# MATH 221 Brief Solution to Sample Final

**Page 1**

P1. (1) (i) $h \neq 0$; (ii) $h = 0,\ k = 0$; (iii) $h = 0,\ k \neq 0$.

(2) when $h = 1$ and $k = 0$, the equation has a unique solution:

$$\mathbf{x} = \begin{bmatrix} \frac{19}{6} \\ -\frac{5}{3} \\ 0 \end{bmatrix}.$$

P2. (1) The set of the two vectors is orthogonal and each of them is a *unit* vector. Thus the set has no zero vector. Therefore, the set is linearly independent and is a basis of $\mathbb{R}^2$.

(2) See the text.

(3) The set is a null space of $A$ where

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \end{bmatrix}.$$

(4) The eigenspace of $A$ associated with $\lambda = 2$ is $\mathrm{Nul}(A - 2I)$. By the Rank Theorem, its dimension is 1.

P3. (1) Its reduced echelon form is $I_{3\times 3}$.

(2) Yes.

(3) $\det A = 4$

(4) No, since $A$ is invertible.

P4. (1) Its characteristic equation is $\det(A - \lambda I) = \lambda\,(\lambda - 2)\,(3 - \lambda)$

(2) $\lambda = 0, 2, 3$

(3) Three bases are

$$\lambda = 0 : \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\}; \quad \lambda = 2 : \left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \right\}; \quad \lambda = 3 : \left\{ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}.$$

(4) Yes, since $A$ has three distinct eigenvalues, and

$$P = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 0 & & \\ & 2 & \\ & & 3 \end{bmatrix}$$

P5. $\det P = 2$ and $P$ is invertible.

P6. (1) Using the Gram-Schmidt process, we obtain an orthogonal basis for $H$:

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} \frac{1}{2} \\ 0 \\ -\frac{1}{2} \\ 0 \end{bmatrix}$$

(2) The closed vector in $H$ is

$$\hat{\mathbf{y}} = \begin{bmatrix} 2 \\ 0 \\ -1 \\ 0 \end{bmatrix}$$

P7. (1) False

(2) False

(3) True

(4) True

(5) False

(6) True

(7) False