Math 430/603 Spring 2017 Homework #12

Due May 16, Tue in class

1. Textbook, Section 7.2, page 520: 4, 5;
2. Textbook, Section 7.5, page 556: 3, 4, 10;
3. Let A be an $n \times n$ symmetric positive definite (P.D.) matrix with the smallest positive eigenvalue $\lambda_n > 0$. Show that for any $\mu > -\lambda_n$, $A + \mu I$ is positive definite.
4. Let $A \in \mathbb{R}^{n\times n}$ be symmetric positive semidefinite, and $Q \in \mathbb{R}^{n\times n}$ be symmetric positive definite. Show that $A + \varepsilon Q$ is positive definite for any $\varepsilon > 0$.

   $\star$ **Don't turn in Problems 3 and 4.**

The following extra problem(s) are for **Math 603** *students only:*

5. Textbook, Section 7.2, page 520: 8;
6. Let A be an $n \times n$ matrix such that $A = c \cdot d^T$, where $c, d \in \mathbb{R}^n$ are two nonzero column vectors. Show that A is diagonalizable if and only if $c^T d \neq 0$.
7. Let $A \in \mathbb{R}^{n\times n}$ be symmetric and positive definite, and let $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ be the minimal and maximal real eigenvalues of A respectively. Show that

$$\lambda_{\min}(A)\|x\|_2^2 \leq x^T A x \leq \lambda_{\max}(A)\|x\|_2^2, \quad \forall\, x \in \mathbb{R}^n.$$