

## Math 430/603 Spring 2017 Homework #10

Due April 25, Tue in class

1. Suppose that  $\mathcal{V} = \mathcal{X} \oplus \mathcal{Y}$ , and let  $P$  be the projection operator onto  $\mathcal{X}$  along  $\mathcal{Y}$ . Show that  $R(P) = N(I - P)$ .
2. Textbook, Section 5.11, page 409: 5(a,b), 14;
3. Textbook, Section 5.13, page 439: 3, 4, 5;
4. Let a basis for the subspace  $\mathcal{M}$  of  $\mathbb{R}^4$  be

$$\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ 1 \\ -1 \end{pmatrix} \right\}.$$

- (1) Construct an orthonormal basis for  $\mathcal{M}$ . (*Hint:* use the Gram-Schmidt procedure.)
- (2) What is the dimension of  $\mathcal{M}^\perp$ ? Construct a basis for  $\mathcal{M}^\perp$ .
- (3) Find the matrix representation of the orthogonal projector onto  $\mathcal{M}$ .

*The following extra problem(s) are for Math 603 students only:*

5. Textbook, Section 5.9, page 390: 12(a), 13;
6. Textbook, Section 5.13, page 442: 18(a, b).