Math 650 Fall 2016 Homework #8

Do not turn in

1. Consider the optimization problem on $\mathbb{R}^n$:

$$\min_{(x_1,\ldots,x_n)\in\mathbb{R}^n} \sum_{i=1}^n \frac{c_i}{x_i} \qquad \text{subject to} \qquad \sum_{i=1}^n a_i x_i = b, \quad \text{and} \quad x_i \geq 0, \ \ \forall\, i = 1,\ldots,n,$$

where a_i, c_i, b are all positive constants. Find all the Fritz John point(s), KKT point(s), and optimal solution(s).

2. Consider the optimization problem on $\mathbb{R}^2$:

$$\min_{(x,y)\in\mathbb{R}^2} \ln x - y \qquad \text{subject to} \qquad x^2 + y^2 \leq 4, \quad \text{and} \quad x \geq 1.$$

   (1) Show that the constraint set is convex and has nonempty interior.

   (2) Use a suitable constraint qualification to show that any Fritz-John point is a KKT point.

   (3) Find all the KKT points, and use the second-order optimality condition to determine (local) minimizer(s).

3. Consider the optimization problem on $\mathbb{R}^2$:

$$\min\ x_1^2 + x_2^2, \quad \text{subject to} \quad -x_1 - x_2 + 4 \leq 0, \quad (x_1, x_2) \in C \equiv \mathbb{R}^2_+.$$

Solve the following problems:

   (1) Show that the optimization problem has a unique global minimizer.

   (2) Show that any Fritz-John point is a KKT point (i.e. $\lambda_0 \neq 0$).

   (3) Find the KKT point and show that it is the global minimizer using the 2nd-order optimality condition. Determine the optimal value of the objective function.

   (4) (Optional) Find the Lagrangian dual function.

   (5) (Optional) Solve the corresponding Lagrangian dual problem and show that there is no duality gap.

More practice problems: *Do not submit*

1. Consider the optimization problem on $\mathbb{R}^2$:

$$\min_{(x,y)\in\mathbb{R}^2} (x+1)^2 - y^2 \qquad \text{subject to} \qquad x + y \leq 0, \quad \text{and} \quad x^2 + y^2 = 1.$$

The following points $(x, y) \in \mathbb{R}^2$ are candidates for a local minimizer:

$$\Big(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\Big); \qquad (-1, 0); \qquad (0, -1); \qquad \Big(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\Big).$$

   (1) Determine which of the above points are KKT points.

   (2) Determine which of the KKT points satisfy the second order necessary conditions.

   (3) Determine which of the KKT points satisfy the second order sufficient conditions.