MATH 487 Fall 2013 Homework #6

Due Dec. 10, Tue in class

- Consider the planar system on $\mathbb{R}^2$:

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= -x + 4x^3 \end{aligned}$$

  - Show that the above system is a Hamiltonian system, find the Hamiltonian $H(x, y)$, and verify $H(0, 0) = 0$.
  - Find all the equilibria of the system, and determine the pattern of linearized dynamics at each equilibrium (e.g., node, saddle, or center).
  - Suppose the initial condition (x_0, y_0) is such that $H(x_0, y_0) \neq 0$. Can $\psi_t(x_0, y_0) \to (0, 0)$ as $t \to \infty$? Explain why.

- Consider the planar system on $\mathbb{R}^2$:

$$\begin{aligned} \dot{x} &= -x + \cos y - 1 \\ \dot{y} &= y \end{aligned}$$

  Find the stable and unstable stes of $(0, 0)$.

  Do not turn in the following problem

- Consider the planar system with the parameter μ:

$$\begin{aligned} \dot{x} &= \mu - x^2 \\ \dot{y} &= -1 + x + y - xy \end{aligned}$$

  Find all the equilibria of the system (for different μ's). Show that $\mu = 0$ is a bifurcation point and determine what type of bifurcation occurs at $\mu = 0$.