

Math 302/401/600 Fall 2010 Homework #6

Due Oct. 20, Wed. in class

1. Textbook, page 181, Section 4.1, 1(b), 2, and 3.
2. Let $\|\cdot\|$ be a norm on \mathbb{R}^n . Show that $f : \mathbb{R}^n \rightarrow \mathbb{R}$ defined by $f(x) := \|x\|$ is continuous on \mathbb{R}^n .
3. Let (M, d) be a metric space. Given $z \in M$, define $f : M \rightarrow \mathbb{R}$ by $f(x) := d(z, x)$. Show that f is continuous on M .
4. Define $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ as follows:

$$f(x, y) := \begin{cases} 0, & \text{if } x \text{ is rational and } y \text{ is irrational} \\ 1, & \text{otherwise} \end{cases}$$

Show that f is discontinuous at any point of \mathbb{R}^2 . (*Hint:* for any real a , there is a rational (resp. irrational) sequence converging to a .)