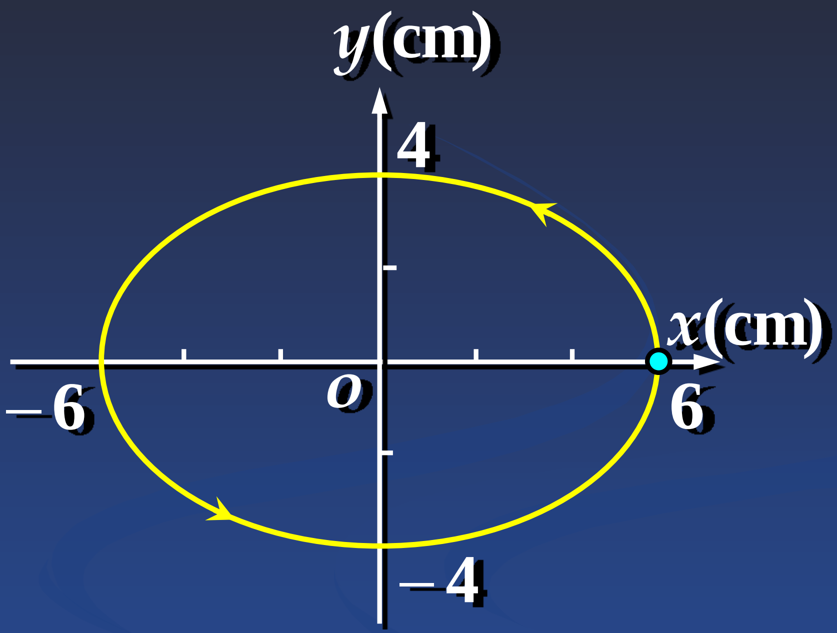
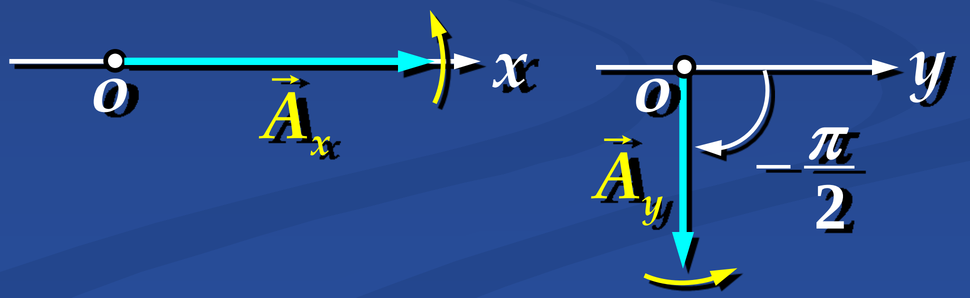


解 由旋转矢量图可知:

$$\varphi_2 - \varphi_1 = -\frac{\pi}{2}$$

$$\varphi_1 = 0$$

$$\therefore \quad y = 4\cos(\pi t - \frac{\pi}{2})$$



(the end)

归纳：

1. 谐振振动合成：

重点：同方向同频率谐振动合成

$$x = x_1 + x_2 = A\cos(\omega t + \varphi)$$

$$\begin{cases} A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\varphi_2 - \varphi_1)} \\ tg\varphi = \dfrac{A_1\sin\varphi_1 + A_2\sin\varphi_2}{A_1\cos\varphi_1 + A_2\cos\varphi_2} \end{cases}$$

归纳:

2. 拍及拍频: $\Delta\nu = \nu_2 - \nu_1$

(The end)