

Math 302 Fall 2011 Homework #6

Due Oct. 24, Mon. in class

1. Prove the following sets in \mathbb{R}^2 is *not* sequentially compact (using the standard metric on \mathbb{R}^2):
 - (1) $\{(x, y) \in \mathbb{R}^2 \mid -1 \leq x \leq 1, 0 \leq y < 1\}$;
 - (2) $\{(x, y) \in \mathbb{R}^2 \mid x \geq 0, y \leq 0\}$
2. Let A be a sequentially compact set in the metric space (M, d) . Show that any closed subset of A is also sequentially compact.
3. Let (M, d) be a metric space. Show that $A \subseteq M$ is sequentially compact if and only if every infinite subset of A (i.e. a subset that contains infinitely many elements) has an accumulation point in A .
4. Let (x_n) be a sequence in a metric space that converges to x^* . Let $A := \{x_1, x_2, \dots, x_n, \dots\} \cup \{x^*\}$. Show that A is sequentially compact.