

## Math 411 Spring 2016 Homework #8

Due March 29, Tue in class

1. Textbook, 3.D, page 88: 1, 9, 10;
2. Recall that  $C[a, b]$  denotes the vector space of real-valued continuous functions on the real interval  $[a, b]$ . Let  $T : C[a, b] \rightarrow C[a, b]$  be defined by  $T(f(t)) = e^t \cdot f(t)$ . Show that  $T$  is linear and bijective, and thus an isomorphism. (You may assume that  $e^t$  is continuous on  $\mathbb{R}$ .)
3. Let  $T : V \rightarrow W$  be a linear map, where  $V$  and  $W$  are finite-dimensional vector spaces with  $\dim(V)$  being odd. Prove that the null space of  $T$  and the range of  $T$  are not isomorphic.
4. Let the map  $T : \mathbb{P}_2(\mathbb{R}) \rightarrow \mathbb{P}_2(\mathbb{R})$  be  $T(p) = p' - 2p$ , where the prime represents the derivative. Show that  $T$  is a linear map, and find the matrix representation of  $T$  with respect to the standard basis  $\{1, t, t^2\}$ . Also determine the null space, range, nullity, and rank of  $T$ .

**More practice problems:** *Do not submit*

1. Textbook, 3.C, page 78: 6;
1. Textbook, 3.D, page 88: 3, 11, 12;