Math 411 Spring 2016 Homework #6

Due March 8, Tue in class

1. Textbook, 3.A, page 57: 1, 4;

2. Let $a \in F$ (where F is a field). Consider the mapping $T : F^\infty \to F^\infty$ given by

$$T\big((x_1, x_2, x_3, \ldots)\big) = (a, x_1, x_2, x_3, \ldots).$$

   Show that T is a linear map if and only if $a = 0$.

3. Let $\mathcal{L}(V, W)$ be the set of all linear maps from the vector space V to the vector space W (over the field F). Show that for any $T \in \mathcal{L}(V, W)$, there exists a linear map $T' \in \mathcal{L}(V, W)$ such that $T + T' = \mathbf{0}$, where $\mathbf{0} \in \mathcal{L}(V, W)$ is the zero map.
   (*Hint*: for a given $T \in \mathcal{L}(V, W)$, construct a function $T' : V \to W$ such that $T + T' = \mathbf{0}$, and show that T' is a linear map, i.e., $T' \in \mathcal{L}(V, W)$.)

More practice problems: *Do not submit*

1. Textbook, 3.A, page 57: 7, 8, 9, 12;

2. Show that $\mathcal{L}(V, W)$ satisfies the axioms A1-A3 and M1-M4 of the definition of vector space (A4 has been shown in P.3 above) and thus is a vector space.