CoPPo: Hand-in assignment 1

Group 15
Yingjie Huang, Jingkai Zhou

Task I: The FIAT-Chrysler case

In theory, we only need to set the production of each type of cars as an integer decision variable $\mathbf{x}$, however, it turns out that we can not properly limit the number of employees to be an integer in constraints, so we can either ignore this trivial fact or instead set the number of employees for each type of cars as an integer decision variable $\mathbf{y}$, and we chose the latter solution for this task.

If we denote budget as b, tax as t, price as p, material cost as mc, salary as s, manhour as mh, maximum manhour as max_mh, minimum production as mp and index i for different type of cars (Panda, FIAT 500, Musa and Giulia in order), the equations corresponding to four constraints can be formed as below:

1. $\sum_{i=1}^{4}(\mathbf{x}_i \cdot p_i \cdot mc_i + \mathbf{y}_i \cdot s_i \cdot 12) \leq b$;
2. $\mathbf{y}_i \cdot max\_mh \cdot 12 \geq \mathbf{x}_i \cdot mh_i, \quad \forall\ i\ \in\ \{1, 2, 3, 4\}$;
3. $\mathbf{x}_1 + \mathbf{x}_3 \leq 300000$;
4. $\mathbf{x}_i \geq mp_i, \quad \forall\ i\ \in\ \{1, 2, 3, 4\}$.

Our objective is to maximize total net profit, which can be organized as below:

$$\mathbf{max}\left\{\sum_{i=1}^{4}((\mathbf{x}_i \cdot p_i \cdot (1 - mc_i) - \mathbf{y}_i \cdot s_i \cdot 12) \cdot (1 - t_i))\right\}.$$

As a result, we obtain the optimal objective as 24047469955 with the following production plan illustrated as table 1:

Type	Panda	FIAT 500	Musa	Giulia
Production	120000	100009	80025	80121

Table 1: Optimal production plan for task I

Task II: The Global Market

We set up three integer decision variables for this task, which are production of each type of cars $\mathbf{x}$, number of employees for each type of cars $\mathbf{y}$ and number of each type of cars to be sold in each country $\mathbf{z}$ respectively.

If we denote budget as b, sales tax as st, export tax as et, sales price as sp, home price as hp material cost as mc, salary as s, manhour as mh, maximum manhour as max_mh minimum production as mp, dilivery cost as DI, demand of cars as Dm, index i for different type of cars (Panda, FIAT 500, Musa and Giulia in order) and index j for different countries (Poland, Italy, the US and Sweden in order), the equations corresponding to five constraints can be briefly illustrated as below:

1. $\sum_{i=1}^{4}(\mathbf{x}_i \cdot hp_i \cdot mc_i + \mathbf{y}_i \cdot s_i \cdot 12) + \sum_{i=1}^{4}\sum_{j=1}^{4}(\mathbf{z}_{ij} \cdot DI_{ij}) + \sum_{i=1}^{4}(\mathbf{z}_{i3} \cdot sp_{i3} \cdot et) \leq b$;
2. $\mathbf{y}_i \cdot max\_mh \cdot 12 \geq \mathbf{x}_i \cdot mh_i, \quad \forall\ i\ \in\ \{1,2,3,4\}$;
3. $\mathbf{z}_{ij} \geq Dm_{ij}, \quad \forall\ i,j\ \in\ \{1,2,3,4\}$;
4. $\sum_{j=1}^{4}\mathbf{z}_{ij} = \mathbf{x}_i, \quad \forall\ i\ \in\ \{1,2,3,4\}$;
5. $\mathbf{x}_i \geq mp_i, \quad \forall\ i\ \in\ \{1,2,3,4\}$.

On top of that, another two constraints that Musa is not available in the US market and Giulia is not available in Poland market, can be achieved be simply setting corresponding market demand for these two types and their sales prices to zero, i.e., $Dm_{33} = Dm_{41} = sp_{33} = sp41 = 0$.

Then our objective is also to maximize total net profit, which can be written as following:

$$\mathbf{MAX}\left\{\sum_{i=1}^{4}(\sum_{j=1}^{4}(\mathbf{z}_{ij} \cdot sp_{ij}) - \mathbf{x}_i \cdot hp_i \cdot mc_i - \mathbf{y}_i \cdot s_i \cdot 12 - \sum_{j=1}^{4}(\mathbf{z}_{ij} \cdot DI_{ij}) - \mathbf{z}_{i3} \cdot sp_{i3} \cdot et) \cdot (1 - st_i)\right\}.$$

Feed all above to the solver, the optimal objective is 24775090021 with marketing and production plans as table 2 shows:

Type / **Country**	Panda	FIAT 500	Musa	Giulia
Poland	75000	20000	10000	0
Italy	35012	40029	80005	8000
US	40003	50000	0	42340
Sweden	2001	5000	1001	1000
Production	152016	115029	91006	51340

Table 2: Optimal marketing and production plans for task II

Task III: Multi-line production

In Task-III, the problem is to solve a MAX-MIN optimization. According to the description, the amount of foam is determined by the minimum number among those three components.

First we need to first set the time that each production line spends on each component as an integer decision variable $\mathbf{x}$, the produced number of each component as an integer decision variable $\mathbf{y}$, and the minimum number of produced components as $\mathbf{y_{min}}$ due to Gurobi syntax requirement.

And also, we denote capacity as c, production rate as pr, and index i for different production line, index j for different component, therefore, three constraints can be created as following:

1. $\sum_{i=1}^{4}(\mathbf{x}_{ij} \cdot pr_{ij}) = \mathbf{y}_j, \quad \forall\, j \;\in\; \{1, 2, 3\};$
2. $\sum_{j=1}^{3}\mathbf{x}_{ij} \leq c_i, \quad \forall\, i \;\in\; \{1, 2, 3, 4\};$
3. $\mathbf{y_{min}} = \min\_(\mathbf{y}_j \quad \forall\, j \;\in\; \{1, 2, 3\}).$

Note that "min_" in the third constraint is a Gurobi syntax which compare multiple Gurobi variables and return the minimum one.

Thus the objective function that we want to maximize becomes follows:

$$\mathbf{MAX}\left\{\mathbf{y_{min}}\right\}.$$

After feeding all the constraints to solver, the maximum number of foam is 2920 with the optimal production plan illustrated as table 3.

Line \ **Component**	Component 1	Component 2	Component 3
Line 1	0	100	0
Line 2	88	62	0
Line 3	80	0	0
Line 4	0	54	146

Table 3: Optimal Production plan for task III

Task IV: Retailers Localization

An easy way to solve this task is to set up two decision variables, where one is a binary variable $\mathbf{x}$ representing whether the retailer is open or not, another one is an integer variable $\mathbf{y}$ representing the size.

If we denote the total number of possible locations as N, and reuse all the notations in the description, the following four constraints can be built:

1. $\sum_{i=1}^{N}\mathbf{x}_i \leq K;$
2. $\mathbf{y}_i \leq U_i \cdot \mathbf{x}_i, \quad \forall\, i \in \{1, 2, \ldots, N\};$
3. $\mathbf{y}_i \geq L_i \cdot \mathbf{x}_i, \quad \forall\, i \in \{1, 2, \ldots, N\};$
4. $\sum_{i=1}^{N}(F_i \cdot \mathbf{x}_i + \frac{C_i \cdot \mathbf{y}_i}{100}) \leq W.$

The objective function that needs to be maximized becomes follows:

$$\textbf{MAX}\left\{\sum_{i=1}^{N}\frac{R_i\cdot\mathbf{y}_i}{100}\right\}.$$

Location / **Parameter**	A	B	C	D	E	F
Fixed cost	10	20	15	25	5	30
Variable cost	0.1	0.2	0.3	0.4	0.5	0.6
Lower bound/m^2	100	110	120	130	140	150
Upper bound/m^2	200	210	220	230	240	250
Revenue rate	1.1	1.2	1.3	1.4	1.5	1.6

Table 4: Test Data Sheet

Adding above constraints in table 4 together with $W = 55$ and $K = 4$, the optimal solution in this case is shown as table 5:

Location	A	B	C	E
Size/m²	200	210	220	240

Table 5: Optimal retailer strategy for test case in task IV

Task V: Investments optimization

For Task 5, we first set up two decision variables, which are a continuous variable $\mathbf{x}$ representing the amount of money in each investment, a binary variable $\mathbf{y}$ representing whether to choose C (1) or D (0), and another binary variable $\mathbf{z}$ representing whether the investment on A is more than 1 billion (1) or not (0).

Then we denote the budget as b, Moody's rateing as mr, duration as d, revenue as r, taxation as t (30% for government investments and 0 for others) and index i for different investments (A, B, C, D and E in order).

Note that in Gurobi, `if` statement can be constructed by two constraints consisting of a binary decision variable, in this case $\mathbf{y}$, a big-$\mathbf{M}$ constant, which is equal to b for this task, and a small enough constant ϵ, which is selected as 0.01.

Afterwards, the seven constraints can be set as following:

1. $\sum_{i=1}^{5}\mathbf{x}_i \leq b$;

2. $\sum_{i=2}^{4}\frac{\mathbf{x}_i}{b} \geq 0.4$;

3. $\sum_{i=1}^{5}\frac{\mathbf{x}_i\cdot d_i}{b} \leq 5$;

4. $\sum_{i=1}^{5} \frac{\mathbf{x}_i \cdot mr_i}{b} \leq 1.5$;

5. $\mathbf{x}_3 \leq \mathbf{M} \cdot \mathbf{z}$;

6. $\mathbf{x}_4 \leq \mathbf{M} \cdot (1 - \mathbf{z})$;

7. $\mathbf{x}_1 \geq 10^6 + \epsilon - \mathbf{M} \cdot (1 - \mathbf{z})$;

8. $\mathbf{x}_1 \leq 10^6 + \mathbf{M} \cdot \mathbf{z}$.

Finally, the objective function to be maximized is formed as following:

$$\mathbf{MAX} \left\{ \sum_{i=1}^{5} \mathbf{x}_i \cdot r_i \cdot (1 - t_i) \right\}.$$

The optimal objective is 39538636.36 crowns, which can be achieved by the following investment strategy as shown in table 6:

Investment	A	C	E
Amount of money/SEK	227272727.27	704545454.55	68181818.18

Table 6: Optimal investment strategy for task V