

## MATH 650 Spring 2007 Section 0101 Homework 2

Due Feb. 26, Monday, in class

**Page 1** P1. Let  $\{x_n\}_{n=1}^{\infty}$  and  $\{y_n\}_{n=1}^{\infty}$  be two sequences in  $\mathbb{R}$ . Prove the following statements.

(1)

$$\inf\{x_n\} \leq \liminf_{n \rightarrow \infty} x_n \leq \limsup_{n \rightarrow \infty} x_n \leq \sup\{x_n\};$$

(2) If  $x_n \leq y_n$  for all  $n$ , then

$$\liminf_{n \rightarrow \infty} x_n \leq \liminf_{n \rightarrow \infty} y_n, \quad \text{and} \quad \limsup_{n \rightarrow \infty} x_n \leq \limsup_{n \rightarrow \infty} y_n;$$

(3)

$$\liminf_{n \rightarrow \infty} x_n + \liminf_{n \rightarrow \infty} y_n \leq \liminf_{n \rightarrow \infty} (x_n + y_n), \quad \text{and} \quad \limsup_{n \rightarrow \infty} x_n + \limsup_{n \rightarrow \infty} y_n \geq \limsup_{n \rightarrow \infty} (x_n + y_n).$$

P.2 Guler's Manuscript, Section 2.11 (p.40): Problems 6, 7, 8, 9.

Hint to 6(a): use the continuity of  $f$  and Rolle's Theorem.