Math 430/603 Spring 2017 Homework #9

Due April 18, Tue in class

1. Textbook, Section 5.6, page 335: 3(a, b), 4(a), 5;
2. Textbook, Section 5.9, page 390: 3, 4, 5;
3. Let P be an $n \times n$ orthogonal matrix.
   (1) Show that $\langle x, y\rangle = \langle Px, Py\rangle$ for any $x, y \in \mathbb{R}^n$, where $\langle \cdot, \cdot \rangle$ is the standard inner product on $\mathbb{R}^n$.
   (2) Show that $x, y \in \mathbb{R}^n$ are orthogonal if and only if Px, Py are orthogonal.
4. Let u, v, w, z be four vectors in $\mathbb{R}^4$ such that they span $\mathbb{R}^4$. Let the subspaces $\mathcal{X} = \text{span}\{u, v\}$ and $\mathcal{Y} = \text{span}\{w, z\}$.
   (1) Show that $\{u, v, w, z\}$ is linearly independent and $\mathbb{R}^4 = \mathcal{X} \oplus \mathcal{Y}$.
   (2) Let $x = u - v + 2w - 3z$. Find the projection of x onto $\mathcal{X}$ (along $\mathcal{Y}$), and the projection of x onto $\mathcal{Y}$ (along $\mathcal{X}$).

The following extra problem(s) are for **Math 603** *students only:*

5. Textbook, Section 5.9, page 390: 12(a), 13.