Badass Title

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What is a Fractal String

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The fractal string we studied the most in particular is the **Cantor String**

Cantor String

Inner-Tubular Neighborhood

Definition 2

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Counting function: $N_\Omega(x) = \sum_{\ell_j^{-1} \leq x} 1$

More Notation: $\mathcal{W}_\Omega(x) = \sum_{j:\ell_j^{-1} > x} \ell_j$

Now we can write: $V(\epsilon) = 2\epsilon \cdot N_\Omega\left(\frac{1}{2\epsilon}\right) + \mathcal{W}_\Omega\left(\frac{1}{2\epsilon}\right)$

Cantor String Volume

Inner Minkowski Dimension of a Fractal String

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Dimension of Ω: $D_\Omega = \inf\left\{\alpha \geq 0 \mid V(\epsilon) = O\left(\epsilon^{1-\alpha}\right) \text{ as } \epsilon \to 0^+\right\}$

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Cantor String Dimension: $D_{cs} = \log_3 2$

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