

# Badass Title

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# What is a fractal string

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Introduce the Cantor string so that the audience can see an example right away. A movie would be nice!

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Lapidus, van Frankenhuijsen, [9, 10, 11]

$$D_{\mathcal{L}} = \inf\{\alpha \in \mathbb{R} \mid \sum_{j=1}^{\infty} \ell_j^\alpha < \infty\} \implies 0 < D_{\mathcal{L}} < 1$$





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