Math 411 Spring 2016 Homework #10

Due April 12, Tue in class

1. Textbook, 5.A, page 138: 25, 29;
2. Textbook, 5.B, page 153: 3;
3. Let A be an $n \times n$ invertible matrix.
   (1) Let λ be a nonzero scalar. Then λ is an eigenvalue of A if and only if $1/\lambda$ is an eigenvalue of A^{-1};
   (2) Suppose, in addition, that A is upper triangular. Then the eigenvalues of A^{-1} are $1/a_{ii}$, $\forall\ i = 1, \ldots, n$.
4. Let λ_1 and λ_2 be two distinct eigenvalues of a linear operator T. Show that $\text{Null}(T - \lambda_1 I) \cap \text{Null}(T - \lambda_2 I) = \{0\}$.

More practice problems: *Do not submit*

1. Textbook, 5.A, page 138: 23;
2. Let the linear operator $T \in \mathcal{L}(V)$ be such that $T^2 = T$.
   (1) Show that the range of $I - T$ equals the null space of T (*Hint*: $x = T(x) + (I - T)(x)$ for any $x \in V$);
   (2) Show that V is a direct sum of the null space of T and the range of T.
3. Show that the product of two $n \times n$ upper triangular matrices is upper triangular, and the inverse of an $n \times n$ upper triangular matrix is upper triangular.