Homework 2

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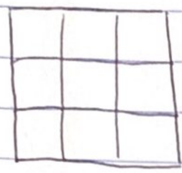
Exercise 1



$\bar{x} = \frac{0+3+2(3)}{9} = 1$ $\qquad x' = x - \bar{x}$

$\bar{y} = \frac{0+3+2(3)}{9} = 1$ $\qquad y' = y - \bar{y}$

$a = \iint x'^2 \, b(x', y') \, dx' \, dy'$ $\qquad b = 2\iint x'y' \, b(x', y') \, dx' \, dy'$

$c = \iint y'^2 \, b(x', y') \, dx' \, dy^2$

$a = (-1)^2 3 + (1^2) 3 = 6$

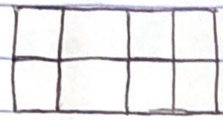
$b = -1 \cdot -1 + -1 \cdot (0) + (-1) \cdot 1 + 0 + 1 \cdot (-1) + 0 + 1 \cdot 1 = 0$

$c = (-1)^2 3 + 3(0)^2 + 3(1)^2 = 6$

circularity $= 1$

since $a = c$, and $b = 0$, E is independent of θ

The object is too symmetric to define an axis



$$\bar{x} = \frac{0+2+6+8}{8} = 2$$

$$\bar{y} = \frac{0+4}{8} = \frac{1}{2} \approx 1$$

centroid $= 2, 1$

$$a = (-2)^2 \cdot 2 + (-1)^2 \cdot 2 + 0 + (1)^2 2 = 12$$

$$b = (-2 \cdot -1) + (-1 \cdot -1) + (0 \cdot -1) + (1 \cdot -1) = 2$$

$$c = (-1^2)4 = 4$$

$$E_{min} = \frac{a+c}{2} - \frac{a-c}{2}\left(\frac{a-c}{\sqrt{(a-c)^2+b^2}}\right) - \frac{b}{2} \cdot \frac{b}{\sqrt{(a-c)^2+b^2}}$$

$$= \frac{12+2}{2} - \frac{12-2}{2}\left(\frac{12-2}{\sqrt{10^2+2^2}}\right) - \frac{2}{\sqrt{10^2+2^2}}$$

$$= 7 - 5\left(\frac{10}{\sqrt{104}}\right) - \frac{2}{\sqrt{104}}$$

$$E_{max} = \frac{a+c}{2} - \frac{a-c}{2}\left(\frac{a-c}{-\sqrt{(a-c)+b^2}}\right) - \frac{b}{2} \cdot \frac{-b}{\sqrt{(a-c)^2+b^2}}$$

$$= \frac{12+2}{2} - \frac{12-2}{2}\left(\frac{12-2}{-\sqrt{10^2+2^2}}\right) + \frac{2}{\sqrt{10^2+2^2}}$$

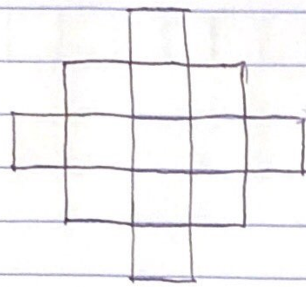
$$= 7 - 5\left(\frac{10}{-\sqrt{104}}\right) + \frac{2}{\sqrt{104}}$$

$$\text{circularity} = \frac{E_{min}}{E_{max}} \approx 0.156$$

$$\sin 2\theta = \pm \frac{b}{\sqrt{b^2 + (a-c)^2}}$$

$$\sin 2\theta = \frac{2}{\sqrt{4 + 10^2}}$$

$$\theta = \frac{1}{2} \arcsin\left(\frac{2}{\sqrt{104}}\right) = 0.1$$



$$\bar{x} = \frac{0+3+10+9+4}{13} = 2$$

$$\bar{y} = \frac{0+3+10+9+4}{13} = 2$$

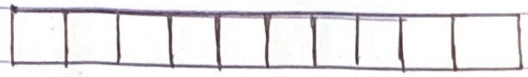
$$a = [(-2)^2 + (-1)^2 + 0 + 1^2 + 2^2] \cdot 5 = 45$$

$$\begin{aligned} b = {} & [(-2 \cdot -2) + (-1 \cdot -2) + 0 + 1 \cdot (-2) + 2 \cdot (-2)] + \\ & [(-2 \cdot -1) + (-1 \cdot -1) + 0 + 1 \cdot (-1) + 2 \cdot (-1)] + 0 \\ & + [-2 \cdot 1 + (-1 \cdot 1) + 0 + 1 \cdot 1 + 2 \cdot 1] + \\ & [-2 \cdot 2 + (-1 \cdot 2) + 0 + 1 \cdot 2 + 2 \cdot 2] \\ = {} & 0 \end{aligned}$$

$$c = [(-2)^2 + (-1)^2 + 0 + 1^2 + 2^2] \cdot 5 = 45$$

since $a = c$ and $b = 0$,

circularity is 1, and E is indepent of θ.
The object is too symmetric to define an axis



$$\bar{x} = \frac{0+1+2+3+4+5+6+7+8+9}{10}$$

$$= 4.5 \approx 5$$

$$\bar{y} = 0$$

$$a = (-5)^2 + (-4)^2 + (-3)^2 + (-2)^2 + (-1)^2 + 0 + 1^2 + 2^2$$
$$+ 3^2 + 4^2$$
$$= 25 + 16 + 9 + 4 + 1 + 1 + 4 + 9 + 16$$
$$= 85$$

$$b = 0$$

$$c = 0$$

$$E_{min} = \frac{85}{2} - \frac{85}{2}\left(\frac{85}{\sqrt{85^2}}\right) \cancel{0} - 0$$

$$E_{max} = \frac{85}{2} - \frac{85}{2}\left(\frac{85}{-\sqrt{85^2}}\right) - 0$$

$$\text{circularity} = 0$$

$$\sin 2\theta = 0 \qquad \theta = 0$$

2.

The change of coordinate origin of an image will not affect by ~~ans~~ anser in problem 1, because the centroid is a distribution of mass in space. Thus, the change of coordinate of origin cannot affect the centroid.

3.

No. I think the measures will ~~verty~~ vary as size become larger because when calculating the variable a and c, we take the square of x' and y'. Therefore, the final values should ~~chag~~ change

Exercise 2

1.

Given a random variable X, we know the equation

$$\sigma_{aX+b} = a \cdot \sigma_X$$

Therefore,

$$r(M, aS+b) = \frac{1}{n} \sum^{n} \frac{((a \cdot s_i + b) - (a \cdot \mu_S + b))(m_i - \mu_M)}{\sigma_{aS+b} \sigma_M}$$

$$= \frac{1}{n} \sum_{i=1}^{n} \frac{a \cdot (s_i - \mu_S)(m_i - \mu_M)}{a \cdot \sigma_S \sigma_M}$$

$$= \frac{1}{n} \sum_{i=1}^{n} \frac{(s_i - \mu_S)(m_i - \mu_M)}{\sigma_S \sigma_M}$$

$$= r(M, S)$$

2.

NCC method can effectively reduce the influence of illumination on ~~image~~ the result of image comparison. Therefore, ~~[illegible]~~ ~~result~~ when we compare the image, we do not need to take the ~~[illegible]~~ brightness into account.

3.

Since the range is between -1 and 1, it ~~[illegible]~~ is very easy to quantify and compare the result. ~~we can~~ ~~[illegible]~~ Given a threshold, we can judge whether the result are good or bad

4.

It is easier to compute ~~v~~ the variance and the stand deviation of r, if $E[r] = 0$.

Exercise 3.

64	24
56	56

1. x is ~~64~~ 56
2. y is 56
3. ~~5~~ $\frac{64}{120} = \frac{8}{15}$
4. $\frac{64}{88}$ precision is $\frac{8}{11} \approx 0.73$
5. accuracy is $\frac{64+56}{200} = \frac{3}{5} = 0.6$
6. $\frac{56}{80} = 0.7$. The classifier is 70% specific for pneumonia
7. $\frac{64}{120} = 0.53$ The classifier is 53% ~~specif~~ sensitive to pneumonia
8. I think the sensitivity is the most important property needed to improve