Math 650 Fall 2011 Homework #8

Due Dec. 7, Wed. in class

P.1 Textbook, Chapter 9, page 241: 4, 6, 8.

P.2 Consider the optimization problem:

$$\min \; x_1^2 + x_2^2, \quad \text{subject to} \quad -x_1 - x_2 + 4 \leq 0, \quad (x_1, x_2) \in C \equiv \mathbb{R}_+^2.$$

Solve the following problems:

(1) Show that the optimization problem has a unique global minimizer. (*Hint*: use the Extreme Value Theorem and convexity.)

(2) Show that any Fritz-John point is a KKT point (i.e. $\lambda_0 \neq 0$).

(3) Find the KKT point and show that it is the global minimizer using the 2nd-order optimality condition. Determine the optimal value of the objective function.

(4) (Optional) Find the Lagrangian dual function.

(5) (Optional) Solve the corresponding Lagrangian dual problem and show that there is no duality gap.