Math 302/600 Spring 2017 Homework #1

Due Feb. 7, Tue. in class

1. Let V be an inner product space with the induced norm $\|\cdot\|$. Show that for any $x, y \in V$, $\|x+y\|^2 + \|x-y\|^2 = 2(\|x\|^2 + \|y\|^2)$.

2. On the Euclidean space $\mathbb{R}^n$, define for each $x = (x_1, \cdots, x_n) \in \mathbb{R}^n$,

$$\|x\|_\star := c_1|x_1| + c_2|x_2| + \cdots + c_n|x_n|,$$

   where c_i is a positive real number for each $i = 1, \cdots, n$. Show that $\|\cdot\|_\star$ is a norm on $\mathbb{R}^n$.

3. Recall that $C([0,1])$ is the vector space of all continuous functions $f : [0,1] \to \mathbb{R}$ on the interval $[0,1]$. For any $f \in C([0,1])$, define

$$\|f\|_\infty := \max\Big\{|f(t)| : t \in [0,1]\Big\}.$$

   (1) Explain why $\|f\|_\infty$ exists for any $f \in C([0,1])$. (*Hint*: think of Theorem 5.3.4 of Bartle and Sherbert's book.)

   (2) Prove that $\|\cdot\|_\infty$ is a norm on $C([0,1])$.

The following extra problems are for **Math 600** *students only:*

4. Let V be an inner product space with the induced norm $\|\cdot\|$. Show that for any $x, y \in V$, $\|x+y\| \cdot \|x-y\| \le \|x\|^2 + \|y\|^2$.

5. On the Euclidean space $\mathbb{R}^n$, define for each $x = (x_1, \ldots, x_n) \in \mathbb{R}^n$,

$$\|x\|_\infty := \max(|x_1|, |x_2|, \ldots, |x_n|)$$

   Show that $\|\cdot\|_\infty$ is a norm on $\mathbb{R}^n$.