Error propagation

- We have seen that the result of our aperture photometry is the *counts* **C** of a given star and the associated *error* σ_C.
- In AstroImageJ these quantities are called **Source – Sky_T$_x$** and **Source_Error_T$_x$** respectively
- We have also seen that in the first step of our analysis, we convert the *counts* to ***instrumental magnitude***.
- The question now is: what is the error on the instrumental magnitude?
- To correctly calculate the error on the instrumental magnitude we need to go through the process of ***error propagation***

Error propagation

- For a quantity y that is a function of a single variable x, y = f(x), the general formula is:

$$\sigma_y = \sqrt{\left(\frac{dy}{dx}\right)^2 * \sigma_x^2}$$

- The equation for the instrumental magnitude is:

$$m_{inst} = -2.5 * \log\left(\frac{C}{t_{\exp}}\right)$$

- So we need to calculate:

$$\sigma_{m_{inst}} = \sqrt{\left(\frac{dm_{inst}}{dC}\right)^2 * \sigma_C^2}$$

$$\boxed{m_{inst} = -2.5 * \log\left(\frac{C}{t_{\exp}}\right)}$$

and

$$\boxed{\sigma_{m_{inst}} = \sqrt{\left(\frac{dm_{inst}}{dC}\right)^2 * \sigma_C^2}}$$

- We set $C' = \dfrac{C}{t_{\exp}}$ and we make use of the chain rule of derivatives:

$$\sigma_{m_{inst}} = \sqrt{\left(\frac{dm_{inst}}{dC}\right)^2 * \sigma_C^2} = \sqrt{\left(\frac{dm_{inst}}{dC'}\right)^2 * \left(\frac{dC'}{dC}\right)^2 * \sigma_C^2} = \sqrt{\left(\frac{d}{dC'}\left[-2.5 * \log\left(C'\right)\right]\right)^2 \left(\frac{d}{dC}\left[\frac{C}{t_{\exp}}\right]\right)^2 \sigma_C^2}$$

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- We can now simplify a little bit:

$$\sigma_{m_{inst}} = \sqrt{(-2.5)^2 \left(\frac{1}{t_{\exp}}\right)^2 \left(\frac{d}{dC'}\left[\log(C')\right]\right)^2 \sigma_C^2}$$

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$$\sigma_{m_{inst}} = \sqrt{(-2.5)^2\left(\frac{1}{t_{\exp}}\right)^2\left(\frac{d}{dC'}\left[\log(C')\right]\right)^2\sigma_C^2} = \sqrt{(-2.5)^2\left(\frac{1}{t_{\exp}}\right)^2\left(\frac{d}{dC'}\left[\frac{\ln(C')}{\ln 10}\right]\right)^2\sigma_C^2}$$

$$\sigma_{m_{inst}} = \sqrt{(-2.5)^2\left(\frac{1}{t_{\exp}}\right)^2\left(\frac{d}{dC'}\left[\log\left(C'\right)\right]\right)^2\sigma_C^2}$$

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- We can again simplify a little bit:

$$\sigma_{m_{inst}} = \sqrt{\left(\frac{-2.5}{\ln 10}\right)^2\left(\frac{1}{t_{\exp}}\right)^2\left(\frac{d}{dC'}\left[\ln\left(C'\right)\right]\right)^2\sigma_C^2}$$

$$\sigma_{m_{inst}} = \sqrt{\left(\frac{-2.5}{\ln 10}\right)^2 \left(\frac{1}{t_{\exp}}\right)^2 \left(\frac{d}{dC'}\left[\ln\left(C'\right)\right]\right)^2 \sigma_C^2}$$

- Finally we can calculate the remaining derivative:

$$\sigma_{m_{inst}} = \sqrt{\left(\frac{-2.5}{\ln 10}\right)^2 \left(\frac{1}{t_{\exp}}\right)^2 \left(\frac{1}{C'}\right)^2 \sigma_C^2} = \sqrt{\left(\frac{-2.5}{\ln 10}\right)^2 \left(\frac{1}{t_{\exp}}\right)^2 \left(\frac{t_{\exp}}{C}\right)^2 \sigma_C^2}$$

- And so we have our result for the error on the instrumental magnitude:

$$\boxed{\sigma_{m_{inst}} = \sqrt{\left(\frac{-2.5}{\ln 10}\right)^2 \left(\frac{\sigma_C}{C}\right)^2}}$$

Error propagation #2

- For a quantity y that is a function of a multiple variables, $y = f(x_1, x_2, \ldots, x_N)$, the general formula becomes:

$$\sigma_y = \sqrt{\sum_{i=1}^{N} \left(\frac{\partial y}{\partial x_i} \right)^2 \sigma_{x_i}^2}$$

- The equation for the colour index g-r:

$$g - r = gmag - rmag$$

- So we need to calculate:

$$\sigma_{g-r} = \sqrt{\left(\frac{\partial (g-r)}{\partial gmag} \right)^2 * \sigma_{gmag}^2 + \left(\frac{\partial (g-r)}{\partial rmag} \right)^2 * \sigma_{rmag}^2} = \sqrt{\sigma_{gmag}^2 + \sigma_{rmag}^2}$$