用松弛迭代法求解如下二维椭圆偏微分方程:

Contents

问题

$$-[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}]\varphi = S(x,y)$$
$$S(x,y) = 2xy$$
$$\varphi(0,y) = \varphi(x,0) = 0$$
$$\varphi(1,y) = \varphi(x,1) = 0$$

```
clear;clc;
N=100;
h=1./N;
w=1.5;
for i=1:N+1
    for j=1:N+1
        x=(i-1)*h;
         y=(j-1)*h;
         s(i,j)=2.*x*y;
         phi(i,j)=0.;
    end
end
 for  ite=1:500
     for i=2:N  % notice that this time the range of i is different fro
m the previous one due to the boundary limit.
         for j=2:N
             phi(i,j)=(1-w)*phi(i,j)+w/4.*(phi(i+1,j)+phi(i-1,j)+phi(i,
j+1)+phi(i,j-1)+h^2.*s(i,j));
         end
     end
     if mod(ite-1,20)==0
         disp('Iteration');
         disp(ite);
         E=0;
         for i=2:N
             for j=2:N
                 E=E+((phi(i,j)-phi(i-1,j))^2.+(phi(i,j)-phi(i,j-1))^2.
)/2.-h^2.*s(i,j)*phi(i,j);
             end
         end
         disp('Energy');
         disp(E);
     end
 end
```

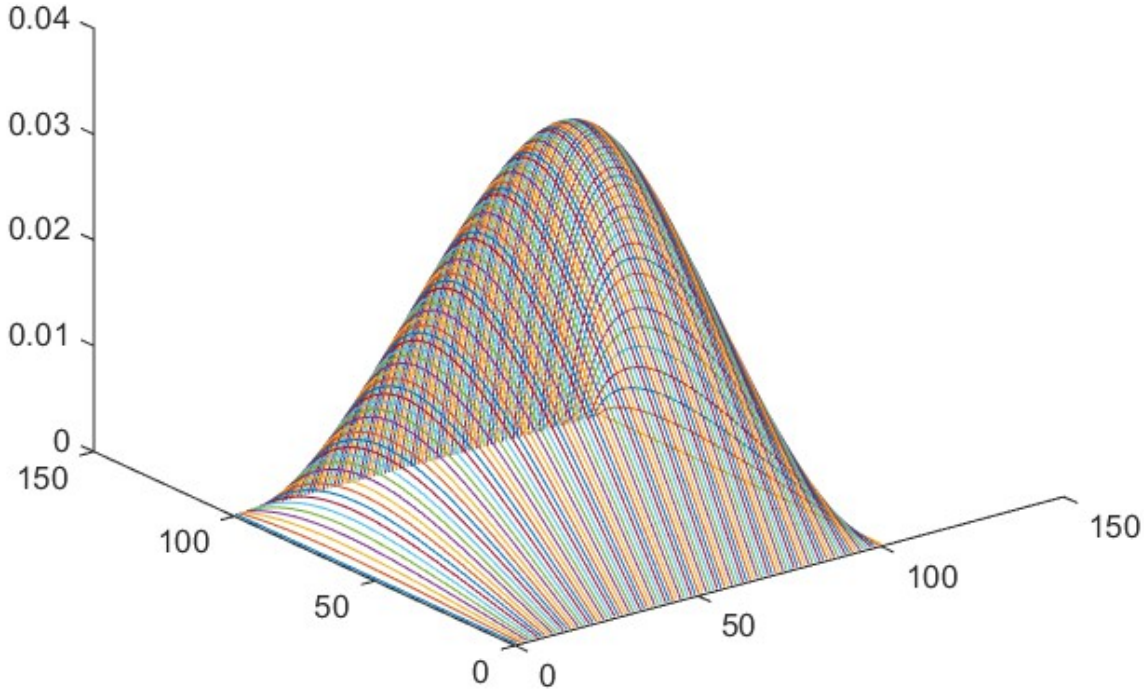
```
Iteration
     1
Energy
    -6.180354803184013e-05
Iteration
    21
```

```
Energy
    -9.927892855066244e-04
Iteration
    41
Energy
  -0.001674593415556
Iteration
    61
Energy
  -0.002221681270674
Iteration
    81
Energy
  -0.002675487634143
Iteration
   101
Energy
  -0.003059081145330
Iteration
   121
Energy
  -0.003387382953717
Iteration
   141
Energy
  -0.003670873065084
Iteration
   161
Energy
  -0.003917310709671
Iteration
   181
Energy
  -0.004132655915927
Iteration
   201
Energy
  -0.004321612791071
Iteration
   221
Energy
  -0.004487972647934
Iteration
   241
Energy
  -0.004634842292158
Iteration
   261
Energy
  -0.004764802203179
Iteration
   281
Energy
  -0.004880019767170
Iteration
   301
Energy
  -0.004982332525642
Iteration
   321
```

```
Energy
  -0.005073310757532
Iteration
   341
Energy
  -0.005154305423467
Iteration
   361
Energy
  -0.005226485501666
Iteration
   381
Energy
  -0.005290867484433
Iteration
   401
Energy
  -0.005348338983828
Iteration
   421
Energy
  -0.005399677845824
Iteration
   441
Energy
  -0.005445567795256
Iteration
   461
Energy
  -0.005486611369449
Iteration
   481
Energy
  -0.005523340709451
```

plot the figure of φ

```
[xx,yy] = meshgrid(1:N+1,1:N+1);
 plot3(xx,yy,phi)
```



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