Badass Title

Isaac Ashton, Joseph Ingenito, Peter Tonuzi

The College of New Jersey

What is a fractal string

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Introduce the Cantor string so that the audience can see an example right away. A movie would be nice!

Cantor Lawn GIF

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Lapidus, van Frankenhuijsen, [9, 10, 11]

$$D_{\mathcal{L}} = \inf\{\alpha \in \mathbb{R} \mid \sum_{j=1}^{\infty} \ell_j^{\alpha} < \infty\} \implies 0 < D_{\mathcal{L}} < 1$$

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