Math 302/600 Spring 2017 Homework #7

Due April 4, Tue in class

Note: For any Euclidean space $\mathbb{R}^n$, consider the usual metric induced by the Euclidean norm $\|\cdot\|_2$ on $\mathbb{R}^n$, unless otherwise stated.

1. Use the definition of compactness (i.e. the open cover definition) to show that the union of two compact sets in a topological space is compact.

2. Use the definition of compactness (i.e. the open cover definition) to show that the following sets are *not* compact, by exhibiting an open cover with no finite sub-cover:

   (1) The open ball $B(x, 1)$ centered at a given $x \in \mathbb{R}^n$ with the radius 1 in the Euclidean space $\mathbb{R}^n$;

   (2) The set $A = \{(x_1, x_2) \in \mathbb{R}^2 : \ 0 \leq x_1 \leq 1, \ x_2 \geq 0\}$ in $\mathbb{R}^2$;

   (3) An infinite set in the metric space (M, d) with the discrete metric d.

The following extra problem(s) are for **Math 600** *students only:*

3. Use the definition of compactness (i.e. the open cover definition) to show that the intersection of two compact sets in a topological space is compact.