Math 430/603 Spring 2017 Homework #5

Due March 9, Thu in class

1. Textbook, Section 4.4, page 206: 4(b-c), 8, 9;
2. Let A be an $m \times n$ real matrix.
   (1) Show that if $Ax = 0$ for any $x \in \mathbb{R}^n$, then $A = 0$.
   (2) Show that if $A^T A = 0$, then $A = 0$. (*Hint*: use (1).)
3. Let V be the vector space of all 2×2 lower triangular matrices.
   (1) Find a basis of V, and prove that your finding is indeed a basis of V.
   (2) Find the dimension of V.
4. Let V be an n-dimensional vector space. Show that
   (1) if $S = \{u_1, \ldots, u_n\}$ spans V, then S is linearly independent;
   (2) if $S' = \{v_1, \ldots, v_n\}$ in V is linearly independent, then S' spans V.

The following extra problem(s) are for **Math 603** *students only:*

5. Let A be an $m \times n$ matrix.
   (1) What is the largest possible rank of A?
   (2) Suppose $A \neq 0$. What is the largest possible dimension of $N(A)$? What is the smallest possible dimension of $N(A)$?