Badass Title

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Introduce the Cantor string so that the audience can see an example right away. A movie would be nice!



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Counting function:
$$N_{\Omega}(x) = \sum_{\ell_j^{-1} \leq x}$$

$$D_{\mathcal{L}} = \inf\{\alpha \geq 0 \mid V(\epsilon) = O(\epsilon^{1-\alpha}) \text{ as } \epsilon \to 0^+\}$$



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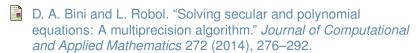
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Lapidus, van Frankenhuijsen, [9, 10, 11]

$$D_{\mathcal{L}} = \inf\{\alpha \in \mathbb{R} \mid \sum_{j=1}^{\infty} \ell_j^{\alpha} < \infty\} \implies 0 < D_{\mathcal{L}} < 1$$



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