MATH 650 Foundations of Optimization
Section 0101 Fall 2016

- Instructor: Dr. Jinglai Shen
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- Office: Math/Psyc 417
- Phone: (410) 455-2402
- Lectures: Tue and Thu, 5:30–6:45 pm, at Math/Psyc 401
- Office hours: Tue and Thu, 4:30–5:30 pm or by appointment
- Course web-page: http://www.math.umbc.edu/~shenj
- Prerequisites: real analysis, linear algebra, multi-variable calculus
- Recommended textbook (but not required): *Foundations of Optimization* by Osman Güler, Springer, 2010
- Other references:
  - D.P. Bertsekas. *Nonlinear Programming*, Athena Scientific, Belmont, MA, 2nd edition, 1999;
  - J.M. Borwein and A.S. Lewis. *Convex Analysis and Nonlinear Optimization*, CMS Books in Mathematics, Springer-Verlag, New York, 2000;
  - S. Boyd and L. Vandenberghe. *Convex Optimization*, Cambridge University Press, 2004.
- Important dates:
  - No class on Nov. 24 (Thanksgiving Day)
  - Last class: Dec. 13, Tue
  - Final exam: Dec. 15, Thu

Course Description Optimization techniques are widely used in diverse areas, including engineering, operations research, economics and finance, statistics, and social science. They have also stimulated a deep and elegant mathematical theory which becomes the foundation of various efficient numerical optimization algorithms. This course treats the basic theory of continuous optimization in finite dimensions. Fundamental techniques from multivariable calculus (e.g. G/F-differentiability, the Implicit Function Theorem) and convex analysis (e.g. convex sets/functions, separation theorems, polyhedra theory) will be presented in this course. By exploring these techniques, unconstrained and constrained optimization problems will be discussed. Focused topics include the Lagrange multiplier approach, Fritz John and Karush-Kuhn-Tucker conditions, local and global optimality conditions, and duality theory.

Specific topics are:

- Existence Theory of Finite-dimensional Continuous Optimization
- Unconstrained Optimization
  - Review of multi-variable calculus
  - Optimality conditions: first-order and second-order optimality conditions
- Equality Constrained Optimization
  - Implicit function theorem
  - Optimality conditions: first-order and second-order optimality conditions
- Fundamentals of Convex Analysis
  - Convex sets and convex functions, optimization on a convex set
  - Separation of convex sets: Euclidean projection and separation theorems
  - Convex polyhedra and polyhedral cones
  - Farkas' lemma and transposition theorems
- Inequality Constrained Optimization
  - Fritz John and KKT optimality conditions, constrained qualification
  - Second-order optimality conditions
- Duality Theory
  - Saddle point, and strong duality
- Convex Optimization
  - Sub-differential and optimality conditions
  - Optimization algorithm: gradient projection methods

Please note that these topics are subject to change, depending on class progress.

Homework Weekly or biweekly homework will be assigned; each homework set contains some theoretic problems (i.e., proofs) and hand-computation problems. Homework must be turned in by a specified date; *no late due will be accepted*. Please present your answers neatly and show all your work; answers without supporting work may not receive full credit.

Exams There will be two in-class exams, and one final exam. Each mid-term exam mainly focuses on topics covered in that month, but the final exam is comprehensive. Please be aware that (i) there will be *no* optional final exam, and all the exams are closed book; and (ii) calculators and other computing devices are *not* allowed for any exam.

Grading Policy The grading scheme is as follows:

- homework: 30%
- mid-term exams: 40% (20% for each)
- final exam: 30% (the final is comprehensive)

The letter grade will be computed based upon the numerical grade:

$A : \geq 85; \quad B : 84 - 73; \quad C : 72 - 60; \quad D : 59 - 50; \quad F :< 50$

Academic Integrity

- The UMBC Academic Integrity Statement:

  "*By enrolling in this course, each student assumes the responsibilities of an active participant in UMBC's scholarly community in which everyone's academic work and behavior are held to the highest standards of honesty. Cheating, fabrication, plagiarism, and helping others to commit these acts are all forms of academic dishonesty, and they are wrong. Academic misconduct could result in disciplinary action that may include, but is not limited to, suspension or dismissal. To read the full Student Academic Conduct Policy, consult the UMBC Student Handbook, the Faculty Handbook, or the UMBC Policies section of the UMBC Directory [or for graduate courses, the Graduate School website]* ".

- All work in homework or an exam must be your own; collaborating on an exam is *not* permitted. Discussions with other students on homework problems are allowed and encouraged, but you should present your own work in the final turn-in; simply copying other people's work is a violation of UMBC's academic integrity code.
- If you wish to contest a graded exam, you must make an appeal within *one week* of the return date to the class. If you must miss an exam due to a prior obligation, you must speak to the instructor *in advance* of the exam. If you must miss an exam due to an unforseen but valid reason (e.g. illness), you must submit a written excuse. Failing to do so may result in loss of substantial points off the top in your make-up.