MATH 603 Fall 2013 Homework #2

Due Oct. 3, Thu in class

- Textbook, Section 4.1, p.167: 4.1.2(a, c, d, g, i), 4.1.11;
- Textbook, Section 4.2, p.178: 4.2.7, 4.2.8 (*Hint*: use $N(A^T) = R(P_2^T)$ and $R(A) = N(P_2)$), 4.2.11, 4.2.13;
- Let S_α be a family of subspaces of the vector space V indexed by α.
  - Show that the intersection of all S_α (i.e. $\bigcap_\alpha S_\alpha$) is a subspace.
  - Let U be a nonempty subset of V. Show that there exists a unique smallest subspace containing U. Note that the subspace W is a smallest subspace containing U if for any subspace X containing U, $W \subseteq X$. (*Hint*: consider the intersection of all subspaces of V containing U.)
  - Is the union of two subspaces a subspace? If so, prove it; otherwise, give a counterexample.
- Solve the following problems:
  - Construct a set A in $\mathbb{R}^2$ such that $A + A \neq 2A$.
  - Let V be a vector space. Show that $V + V = 2V = V$ and $V - V = V$.