Math 302/401/600 Fall 2010 Homework #2

Due Sept. 20, Mon. in class

1. Let (M,d) be a metric space. Show the following inequality:

$$|d(x,y) - d(z,y)| \leq d(x,z), \quad \forall\ x,y,z \in M$$

2. Textbook, page 108, Ex. 3, 4. (*note*: each problem carries the standard metric of $\mathbb{R}^n$ induced by the Euclidean norm $\|\cdot\|_2$.)

3. Consider $\mathbb{R}^2$ and the metric induced by the 1-norm: $d(x,y) = |x_1 - y_1| + |x_2 - y_2|$, $\forall\ x,y \in \mathbb{R}^2$. Let the set $A = \{x = (x_1,x_2) \in \mathbb{R}^2 \ : \ x_1 \geq 1 \text{ and } x_2 \leq 1\}$. Find the interior of A and prove your answer.

4. Let (M,d) be a metric space and A, B be two subsets of M. Show the following:

   (1) if $A \subseteq B$, then $\text{int}A \subseteq \text{int}B$;

   (2) $\text{int}(A \cap B) = (\text{int}A) \cap (\text{int}B)$.

The following extra problem is for **Math 401/600** *students only:*

5. Let M be a set endowed with two metrics d_1 and d_2, namely, both (M,d_1) and (M,d_2) are metric spaces. Suppose that there exist positive real numbers α and β such that

$$\beta d_1(x,y) \leq d_2(x,y) \leq \alpha d_1(x,y), \quad \forall\ x,y \in M$$

   Show that a set $A \subseteq M$ is open with respect to d_1 if and only if it is open with respect to d_2.