Math 302 Fall 2011 Homework #3

Due Oct. 3, Mon. in class

1. Consider the metric induced by the 1-norm on $\mathbb{R}^2$: $d(x,y) = |x_1 - y_1| + |x_2 - y_2|$, $\forall\ x, y \in \mathbb{R}^2$. Let the set $A = \{x = (x_1, x_2) \in \mathbb{R}^2 \ : \ x_1 \geq 1 \text{ and } x_2 \geq 1\}$. Prove that the set A is closed via the definition, namely, by showing that the complement of A is open.

2. Let $\mathbb{Q}$ denote the set of rational numbers, and $\mathbb{I}$ denote the set of irrational numbers. Use the standard metric on $\mathbb{R}^2$.

   (1) Find the accumulation points of the set $\mathbb{Q} \times \mathbb{I} := \{(x_1, x_2) \ : \ x_1 \in \mathbb{Q},\ x_2 \in \mathbb{I}\} \subseteq \mathbb{R}^2$.

   (2) Is the set $\mathbb{Q} \times \mathbb{I}$ closed? Justify your answer.

   (3) Determine the interior, closure, and boundary of the set $\mathbb{Q} \times \mathbb{I}$.

3. Find the closure of the set $\{(-1)^n/n \ : \ n \in \mathbb{N}\}$ in $\mathbb{R}$. Use the standard metric on $\mathbb{R}$ induced by the absolute value function.

4. Let (M, d) be a metric space, and A, B be two subsets of M. Show the following:

   (1) $M \setminus \text{int}A = \text{cl}(M \setminus A)$.

   (2) $\text{cl}(A \cap B) \subseteq (\text{cl}A) \cap (\text{cl}B)$.