

Math 430/603 Spring 2017 Homework #8

Due April 11, Tue in class

1. Textbook, Section 5.1, page 276: 5, 6;
2. Textbook, Section 5.4, page 303: 1(d), 2, 8, 12;
3. Let $x, y \in \mathbb{R}^n$ and $\|\cdot\|_2$ be the Euclidean norm on \mathbb{R}^n . Show that $\|x + y\|_2^2 = \|x\|_2^2 + \|y\|_2^2$ if and only if x is orthogonal to y (i.e. $x^T y = 0$).
4. Let A be an $m \times n$ real matrix. Show that for any vectors $x \in R(A)$ and $y \in N(A^T)$, x is orthogonal to y .
5. Let A be an $n \times n$ real matrix such that $AA^T = A^T A$. Show that for any vectors $x \in R(A)$ and $y \in N(A)$, x is orthogonal to y . (*Hint:* use equations (4.5.5) and (4.5.6) in the textbook.)
6. Use the Gram-Schmidt procedure to construct an orthonormal basis from the following basis for \mathbb{R}^3 :

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}.$$

The following extra problem(s) are for Math 603 students only:

7. Textbook, Section 5.3, page 292: 4;
8. Textbook, Section 5.4, page 303: 13(a).