Math 430/603 Spring 2017 Homework #3

Due Feb. 21, Tue in class

1. Textbook, Section 4.1, page 167: 1(a, d, e), 2(b, c, i), 9;
2. Textbook, Section 4.2, page 178: 7 (*Hint*: show that $Ax = 0$ implies $x = 0$.)
3. Show that the set of all $n \times n$ lower triangular matrices is a subspace of $\mathcal{M}^{n\times n}$.
4. Let S be a subspace of a vector space V, and z be a vector in V. Define the set $S + z := \{x + z : x \in S\}$.
   (1) Show that $z \in S$ if and only if $-z \in S$.
   (2) Show that $S + z$ is a subspace of V only if $z \in S$. (*Hint*: use $0 \in S + z$ and (1).)
   (3) Show that if $z \in S$, then $S + z$ is a subspace of V. (*Hint*: show that $S + z$ is closed under vector addition and scalar multiplication.)

The following extra problem(s) are for **Math 603** *students only:*

5. Textbook, Section 4.2, page 178: 9;
6. Let V be a vector space. Show that $V - V = V$ and $\alpha V = V$ for any nonzero scalar α, where $V - V := \{x - y : x \in V,\ y \in V\}$ and $\alpha V := \{\alpha x : x \in V\}$.