Math 302 Fall 2011 Homework #8

Due Nov. 7, Mon. in class

1. Textbook, page 181, Section 4.1, 1, 2, and 3.
2. Let $\|\cdot\|$ be a norm on $\mathbb{R}^n$. Show that $f : \mathbb{R}^n \to \mathbb{R}$ defined by $f(x) := \|x\|$ is continuous on $\mathbb{R}^n$.
3. Let (M, d) be a metric space. Given $z \in M$, define $f : M \to \mathbb{R}$ by $f(x) := d(z, x)$. Show that f is continuous on M.
4. Define $f : \mathbb{R}^2 \to \mathbb{R}$ as follows:

$$f(x, y) := \begin{cases} 0, & \text{if } x \text{ is rational and } y \text{ is irrational} \\ 1, & \text{otherwise} \end{cases}$$

   Show that f is discontinuous at any point of $\mathbb{R}^2$. (*Hint*: recall that for any real number a, there is a rational (resp. irrational) sequence converging to a.)

5. Let $f, g : (M, d) \to (V, \|\cdot\|)$ be two functions, where (M, d) is a metric space and $(V, \|\cdot\|)$ is a normed vector space.
   (1) Use the sequential argument to show that if f and g are continuous at $x_0 \in M$, so is $f + g$;
   (2) Let λ be a scalar. Use the $\varepsilon - \delta$ definition to show that if f is continuous at $x_0 \in M$, so is λf.