Math 302 Fall 2011 Homework #10

Due Nov. 21, Mon. in class

Use the standard Euclidean metric for a Euclidean space unless otherwise specified.

1. Let A be a path connected set in a metric space (M, d) and f be a continuous function on M. Show that $f(A)$ is path connected.

2. Show that the unit circle in $\mathbb{R}^2$ is path connected, i.e. $\mathbb{S}^1 := \{(x, y) : x^2 + y^2 = 1\}$ is path connected. (*Hint*: you may assume that the functions sin and cos are continuous on $\mathbb{R}$.)

3. (1) Let f be a continuous function on a metric space (M, d) and A be a nonempty set in M. Show that if f is uniformly continuous on the closure of A, so is on A.

   (2) Let $g : \mathbb{R}^2 \to \mathbb{R}$ be continuous on $\mathbb{R}^2$. Let $(a, b]$ and (c, d) be two intervals in $\mathbb{R}$. Use (1) to show that g is uniformly continuous on $(a, b] \times (c, d)$.

4. Use the negation of uniform continuity to show that the function $f(x) := x^{-2}$ is *not* uniformly continuous on the interval $(0, 2]$. (*Hint*: find sequences (x_n) and (y_n) in $(0, 2]$ such that $(x_n - y_n) \to 0$ but $(f(x_n) - f(y_n)) \ldots$.)