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Hot Topics

The Generalized Gamma Distribution and Reliability Analysis

The Weibull life distribution has long been used for most life data analysis problems. The main reason for the distribution's popularity is the versatility it provides, *i.e.* the Weibull distribution can take on the characteristics of other types of distributions, based on the value of the shape parameter, β. The generalized gamma distribution is also a flexible distribution, and in fact **contains the exponential, Weibull, lognormal and gamma distributions** as special cases. ReliaSoft's Weibull++ software also includes a Generalized Gamma distribution option (when used in the Expert mode). We will use this article to explore the generalized gamma distribution and its relation to other lifetime distributions.

The Generalized Gamma Distribution

The generalized gamma distribution is a three-parameter distribution. One parameterization of the generalized gamma distribution uses the parameters k, β, and θ. The *pdf* for this form of the generalized gamma distribution is given by:

$$f(t)=\frac{\beta}{\Gamma(k)\cdot\theta}\left(\frac{t}{\theta}\right)^{k\beta-1}e^{-\left(\frac{t}{\theta}\right)^{\beta}}$$

while another parameterization (as used by ReliaSoft's Weibull++ software and as used in this article) uses the parameters μ , σ and λ, where:

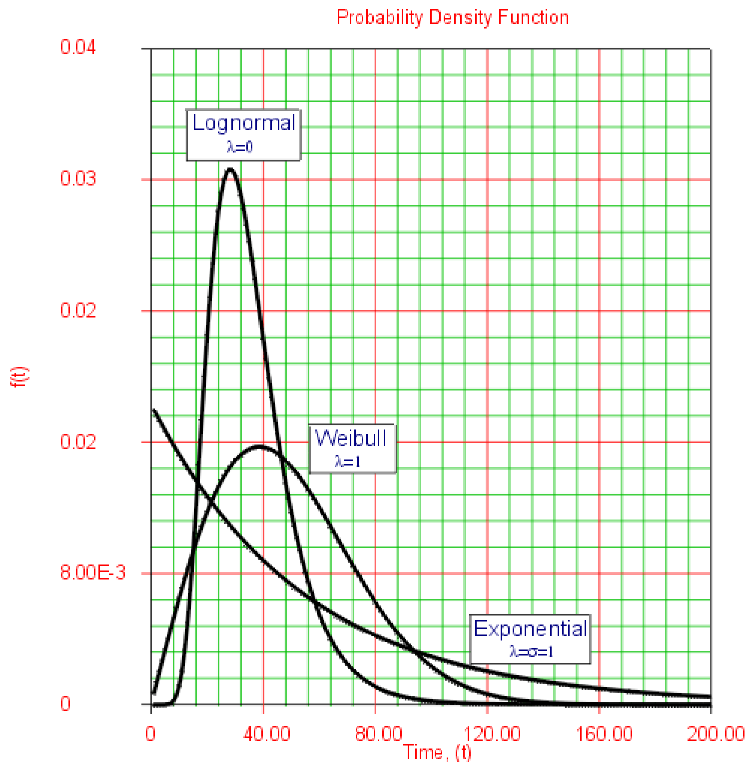
$$\mu = ln(\theta)+\frac{1}{\beta}\cdot ln\left(\frac{1}{\lambda^2}\right), \quad \sigma = \frac{1}{\beta\sqrt{k}}, \quad \lambda = \frac{1}{\sqrt{k}}$$

or

$$f(t)=\begin{cases}\frac{|\lambda|}{\sigma\cdot t}\cdot\frac{1}{\Gamma\left(\frac{1}{\lambda^2}\right)}\cdot e^{\left[\frac{\lambda\cdot\frac{\ln(t)-\mu}{\sigma}+\ln\left(\frac{1}{\lambda^2}\right)-e^{\lambda\cdot\frac{\ln(t)-\mu}{\sigma}}}{\lambda^2}\right]} & \text{if } \lambda\neq 0\\ \frac{1}{t\cdot\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{\ln(t)-\mu}{\sigma}\right)^2} & \text{if } \lambda=0\end{cases}$$

As can be seen, the *pdf* of the generalized gamma distribution is complex, and parameter evaluation is by no means trivial. This is one of the reasons that the distribution has not been widely used or discussed in reliability and life data analysis. However, this complexity aside (given the fact that software can be utilized), the generalized gamma can prove quite useful since (and as mentioned previously) the generalized gamma distribution includes other distributions as special cases based on the values of the parameters.

As can be seen from the following graph, the Weibull distribution is a special case of the generalized gamma when λ = 1. The exponential distribution is a special case when λ =1 and σ=1. The lognormal distribution is a special case when λ = 0. The gamma distribution is a special case when λ = σ.



Furthermore, by allowing λ to take negative values, the generalized gamma distribution can be further extended to include additional distributions as special cases. For example, the Frechet distribution of maxima (also known as a reciprocal Weibull) is a special case when λ = -1.

Given this flexibility and the fact that the distribution is a superset of the commonly used life distributions, one may want to consider the generalized gamma distribution in lieu of another candidate (*i.e.* Weibull, lognormal, etc.). While this is appropriate, one should also consider the data requirement drawback present when dealing with more complex distributions. In other words, the more parameters present, the more data points that are needed.