1. the gradient vector for $Z = f(x,y) = ax + by + c$ is

$$\nabla Z = \nabla f(x,y) = \cancel{\frac{\partial f(x,y)}{\partial x} + \frac{\partial f(x,y)}{\partial y} = a+b} \begin{bmatrix} \frac{\partial f(x,y)}{\partial x} \\ \frac{\partial f(x,y)}{\partial y} \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$$

2. the gradient vector for a hyperplane $z = f(x) = a_1x_1 + a_2x_2 + \cdots a_Nx_N + d$ is

$$\nabla Z = \nabla f(x) = \cancel{\frac{\partial f(x)}{\partial x_1} + \frac{\partial f(x)}{\partial x_2} + \cdots \frac{\partial f(x)}{\partial x_N} = a_1 + a_2 + \cdots a_N + d} = \begin{bmatrix} \frac{\partial f(x)}{\partial x_1} \\ \vdots \\ \frac{\partial f(x)}{\partial x_N} \end{bmatrix} = \begin{bmatrix} a_1 \\ \vdots \\ a_N \end{bmatrix}$$

3. $Z = f(x,y) = A(x - x_0)^2 + B(y - y_0)^2 + C$

$$f_x(x,y) = \left(\frac{\partial f(x,y)}{\partial x}\right)_y = 2Ax - 2Ax_0$$

$$f_y(x,y) = \left(\frac{\partial f(x,y)}{\partial y}\right)_x = 2By - 2By_0$$

4. $x^T = (3, 1, 4)$ $[1 \times 3]$ $y^T = \begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix}$ $[3 \times 1]$ $B^T = \begin{pmatrix} 3 & 5 & 1 \\ 5 & 2 & 4 \end{pmatrix}$ $[2 \times 3]$

$x \cdot x = 9 + 1 + 16 = 26$ $x \cdot y^T = 6 + 5 + 4 = 15$

$$\cancel{x \times y = \begin{vmatrix} i & j & k \\ 3 & 1 & 4 \\ 2 & 5 & 1 \end{vmatrix} = (1-20)i - (3-8)j + (15-2)k = -19i + 5j - 13k.}$$

$$x \cdot y = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}(2\ 5\ 1) = \begin{pmatrix} 6 & 15 & 3 \\ 2 & 5 & 1 \\ 8 & 20 & 4 \end{pmatrix} \quad y \cdot x = (2\ 5\ 1)\begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} = 6 + 5 + 4 = 15$$

$$\cancel{y \times x = \begin{vmatrix} i & j & k \\ 2 & 5 & 1 \\ 3 & 1 & 4 \end{vmatrix} = (20-1)i - (8-3)j + (2-15)k = 19i - 5j + 13k.}$$

$$A \times x = \begin{pmatrix} 12 + 5 + 8 \\ 9 + 1 + 20 \\ 18 + 4 + 12 \end{pmatrix} = \begin{pmatrix} 25 \\ 30 \\ 34 \end{pmatrix}$$

$$A \times B = \begin{pmatrix} 12 + 25 + 2 & 20 + 10 + 8 \\ 9 + 5 + 5 & 15 + 2 + 20 \\ 18 + 20 + 3 & 30 + 8 + 12 \end{pmatrix} = \begin{pmatrix} 39 & 38 \\ 19 & 37 \\ 41 & 50 \end{pmatrix}$$

B. reshape(1,6) = [3, 5, 5, 2, 1, 4]

Date. /

LLS - single variable

$$L(p) = \sum_{i=1}^{N} (\hat{y}_i - m x_i - b)^2 = L(m, b)$$

$$\frac{\partial L(m,b)}{\partial m} = -2\sum_{i=1}^{N} x_i y_i + 2m\sum_{i=1}^{N} x_i^2 + 2b\sum_{i=1}^{N} x_i = 0 \quad ①$$

$$\frac{\partial L(m,b)}{\partial b} = -2\sum_{i=1}^{N} y_i + 2m\sum_{i=1}^{N} x_i + 2nb = 0 \quad ②$$

$$① \Rightarrow \sum_{i=1}^{N} x_i y_i = m\sum_{i=1}^{N} x_i^2 + b\sum_{i=1}^{N} x_i \Rightarrow m = \frac{\sum(x_i y_i - \bar{y} x_i)}{\sum(x_i^2 - \bar{x} x_i)} = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sum(x_i - \bar{x})^2}$$

$$= \frac{Cov(x, y)}{Var(x)} \quad ③$$

$$② \Rightarrow \sum y_i = m \cdot \sum x_i + nb \Rightarrow b = \bar{y}_i - m\bar{x}_i \quad ④$$

$$③④ \Rightarrow b = \bar{y} - \frac{cov(x, y)}{Var(x)}\bar{x}$$

LLS - Multi-Variable

set $X = \{(x_1, y_1) \cdots (x_m, y_m)\}$. $y = x\beta + \beta_0$

$$SSR \Rightarrow S(\beta) = \sum (y_i - x_i\beta - \beta_0)^2$$

$$\frac{\partial S(\beta)}{\partial \beta} = \sum 2(y_i - x_i\beta - \beta_0)^2(-x_i) = -2 \cdot \sum (x_i^2\beta + \beta_0 x_i - x_i y_i) \quad ①$$

$$\frac{\partial S(\beta)}{\partial \beta_0} = 2\sum (x_i\beta + \beta_0 - y_i) = 2\left(m\beta\frac{\sum x_i}{m} + m\beta_0 - m\frac{\sum y_i}{m}\right) = 0 \quad ②$$

$$\Rightarrow \beta_0 = \bar{y} - \beta\bar{x}$$

let $① = 0 \Rightarrow \beta = \frac{\sum x_i y_i - y\sum x_i}{\sum x_i^2 - \bar{x}\sum x_i} = \frac{\sum(x_i y_i - \bar{y} x_i - y_i\bar{x} + \bar{y}\bar{x})}{\sum(x - \bar{x})^2} =$

the

$S(\beta) = \sum_{i=1}^{m} \| y_i - \sum_{j=1}^{N} x_{ij}\beta_j \|^2 = \| y - X\beta^T \|^2$ $\quad y - X\beta^T = 0 \Rightarrow X^T X\beta^T = X^T y \Rightarrow \beta^T = (X^T X)^{-1} X^T y$