MATH 487 Fall 2013 Homework #5

Due Nov. 21, Thu in class

- Consider the ODE system on $\mathbb{R}^3$:

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= -\sin x_1 - 2x_1 - x_2 - x_3\\ \dot{x}_3 &= -x_3 + x_2\end{aligned}$$

  Use the Lyapunov function

$$V(x) = 2(1-\cos x_1) + 2x_1^2 + x_2^2 + p\,x_3^2$$

  and choose an appropriate parameter $p > 0$ to show that the origin of the system is (globally) asymptotically stable.

- Let the ODE system on $\mathbb{R}^2$ be:

$$\begin{aligned}\dot{x}_1 &= -x_1 + x_2 - x_2^2 - x_1^3\\ \dot{x}_2 &= x_1 - x_2 + x_1 x_2\end{aligned}$$

  Use the Lyapunov function

$$V(x) = (x_1^2 + x_2^2)/2$$

  to show that the origin of the system is (globally) asymptotically stable. (*Hint*: use LaSalle's invariance principle.)

- Let

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}.$$

  - Which of the linear ODEs defined by the following matrices are topologically conjugate to the one defined by A?

$$B = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}, \quad C = \begin{bmatrix} 3 & 1 \\ -1 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 0 \\ -2 & -1 \end{bmatrix}.$$

  - Does there exist a 2×2 matrix $E \neq A$ such that its corresponding flow is diffeomorphic to the flow defined by A? If so, find such an E; otherwise, justify why.