Math 302 Fall 2011 Homework #1

Due Sept. 14, Wed. in class

1. Textbook, page 98, Ex. 12 (a), (b).

2. On the Euclidean space $\mathbb{R}^3$, define for each $x = (x_1, x_2, x_3) \in \mathbb{R}^3$,

$$\|x\|_\star := |x_1| + 2|x_2| + 3|x_3|, \qquad \|x\|_\infty := \max(|x_1|, |x_2|, |x_3|)$$

   (1) Show that $\|\cdot\|_\star$ is a norm on $\mathbb{R}^3$.

   (2) Show that $\|\cdot\|_\infty$ is a norm on $\mathbb{R}^3$.

3. Recall that $C([0,1])$ is the vector space of all continuous functions $f : [0,1] \to \mathbb{R}$ on the interval $[0,1]$. Let $c > 0$ be a given positive real number. For any $f \in C([0,1])$, define

$$\|f\| := \max\left\{c \cdot |f(t)| : t \in [0,1]\right\}$$

   (1) Explain why $\|f\|$ exists for any $f \in C([0,1])$. (*hint*: think of Theorem 5.3.4 of Bartle and Sherbert's book.)

   (2) Prove that $\|\cdot\|$ is a norm on $C([0,1])$.

4. Prove that the discrete metric d discussed in class satisfies the triangle inequality, i.e. $d(x,y) \leq d(x,z) + d(y,z), \forall\, x, y, z \in M$.

5. Let d_1 and d_2 be two metrics on a set M. Define the sum $d_1 + d_2$ on M as

$$(d_1 + d_2)(x,y) := d_1(x,y) + d_2(x,y), \quad \forall\ x, y \in M$$

   Show that $d_1 + d_2$ is a metric on M.