Badass Title

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What is a fractal string

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Introduce the Cantor string so that the audience can see an example right away. A movie would be nice!

Cantor String

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Inner-tubular neighborhood: $V(\epsilon) = \mathrm{vol}_1\{x \in \Omega \mid d(x, \partial\Omega) < \epsilon\}$

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Counting function: $N_\Omega(x) = \sum_{\ell_j^{-1} \leq x}$

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Lapidus, van Frankenhuijsen, [9, 10, 11]

$$D_{\mathcal{L}} = \inf\{\alpha \in \mathbb{R} \mid \sum_{j=1}^{\infty} \ell_j^{\alpha} < \infty\} \implies 0 < D_{\mathcal{L}} < 1$$

Cantor string volume

Fractal lawn

Because a fractal string is a bounded open subset of $\mathbb{R}$, the Cartesian product of two fractal strings is a bounded open subset of $\mathbb{R}^2$.

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A **fractal lawn** $\mathcal{L}^2$ is a bounded open subset $\Omega = \{(x, y) : x \in \Omega_1$ and $y \in \Omega_2\}$ of $\mathbb{R}^2$.

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$$\Omega_1 = \bigcup_{j=1}^{\infty} \ell_j,\, \Omega_2 = \bigcup_{i=1}^{\infty} l_i$$

$$\ell_1 \geq \ell_2 \geq \ell_3 \geq \cdots \geq 0,\, l_1 \geq l_2 \geq l_3 \geq \cdots \geq 0$$

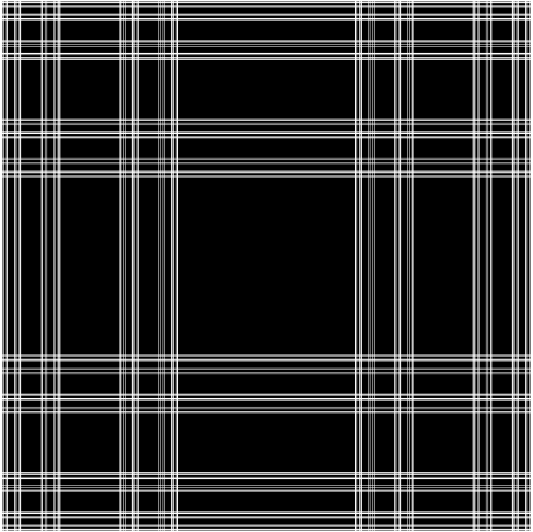
Cantor lawn

Dimension of fractal lawn

Inner-tubular neighborhood: $V(\epsilon) = \mathrm{vol}_2\{x \in \Omega \mid d(x, \partial\Omega) < \epsilon\}$

Counting functions: $N_{\Omega_1}(x) = \sum_{\ell_j^{-1} \leq x}$, $N_{\Omega_2}(x) = \sum_{l_j^{-1} \leq x}$

Cantor Lawn Volume GIF



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