MATH 710 Special Topics in Applied Optimization
Section 0101 Fall 2007

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- Lectures: Mon and Wed, 5:30–6:45 pm at ACIV 011
- Office hours: Mon and Wed, 3:00–4:00 pm or by appointment
- Course web-page: http://www.math.umbc.edu/~shenj
- Prerequisites: Math 650 or its equivalence, basic courses in ODE/ dynamical systems/systems & control theory
- Reference books:
  1. R.W. Cottle, J.S. Pang, and R.E. Stone. *The Linear Complementarity Problem*, Academic Press Inc., Cambridge, 1992;
  2. F. Facchinei and J.S. Pang. *Finite-Dimensional Variational Inequalities and Complementarity Problem*, Springer-Verlag, New York, 2003;
  3. R.W. Cottle. http://www.stanford.edu/class/msande316/slide.shtml
  4. H. Khalil. *Nonlinear Systems*, 2nd Edition, Prentice Hall, 1996;
  5. C.T. Chen. *Linear System Theory and Design*, Oxford University Press, 1984;
  6. E.D. Sontag. *Mathematical Control Theory: Deterministic and Finite Dimensional Systems*. 2nd edition, Springer-Verlag, New York, 1998;
  7. G.V. Smirnov. *Introduction to the Theory of Differential Inclusions*, Graduate Studies in Mathematics, Vol.41, American Math Society, Providence, 2002;
  8. K. Deimling. *Multivalued Differential Equations*. Walter de Gruyter, Berlin, 1992.

Course Description This special topic course focuses on fundamental issues of complementarity systems and their generalizations, e.g., differential variational inequalities. A complementarity system is a dynamical system defined by an ODE coupled with a finite-dimensional complementarity problem. Such systems and their generalizations provide a powerful and unified modeling paradigm for a wide spectrum of applied nonsmooth/switched/hybrid dynamical systems, particularly those subject to unilateral/inequality constraints and possessing state dependent mode switchings. Typical examples include contact mechanical and robotic systems, switched electrical networks and control systems, constrained economic and traffic systems, and biochemical reaction networks. Belonging to the general framework of differential inclusions, complementarity systems enjoy specific structures that can be exploited via mathematical programming and nonsmooth analytic tools. This course aims at an introduction to the recent advancement in analysis and control of complementarity systems and their applications. The course will begin with a review of basic results and analytic tools in complementarity theory, linear/nonlinear systems theory, and stability theory. With these techniques in hand, fundamental solution properties, such as solution existence and uniqueness and Zeno analysis, and control theoretical issues, for example, stability and observability, as well as certain numerical aspects of complementarity systems will be addressed.

Objectives This course will offer a solid introduction to basic theoretical results and wide applications of complementarity systems and differential variational inequalities.

Homework Homework will be assigned about every two weeks. Homework must be turned in at specified time; no late due will be accepted. Please present your answers neatly and show all your work; answers without supporting work may not receive full credit.

Exam There will be one in-class mid-term exam that will cover the topics taught before the test. Please be alerted that (a) the exam is closed-book but you can bring one crib sheet to the exam; (b) calculators and other computing devices are *not* allowed for the exam.

Project Each student in the class must select a course-relevant topic, carry out a course project, which will be due at the end of the term, and present his/her work in class. The detailed project and presentation policy will be announced in the middle of the term.

Grading Policy The grading scheme is as follows:

- 30% homework
- 25% mid-term exam
- 40% project
- 5% class participation and involvement

The letter grade will be computed based upon the numerical grade:

$$A : \geq 85; \quad B : 84 - 73; \quad C : 72 - 60; \quad D : 59 - 50; \quad F :< 50$$

Academic Integrity

- The UMBC Academic Integrity Statement:

  "*By enrolling in this course, each student assumes the responsibilities of an active participant in UMBC's scholarly community in which everyone's academic work and behavior are held to the highest standards of honesty. Cheating, fabrication, plagiarism, and helping others to commit these acts are all forms of academic dishonesty, and they are wrong. Academic misconduct could result in disciplinary action that may include, but is* *not limited to, suspension or dismissal. To read the full Student Academic Conduct Policy, consult the UMBC Student Handbook, the Faculty Handbook, or the UMBC Policies section of the UMBC Directory [or for graduate courses, the Graduate School website]*".

- All work in homework, exam, or project must be your own. Discussions with other students on homework problems are allowed, but you should present your own work in the final turn-in; simply copying other people's results is a violation of UMBC's academic integrity code.
- If you wish to contest a graded homework or exam, you must make the appeal within *one week* of the return date. All appeals should be made in writing to the instructor with a signed and dated note on the homework or exam.

Tentative Topics

1. Introduction to Linear Complementarity Problem:
   - feasibility and global existence/uniqueness, P-matrix class
   - general solution properties: polyhedral multi-functions, piecewise affine functions, upper Lipschitz property
   - singleton property of polyhedral multi-functions
2. Introduction to ODE and Control Theory:
   - local Lipschitz property and well-posedness (i.e. solution existence and uniqueness)
   - Lyapunov stability
   - linear systems theory: controllability, observability, passivity
3. Fundamental Dynamic Properties of (Linear) Complementarity Systems:
   - well-posedness: singleton case and multi-valued case
   - solution dependence property
   - hybrid system formulation
   - Zeno analysis and switching properties
4. Stability and Control Theoretical Issues of (Linear) Complementarity Systems:
   - Lyapunov stability
   - observability
5. Numerical Analysis of (Linear) Complementarity Systems:
   - numerical schemes
   - convergence analysis

Please be aware that the topics in the list are subject to change, depending on the class progress.