Math 411 Spring 2016 Homework #5

Due March 1, Tue in class

1. Textbook, 2.B, page 43: 8;

2. Textbook, 2.C, page 48: 1, 11;

3. Let $U = \{p \in \mathbb{P}_2(\mathbb{R}) : p(2) = p(5)\}$ be a subspace of $\mathbb{P}_2(\mathbb{R})$.

   (1) Find a basis of U.

   (2) Extend the basis in (1) to a basis of $\mathbb{P}_2(\mathbb{R})$.

   (3) Find a subspace W of $\mathbb{P}_2(\mathbb{R})$ such that $\mathbb{P}_2(\mathbb{R}) = U \oplus W$.

4. Let M, N be two subspaces of V with $M \cap N = \{0\}$. Let $B_M = \{v_1, \ldots, v_m\}$ be a basis of M, and $B_N = \{u_1, \ldots, u_p\}$ be a basis of N.

   (1) Show that $B_M \cap B_N = \{0\}$. (This shows that $B_M \cup B_N = \{v_1, \ldots, v_m, u_1, \ldots, u_p\}$.)

   (2) Show that $B_M \cup B_N$ is linearly independent and spans $M + N$.

   (3) Use the above results to show that $\dim(M \oplus N) = \dim M + \dim N$.

More practice problems: *Do not submit*

1. Textbook, 2.C, page 48: 8, 10;

2. Let $S = \{v_1, \ldots, v_n\}$ be a finite set in the vector space V, and $T = \{v_1,\ v_2 + \alpha_{2,1}v_1,\ v_3 + \alpha_{3,1}v_1 + \alpha_{3,2}v_2,\ \ldots,\ v_n + \sum_{s=1}^{n-1} \alpha_{n,s}v_i\}$ for some scalars $\alpha_{i,j}$.

   (1) Show that S spans V if and only if T spans V.

   (2) Show that S is linearly independent if and only if T is linearly independent.

   (3) Use the above results to show that if S is linearly independent, then for any given $z \in V$ and any scalars $\beta_1, \beta_2, \ldots, \beta_n$ and $\gamma_{i,j}$,

$$\dim\left\{\operatorname{span}\left(v_1 + \beta_1 z,\ v_2 + \gamma_{2,1}v_1 + \beta_2 z,\ \ldots,\ v_n + \sum_{s=1}^{n-1} \gamma_{n,s}v_i + \beta_n z\right)\right\} \geq n - 1.$$