Math 430/603 Spring 2017 Homework #2

Due Feb. 14, Tue in class

1. Textbook, Section 3.5, page 103: 8(a-b);
2. Textbook, Section 3.6, page 113: 4, 7;
3. Textbook, Section 3.7, page 122: 3(b-c), 5;
4. Let $A = [a_{ij}]$ be an $m \times n$ matrix.
   (1) Find the trace of AA^T in terms of a_{ij}'s.
   (2) Show that trace$(AA^T) = 0$ if and only if A is the zero matrix.
5. Let A be an $m \times m$ invertible matrix, B be an $n \times n$ invertible matrix, and C be an $m \times n$ matrix.
   (1) Use the block matrix multiplication rule to show that
   $$\begin{bmatrix} A & C \\ 0 & B \end{bmatrix} \cdot \begin{bmatrix} A^{-1} & -A^{-1}CB^{-1} \\ 0 & B^{-1} \end{bmatrix} = \begin{bmatrix} I_m & 0 \\ 0 & I_n \end{bmatrix}.$$
   (2) Is the matrix $\begin{bmatrix} A & C \\ 0 & B \end{bmatrix}$ invertible? If so, what is its inverse?
6. Suppose that A, B and $A + B$ are all invertible. Show that $A(A + B)^{-1}B = (A^{-1} + B^{-1})^{-1}$.

The following extra problem(s) are for **Math 603** *students only:*

7. Recall that for a skew symmetric matrix S and any scalar α, we have shown that $I + \alpha S$ is invertible. Let $A = (I - S)(I + S)^{-1}$, where S is skew symmetric.
   (1) Show that $A^T = (I - S)^{-1}(I + S)$;
   (2) Show that $(I - S)^{-1}(I + S) = (I + S)(I - S)^{-1}$, and use it to show $A^{-1} = A^T$.