Math 302 Fall 2011 Homework #2

Due Sept. 21, Wed. in class

1. Let (M, d) be a metric space. Show the following inequality:

$$|d(x, y) - d(z, y)| \leq d(x, z), \quad \forall\ x, y, z \in M$$

2. Textbook, page 108, Ex. 3, 4. (*note*: each problem carries the standard metric of $\mathbb{R}^n$ induced by the Euclidean norm $\| \cdot \|_2$.)

3. Consider the metric induced by the 1-norm on $\mathbb{R}^2$: $d(x, y) = |x_1 - y_1| + |x_2 - y_2|, \forall\ x, y \in \mathbb{R}^2$. Let the set $A = \{(x_1, x_2) \in \mathbb{R}^2 : x_1 \geq 1 \text{ and } x_2 \geq 1\}$. Find the interior of A (using the given metric), and prove your answer.

4. Let (M, d) be a metric space and A, B be two subsets of M. Show the following:

   (1) if $A \subseteq B$, then $\text{int} A \subseteq \text{int} B$;

   (2) $\text{int}(A \cap B) = (\text{int} A) \cap (\text{int} B)$.