MATH 487 Fall 2013 Homework #4

Due Nov. 14, Thu in class

- Textbook, Section 4.13, p.159: 1(a).
- Find all equilibria of each of the following systems on $\mathbb{R}^2$, and determine local stability/asympotitic stability at each equilibrium via linearization:
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  $$\begin{aligned} \dot{x} &= x^2 - y^2 - 1 \\ \dot{y} &= 2y \end{aligned}$$
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  $$\begin{aligned} \dot{x} &= -4x - 2y + 4 \\ \dot{y} &= xy \end{aligned}$$
- We prove one version of Gronwall's inequality as follows. Let $g(t)$ be a real-valued differentiable function on $[0, T]$, and $\lambda \in \mathbb{R}$. Suppose that $\dot{g}(t) + \lambda g(t) \geq 0$ for all $t \in [0, T]$. Show that
  $$g(t) \geq e^{-\lambda t} \cdot g(0), \quad \forall\, t \in [0, T].$$
  (*Hint*: consider the derivative of $e^{\lambda t} \cdot g(t)$.)