# MATH 650 Foundations of Optimization
# Section 0101 Spring 2008

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- Lectures: Mon and Wed, 5:30–6:45 pm, at Math/Psyc 401
- Office hours: Mon and Wed, 4:30–5:30 pm or by appointment
- Course web-page: http://www.math.umbc.edu/~shenj
- Prerequisites: Linear algebra and multi-variable calculus
- Text: Manuscript of *Foundations of Optimization* by Osman Guler, 2008
- Other reference books:
  - M.S. Bazaraa, H.D. Sherali, and C.M. Shetty. *Nonlinear Programming: Theory and Applications*, John Wiley & Sons Inc., New York, 2nd edition, 1993;
  - D.P. Bertsekas. *Nonlinear Programming*, Athena Scientific, Belmont, MA, 2nd edition, 1999;
  - J.M. Borwein and A.S. Lewis. *Convex Analysis and Nonlinear Optimization*, CMS Books in Mathematics, Springer-Verlag, New York, 2000;
  - S. Boyd and L. Vandenberghe. *Convex Optimization*, Cambridge University Press, 2004.
- **Tentative exam dates**:
  - Exam I: Wed, Feb. 27 (in class)
  - Exam II: Mon, Apr. 7 (in class)
  - Final Exam: Mon, May 19

  These dates are subject to change.

**Course Description** Optimization techniques are widely used in diverse areas, ranging from engineering, operations research, economics, business/finance to biological systems, management systems, government/military activities, and social systems. They have also stimulated a deep and elegant mathematical theory which becomes the foundation of various optimization methods and algorithms. This course treats the basic theory of continuous optimization in finite dimensions. Fundamental techniques from multivariable calculus (e.g. G/F-differentiability, multivariate Taylor series expansions, the Implicit Function Theorem) and convex analysis (such as convex sets/functions, separation theorems, convex polyhedra, linear inequalities, the theorems of alternative) will be presented in this course. Based on these techniques, unconstrained and constrained optimization problems, i.e., those subject to equality and/or inequality constraints, will be discussed. Typical topics include the Lagrange multiplier approach, Fritz John and Karush-Kuhn-Tucker (KKT) conditions, constraint qualifications, necessary and sufficient optimality conditions, duality theory, and general methods for various linear and nonlinear programming problems.

**Objectives** This course will equip you with basic concepts and theoretical tools of continuous optimization.

**Homework** Homework will be assigned about every two weeks, and each homework set contains some proofs and computation problems. Homework must be turned in by specified time; no late due will be accepted. Please present your answers neatly and show all your work; answers without supporting work may not receive full credit.

**Exams** There will be two (2) in-class exams, and one (1) final exam (see the tentative dates on the front page). Each mid-term exam mainly focuses on topics covered in that month, but the final exam will be comprehensive. Please be alerted that (i) there will be *no* optional final exam, and all the exams are closed-book but you will be allowed to bring one or two crib sheets to the exams; (ii) calculators and other computing devices are *not* allowed for any exam.

**Grading Policy** The grading scheme is as follows:

- homework: 30%
- mid-term exams: 40% (18% for the first and 22% for the second)
- final exam: 30% (the final is comprehensive)

The letter grade will be computed based upon the numerical grade:

$A: \geq 85$; $B: 84-73$; $C: 72-60$; $D: 59-50$; $F: < 50$

**Academic Integrity**

- The UMBC Academic Integrity Statement:

  "*By enrolling in this course, each student assumes the responsibilities of an active participant in UMBC's scholarly community in which everyone's academic work and behavior are held to the highest standards of honesty. Cheating, fabrication, plagiarism, and* *helping others to commit these acts are all forms of academic dishonesty, and they are wrong. Academic misconduct could result in disciplinary action that may include, but is not limited to, suspension or dismissal. To read the full Student Academic Conduct Policy, consult the UMBC Student Handbook, the Faculty Handbook, or the UMBC Policies section of the UMBC Directory [or for graduate courses, the Graduate School website]*".

- All work in homework or an exam must be your own; collaborating on an exam is *not* permitted. Discussions with other students on homework problems are allowed and encouraged, but you should present your own work in the final turn-in; simply copying other people's work is violation of UMBC's academic integrity code.

- If you wish to contest a graded exam, you must make the appeal within *one week* of the return date to the class. All appeals should be made in writing to the instructor with a signed and dated note on the exam. End of the semester appeals for earlier exams will be ignored.

**Tentative Schedule and Topics**

Here is a list of the tentative schedule and topics we will be covering in chronological order with text citations:

| No. | Week | Topic | Section(Guler) |
|---|---|---|---|
| 1 | Jan. 28–Feb. 1 | Introduction, one-variable Taylor's formula, Multi-variable differentiation | 1.1, 1.2, 1.3 |
| 2 | Feb. 4–Feb. 8 | Chain rule, multi-variable Taylor's formula, Unconstrained optimization, examples | 1.4, 1.5, 2.3–2.5 |
| 3 | Feb. 11–Feb. 15 | Lyusternik's theorem, Equality constrained optimization | 1.6 |
| 4 | Feb. 18–Feb. 22 | Weierstrass' theorem and generalization | 2.1, 2.2 |
| 5 | Feb. 25–Feb. 29 | Affine geometry, Exam I | 3.1 |
| 6 | Mar. 3–Mar. 7 | Convex sets, convex functions | 3.2, 3.3 |
| 7 | Mar. 10–Mar. 14 | Differentiable convex functions, Optimization on a convex set | 3.4, 3.5, 3.6 |
| 8 | Mar. 17–Mar. 21 | No class (spring break) | |
| 9 | Mar. 24–Mar. 28 | Projection onto a convex set, Separation of convex sets | 4.1, 4.2 |
| 10 | Mar. 31–Apr. 4 | The D-M lemma, polyhedral sets and cones | 4.3, 6.1, 6.2 |
| 11 | Apr. 7–Apr. 11 | Exam II, Motzkin's transposition theorem | 6.3 |
| 12 | Apr. 14–Apr. 18 | Affine Farkas' lemma, Fritz John conditions | 6.4, 9.1, 9.2, |
| 13 | Apr. 21–Apr. 25 | Constraint qualifications, examples | 9.3, 9.4 |
| 14 | Apr. 28–May 2 | 2nd-order optimality conditions, saddle points | 9.6, 11.1 |
| 15 | May 5–May 9 | Constrained minimization, strong duality | 11.2–11.4 |
| 16 | May 12–May 16 | Review | |

Please notice that the topics and dates in the list are subject to change, depending on the class progress.