MATH 221 Solution to HW #14

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4.
$$\widehat{\mathbf{y}} = \begin{bmatrix} 6 \\ 3 \\ 0 \end{bmatrix}.$$

5.
$$\widehat{\mathbf{y}} = \begin{bmatrix} -1 \\ 2 \\ 6 \end{bmatrix}.$$

8.
$$\widehat{\mathbf{y}} = \begin{bmatrix} \frac{3}{2} \\ \frac{7}{2} \\ 1 \end{bmatrix}, \quad \mathbf{z} = \begin{bmatrix} -\frac{5}{2} \\ \frac{1}{2} \\ 2 \end{bmatrix}.$$

9.
$$\widehat{\mathbf{y}} = \begin{bmatrix} 2 \\ 4 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{z} = \begin{bmatrix} 2 \\ -1 \\ 3 \\ -1 \end{bmatrix}.$$

11. The closest point in W is
$$\widehat{\mathbf{y}} = \begin{bmatrix} 3 \\ -1 \\ 1 \\ -1 \end{bmatrix}.$$

21. (a) True, since $\mathbf{z}$ is orthogonal to W, thus it is in $W^{\perp}$;
    (b) True, due to the Orthogonal Decomposition Theorem (Theorem 8);
    (c) False, due to the uniqueness of the orthogonal projection stated in the Orthogonal Decomposition Theorem (Theorem 8);
    (d) True, because of the Best Approximation Theorem (Theorem 9) or Theorem 5 of Section 6.2.

Section 6.4

2. The orthogonal basis is
$$\mathbf{v_1} = \begin{bmatrix} 0 \\ 4 \\ 2 \end{bmatrix}, \quad \mathbf{v_2} = \begin{bmatrix} 5 \\ 4 \\ -8 \end{bmatrix}.$$

7. The orthonormal basis is
$$\mathbf{v_1} = \frac{1}{\sqrt{30}} \begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix}, \quad \mathbf{v_2} = \frac{1}{\sqrt{6}} \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}.$$

10. The orthogonal basis is

$$\mathbf{v_1} = \begin{bmatrix} -1 \\ 3 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{v_2} = \begin{bmatrix} 3 \\ 1 \\ 1 \\ -1 \end{bmatrix}, \quad \mathbf{v_3} = \begin{bmatrix} -1 \\ -1 \\ 3 \\ -1 \end{bmatrix}.$$

17. (a) False. The statement is true when $c \neq 0$. However, if $c = 0$, it does not hold because the new set is linearly dependent.

18. (a) False. Let

$$\mathbf{x}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{x}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{x}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

such that $W = \text{span}\{\ \mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3\}$. Thus $\{\ \mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3\}$ is linearly independent (check it). Let

$$\mathbf{v}_1 = \mathbf{x}_1, \qquad \mathbf{v}_2 = \mathbf{x}_2, \qquad \mathbf{v}_3 = \mathbf{0}.$$

Therefore $\{\ \mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is an orthogonal set in W (verify it!). However, $\{\ \mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is not linearly independent (why?), thus it is *not* a basis.

Comment: if $\{\ \mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ has no zero vector, then it is linearly independent and must be a basis for W (Think of why?).

(b) True. Note that $\text{proj}_W \mathbf{x}$ is in W. So if $\mathbf{x} - \text{proj}_W \mathbf{x} = \mathbf{0}$ or equivalently $\mathbf{x} = \text{proj}_W \mathbf{x}$, then $\mathbf{x}$ is in W, a contradiction.