MATH 603 Fall 2013 Homework #4

Due Nov. 7, Thu in class

- Textbook, Section 5.1, p.276: 5.1.5;
- Textbook, Section 5.2, p.285: 5.2.3;
- Textbook, Section 5.3, p.292: 5.3.4;
- Textbook, Section 5.4, p.304: 5.4.9, 5.4.12, 5.4.16;
- Consider the pseudo-norm on $\mathbb{R}^n$: let $x \in \mathbb{R}^n$,

$$\|x\|_0 := \text{the number of nonzero elements in } x.$$

  - Show that (i) $\|x\|_0 \geq 0, \forall x \in \mathbb{R}^n$, and $\|x\|_0 = 0$ if and only if $x = 0$; and (ii) $\|x+y\|_0 \leq \|x\|_0 + \|y\|_0, \forall\, x, y \in \mathbb{R}^n$;
  - Explain why $\|\cdot\|_0$ is *not* a norm.

- Let $A \in \mathbb{R}^{m\times n}$ be nonzero, and $\|A\|_2$ be the induced 2-norm of A. It is shown that $\|A\|_2 = \max_{\|x\|_2=1,\ \|y\|_2=1} y^T Ax$, where $x \in \mathbb{R}^n$ and $y \in \mathbb{R}^m$, in two steps as follows.
  - Show that $\max_{\|x\|_2=1,\ \|y\|_2=1} y^T Ax \leq \|A\|_2$.
  - Show that

$$\max_{\|x\|_2=1,\ \|y\|_2=1} y^T Ax \geq \|A\|_2.$$

  (*Hint*: show that

$$\|A\|_2 = \max_{\|x\|_2=1,\ Ax\neq 0} \left(\frac{Ax}{\|Ax\|_2}\right)^T Ax.$$

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