Math 302/401/600 Fall 2010 Homework #11

Due Dec. 1, Wed. in class

1. Textbook, page 317, Exercises 4, 6, 8.

2. Let $f_n(x) = (x^2 + n^4)^{-1}$, where $x \in \mathbb{R}$. Use Weierstrass M-test to show uniform convergence of the series $\sum_{n=1}^{\infty} f_n$ on $\mathbb{R}$.

3. Let
$$f_n(x) = \frac{(-1)^{n-1} x^{2n-1}}{(2n-1)!}, \quad x \in \mathbb{R}.$$

   (3.1) Let $A = [-a, a]$ with $a > 0$. Use Weierstrass M-test to show uniform convergence of the series $\sum_{n=1}^{\infty} f_n$ on A.

   (3.2) Let f_* be the limiting function of the series on A, i.e., $f_*(x) = \sum_{n=1}^{\infty} f_n(x)$. Is f_* differentiable on $(-a, a)$? If so, is $f'_*(x) = \sum_{n=1}^{\infty} f'_n(x)$ on $(-a, a)$? Prove your answers.

4. Let $f_n : \mathbb{R} \to \mathbb{R}$ be such that the sequence (f_n) converges uniformly on the set A to f_*. Suppose that each f_n is bounded on A, i.e., for each f_n, there exists $M_n > 0$ (dependent on f_n) such that $|f_n(x)| \leq M_n, \forall\, x \in A$. Show that f_* is bounded on A.

The following extra problem is for **Math 401/600** *students only:*

5. Suppose that each $f_n : \mathbb{R} \to \mathbb{R}$ is continuous on the set A, and (f_n) converges to f_* uniformly on A. Let (x_n) in A converge to $x^* \in A$. Show that $(f_n(x_n))$ converges to $f_*(x^*)$.