MATH 603 Fall 2013 Homework #6

Due Dec. 10, Tue in class

- Textbook, Section 6.1, p.472: 6.1.3(f), 6.1.8, 6.1.11, 6.1.12;
- Textbook, Section 6.2, p.483: 6.2.14(a), 6.2.15(a);
- Textbook, Section 7.1, p.500: 7.1.4, 7.1.9, 7.1.10, 7.1.18(a-b);
- Textbook, Section 7.2, p.520: 7.2.8 (where $\rho(A) := \max_{\lambda\in\sigma(A)} |\lambda|$ is the spectral radius of A), 7.2.14, 7.2.21;

Don't turn in the following problems

- Textbook, Section 7.5, p.556: 7.5.3, 7.5.4, 7.5.10;
- Solve the following problems:
  (1) Let $A \in \mathbb{R}^{n\times n}$ be symmetric and positive semidefinite, and $Q \in \mathbb{R}^{n\times n}$ be symmetric positive definite. Show that $A + \varepsilon Q$ is positive definite for any $\varepsilon > 0$.
  (2) Let $A \in \mathbb{R}^{n\times n}$ be symmetric and positive definite, and let $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ be the minimal and maximal real eigenvalues of A respectively. Show that
  $$\lambda_{\min}(A)\|x\|_2^2 \le x^T A x \le \lambda_{\max}(A)\|x\|_2^2, \quad \forall\ x \in \mathbb{R}^n.$$
  (3) Let $A \in \mathbb{R}^{n\times n}$ be symmetric and positive definite, and $B \in \mathbb{R}^{n\times n}$ be symmetric. Use (2) to show that there exists $\eta > 0$ such that $A + \varepsilon B$ remains positive definite for all $\varepsilon \in [-\eta, \eta]$.