Reinforcement Learning China Summer School

RLChina 2020

Introduction to Reinforcement Learning and Value-based Methods

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Content

- Introduction to Reinforcement Learning
  - About RL
  - RL problem
  - Markov Decision Processes
- Value-based Methods
  - Dynamic Programming
  - Monte Carlo
  - TD Learning
  - Off-policy Learning
  - DQN and its variants

Many Faces of Reinforcement Learning

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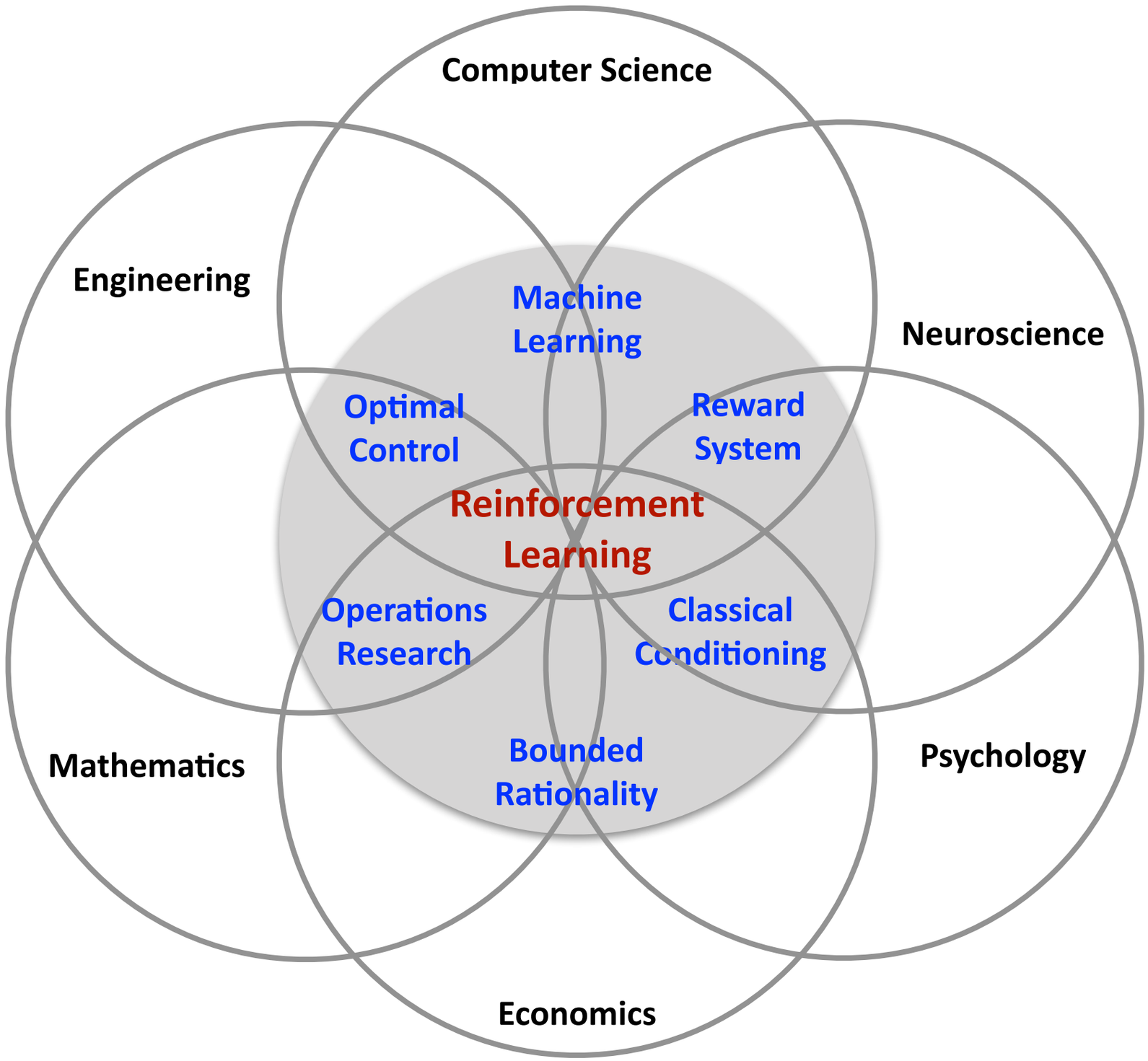

Branches of Machine Learning

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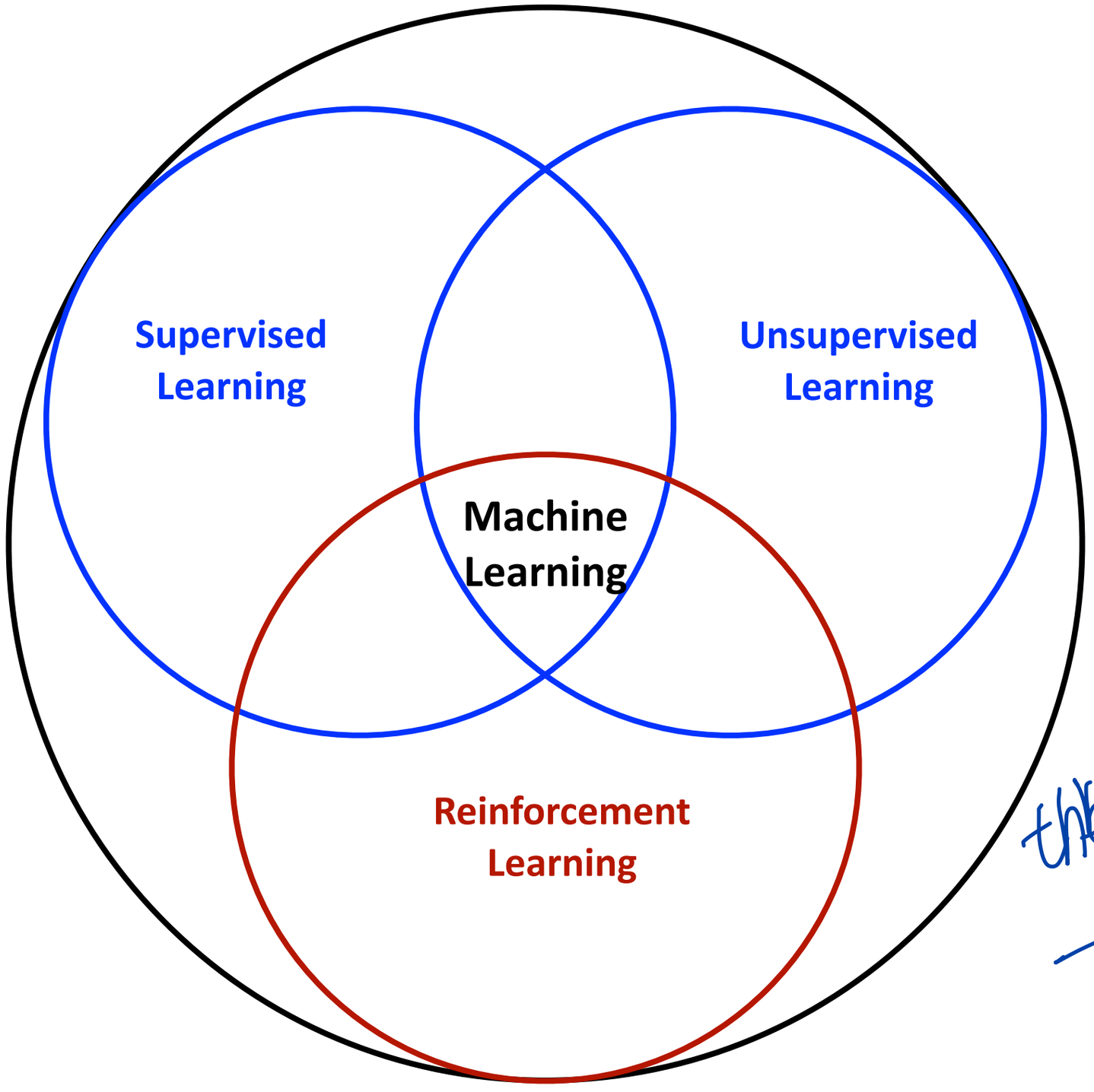

three paradigms

Characteristics of Reinforcement Learning

- What makes reinforcement learning different from other machine learning paradigms?
  - There is no supervisor, only a reward signal, trial-and-error
  - Feedback is delay, not instantaneous
  - Time really matters (sequential, non-iid data)
  - Agent's action affects the subsequent data it receives

The RL Problem

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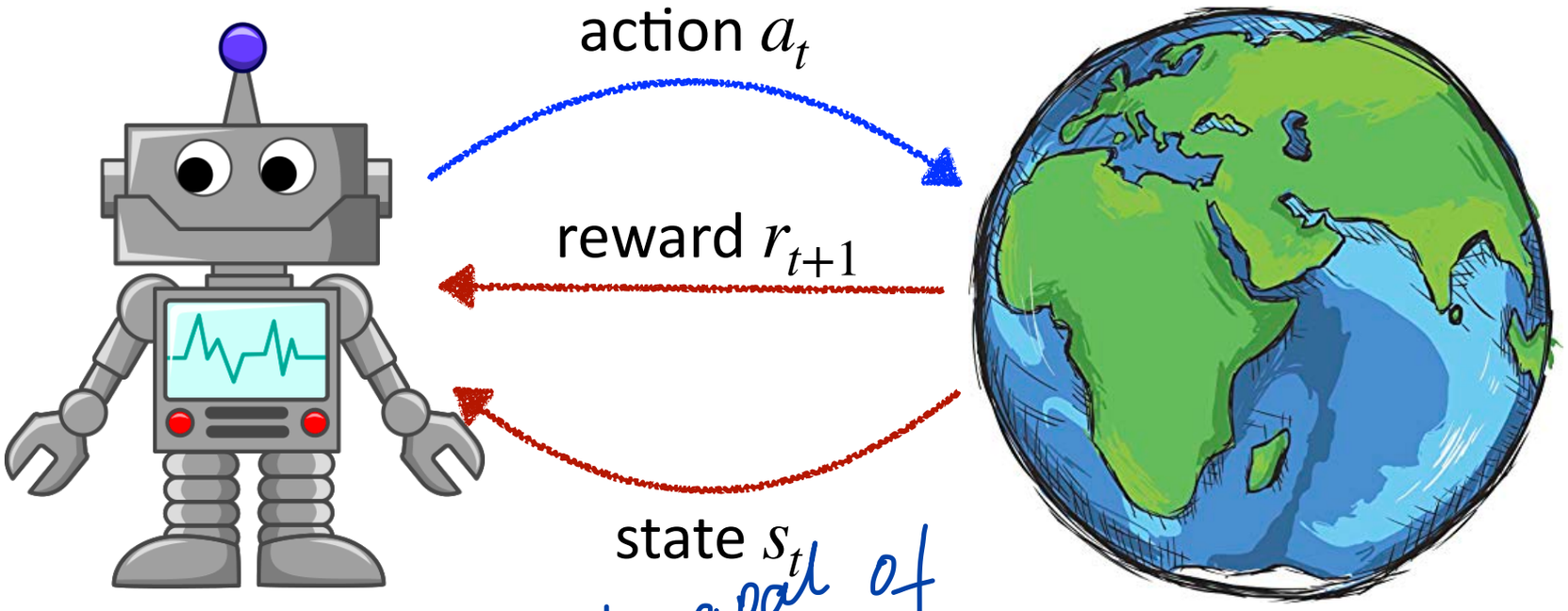

The agent is to maximize the cumulative reward

All goals and purposes can be described by the maximization of the expected cumulative reward

Inside An RL Agent

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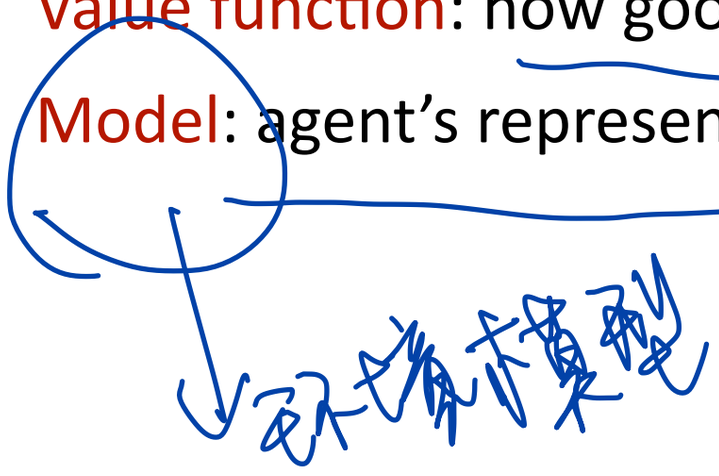
- An RL agent may include one or more of these components
  - Policy: agent's behavior function
  - Value function: how good is a state and/or action
  - Model: agent's representation of the environment



Categorizing RL Agents

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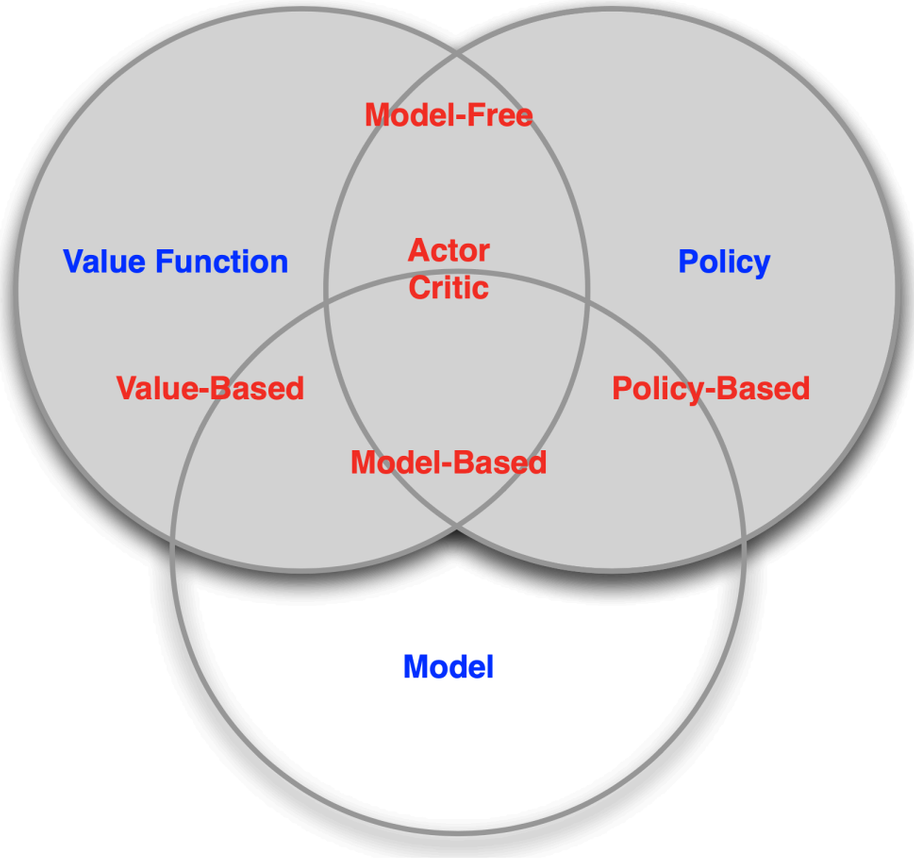
- Value based
  - No policy
  - Value function
- Policy based
  - Policy
  - No value function
- Actor Critic
  - Policy
  - Value function



- Model free
  - Policy and/or value function
  - No model
- Model based
  - Policy and/or value function
  - Model

Markov Decision Processes

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- Markov decision processes formally describe an environment for reinforcement learning
  - The environment is fully observable
- Almost all RL problems can be formalized as MDPs
  - Optimal control primarily deals with continuous MDPs
  - Partially observable problems can be converted into MDPs
  - Bandits are MDPs with one state
- All states in MDP has "Markov" property
  - $\mathbb{P}[S_{t+1} \mid S_t] = \mathbb{P}[S_{t+1} \mid S_1, \ldots, S_t]$
  - Current state encapsulates all the statistics we need to decide the future

MDP

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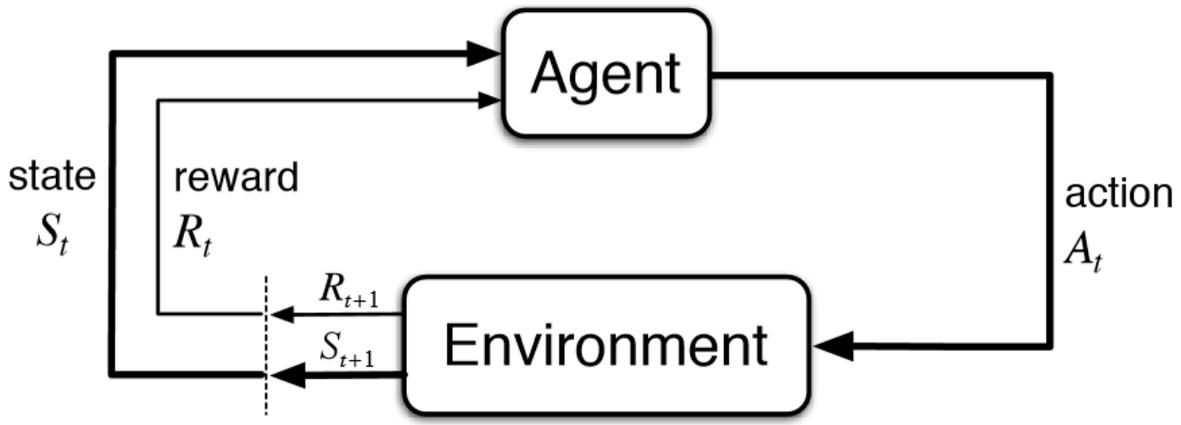

$\langle \mathcal{S}, \mathcal{A}, \mathcal{P}, \mathcal{R}, \gamma \rangle$

- $\mathcal{S}$ - a set of states
- $\mathcal{A}$ - a set of actions
- $\mathcal{P}$ - transition probability function,
  - $\mathcal{P}^a_{ss'} = \mathbb{P}\left[S_{t+1} = s' \,|\, S_t = s, A_t = a\right]$
- $\mathcal{R}$ - reward function,
  - $\mathcal{R}^a_s = \mathbb{E}\left[R_{t+1} \,|\, S_t = s, A_t = a\right]$
- γ - discounting factor for future reward, $\gamma \in [0,1]$

$P^a_{ss'} = P[S_{t+1}=s' \,|\, S_t=s, A_t=a]$

$R^a_s = E[R_{t+1} \,|\, S_t=s, A_t=a]$.

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Policy

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- A policy π is a distribution over actions given states
  - $\pi(a \mid s) = \mathbb{P}[A_t = a \mid S_t = s]$
- Given a MDP $\mathcal{M} = \langle \mathcal{S}, \mathcal{A}, \mathcal{P}, \mathcal{R}, \gamma \rangle$ and a policy π
  - The state sequence $S_1, S_2, \ldots$ is a Markov process $\langle \mathcal{S}, \mathcal{P}^\pi \rangle$
  - The state and reward sequence $S_1, R_2, S_2, \ldots$ is a Markov reward process $\langle \mathcal{S}, \mathcal{P}^\pi, \mathcal{R}^\pi, \gamma \rangle$, where

$$\mathcal{P}^\pi_{s,s'} = \sum_{a \in \mathcal{A}} \pi(a \mid s) \mathcal{P}^a_{ss'}$$

$$\mathcal{R}^\pi_s = \sum_{a \in \mathcal{A}} \pi(a \mid s) \mathcal{R}^a_s$$

Value Function

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- The state-value function $v_\pi(s)$ of an MDP is the expected return starting from state s, and then following policy π

$$
\begin{aligned}
v_\pi(s) &= \mathbb{E}_\pi\left[G_t \mid S_t = s\right] \\
&= \mathbb{E}_\pi\left[R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \ldots \mid S_t = s\right] \\
&= \mathbb{E}_\pi\left[R_{t+1} + \gamma\left(R_{t+2} + \gamma R_{t+3} + \ldots\right) \mid S_t = s\right] \\
&= \mathbb{E}_\pi\left[R_{t+1} + \gamma G_{t+1} \mid S_t = s\right] \\
&= \mathbb{E}_\pi\left[R_{t+1} + \gamma v_\pi\left(S_{t+1}\right) \mid S_t = s\right]
\end{aligned}
$$

A cumulative reward

- The action-value function $q_\pi(s,a)$ of an MDP is the expected return starting from state s, taking action a, and then following policy π

$$
\begin{aligned}
Q_\pi(s,a) &= \mathbb{E}_\pi\left[R_{t+1} + \gamma v\left(S_{t+1}\right) \mid S_t = s, A_t = a\right] \\
&= \mathbb{E}_\pi\left[R_{t+1} + \gamma \mathbb{E}_{a' \sim \pi} Q\left(S_{t+1}, a'\right) \mid S_t = s, A_t = a\right]
\end{aligned}
$$

recursive 迭代的方式

Bellman Expectation Equations

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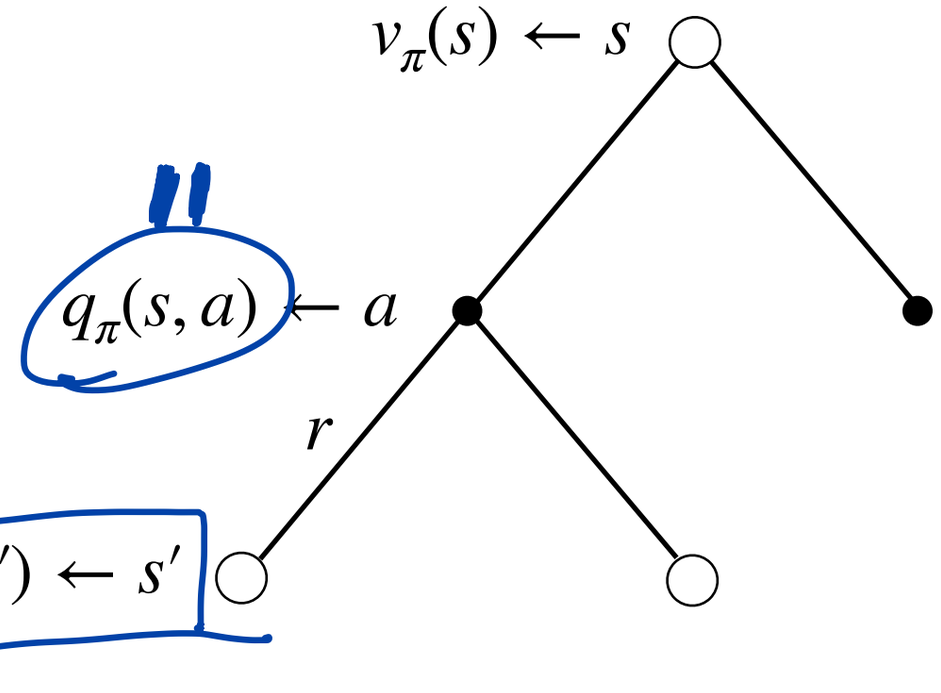
$$v_\pi(s) = \sum_{a \in \mathcal{A}} \pi(a \mid s) q_\pi(s, a)$$

$$q_\pi(s, a) = \mathcal{R}_s^a + \gamma \sum_{s' \in \mathcal{S}} \mathcal{P}_{ss'}^a v_\pi(s')$$



$$v_\pi(s) = \sum_{a \in \mathcal{A}} \pi(a \mid s) \left(\mathcal{R}_s^a + \gamma \sum_{s' \in \mathcal{S}} \mathcal{P}_{ss'}^a v_\pi(s') \right)$$

$$q_\pi(s, a) = \mathcal{R}_s^a + \gamma \sum_{s' \in \mathcal{S}} \mathcal{P}_{ss'}^a \sum_{a' \in \mathcal{A}} \pi(a' \mid s') q_\pi(s', a')$$

Optimal Value Function

- The optimal state-value function $v_*(s)$ is the maximum value function over all policies

$$v_*(s) = \max_\pi v_\pi(s)$$

- The optimal action-value function $q_*(s, a)$ is the maximum value function over all policies

$$q_*(s, a) = \max_\pi q_\pi(s, a)$$

Optimal Policy

- Define $\pi \geq \pi'$ if $v_\pi(s) \geq v_{\pi'}(s), \forall s$
  - There exists an optimal policy π_* that is better tan or equal to all other policies, $\pi_* \geq \pi', \forall \pi$
  - All optimal policies achieve the optimal value function, $v_{\pi_*}(s) = v_*(s)$
  - All optimal policies achieve the optimal action-value function, $q_{\pi_*}(s,a) = q_*(s,a)$
- If we know $q_*(s,a)$, we immediately have the optimal policy

$$a = \arg\max_a q_*(s,a)$$

greedy policy

Bellman Optimality Equations

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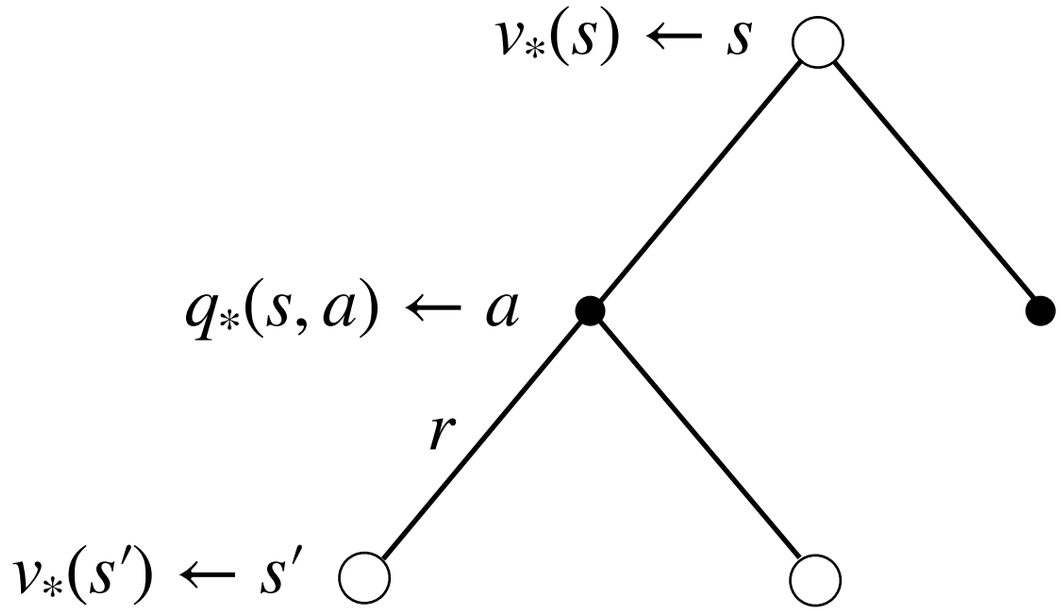
$$v_*(s) = \max_a q_*(s,a)$$

$$q_*(s,a) = \mathcal{R}_s^a + \gamma \sum_{s' \in \mathcal{S}} \mathcal{P}_{ss'}^a v_*(s')$$

$$v_*(s) = \max_a \left(\mathcal{R}_s^a + \gamma \sum_{s' \in \mathcal{S}} \mathcal{P}_{ss'}^a v_*(s') \right)$$

$$q_*(s,a) = \mathcal{R}_s^a + \gamma \sum_{s' \in \mathcal{S}} \mathcal{P}_{ss'}^a \max_{a'} q_*(s',a')$$



Solving the Bellman Optimality Equation

- If we have complete information of the environment, this turns into a planning problem, solvable by Dynamic Programming
- Unfortunately, in most scenarios, we do not know $\mathcal{P}^a_{ss'}$ or $\mathcal{R}^a_s$, we cannot solve MDPs by directly applying Bellman equation

Outline

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Dynamic Programming

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- DP assumes full knowledge of the MDP
- Planning in an MDP
- For prediction:
  - Input: MDP $\langle \mathcal{S}, \mathcal{A}, \mathcal{P}, \mathcal{R}, \gamma \rangle$ and policy π
  - Output: value function v_π
- For control:
  - Input: MDP $\langle \mathcal{S}, \mathcal{A}, \mathcal{P}, \mathcal{R}, \gamma \rangle$
  - Output: optimal value function v_* and π_*

Policy Evaluation

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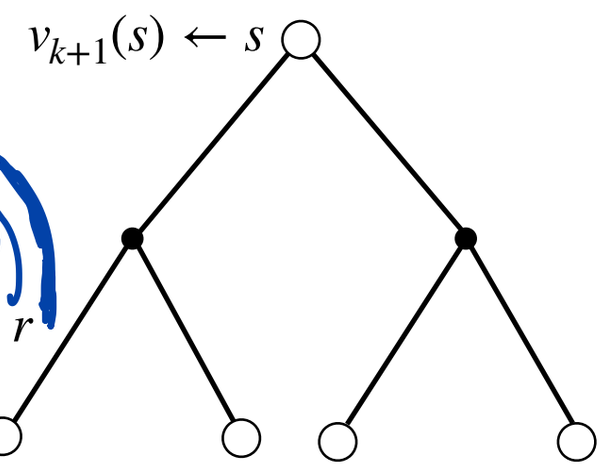
- Problem: compute the value function for a given policy π
- Solution: iteration of Bellman Expectation backup
- $v_1 \rightarrow v_2 \rightarrow \ldots \rightarrow v_\pi$
  - At each iteration $k+1$
  - Synchronous update $v_{k+1}(s)$ from $v_k(s')$, $\forall s \in \mathcal{S}$, where s' is a successor state of s

$$v_{k+1}(s) = \sum_{a \in \mathcal{A}} \pi(a|s) \left(\mathcal{R}_s^a + \gamma \sum_{s' \in \mathcal{S}} \mathcal{P}_{ss'}^a v_k(s') \right)$$

$$v^{k+1} = \mathcal{R}^\pi + \gamma \mathcal{P}^\pi v^k$$



Evaluating a Random Policy

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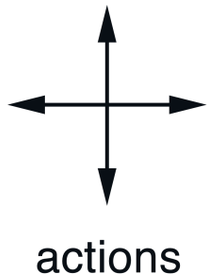
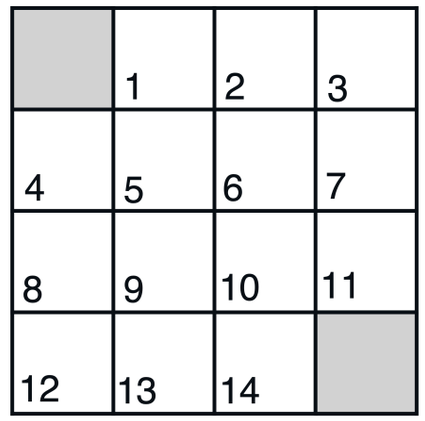
- Undiscounted episodic MDP
- Actions leading out of the grid make state unchanged
- Two terminal states
- A uniform random policy





$R_t = -1$
on all transitions

$$\pi(\text{up}\,|\,\cdot) = \pi(\text{down}\,|\,\cdot) = \pi(\text{right}\,|\,\cdot) = \pi(\text{left}\,|\,\cdot) = 0.25$$

Policy Evaluation in Gridworld

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v_k for the random policy

$k = 0$

0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0

$k = 1$

0.0	-1.0	-1.0	-1.0
-1.0	-1.0	-1.0	-1.0
-1.0	-1.0	-1.0	-1.0
-1.0	-1.0	-1.0	0.0

$k = 2$

0.0	-1.7	-2.0	-2.0
-1.7	-2.0	-2.0	-2.0
-2.0	-2.0	-2.0	-1.7
-2.0	-2.0	-1.7	0.0

$k = 3$

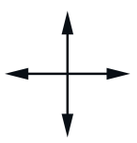
0.0	-2.4	-2.9	-3.0
-2.4	-2.9	-3.0	-2.9
-2.9	-3.0	-2.9	-2.4
-3.0	-2.9	-2.4	0.0

$k = 10$

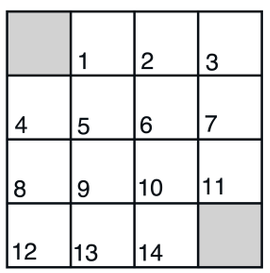
0.0	-6.1	-8.4	-9.0
-6.1	-7.7	-8.4	-8.4
-8.4	-8.4	-7.7	-6.1
-9.0	-8.4	-6.1	0.0

$k = \infty$

0.0	-14.	-20.	-22.
-14.	-18.	-20.	-20.
-20.	-20.	-18.	-14.
-22.	-20.	-14.	0.0



actions



$R_t = -1$
on all transitions

Policy Improvement

$\pi \rightarrow V$

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- Consider a deterministic policy, $a = \pi(s)$
- Improve the policy by acting greedily

$$\pi'(s) = \underset{a \in \mathcal{A}}{\operatorname{argmax}}\, q_\pi(s, a)$$

- This improves the value from any state s over one step

$$q_\pi\left(s, \pi'(s)\right) = \max_{a \in \mathcal{A}} q_\pi(s, a) \geq q_\pi(s, \pi(s)) = v_\pi(s)$$

- It therefore improves the value function, $v_{\pi'}(s) \geq v_\pi(s)$

$$\begin{aligned} v_\pi(s) \leq q_\pi\left(s, \pi'(s)\right) &= \mathbb{E}_{\pi'}\left[R_{t+1} + \gamma v_\pi\left(S_{t+1}\right) \mid S_t = s\right] \\ &\leq \mathbb{E}_{\pi'}\left[R_{t+1} + \gamma q_\pi\left(S_{t+1}, \pi'\left(S_{t+1}\right)\right) \mid S_t = s\right] \\ &\leq \mathbb{E}_{\pi'}\left[R_{t+1} + \gamma R_{t+2} + \gamma^2 q_\pi\left(S_{t+2}, \pi'\left(S_{t+2}\right)\right) \mid S_t = s\right] \\ &\leq \mathbb{E}_{\pi'}\left[R_{t+1} + \gamma R_{t+2} + \ldots \mid S_t = s\right] = v_{\pi'}(s) \end{aligned}$$

Policy Improvement (cont'd)

- If improvement stops,

$$q_\pi\left(s, \pi'(s)\right) = \max_{a \in \mathcal{A}} q_\pi(s, a) = q_\pi(s, \pi(s)) = v_\pi(s)$$

- Then the Bellman optimality equation has been satisfied

$$v_\pi(s) = \max_{a \in \mathcal{A}} q_\pi(s, a)$$

- Therefore $v_\pi(s) = v_*(s), \forall s \in \mathcal{S}$
- π is an optimal policy

Policy Improvement in Gridworld

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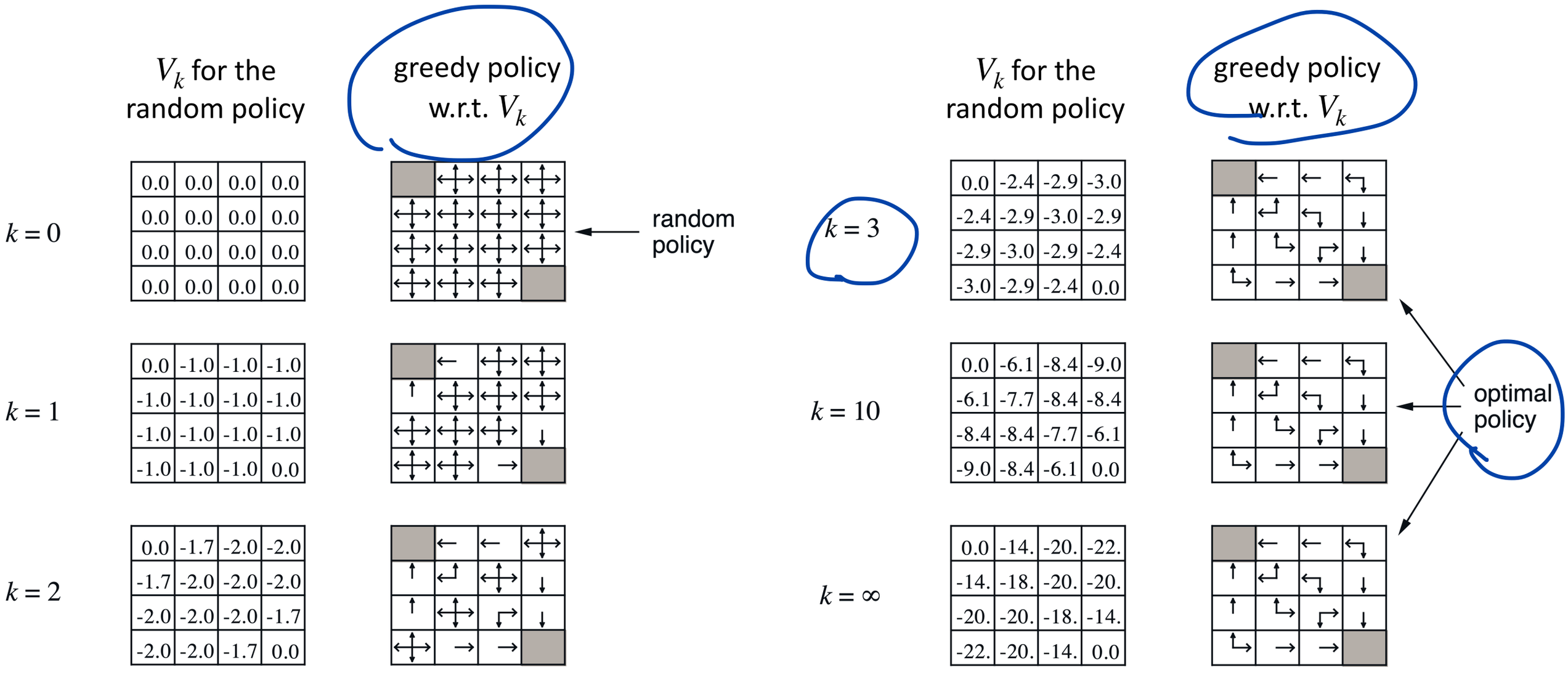

Policy Iteration

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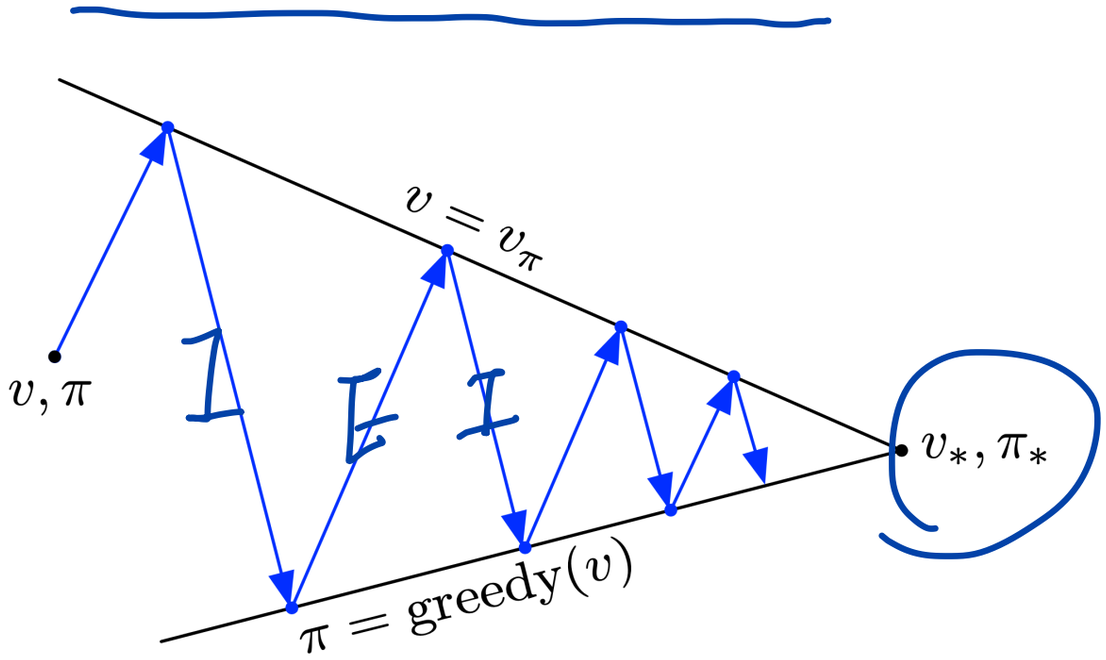
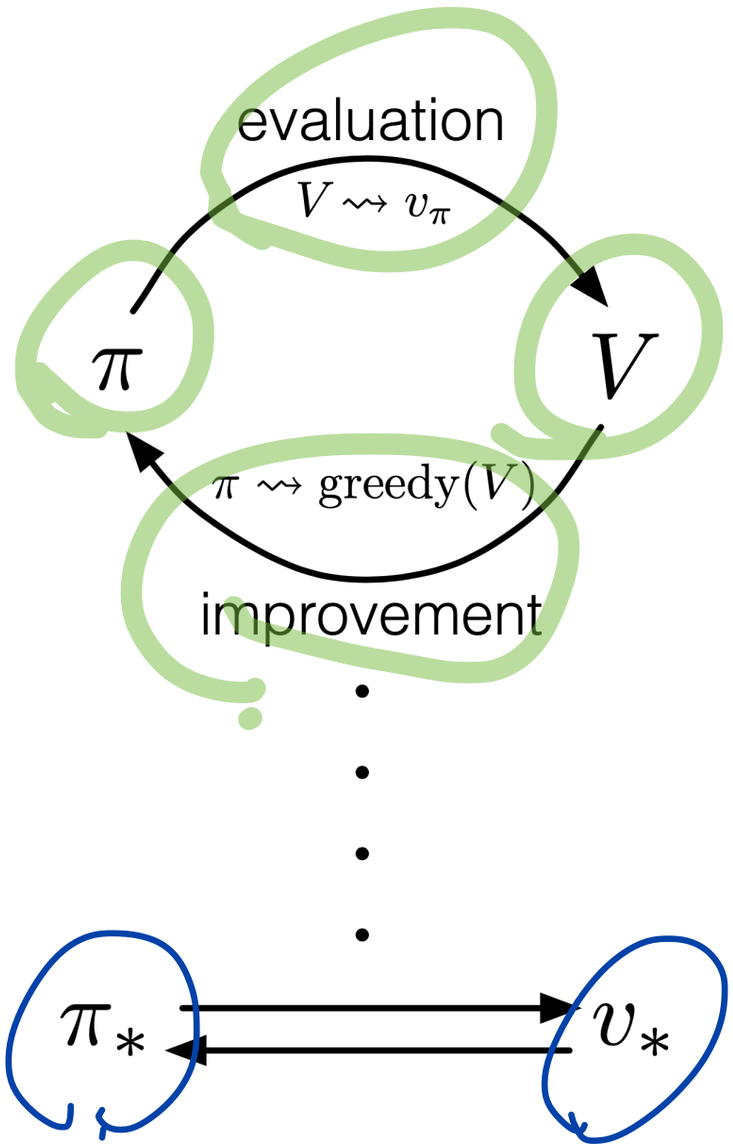
$$\pi_0 \xrightarrow{\text{E}} v_{\pi_0} \xrightarrow{\text{I}} \pi_1 \xrightarrow{\text{E}} v_{\pi_1} \xrightarrow{\text{I}} \pi_2 \xrightarrow{\text{E}} \cdots \xrightarrow{\text{I}} \pi_* \xrightarrow{\text{E}} v_*$$

- Policy evaluation: estimate v_π
  - Iterative policy evaluation
- Policy improvement: generate $\pi' \geq \pi$
  - Greedy policy improvement





Policy Iteration (cont'd)

- Does policy evaluation need to converge to v_π?
- Stop after k iterations of policy evaluation
  - $k = 3$ is sufficient for the gridworld example
- Why not update policy every iteration? i.e., $k = 1$
  - This is *value iteration*

Value Iteration

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- Any optimal policy can be subdivided into two components:
  - An optimal first action A_*
  - Followed by an optimal policy from successor state s'
- Deterministic Value Iteration
  - If we know the solution to subproblems $v_*(s')$
  - Then solution $v_*(s)$ can be found by one-step lookahead

$$v_*(s) \leftarrow \max_{a \in \mathcal{A}} \left(\mathcal{R}_s^a + \gamma \sum_{s' \in \mathcal{S}} \mathcal{P}_{ss'}^a v_*(s') \right)$$

  - Value iteration is applying these updates iteratively
  - Intuition: start with final rewards and work backwards

Value Iteration

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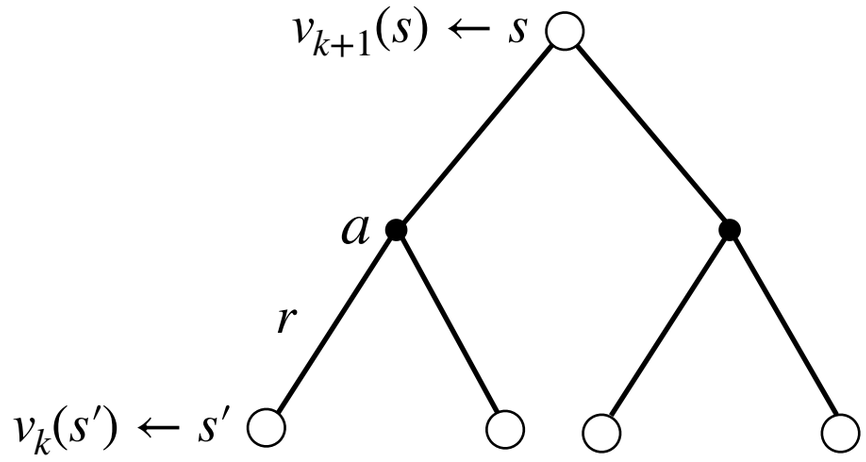
- Problem: find optimal policy π
- Solution: iteration of Bellman optimality backup
- $v_1 \rightarrow v_2 \rightarrow \cdots \rightarrow v_*$
- Each iteration $k+1, \forall s \in \mathcal{S}$, update $v_{k+1}(s)$ from $v_k(s')$
- Convergence to v_*
- Unlike policy iteration, there is no explicit policy

$$v_{k+1}(s) = \max_{a \in \mathcal{A}} \left(\mathcal{R}_s^a + \gamma \sum_{s' \in \mathcal{S}} \mathcal{P}_{ss'}^a v_k(s') \right)$$



Short Path Example

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Problem

0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

V_1

0	-1	-1	-1
-1	-1	-1	-1
-1	-1	-1	-1
-1	-1	-1	-1

V_2

0	-1	-2	-2
-1	-2	-2	-2
-2	-2	-2	-2
-2	-2	-2	-2

V_3

0	-1	-2	-3
-1	-2	-3	-3
-2	-3	-3	-3
-3	-3	-3	-3

V_4

0	-1	-2	-3
-1	-2	-3	-4
-2	-3	-4	-4
-3	-4	-4	-4

V_5

0	-1	-2	-3
-1	-2	-3	-4
-2	-3	-4	-5
-3	-4	-5	-5

V_6

0	-1	-2	-3
-1	-2	-3	-4
-2	-3	-4	-5
-3	-4	-5	-6

V_7

Generalized Policy Iteration

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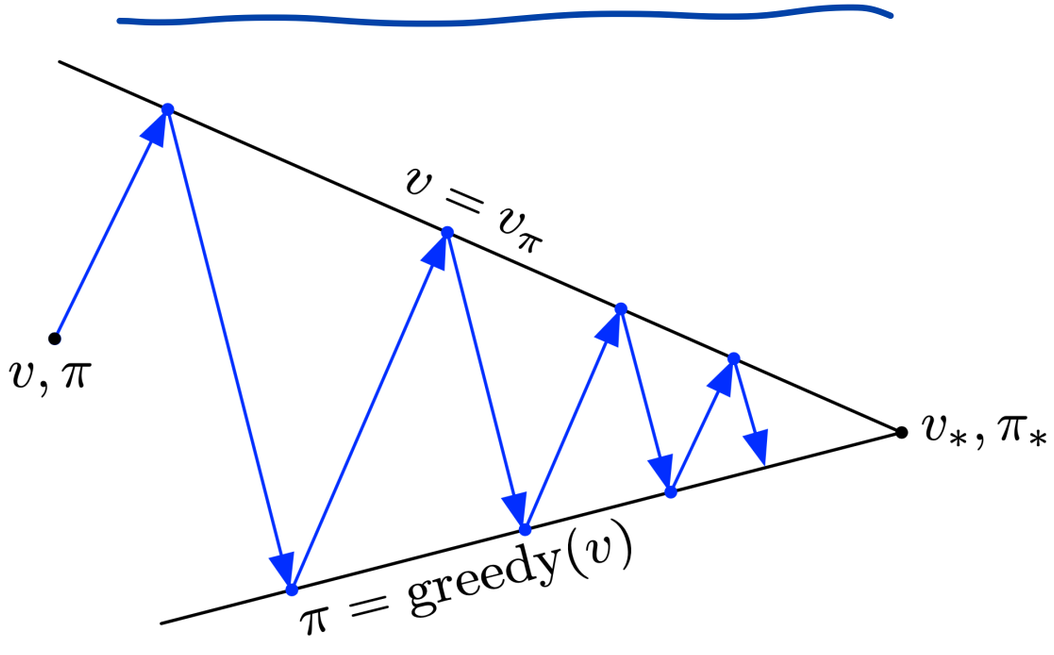
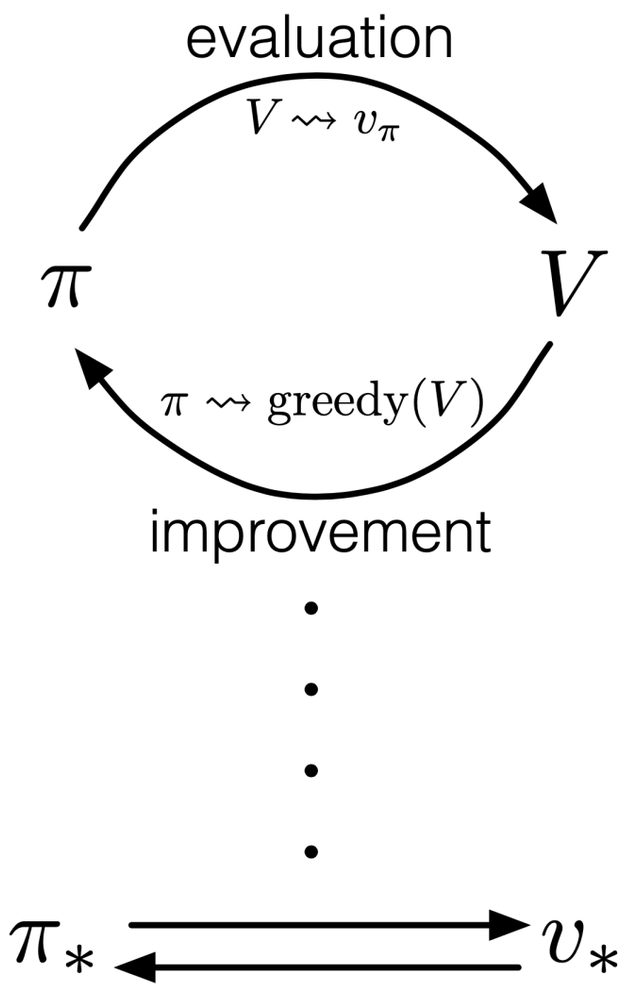
$$\pi_0 \xrightarrow{\text{E}} v_{\pi_0} \xrightarrow{\text{I}} \pi_1 \xrightarrow{\text{E}} v_{\pi_1} \xrightarrow{\text{I}} \pi_2 \xrightarrow{\text{E}} \cdots \xrightarrow{\text{I}} \pi_* \xrightarrow{\text{E}} v_*$$

- Policy evaluation: estimate v_π
  - **Any** policy evaluation algorithm
- Policy improvement: genera $\pi' \geq \pi$
  - **Any** policy improvement algorithm





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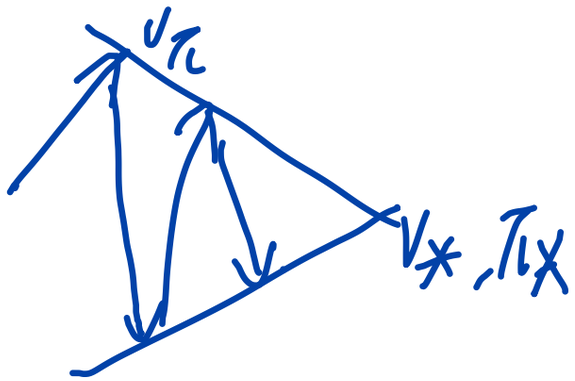
Questions

- Does iterative policy evaluation converges to v_π?
- Does policy iteration converges to v_*?
- Does value iteration converges to v_*?
- Is the solution unique?
- How fast do these algorithms converge?



Yes, can be proved based on contraction mapping theory !

Contraction Mapping

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- Value function space
  - Consider a vector space $\mathcal{V}$ with $|\mathcal{S}|$ dimensions over value functions
  - Each point in this space fully specifies a state-value function v
- Value function ∞-norm

max.-norm

  - The distance between state-value functions u and v is measured by ∞-norm

$$\|u - v\|_\infty = \max_{s \in \mathcal{S}} |u(s) - v(s)|$$

- Define the Bellman expectation backup operator $\mathcal{T}^\pi$

$$\mathcal{T}^\pi(v) = \mathcal{R}^\pi + \gamma \mathcal{P}^\pi v$$

(policy Evaluation 的简写)

- The operator is a γ-contraction, i.e., it makes value functions closer by at least γ

$$\begin{aligned} \left\| \mathcal{T}^\pi(u) - \mathcal{T}^\pi(v) \right\|_\infty &= \left\| \left(\mathcal{R}^\pi + \gamma \mathcal{P}^\pi u\right) - \left(\mathcal{R}^\pi + \gamma \mathcal{P}^\pi v\right) \right\|_\infty \\ &= \left\| \gamma \mathcal{P}^\pi (u - v) \right\|_\infty \\ &\le \left\| \gamma \mathcal{P}^\pi \right\| u - v \|_\infty \Big\|_\infty \\ &\le \gamma \|u - v\|_\infty \end{aligned}$$

Contraction Mapping Theorem

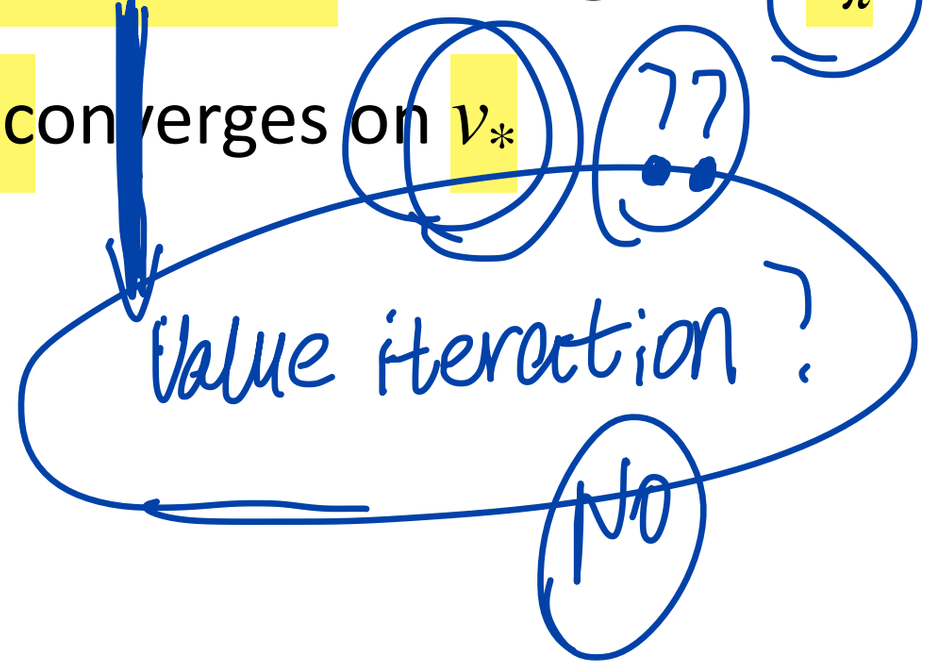
- For any metric space $\mathscr{V}$ that is complete under an operator $\mathscr{T}(v)$, where $\mathscr{T}$ is a γ-contraction
  - $\mathscr{T}$ converges to a unique fixed point
  - At a linear convergence rate of γ

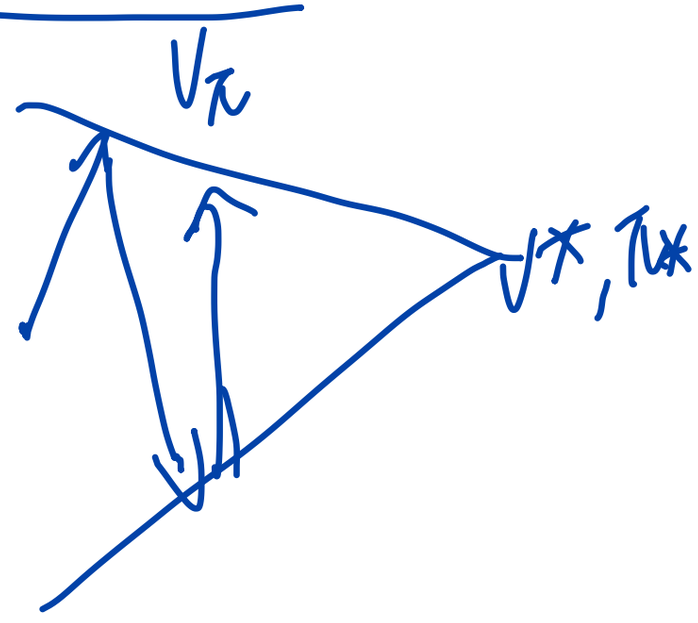
$$\|Tu - Tv\| \leq \gamma \|u - v\|$$

Convergence of Policy Evaluation and Policy Iteration

RLChina 2020

- The Bellman expectation operator $\mathcal{T}^\pi$ has a unique fixed point
- v_π is a fixed point of $\mathcal{T}^\pi$ (by Bellman expectation equation)
- Iterative policy evaluation converges on v_π
- Policy iteration converges on v_*





Convergence of Value Iteration

- Define the Bellman optimality backup operator $\mathcal{T}^*$

$$\mathcal{T}^*(v) = \max_{a \in \mathcal{A}} \mathcal{R}^a + \gamma \mathcal{P}^a v$$

- The operator is a γ-contraction (similar to previous proof)

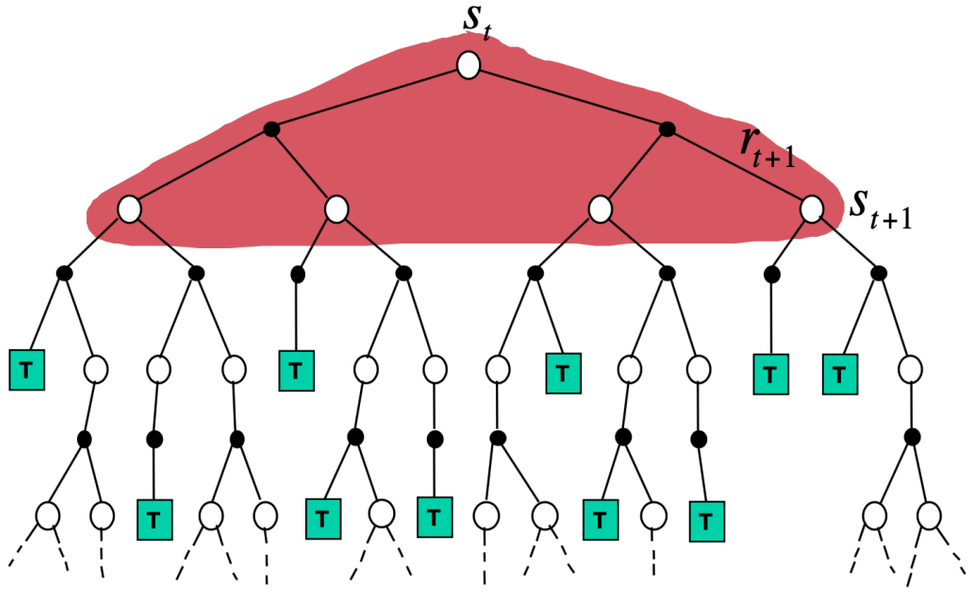
$$\left\| \mathcal{T}^*(u) - \mathcal{T}^*(v) \right\|_\infty \leq \gamma \|u - v\|_\infty$$

- The Bellman optimality operator $\mathcal{T}^*$ has a unique fixed point
- v_* is a fixed point of $\mathcal{T}^*$ (by Bellman optimality equation)
- Value iteration converges on v_*

DP Summary

RLChina 2020

- Algorithms are based on state-value function $v_\pi(s)$ or $v_*(\pi)$
- Complexity $O(mn^2)$ per iteration, for m actions and n states
- DP uses full-width backups, and for each backup
  - Every successor state and action is considered
  - Using knowledge of the MDP transitions and reward function
  - For large problem, **DP suffers from the curse of dimensionality**



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  - Off-policy Learning
  - DQN and its variants

Monte-Carlo Methods

DP需Model.

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- MC methods learn directly from episodes of experience
- MC is model-free: no knowledge of MDP transitions/rewards
- MC learns from complete episodes: no bootstrapping
- MC uses the simplest possible idea: value = mean return
- Can only apply MC to episodic MDPs
  - All episodes must terminate

Monte-Carlo Policy Evaluation

- Goal: learn v_π from episodes of experience under policy π

$$S_1, A_1, R_2, \ldots, S_k \sim \pi$$

- Recall that the return is the total discounted rewards:

$$G_t = R_{t+1} + \gamma R_{t+2} + \ldots + \gamma^{T-1} R_T$$

- Recall that the value function is the expected return from s

$$v_\pi(s) = \mathbb{E}_\pi \left[G_t \,|\, S_t = s \right]$$

- Monte-Carlo policy evaluation uses empirical mean return instead of expected return

Monte-Carlo Policy Evaluation (cont'd)

- To evaluate state s
- The **first/every** timestep t that state s is visited in an episode
- Increment counter $N(s) \leftarrow N(s) + 1$
- Increment total return $S(s) \leftarrow S(s) + G_t$
- Value is estimated by mean return $V(s) \leftarrow S(s)/N(s)$
- By law of large numbers, $V(s) \rightarrow v_\pi(s)$ as $N(s) \rightarrow \infty$

Incremental Monte-Carlo Updates

- Incremental mean

$$\mu_k = \frac{1}{k}\sum_{j=1}^{k} x_j = \frac{1}{k}\left(x_k + (k-1)\mu_{k-1}\right) = \mu_{k-1} + \frac{1}{k}\left(x_k - \mu_{k-1}\right)$$

- Update $V(s)$ incrementally after an episode
- For each state s_t with return G_t

$$N(s_t) \leftarrow N(s_t) + 1$$

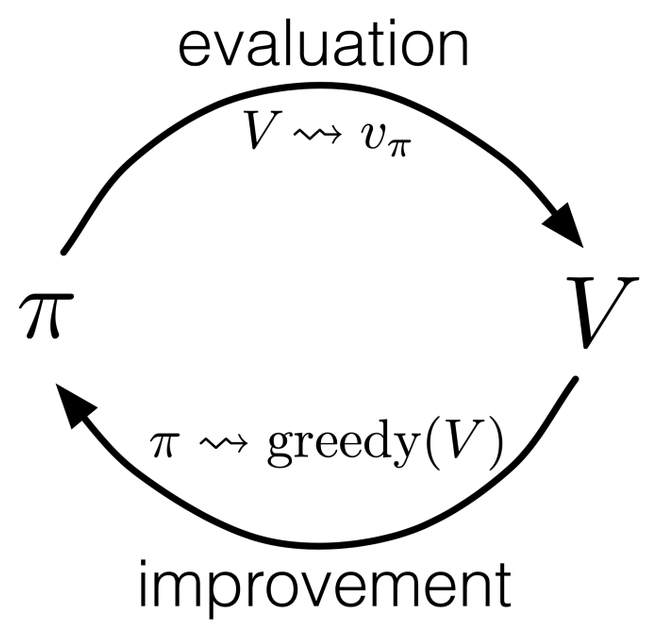
$$V\left(s_t\right) \leftarrow V\left(s_t\right) + \frac{1}{N\left(s_t\right)}\left(G_t - V\left(s_t\right)\right)$$

- In non-stationary problems, it can be used to track a running mean, i.e., forget old episodes

$$V\left(s_t\right) \leftarrow V\left(s_t\right) + \alpha\left(G_t - V\left(s_t\right)\right)$$

Monte-Carlo Policy Iteration

- Generalized Policy Iteration via Monte-Carlo Evaluation



- Policy evaluation: Monte-Carlo policy evaluation, $V = v_\pi$?
- Policy improvement: Greedy policy improvement?

Policy Iteration Using Action-Value Function

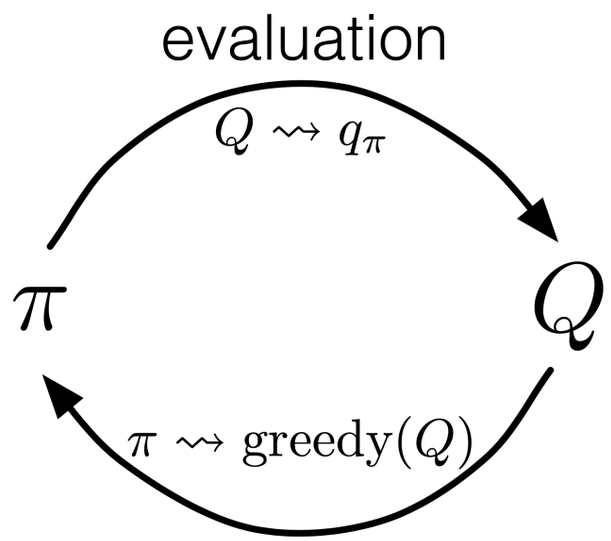
- Greedy policy improvement over $V(s)$ requires model of MDPs

$$\pi'(s) = \arg\max_{a \in \mathcal{A}} \left(\mathcal{R}_s^a + \mathcal{P}_{ss'}^a V(s') \right)$$

- Greedy policy improvement over $Q(s,a)$ is **model-free**

$$\pi'(s) = \arg\max_{a \in \mathcal{A}} Q(s,a)$$

Monte-Carlo Policy Iteration



- Policy evaluation: Monte-Carlo policy evaluation, $Q = q_\pi$
- Policy improvement: Greedy policy improvement?

ϵ-greedy Exploration

- Simplest idea for ensuring continual exploration
- All $|\mathcal{A}|$ actions are tried with non-zero probability
- With probability $1-\epsilon$ choose the greedy action
- With probability ϵ choose an random action

$$\pi(a \mid s) = \begin{cases} \epsilon/m + 1 - \epsilon & \text{if } a^* = \arg\max_{a \in \mathcal{A}} Q(s,a) \\ \epsilon/m & \text{otherwise} \end{cases}$$

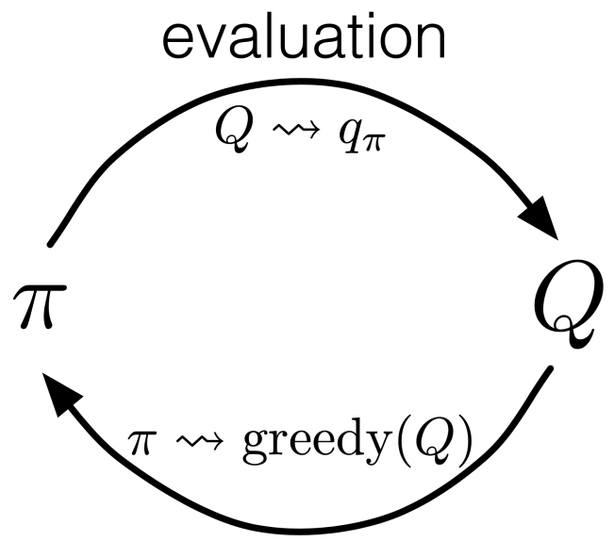
ϵ-greedy Policy Improvement

- For any ϵ-greedy policy π, the ϵ-greedy policy π' with respect to q_π is an improvement, $v_{\pi'} \geq v_\pi$

$$
\begin{aligned}
q_\pi\left(s, \pi'(s)\right) &= \sum_{a \in \mathcal{A}} \pi'(a \mid s) q_\pi(s, a) \\
&= \epsilon / m \sum_{a \in \mathcal{A}} q_\pi(s, a) + (1-\epsilon) \max_{a \in \mathcal{A}} q_\pi(s, a) \\
&\geq \epsilon / m \sum_{a \in \mathcal{A}} q_\pi(s, a) + (1-\epsilon) \sum_{a \in \mathcal{A}} \frac{\pi(a \mid s) - \epsilon / m}{1-\epsilon} q_\pi(s, a) \\
&= \sum_{a \in \mathcal{A}} \pi(a \mid s) q_\pi(s, a) \\
&= v_\pi(s)
\end{aligned}
$$

- From policy improvement theorem, $v_{\pi'} \geq v_\pi$

Monte-Carlo Control



- Every episode:
  - Policy evaluation: Monte-Carlo policy evaluation, $Q = q_\pi$
  - Policy improvement: ϵ-greedy policy improvement

Greedy in the limit with Infinite Exploration

- GLIE
  - All state-action pairs are explored infinitely often

$$\lim_{k\to\infty} N_k(s,a) = \infty$$

  - The policy converges on a greedy policy

$$\lim_{k\to\infty} \pi_k(a\,|\,s) = \mathbf{1}(a = \arg\max_{a'\in\mathcal{A}} Q_k(s,a'))$$

- ϵ-greedy is GLIE if ϵ reduces to zero at $\epsilon_k = \frac{1}{k}$

GLIE Monte-Carlo Control

- Sample kth episode using π: $\{S_1, A_1, R_2, \ldots, S_T\} \sim \pi$
- For each state-action pair in the episode

$$N(S_t, A_t) \leftarrow N(S_t, A_t) + 1$$

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \frac{1}{N(S_t, A_t)}\left(G_t - Q(S_t, A_t)\right)$$

- Improve the policy based on new action-value function

$$\epsilon \leftarrow 1/k$$

$$\pi \leftarrow \epsilon - \text{greedy}(Q)$$

- GLIE Monte-Carlo control converges to the optimal action-value function, $Q(s, a) \rightarrow q_*(s, a)$

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Temporal-Difference Learning

- TD methods learn directly from episodes of experience
- TD is model-free: no knowledge of MDP transitions/rewards
- TD learns from incomplete episodes: by bootstrapping
- TD updates a guess towards a guess

MC and TD Policy Evaluation

- Goal: learn v_π online from experience under policy π
- Incremental every-visit MC
  - $V(S_t)$ is updated toward actual return G_t

$$V(S_t) \leftarrow V(S_t) + \alpha(G_t - V(S_t))$$

- Simplest TD learning
  - $V(S_t)$ is updated toward estimated return $R_{t+1} + \gamma V(S_{t+1})$

$$V(S_t) \leftarrow V(S_t) + \alpha(R_{t+1} + \gamma V(S_{t+1}) - V(S_t))$$

  - $R_{t+1} + \gamma V(S_{t+1})$ is called the *TD target*
  - $\delta_t = R_{t+1} + \gamma V(S_{t+1}) - V(S_t)$ is called the *TD error*

Advantages and Disadvantages of MC and TD

- TD can learn *before* knowing the final outcome
  - TD can learn online after every step
  - MC must wait until end of episode after return is known
- TD can learn *without* the final outcome
  - TD works in continuing (non-terminating) environments
  - MC only works for episodic (terminating) environments

Bias/Variance Tradeoff

- Return $G_t = R_{t+1} + \gamma R_{t+2} + \ldots, \gamma^{T-1} R_T$ is *unbiased* estimate of $v_\pi(S_t)$
- True TD target $R_{t+1} + \gamma v_\pi(S_{t+1})$ is *unbiased* estimate of $v_\pi(S_t)$
- TD target $R_{t+1} + \gamma V(S_{t+1})$ is *biased* estimate of $v_\pi(S_t)$
- TD target is much lower variance than the return
  - Return depends on **many** actions, transitions, rewards
  - TD target depends on **only one** action, transition, reward

TD Control: SARSA

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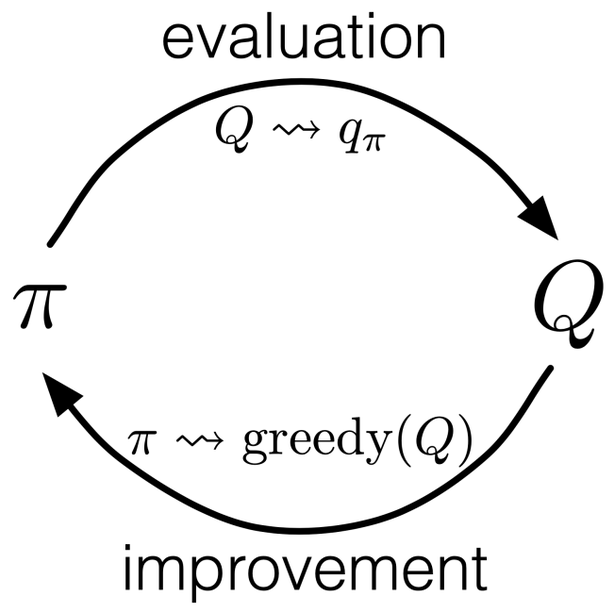
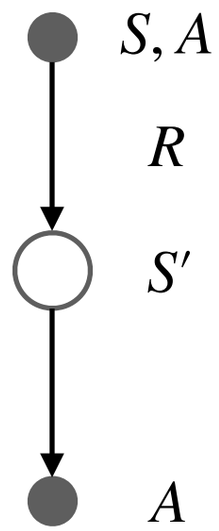
- Every timestep:
  - Policy evaluation: $Q \approx q_\pi$
  - Policy improvement: ϵ-greedy policy improvement



- $Q(S,A) \leftarrow Q(S,A) + \alpha\left(R + \gamma Q(S',A') - Q(S,A)\right)$



SARSA Algorithm

Sarsa (on-policy TD control) for estimating $Q \approx q_*$

Algorithm parameters: step size $\alpha \in (0, 1]$, small $\varepsilon > 0$
Initialize $Q(s, a)$, for all $s \in \mathcal{S}^+, a \in \mathcal{A}(s)$, arbitrarily except that $Q(\mathit{terminal}, \cdot) = 0$

Loop for each episode:
  Initialize S
  Choose A from S using policy derived from Q (e.g., ε-greedy)
  Loop for each step of episode:
    Take action A, observe R, S'
    Choose A' from S' using policy derived from Q (e.g., ε-greedy)
    $Q(S, A) \leftarrow Q(S, A) + \alpha\big[R + \gamma Q(S', A') - Q(S, A)\big]$
    $S \leftarrow S'$; $A \leftarrow A'$;
  until S is terminal

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Off-Policy Learning

❖ Off-policy learning

- Evaluate *target policy* $\pi(a \mid s)$ to compute $v_\pi(s)$ or $q_\pi(s, a)$, while following *behavior policy* $\mu(a \mid s)$

$$\{S_1, A_1, R_2, \ldots, S_T\} \sim \mu$$

❖ Why is this important?

- Learn from observing humans or other agents
- Reuse experience generated from old policies $\pi_1, \pi_2, \ldots, \pi_{t-1}$
- Learn about *optimal policy* while following *exploratory policy*
- Learn about *multiple policies* while following *one policy*

Importance Sampling

- Estimate the expectation of a difference distribution

$$\mathbb{E}_{X\sim P}[f(X)] = \sum P(X)f(X)$$
$$= \sum Q(X)\frac{P(X)}{Q(X)}f(X)$$
$$= \mathbb{E}_{X\sim Q}\left[\frac{P(X)}{Q(X)}f(X)\right]$$

Importance Sampling for Off-Policy MC

- Use returns generated from μ to evaluate π
- Weight return G_t according to similarity between policies
- Multiply importance sampling corrections along whole episode

$$G_t^{\pi/\mu} = \frac{\pi(A_t \mid S_t)}{\mu(A_t \mid S_t)} \frac{\pi(A_{t+1} S_{t+1})}{\mu(A_{t+1} \mid S_{t+1})} \cdots \frac{\pi(A_T \mid S_T)}{\mu(A_T \mid S_T)} G_t$$

- Update values towards corrected return

$$V(S_t) \leftarrow V(S_t) + \alpha \left(G_t^{\pi/\mu} - V(S_t) \right)$$

- Importance sampling can dramatically increase variance

Importance Sampling for Off-Policy TD

- Use TD targets generated from μ to evaluate π
- Weighted TD target $R + \gamma V(S')$ by importance sampling
- Only need a single importance correction

$$V(S_t) \leftarrow V(S_t) + \alpha \left(\frac{\pi(A_t \mid S_t)}{\mu(A_t \mid S_t)} \left(R_{t+1} + \gamma V(S_{t+1}) \right) - V(S_t) \right)$$

- Much lower variance than Monte-Carlo importance sampling

Q Learning

- Now considering off-policy learning of action-values $Q(s, a)$
- **No** importance sampling is required
- Action is chosen using behavior policy $A_t \sim \mu(\cdot \mid S_t)$
- But we consider alternative successor action $A' \sim \pi(\cdot \mid S_{t+1})$
- Update $Q(S_t, A_t)$ towards value of alternative action

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(R_{t+1} + \gamma Q(S_{t+1}, A') - Q(S_t, A_t))$$

Off-Policy Control with Q-Learning

- The target policy π is greedy w.r.t. $Q(s,a)$

$$\pi(S_{t+1}) = \arg\max_{a'} Q(S_{t+1}, a')$$

- The behavior policy μ is e.g. ϵ-greedy w.r.t $Q(s,a)$
- The Q-learning target then simplifies:

$$R_{t+1} + \gamma Q\left(S_{t+1}, A'\right)$$

$$= R_{t+1} + \gamma Q\left(S_{t+1}, \operatorname*{argmax}_{a'} Q\left(S_{t+1}, a'\right)\right)$$

$$= R_{t+1} + \max_{a'} \gamma Q\left(S_{t+1}, a'\right)$$

Off-Policy Control with Q-Learning (cont'd)

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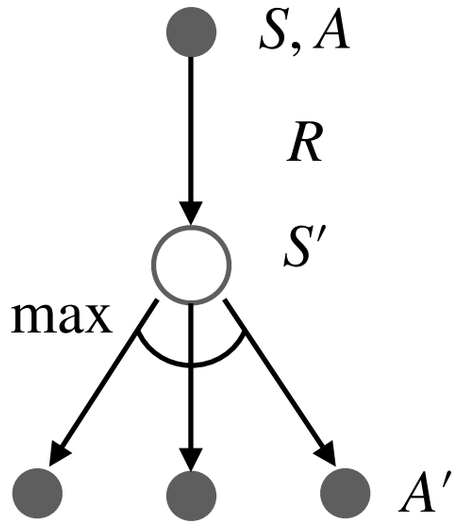

- Q-learning converges to the optimal action-value function

$$Q(s,a) \rightarrow q_*(s,a)$$

Off-Policy Control with Q-Learning (cont'd)

Q-learning (off-policy TD control) for estimating $\pi \approx \pi_*$

Algorithm parameters: step size $\alpha \in (0, 1]$, small $\varepsilon > 0$
Initialize $Q(s, a)$, for all $s \in \mathcal{S}^+, a \in \mathcal{A}(s)$, arbitrarily except that $Q(terminal, \cdot) = 0$

Loop for each episode:
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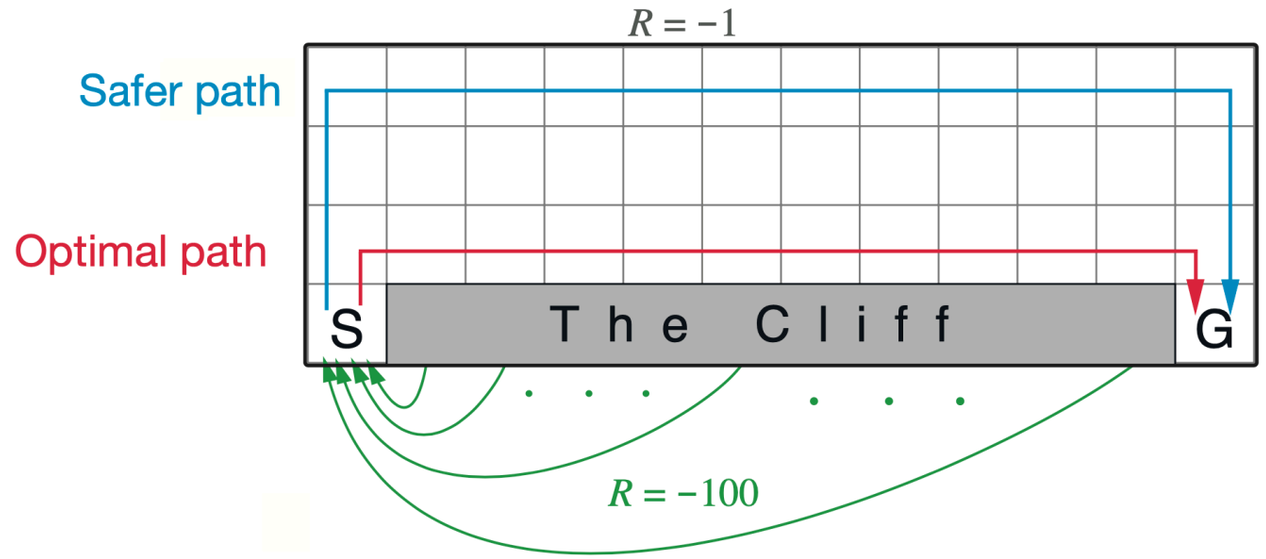
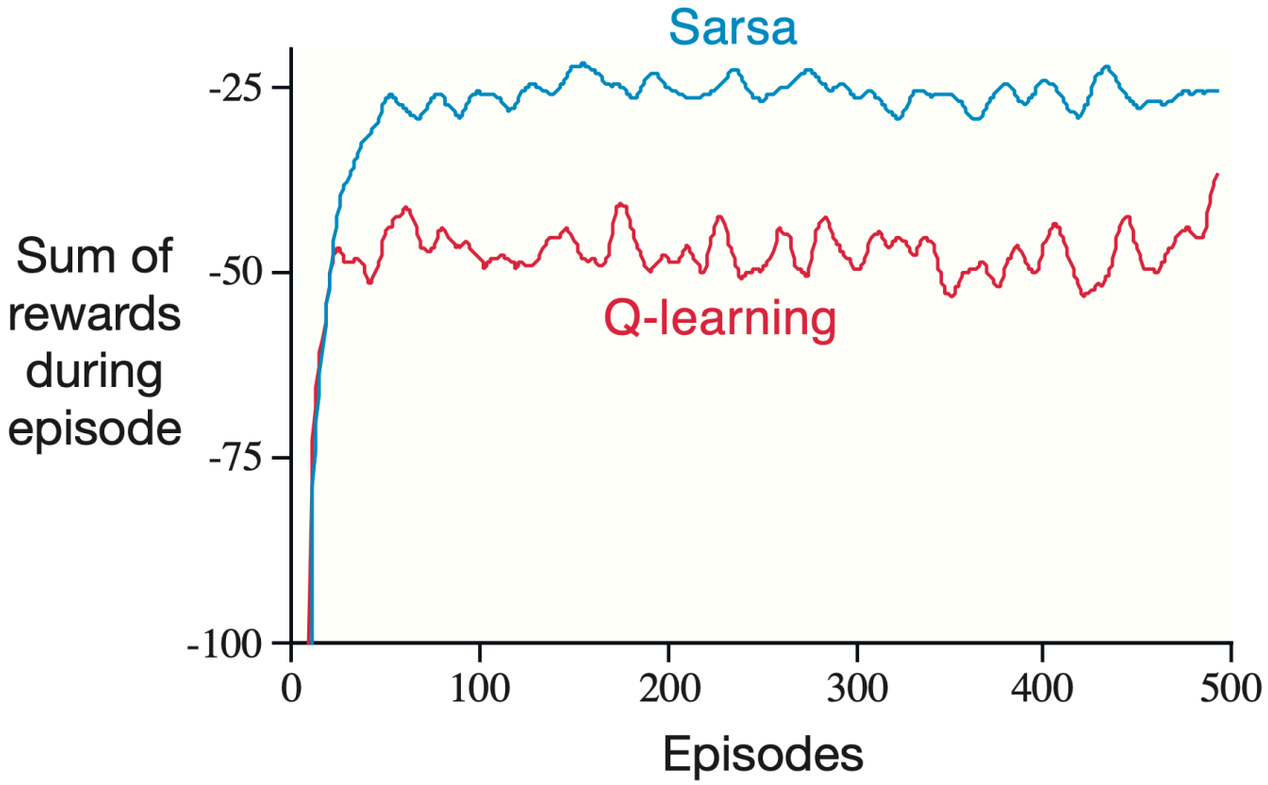
    Take action A, observe R, S'
    $Q(S, A) \leftarrow Q(S, A) + \alpha\big[R + \gamma \max_a Q(S', a) - Q(S, A)\big]$
    $S \leftarrow S'$
  until S is terminal

Cliff Walking Example





Summary on DP, MC, and TD

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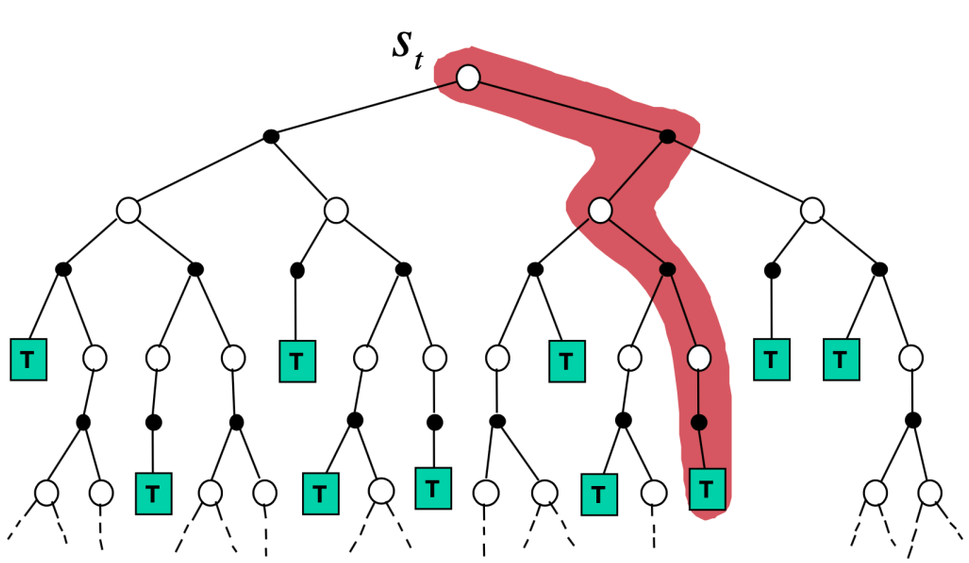
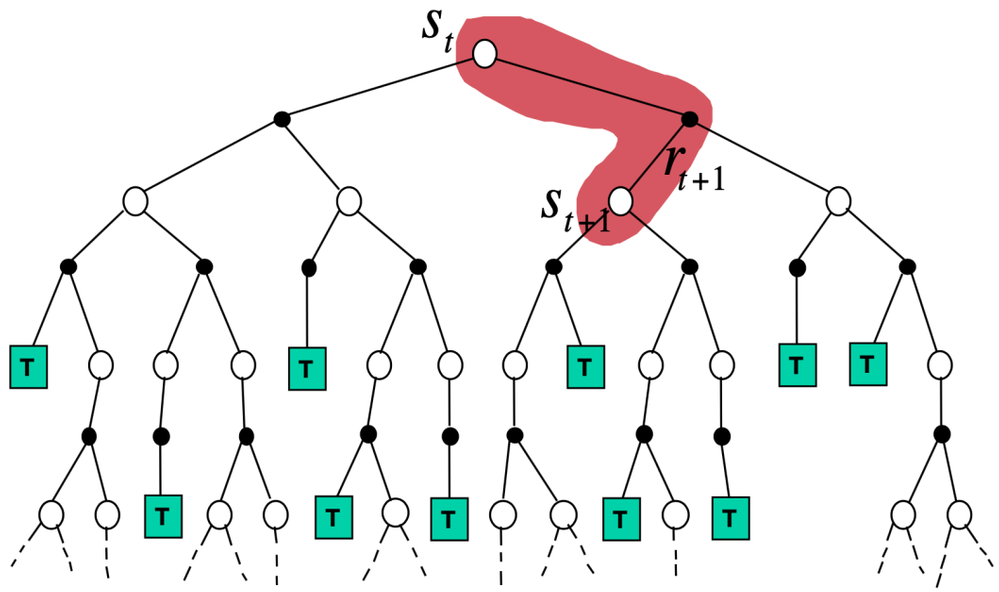
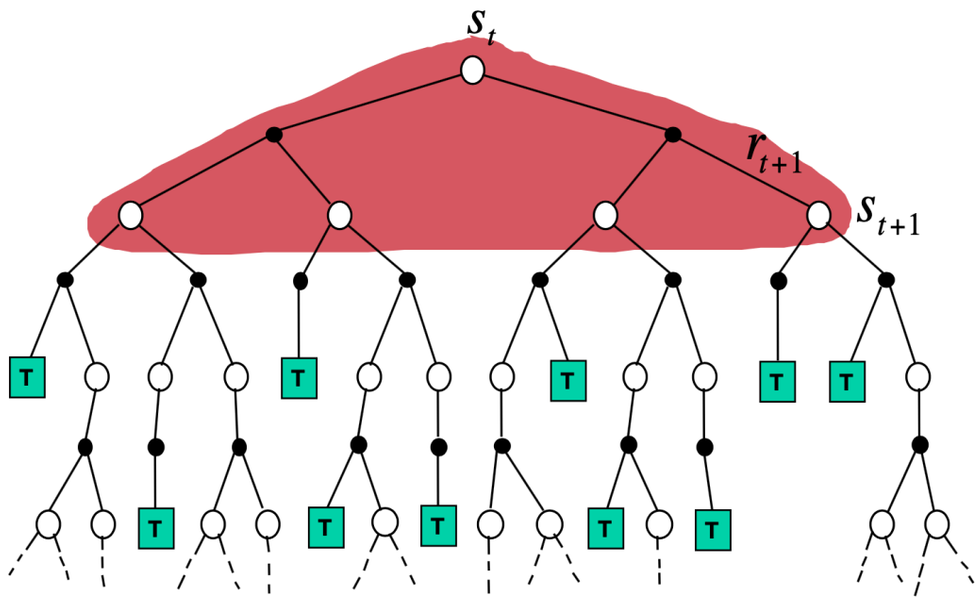


MC backup



TD backup



DP backup

Unified View of Reinforcement Learning

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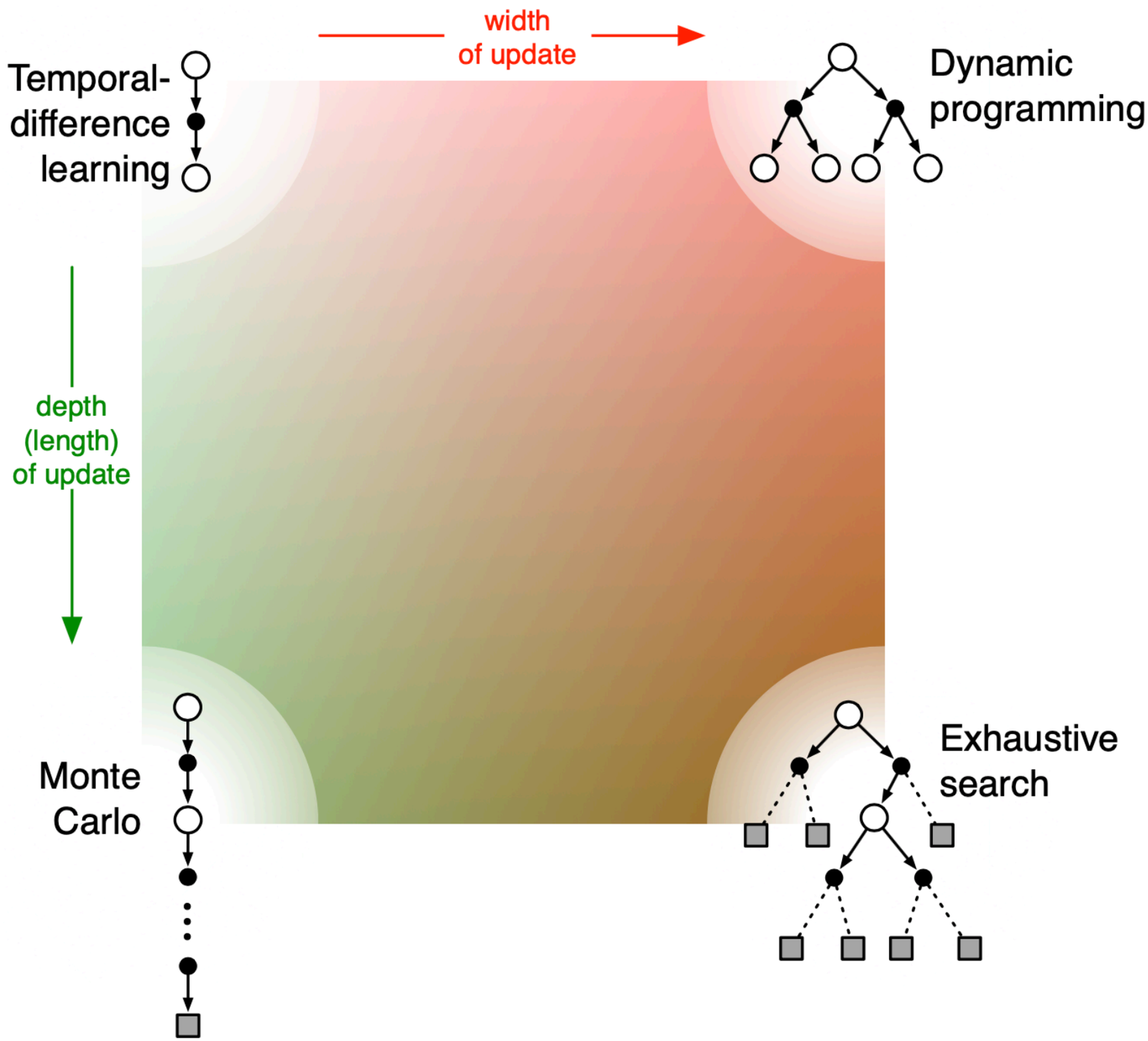

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Reinforcement Learning in Practical

- Reinforcement learning can solve large problems, e.g.,
  - Backgammon: 10^{20} states
  - Computer Go: 10^{170} states
  - Robots: continuous state space
- How can we scale up the model-free methods?

Value Function Approximation

- We have represented value function by a lookup table
  - Every state s has an entry $V(s)$
  - Every state-action pair s, a has an entry $Q(s, a)$
- Problem with large MDPs
  - There are too many states and actions to store in memory
  - It is too slow to learn the value of each state individually
- Solution for large MDPs
  - Estimate value function with function approximation

$$\hat{v}(s, \boldsymbol{w}) \approx v_\pi(s)$$

$$\hat{q}(s, a, \boldsymbol{w}) \approx q_\pi(s, a)$$

  - Generalize from seen states to unseen states
  - Update parameter $\boldsymbol{w}$ using MC or TD learning

Types of Value Function Approximation

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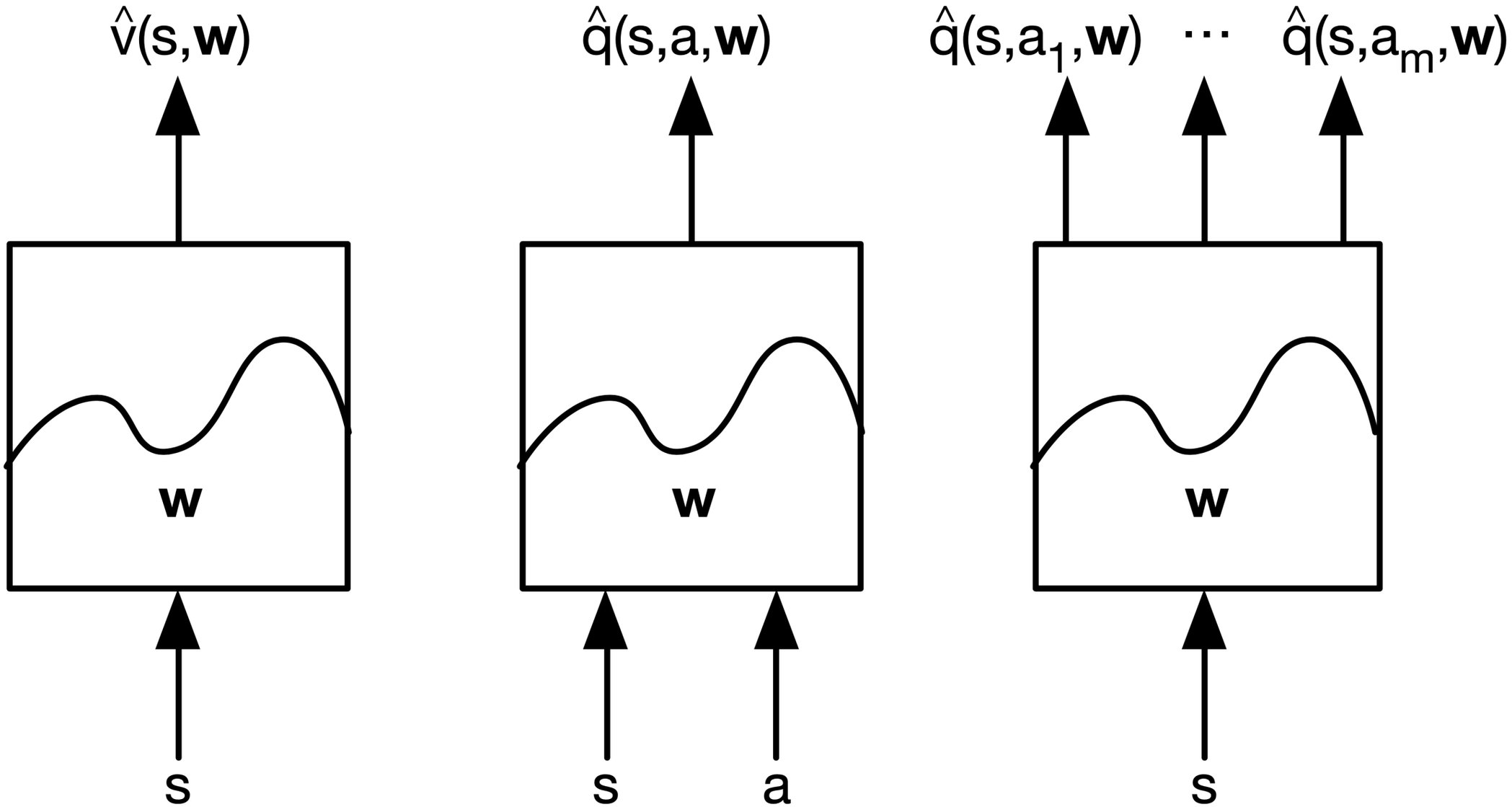

Control with Value Function Approximation

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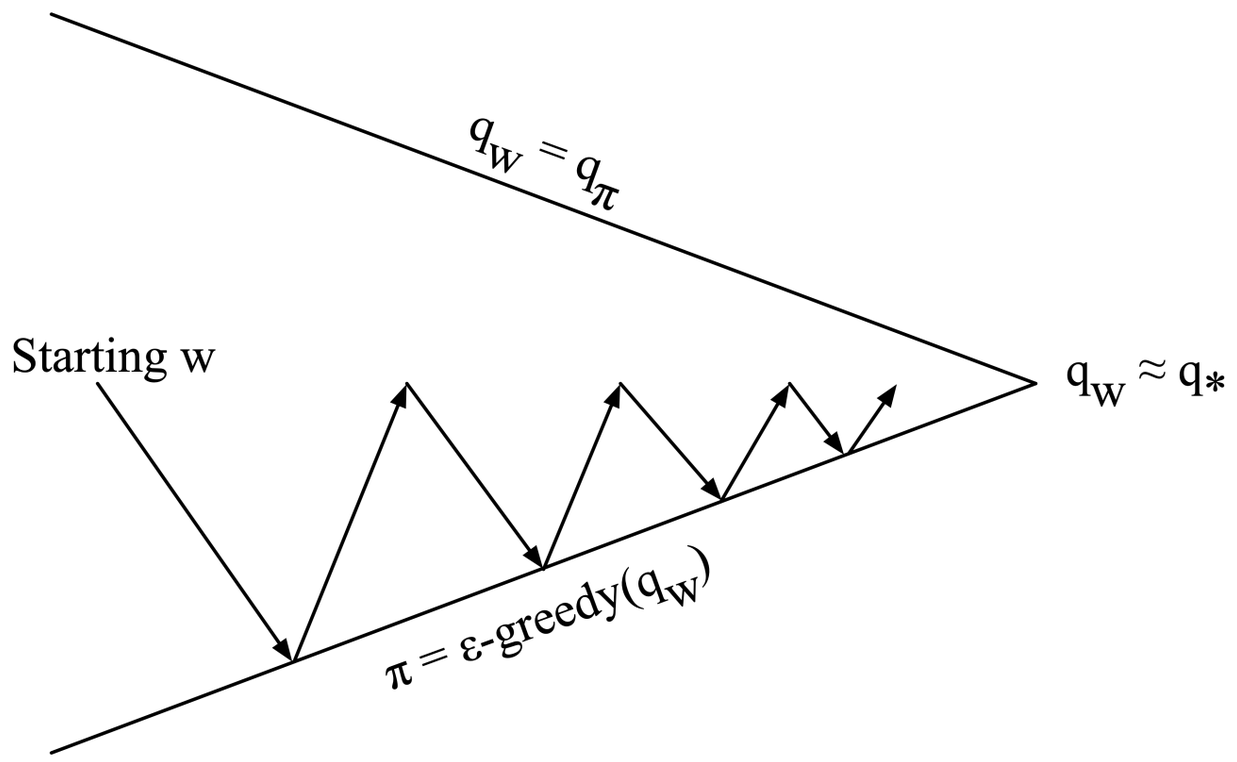
- Policy evaluation: *approximate* policy evaluation, $\hat{q}(\cdot, \cdot, \mathbf{w}) \approx q_\pi$
- Policy improvement: ϵ-greedy policy improvement



Action-Value Function Approximation

- Approximate the action-value function

$$\hat{q}(S, A, \mathbf{w}) \approx q_\pi(S, A)$$

- Minimize mean-squared error between approximate action-value function $\hat{q}(S, A, \mathbf{w})$ and true function $q_\pi(S, A)$

$$J(\mathbf{w}) = \mathbb{E}_\pi[(q_\pi(S, A) - \hat{q}(S, A, \mathbf{w}))^2]$$

- Using stochastic gradient descent to find a local minimum

$$\Delta\mathbf{w} = \alpha(q_\pi(S, A) - \hat{q}(S, A, \mathbf{w}))\nabla_{\mathbf{w}}\hat{q}(S, A, \mathbf{w})$$

Action-Value Function Approximation (cont'd)

- We do not know $q_\pi(S, A)$
- We substitute a target for $q_\pi(S, A)$
  - For MC, the target is the return G_t

$$\Delta \mathbf{w} = \alpha(G_t - \hat{q}(S_t, A_t, \mathbf{w}))\nabla_{\mathbf{w}}\hat{q}(S_t, A_t, \mathbf{w})$$

  - For TD, the target is the TD target $R_{t+1} + \gamma\hat{q}(S_{t+1}, A_{t+1}\mathbf{w})$

$$\Delta \mathbf{w} = \alpha(R_{t+1} + \gamma\hat{q}(S_{t+1}, A_{t+1}, \mathbf{w}) - \hat{q}(S_t, A_t, \mathbf{w}))\nabla_{\mathbf{w}}\hat{q}(S_t, A_t, \mathbf{w})$$

Online Q Learning

1. Take action a according to ϵ-greedy policy and observe (s, a, r, s')
2. $\Delta \mathbf{w} = \alpha(r + \gamma \max_{a'} \hat{q}(s', a', \mathbf{w}) - \hat{q}(s, a, \mathbf{w})) \nabla_{\mathbf{w}} \hat{q}(s, a, \mathbf{w})$

- Sequential states are strongly correlated!

- Target Value is always changing!

Deep Q Network

- DQN uses experience replay and target network
  - Take ϵ-greedy action w.r.t. $Q(s, a; \mathbf{w})$
  - Store transition $(s_t, a_t, r_{t+1}, s_{t+1})$ in replay memory $\mathscr{D}$
  - Sample random mini-batch of transitions (s, a, r, s') from $\mathscr{D}$
  - Optimize MSE between Q-network and Q-learning targets

$$\mathscr{L}(\mathbf{w}) = \mathbb{E}_{s,a,r,s' \sim \mathscr{D}} \left[\left(r + \gamma \max_{a'} Q(s', a'; \mathbf{w}^-) - Q(s, a; \mathbf{w}) \right)^2 \right]$$

  - Using variant of SGD
  - Every N timesteps, $\mathbf{w}^- = \mathbf{w}$

Mnih et al., Human-level control through deep reinforcement learning. Nature, 2015, 518(7540): 529-533.

DQN in Atari

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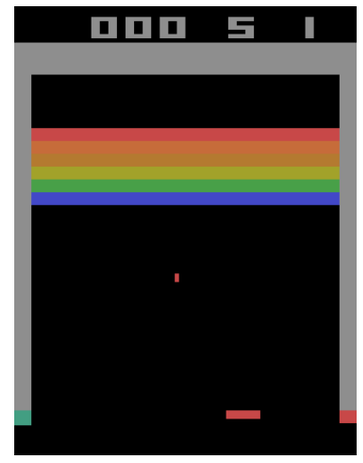
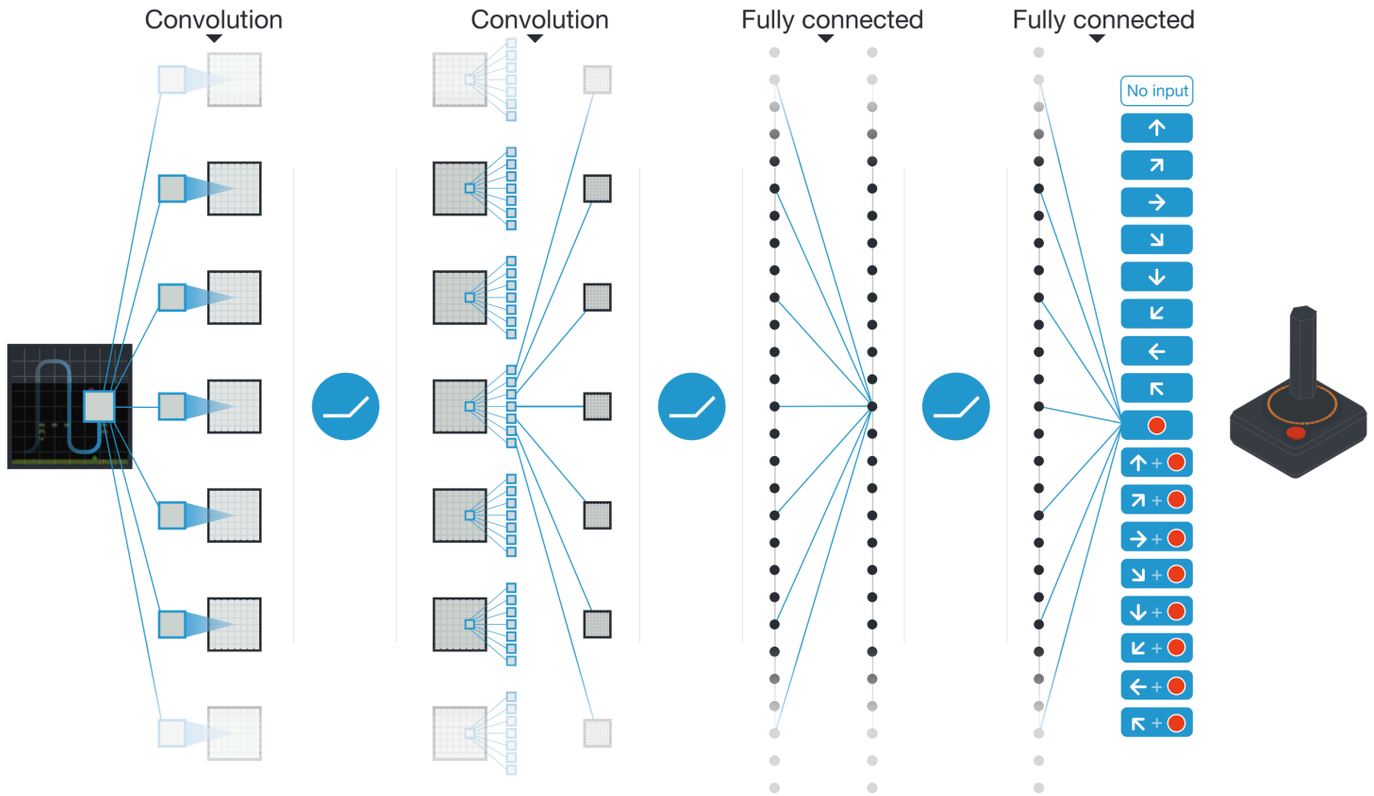
- End-to-end learning of $Q(s, a)$ from pixels s
- Input state s is stack of raw pixels from last 4 frames
- Output is $Q(s, a)$ for 18 joystick/button positions
- Reward is change in score for that step





Mnih et al., Human-level control through deep reinforcement learning. Nature, 2015, 518(7540): 529-533.

DQN Results in Atari

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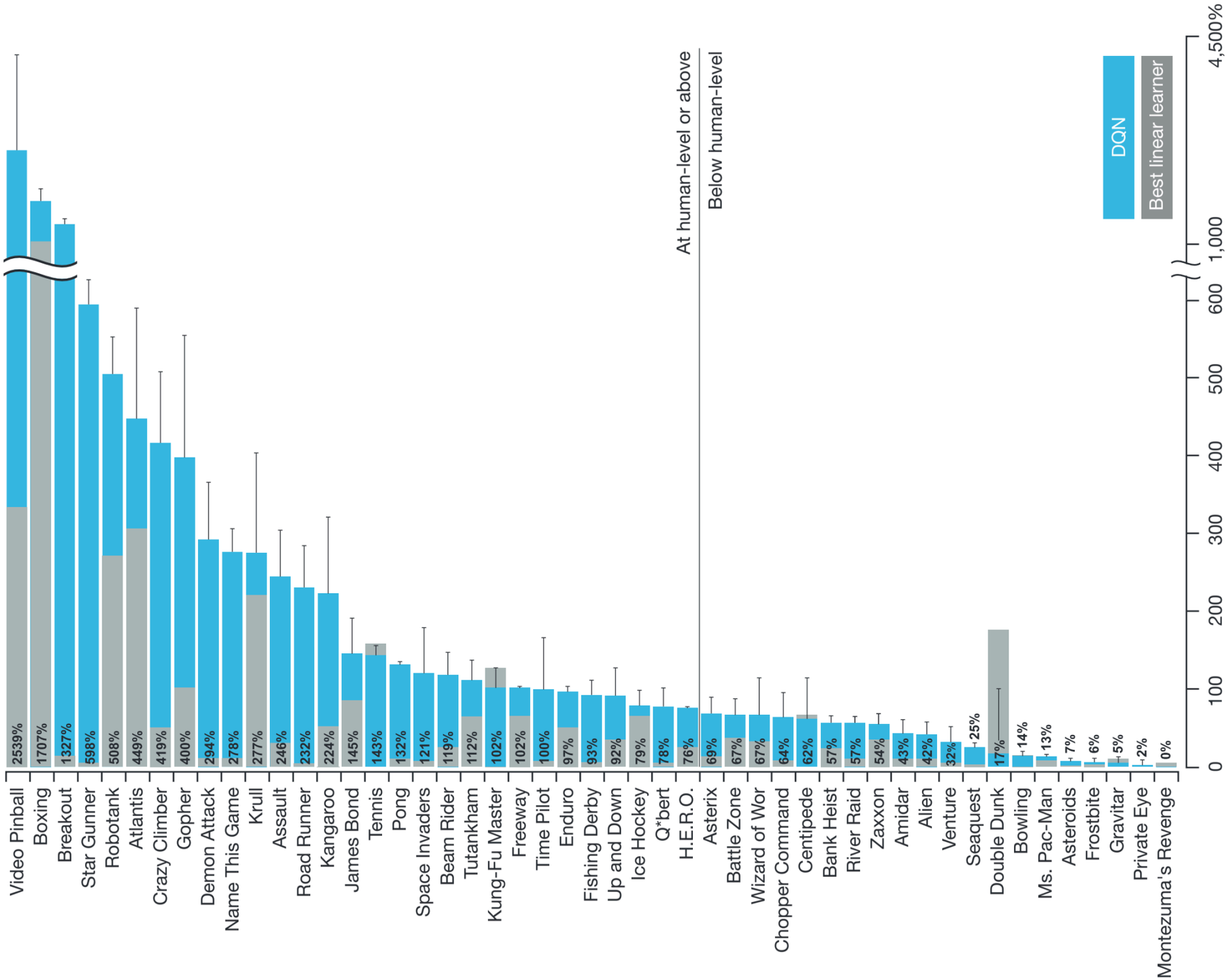

Mnih et al., Human-level control through deep reinforcement learning. Nature, 2015, 518(7540): 529-533.

How much does DQN help?

	Replay Fixed-Q	Replay Q-learning	No replay Fixed-Q	No replay Q-learning
Breakout	316.81	240.73	10.16	3.17
Enduro	1006.3	831.25	141.89	29.1
River Raid	7446.62	4102.81	2867.66	1453.02
Seaquest	2894.4	822.55	1003	275.81
Space Invaders	1088.94	826.33	373.22	301.99

How accurate are the Q-values?

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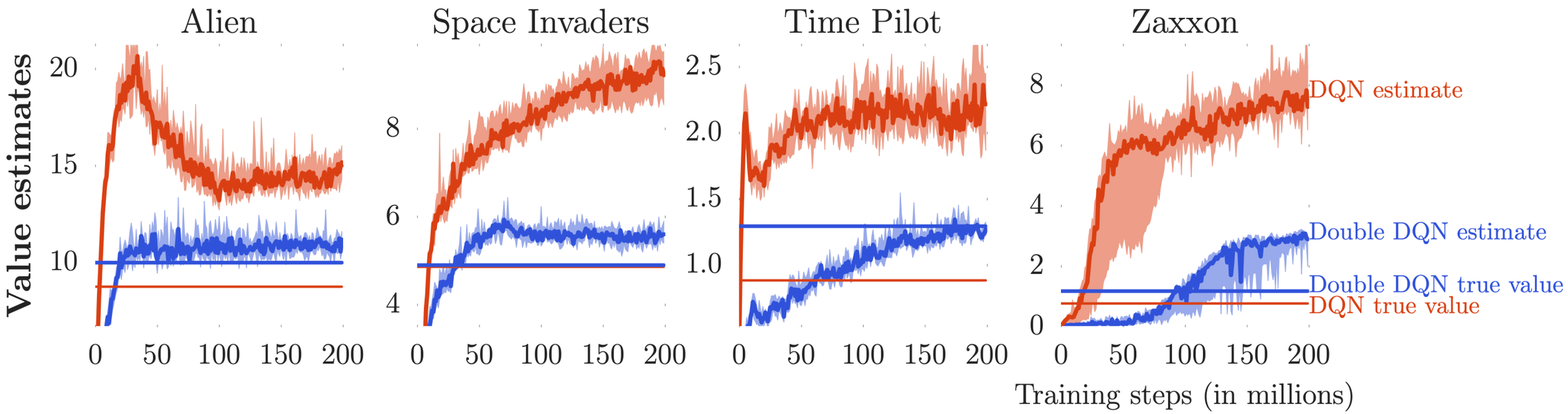

- Overestimation
  - Target value: $r + \gamma \max_{a'} Q(s', a'; \mathbf{w}^-)$

This is the problem!

- $\mathbb{E}[\max(X_1, X_2)] \geq \max\left(\mathbb{E}[X_1], \mathbb{E}[X_2]\right)$
- For bootstrapping (learning estimates from estimates), such overestimation can be problematic.

Van et al., Deep reinforcement learning with double q-learning, AAAI 2016.

Double DQN

- $\max_{a'} Q(s', a'; \mathbf{w}^-) = Q(s', \arg\max_{a'} Q(s', a'; \mathbf{w}^-); \mathbf{w}^-)$
  - Action selected according to $Q(\cdot, \cdot; \mathbf{w}^-)$
  - Value also from $Q(\cdot, \cdot; \mathbf{w}^-)$
- Use different networks to choose action and evaluate value
  - Current network to evaluate action
  - Target network to evaluate value

$$r + \gamma Q(s', \arg\max_{a'} Q(s', a'; \mathbf{w}); \mathbf{w}^-)$$

Van et al., Deep reinforcement learning with double q-learning, AAAI 2016.

Double DQN

- DQN uses experience replay and target networks
  - Take action a_t according to ϵ-greedy policy
  - Store transition $(s_t, a_t, r_{t+1}, s_{t+1})$ in replay memory $\mathscr{D}$
  - Sample random mini-batch of transitions (s, a, r, s') from $\mathscr{D}$
  - Optimize MSE between Q-network and Q-learning targets

$$\mathscr{L}(\mathbf{w}) = \mathbb{E}_{s,a,r,s' \sim \mathscr{D}} \left[\left(r + \gamma Q(s', \arg\max_{a'} Q(s', a'; \mathbf{w}); \mathbf{w}^-) - Q(s, a; \mathbf{w}) \right)^2 \right]$$

  - Using variant of SGD
  - $\mathbf{w}^- = (1 - \tau)\mathbf{w} + \tau \mathbf{w}^-$

Van et al., Deep reinforcement learning with double q-learning, AAAI 2016.

Dueling DQN

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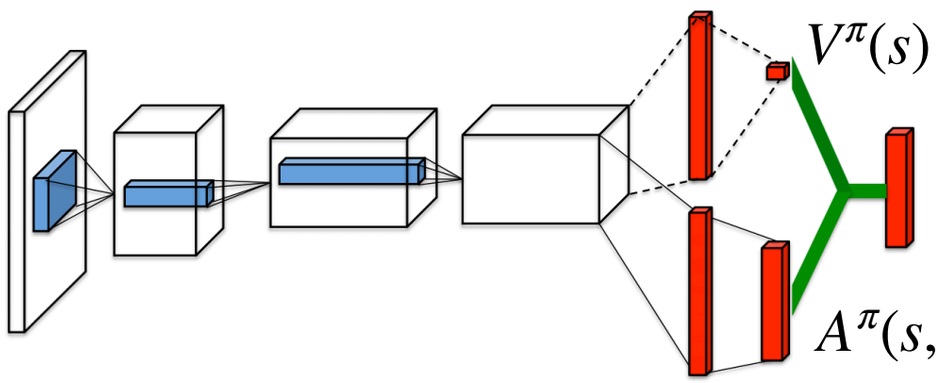
- **Motivation**: it is unnecessary to know the exact value of each action at every timestep

- Dueling DQN learns state value without having to learn the effect of each action for each state



- $Q(s,a;\theta,\alpha,\beta) = V(s;\theta,\beta) + A(s,a;\theta,\alpha)$ "unidentifiable"
- $Q(s,a;\theta,\alpha,\beta) = V(s;\theta,\beta) + (A(s,a;\theta,\alpha) - \max_{a' \in \mathcal{A}} A(s,a';\theta,\alpha))$
- $Q(s,a;\theta,\alpha,\beta) = V(s;\theta,\beta) + (A(s,a;\theta,\alpha) - \frac{1}{|\mathcal{A}|} \sum_{a' \in |\mathcal{A}|} A(s,a';\theta,\alpha))$

Wang et l., Dueling network architectures for deep reinforcement learning, ICML 2016

n-Step DQN

- $R_{t+1} + \gamma R_{t+2} + \cdots + \gamma^{n-1} R_{t+n} + \gamma^n \max_a Q(S_{t+n}, a; \theta)$
  - Less biased target values when Q values are inaccurate
  - Faster learning when early on
  - But, only actually correct when learning on-policy
- How to deal with it?
  - Ignore the problem

    Often works in practice
  - Dynamically choose N to get only on-policy data

    Work well when data is on-policy and action space is small
  - Importance sampling

Distributional DQN

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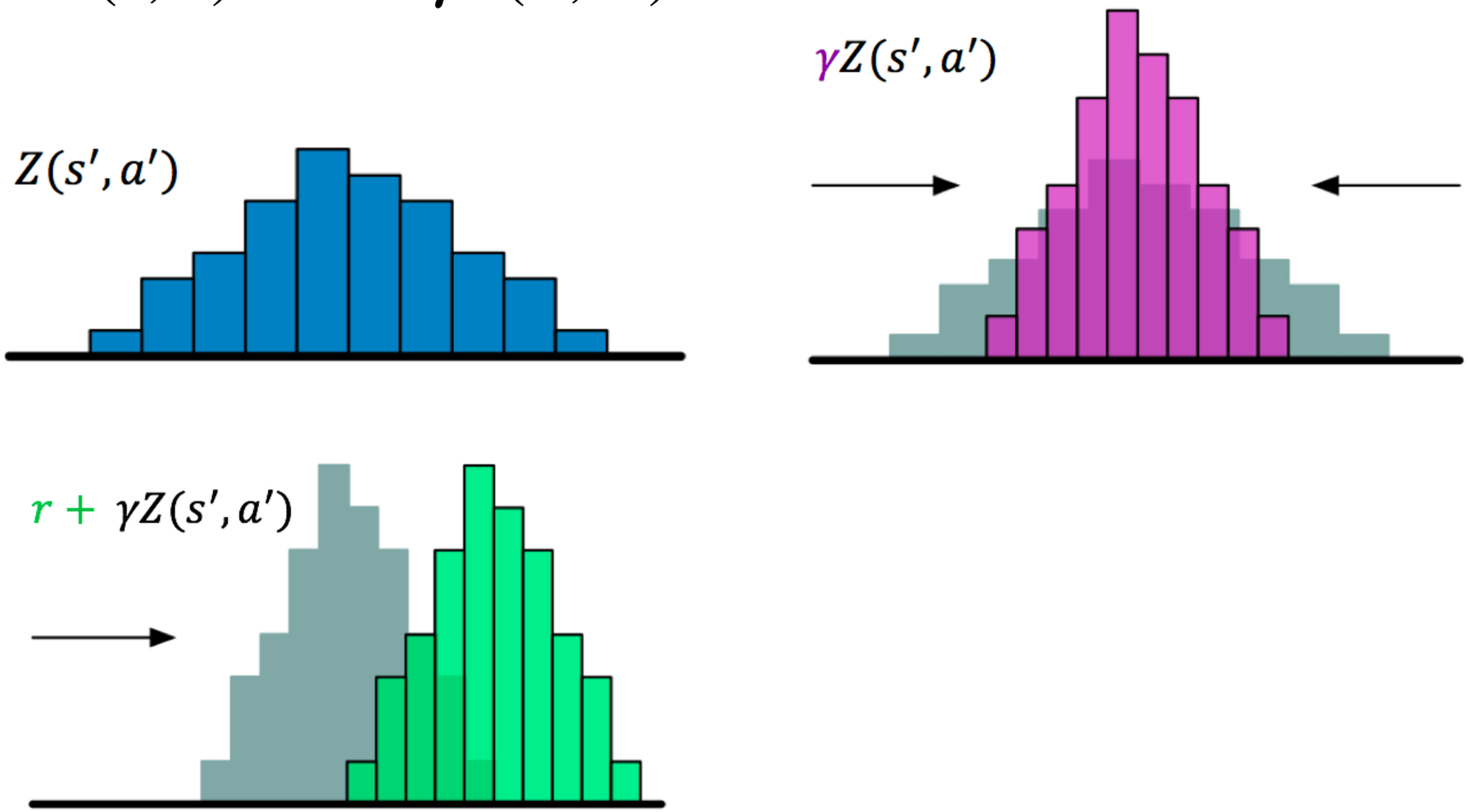
- $Q^{\pi}(s,a) = \mathbb{E}[G_t]$
- $Q^{*}(s,a) = r + \gamma \max_{a'} Q^{*}(s',a')$
- $Z(s,a) = r + \gamma Z(s',a')$



Bellemare et al., A distributional perspective on reinforcement learning, ICML 2017

Prioritized Experience Replay

- Is uniform sampling from reply buffer an efficient way to learn? Does each sample contribute to the learning equally?
- The higher the TD error, the greater the degree of update there is to the neural network weights
- Sampling experiences according to TD error
  - $P(i) = \frac{p_i^\alpha}{\sum_k p_k^\alpha}$, (1) $p_i = 1/\text{rank}(\delta_i)$; (2) $p_i = |\delta_i| + \epsilon$
  - Biased sampling
  - $w_i = \left(\frac{1}{N} \cdot \frac{1}{P(i)}\right)^\beta$
  - Better than DQN in 41 of 49 Atari 2600 games

Schaul et al., Prioritized experience replay, arXiv:1511.05952, 2015

Rainbow

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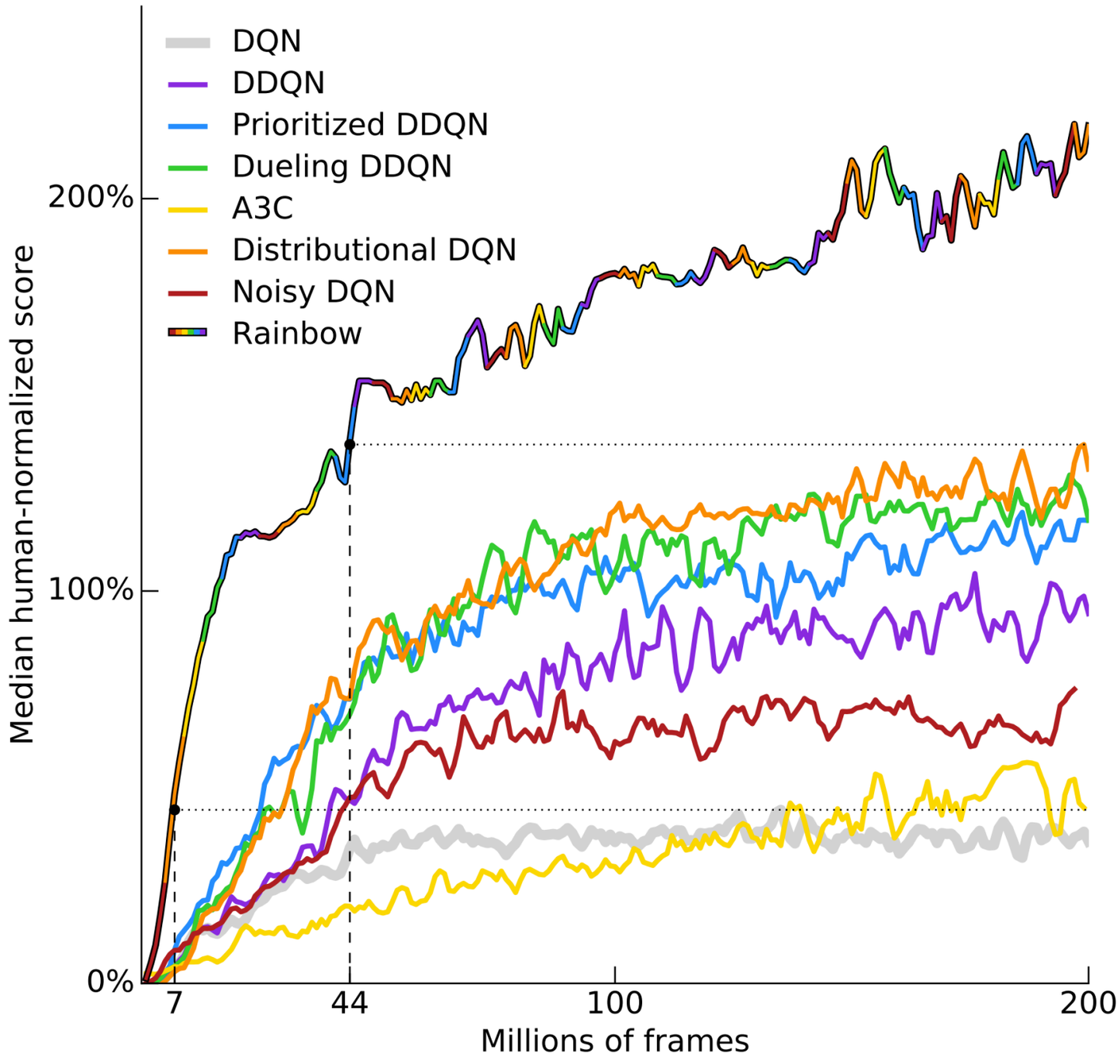

Hessel et al., Rainbow: Combining Improvements in Deep Reinforcement Learning, AAAI 2018

Can DQN work with Continuous Actions?

- DQN uses experience replay and target network
  - Take action $a_t = \arg\max_{a_t} Q(s, a; \mathbf{w})$ or act randomly
  - Store transition $(s_t, a_t, r_{t+1}, s_{t+1})$ in replay memory $\mathscr{D}$
  - Sample random mini-batch of transitions (s, a, r, s') from $\mathscr{D}$
  - Optimize MSE between Q-network and Q-learning targets

$$\mathscr{L}(\mathbf{w}) = \mathbb{E}_{s,a,r,s' \sim \mathscr{D}}\left[\left(r + \gamma \max_{a'} Q(s', a'; \mathbf{w}^-) - Q(s, a; \mathbf{w})\right)^2\right]$$

How to perform the max?

- Using variant of SGD
- Every N timesteps, $\mathbf{w}^- = \mathbf{w}$

DDPG

- Learning an approximate maximizer
  - $\max_a Q(s, a; \phi) = Q(s, \arg\max_a Q(s, a); \phi)$
  - Train another network $\mu(s; \theta)$ such that $\mu(s; \theta) \approx \arg\max_a Q(s, a; \phi)$
  - Solve $\theta = \arg\max_\theta Q(s, \mu(s; \theta); \phi)$
  - By chain rule: $\frac{\mathrm{d}Q_\phi}{\mathrm{d}\theta} = \frac{\mathrm{d}Q_\phi}{\mathrm{d}a}\frac{\mathrm{d}a}{\mathrm{d}\theta}$
  - New target: $r + \gamma Q(s', \mu(s'; \theta^-); \phi^-)$

Lillicrap et al., Continuous control with deep reinforcement learning, ICLR 2016

DDPG

- DDPG uses experience replay and target networks
  - Take action $a_t = \mu(s, \theta) + \mathcal{N}_t$ (exploration)
  - Store transition $(s_t, a_t, r_{t+1}, s_{t+1})$ in replay memory $\mathcal{D}$
  - Sample random mini-batch of transitions (s, a, r, s') from $\mathcal{D}$
  - Compute $\delta = r + \gamma Q\left(s', \mu(s'; \theta^-); \phi^-\right) - Q\left(s, a; \phi\right)$
  - $\Delta\phi \leftarrow \alpha\delta \dfrac{\mathrm{d}Q(s, a; \phi)}{\mathrm{d}\phi}$
  - $\Delta\theta \leftarrow \beta \dfrac{\mathrm{d}Q(s, a; \phi)}{\mathrm{d}a} \dfrac{\mathrm{d}\mu(s; \theta)}{\mathrm{d}\theta}$
  - Soft update ϕ^- and θ^-

Lillicrap et al., Continuous control with deep reinforcement learning, ICLR 2016

TD3: Twin Delayed DDPG

- Clipped double Q learning for overestimation
  - Two Q functions use a single target
  - TD target: $y = r + \gamma \min_{i=1,2} Q_{\theta_i'}(s', \mu_{\phi'}(s'))$
- Delayed policy update
  - Value and policy updates are deeply coupled
  - Policy updates less frequently than Q (stable target)
- Target policy smoothing
  - Deterministic policy can overfit to narrow peaks developed by Q function approximator
  - $a_{\text{clip}}(s') = \mu_{\phi'}(s') + \text{clip}(\mathcal{N}(0,\sigma), -c, +c)$

$$y = r + \gamma \min_{i=1,2} Q_{\theta'}(s', a_{\text{clip}})$$

Fujimoto et al., Addressing function approximation error in actor-critic methods, ICML 2018.

Practical Tips for Deep Q Networks

- DQN takes cares to stabilize
  - Test on easy tasks first, make sure your implementation is correct
- Large replay buffers help improve stability
- It takes time, be patient - might be no better than random for a while
- Start with high exploration (ϵ) and gradually reduce, also for learning rate
- DDQN helps a lot in practice, simple and no downsides
- Run multiple random seeds, it is very inconsistent between runs

Convergence of Control Algorithms

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Algorithm	Table Lookup	Linear	Non-linear
MC Control	✓	(✓)	✗
SARSA	✓	(✓)	✗
Q-learning	✓	✗	✗

(✓) = chatters around near-optimal value function

References

- Richard Sutton and Andrew Barto. Reinforcement learning: An introduction. MIT press, 2018 (**some slides are borrowed from**)
- David Silver, Reinforcement Learning Course, UCL (**some slides are borrowed from**)
- Sergey Levin, Deep Reinforcement Learning Course, UC Berkeley (**some slides are borrowed from**)
- Mnih et al., Human-level control through deep reinforcement learning. Nature, 2015, 518(7540): 529-533.
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