Math 302/600 Spring 2017 Homework #6

Due March 16, Thu. in class

1. Show that the intersection and union of two sequentially compact sets in a metric space (M, d) remain sequentially compact.
2. Show that any (nonempty) closed subset of a sequentially compact set in a metric space is also sequentially compact.
3. Let (M, d) be a metric space. Show that $A \subseteq M$ is sequentially compact if and only if every infinite subset of A (i.e., a subset that contains infinitely many elements) has an accumulation point in A.
4. Use the definition of compactness (i.e. the open cover definition) to show that the union of two compact sets in a topological space is compact.

The following extra problem(s) are for **Math 600** *students only:*

5. Let (M, d) be a metric space such that M is sequentially compact. Show that (M, d) is complete.
6. Let (x_n) be a sequence in a metric space that converges to x_*, and the set $A := \{x_1, x_2, \ldots, x_n, \ldots\} \cup \{x_*\}$. Show that A is sequentially compact.