Math 430/603 Spring 2017 Homework #6

Due March 16, Thu in class

1. Textbook, Section 4.5, page 220: 12, 18;

2. Textbook, Section 4.6, page 237: 9;

3. Let A be an $m \times n$ matrix and B be an $n \times p$ matrix. Show that $\operatorname{rank}(AB) = \operatorname{rank}(A) - \dim(N(B^T) \cap R(A^T))$. (*Hint*: think of the rank of $(AB)^T$.)

4. Let A be an $m \times n$ matrix, and Q be an $n \times n$ invertible matrix. Show that $\operatorname{rank}(AQ) = \operatorname{rank}(A)$. (*Hint*: use the result of Problem 3.)

5. Let A and B be two $m \times n$ real matrices. Show that if $B = EAF$ for an $m \times m$ matrix E and an $n \times n$ matrix F, then $\operatorname{rank} B \leq \operatorname{rank} A$.

The following extra problem(s) are for **Math 603** *students only:*

6. Let A and B be two $m \times n$ real matrices. Suppose that each column of B is a linear combination of the columns of A.

   (1) Show that $\operatorname{rank} B \leq \operatorname{rank} A$.
   (2) Show that $\operatorname{rank}(A + B) \leq \operatorname{rank} A$.