

# Atlas generation

Xiao-Ming Fu

# Outlines

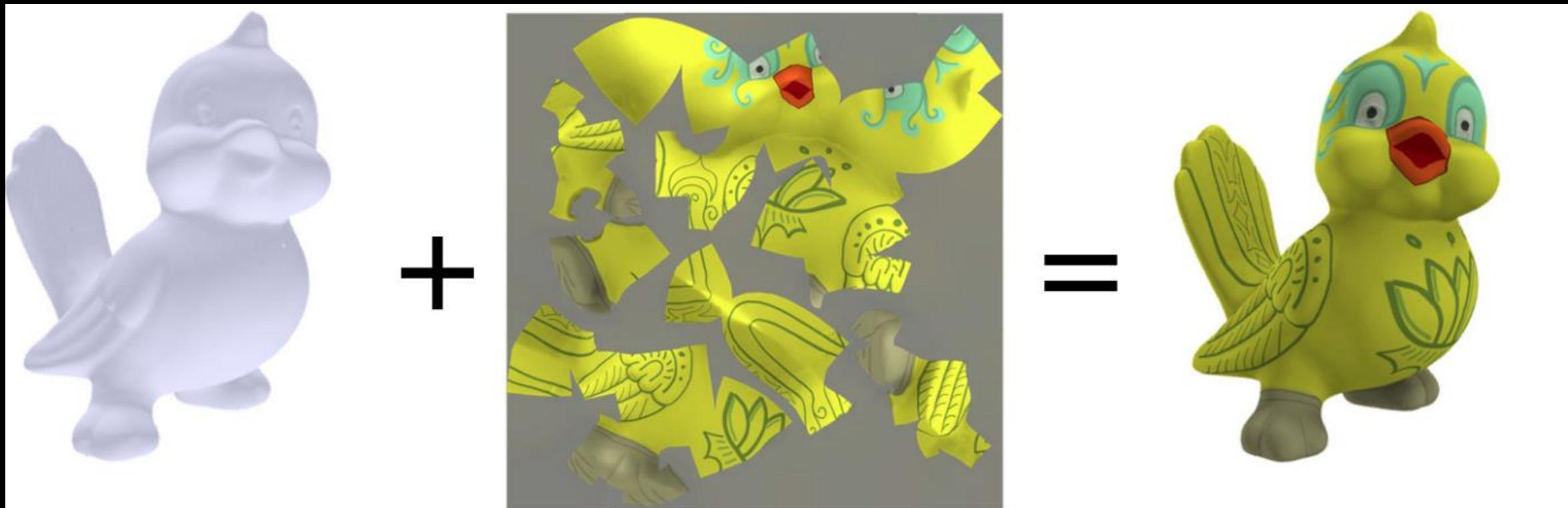
- Definition
- Mesh cutting
- Chart parameterization
  - Bijective, low distortion
- Chart packing

# Outlines

- Definition
- Mesh cutting
- Chart parameterization
  - Bijective, low distortion
- Chart packing

# Texture Mapping

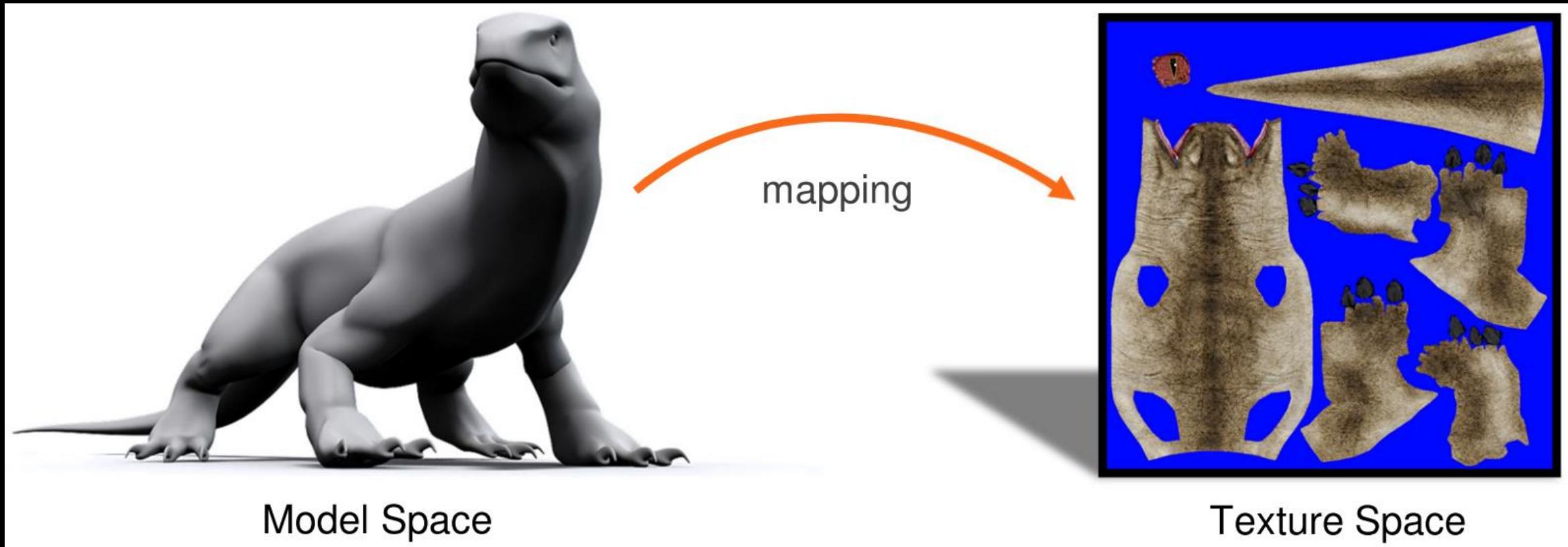
- Texture mapping is a method for defining high frequency detail, surface texture, or color information on a computer-generated graphic or 3D model.



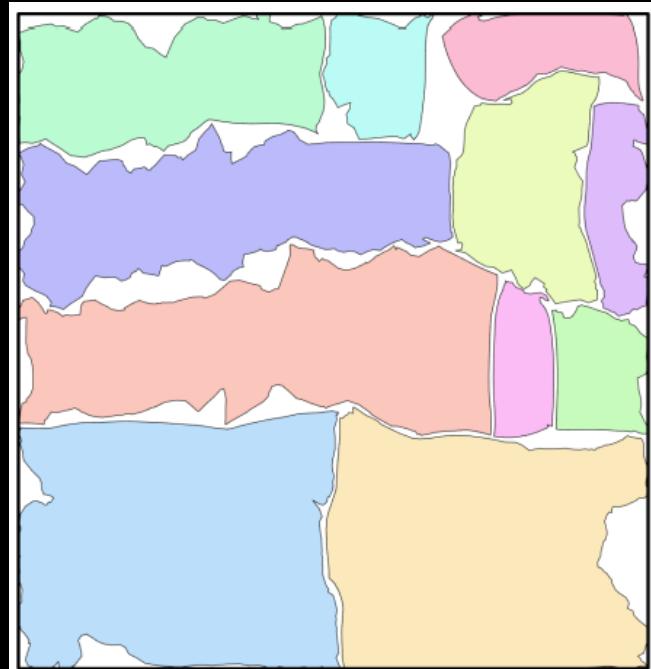
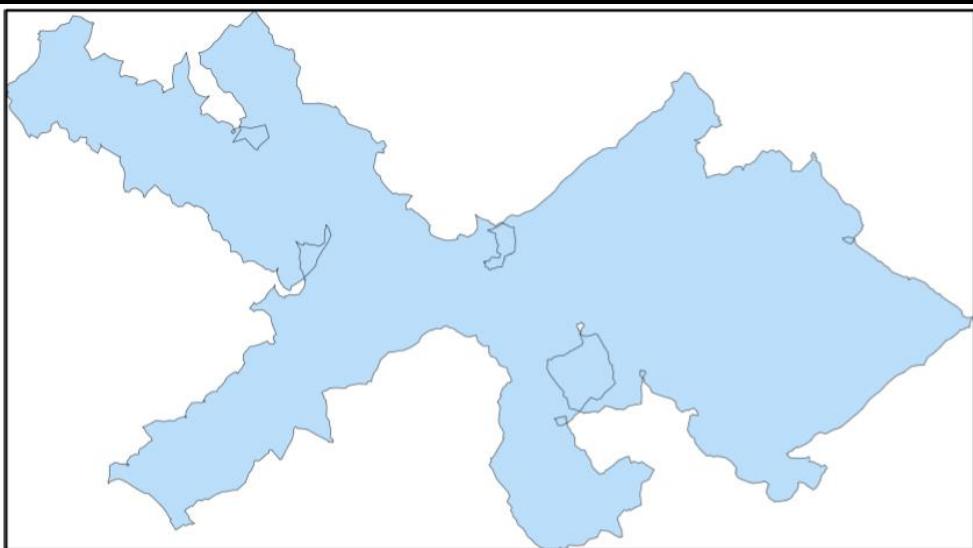
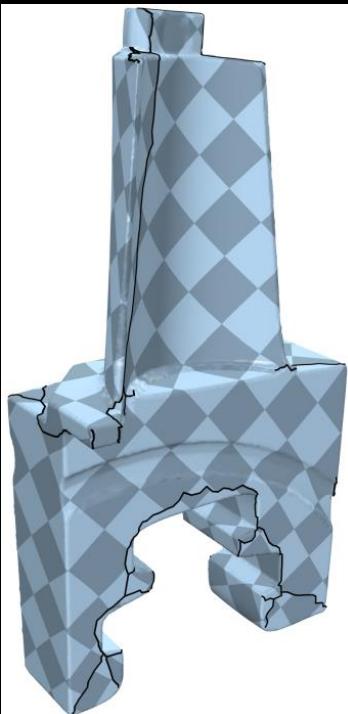
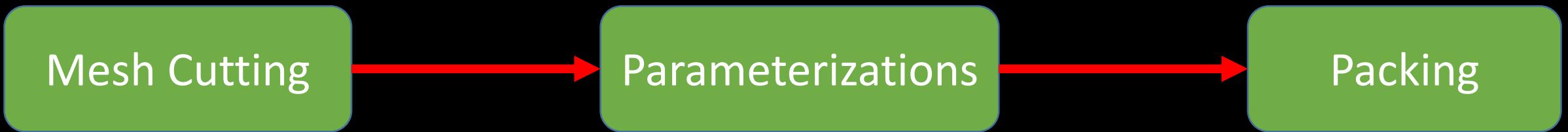


# Atlas

- Requires defining a **mapping** from the model space to the texture space.

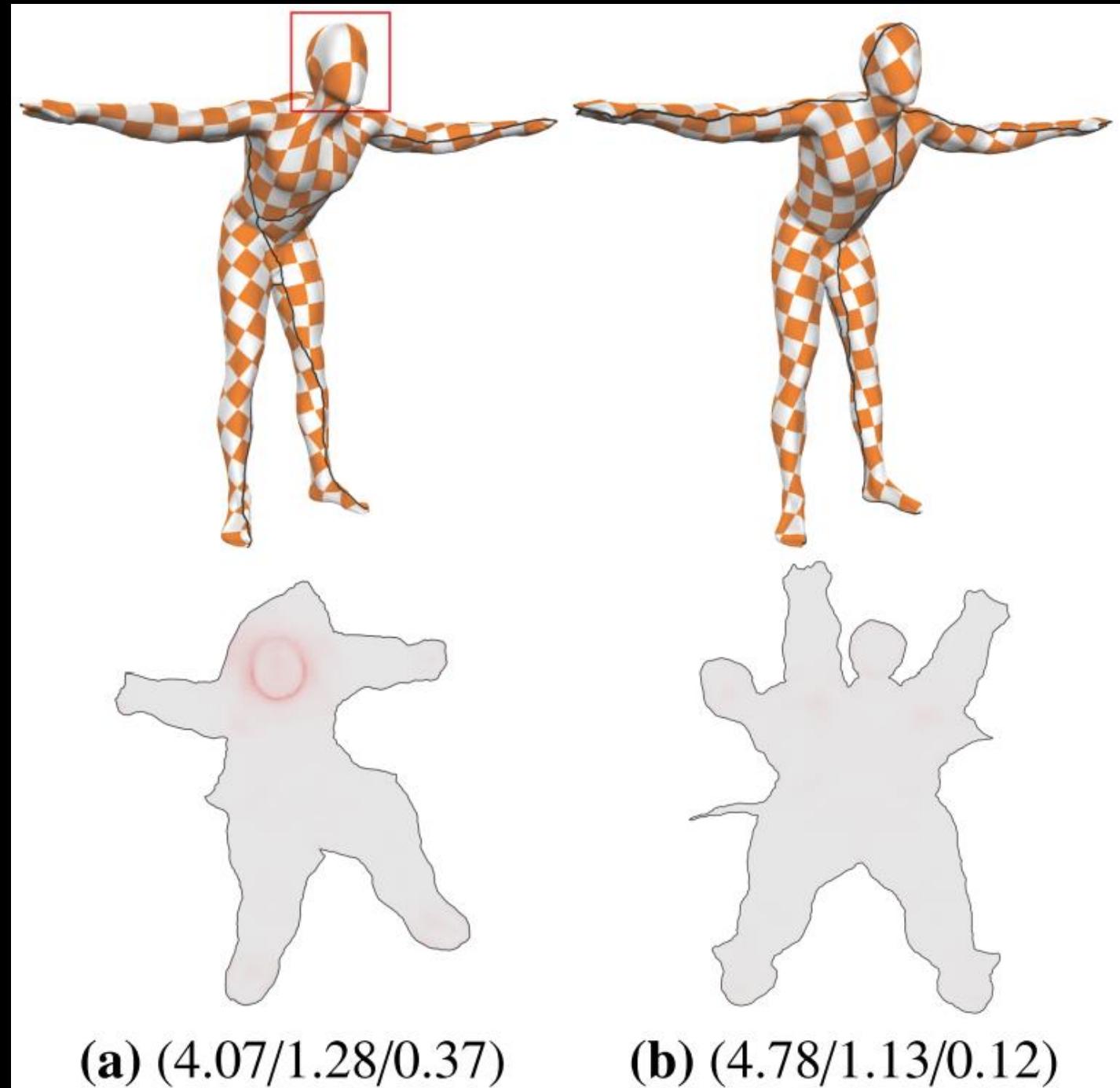


# Generation process



# Mesh Cutting

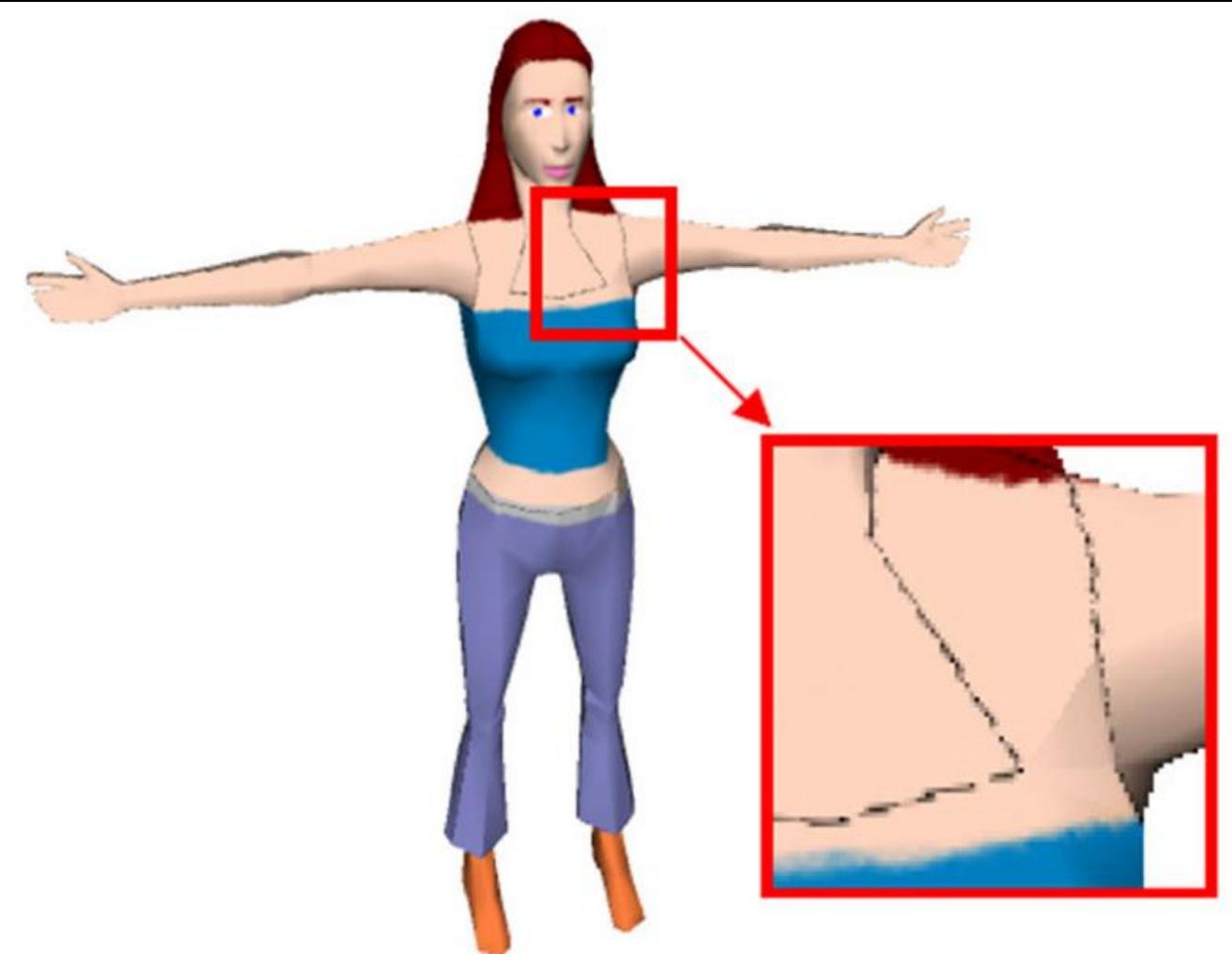
- Low distortion
- As short as possible length



# Seams introduce filtering artifacts

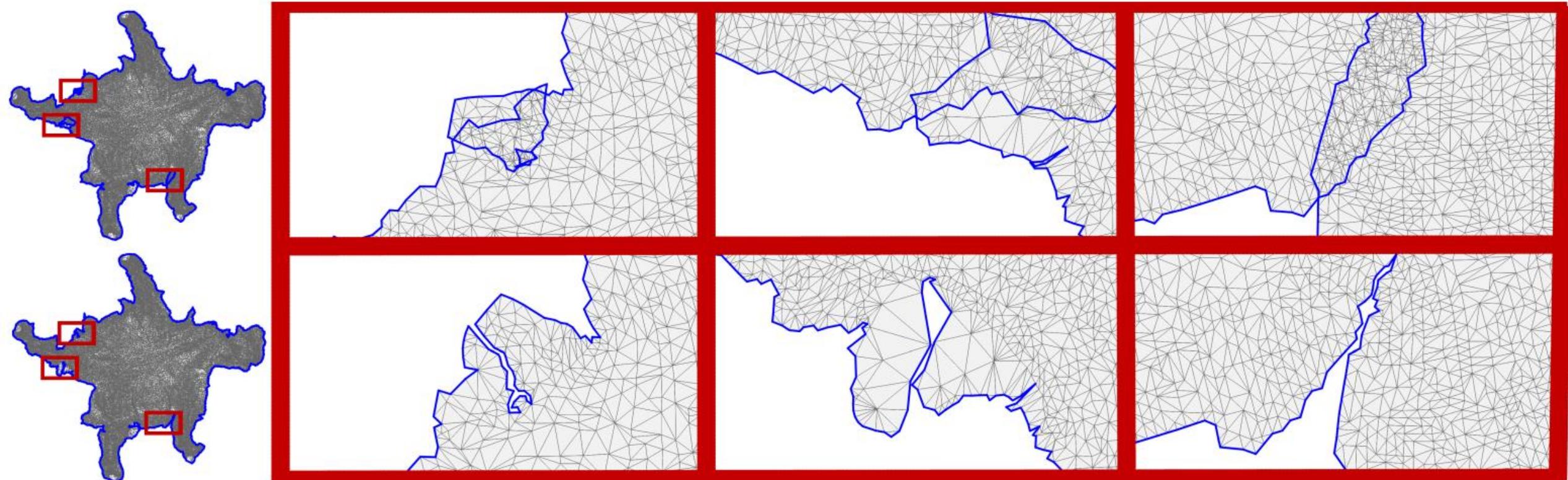


High-resolution texture



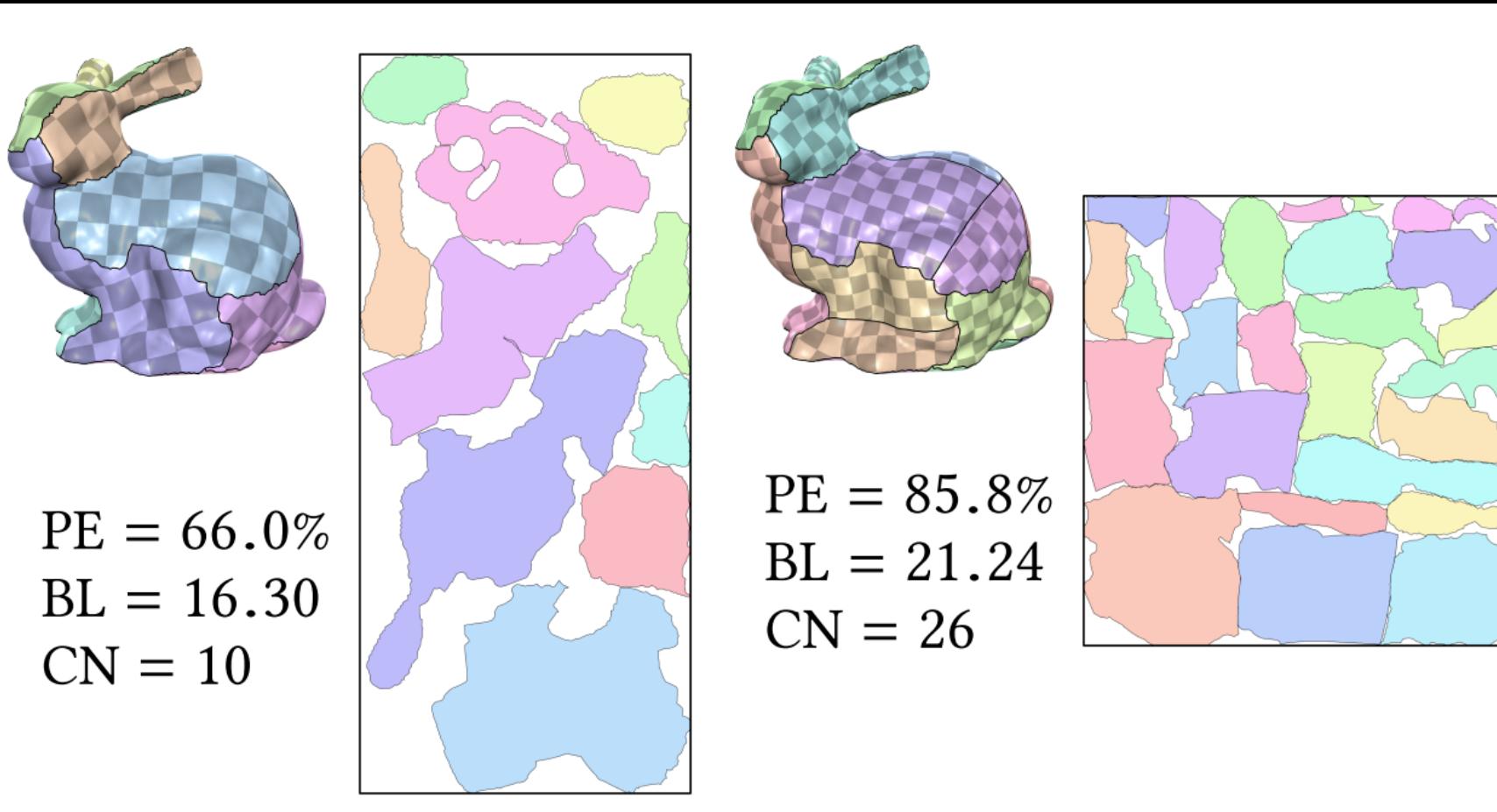
# Parameterizations

- Bijective
- Low isometric distortion



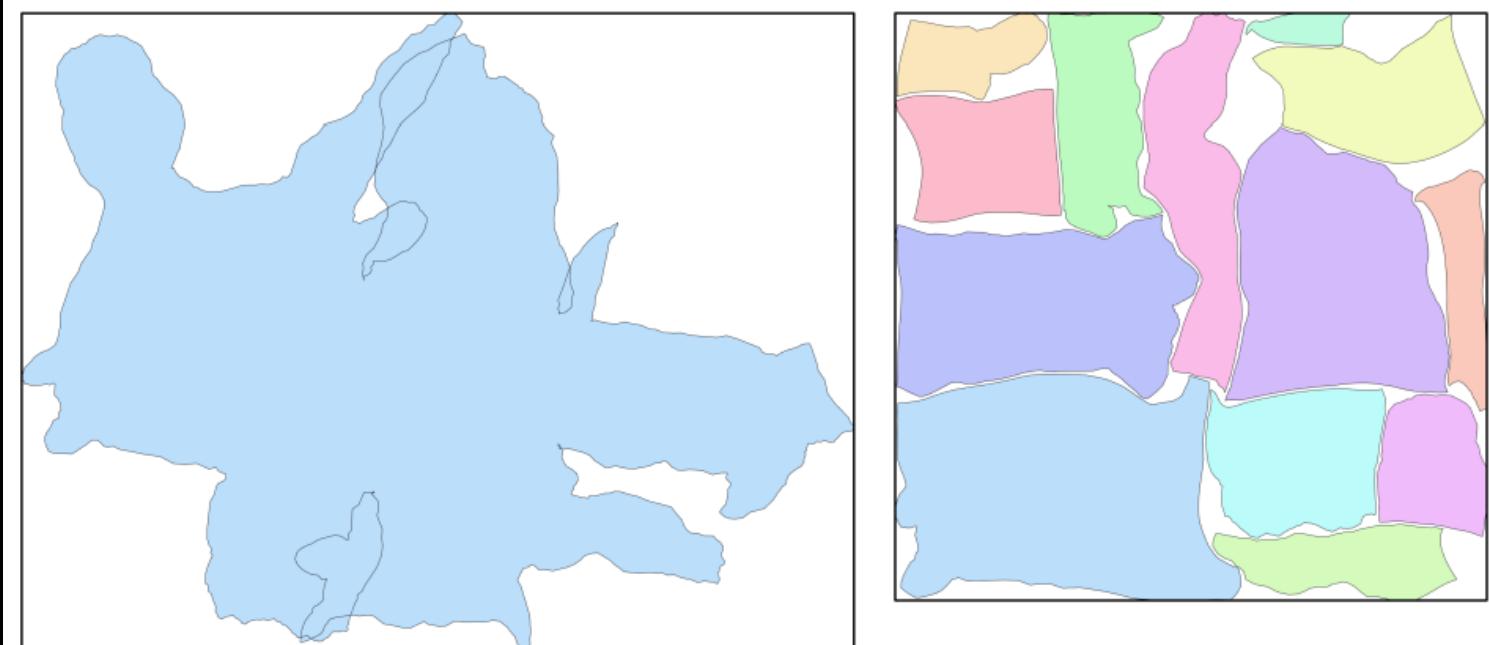
# Packing

- High packing efficiency



# Packing

- High packing efficiency



$BL = 6.34$   
 $E_d = 1.040$

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**Input**



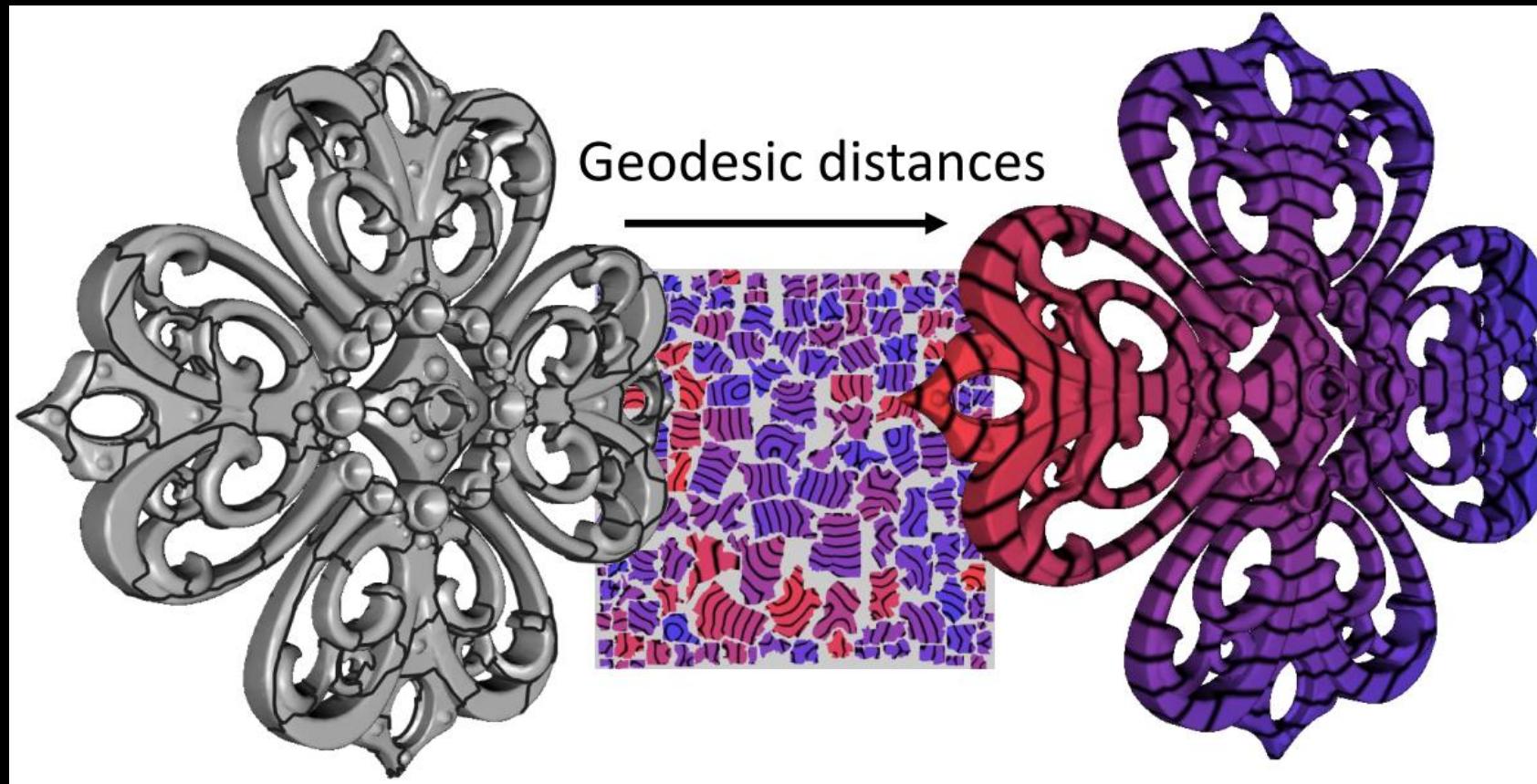
$PE = 85.0\%$   
 $BL = 11.45$   
 $CN = 13$   
 $E_d = 1.030$

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**Result**

# Applications

- Signal storage
- Geometric processing



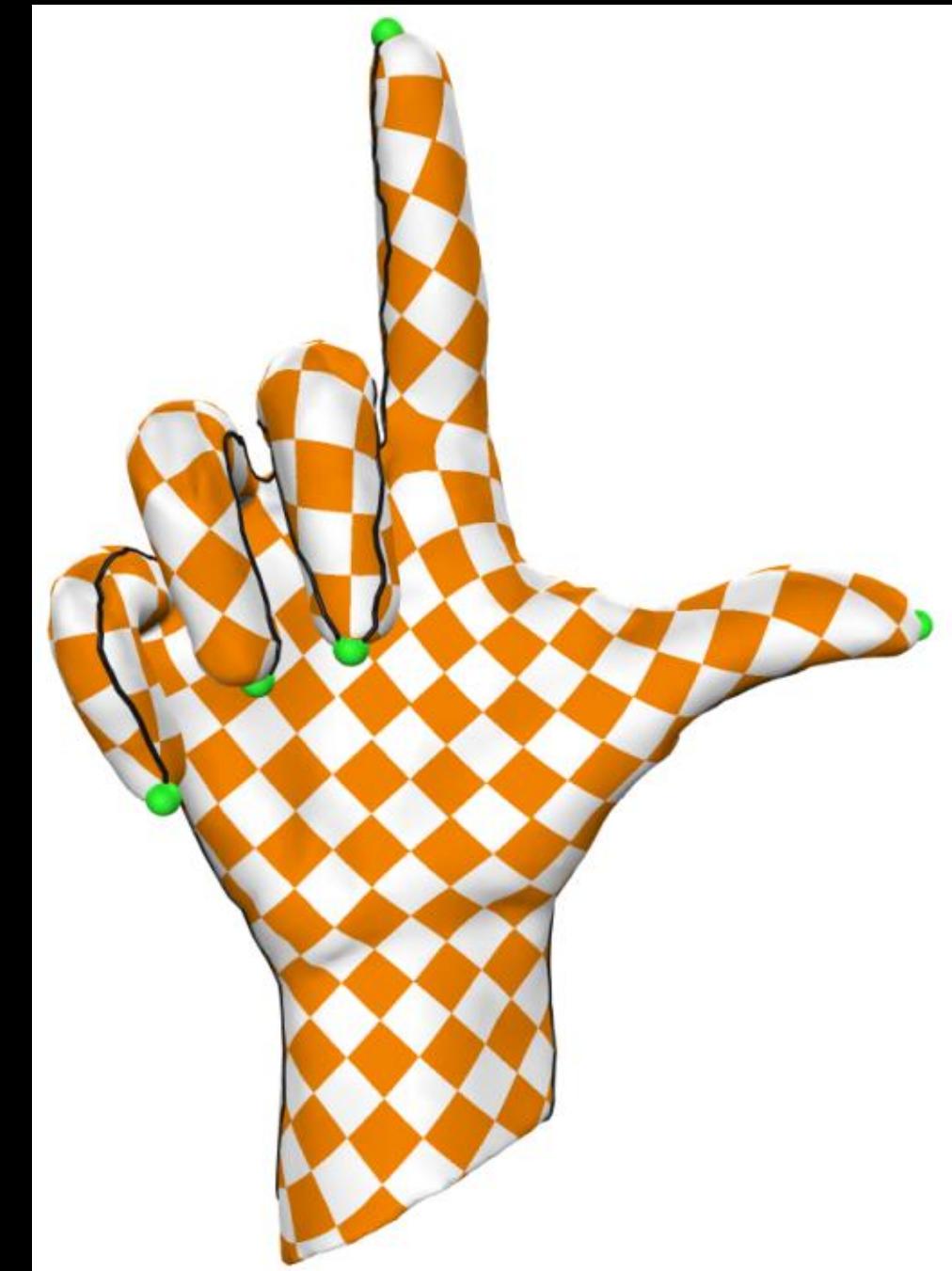
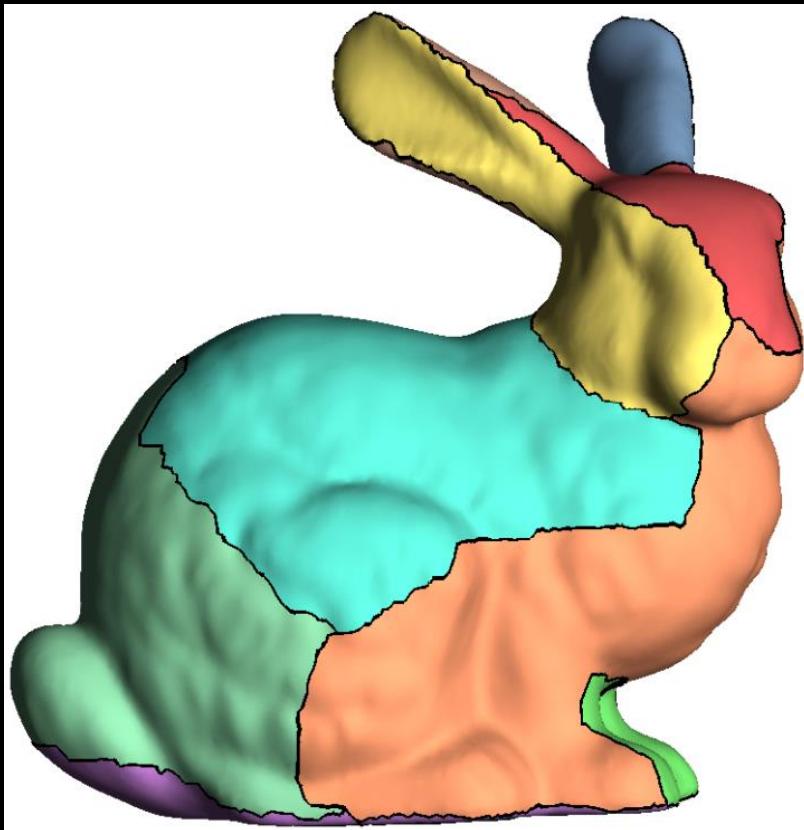
**Gradient-Domain Processing  
within a Texture Atlas**

# Outlines

- Definition
- Mesh cutting
- Chart parameterization
  - Bijective, low distortion
- Chart packing

# Mesh cutting

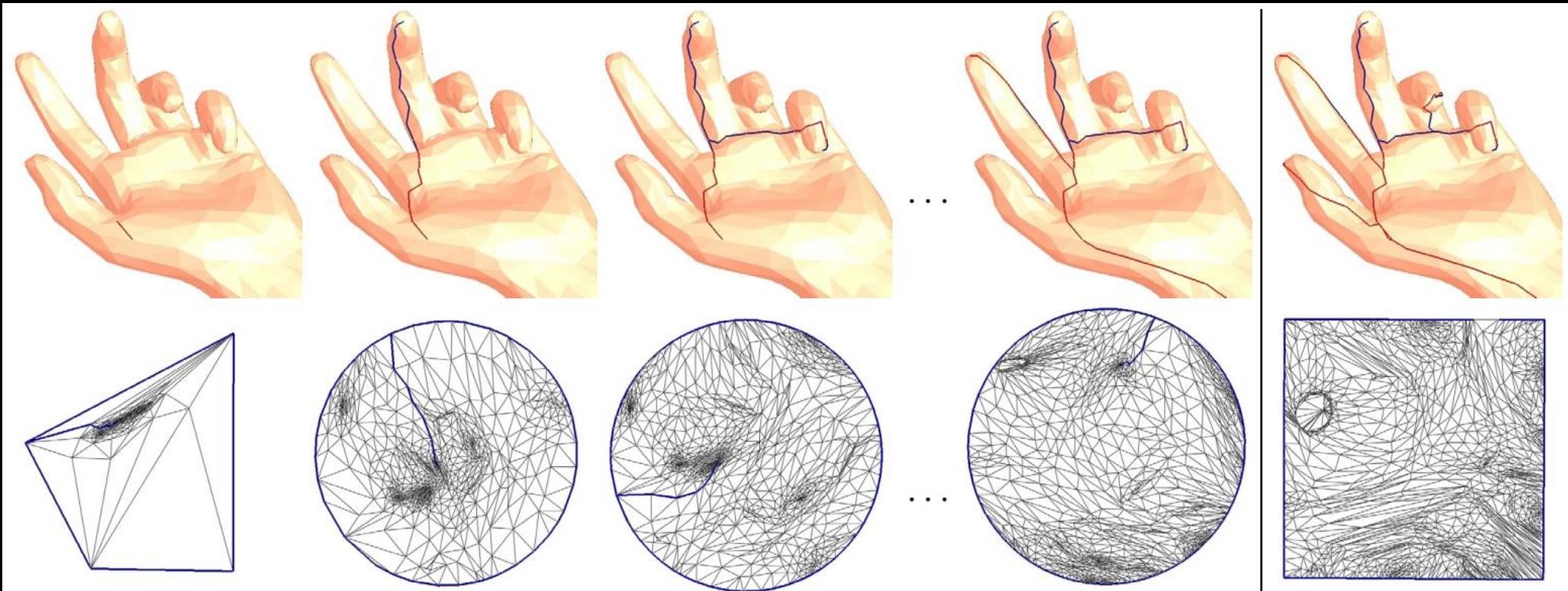
- Points → Paths
- Segmentation



# Distortion points

Geometry Images, SIGGRAPH 2002

- Iterative method
  - Parameterize the mesh to the plane.
  - Add the point of greatest isometric distortion.



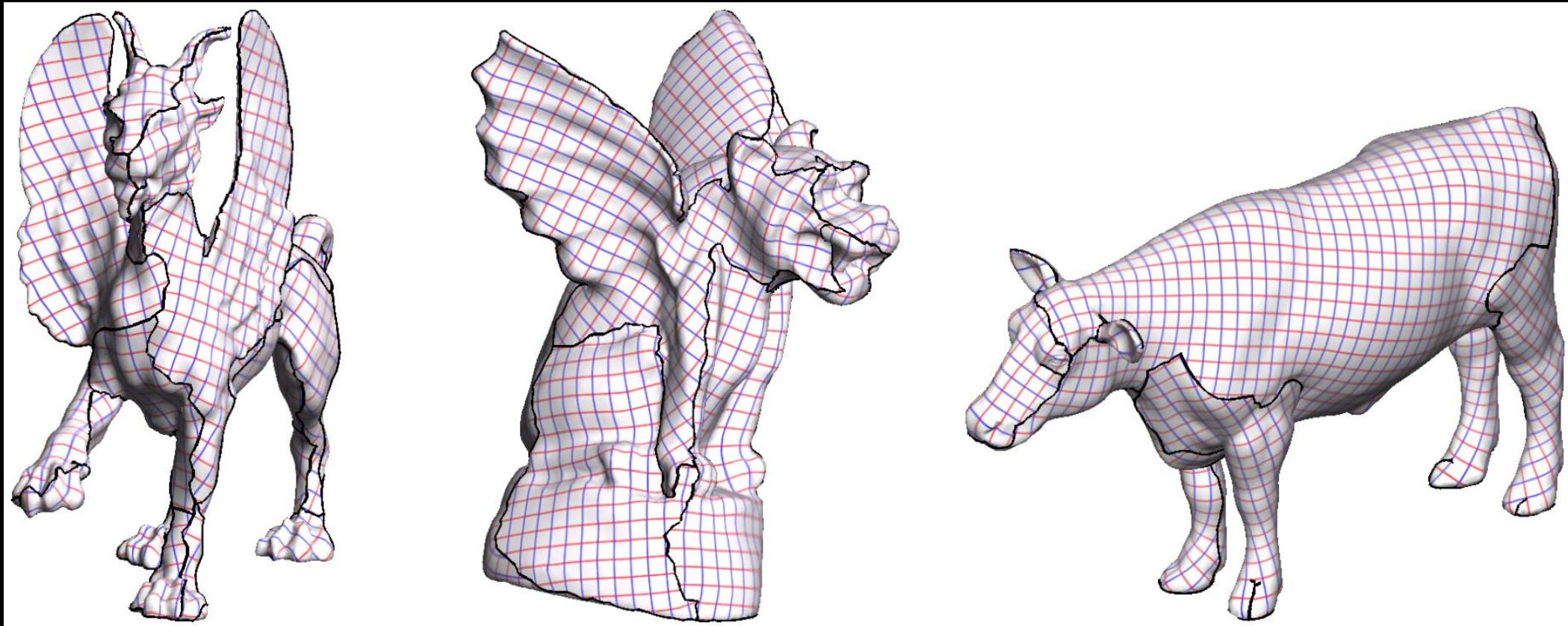
# More methods

- Spanning tree seams for reducing parameterization distortion of triangulated surfaces, 2002
- Sphere-based cut construction for planar parameterizations, 2018

# Segmentation

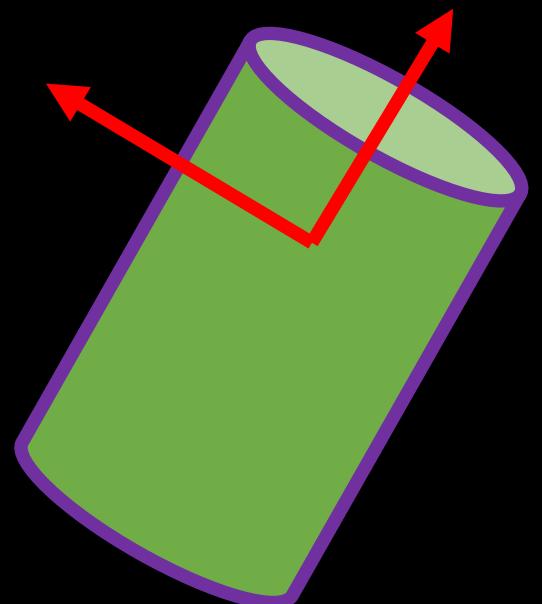
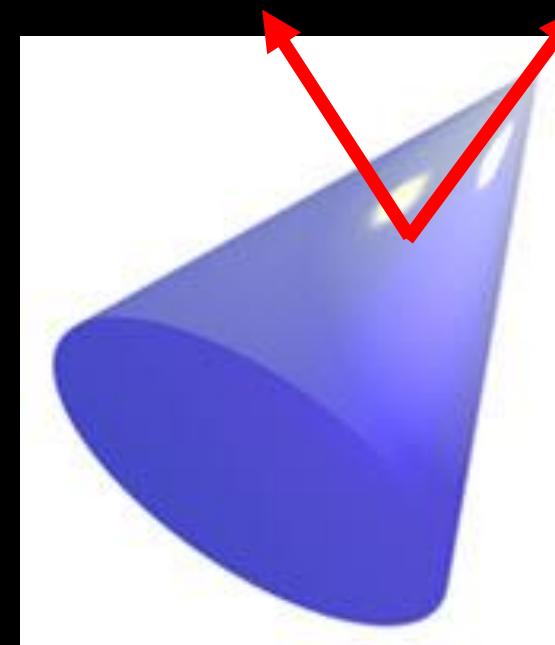
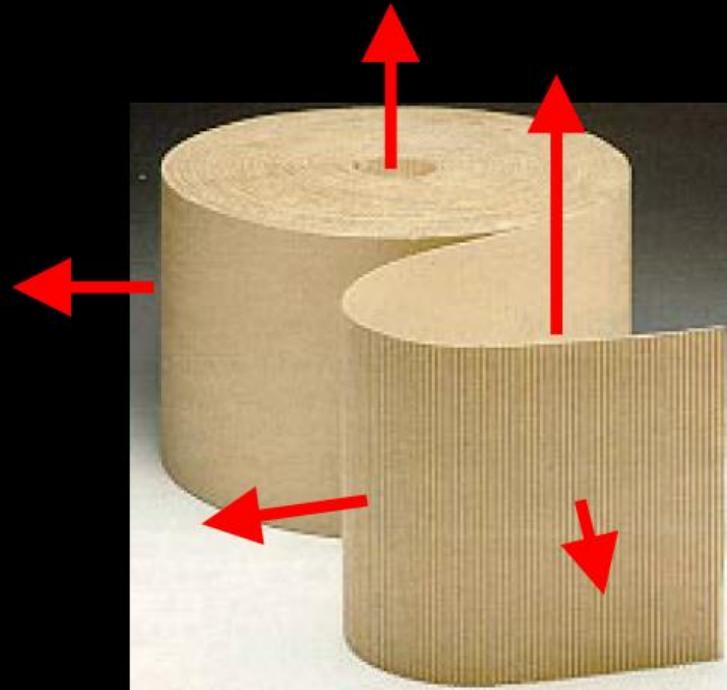
D-Charts: Quasi-Developable Mesh Segmentation, EG 2015

- Goal: mesh segmentation into compact charts that unfold with minimal distortion



# Proxy

- Developable surfaces of constant slope
- Constant angle between surface normal and axis
- Proxy:  $\langle \text{axis}, \text{angle} \rangle, \langle N_c, \theta_c \rangle$



# Fitting error

- Measures how well triangle fits a chart

$$F(C, t) = (N_c \cdot n_t - \cos\theta_c)^2$$

- Combine with compactness

$$C(C, t) = \frac{\pi D(S_c, t)^2}{A_c}$$

✓  $S_c$  is the seed triangle of the given chart

✓  $D(S_c, t)$  is the length of the shortest path (inside the chart) between the two triangles

✓  $A_c$  is the area of chart  $C$

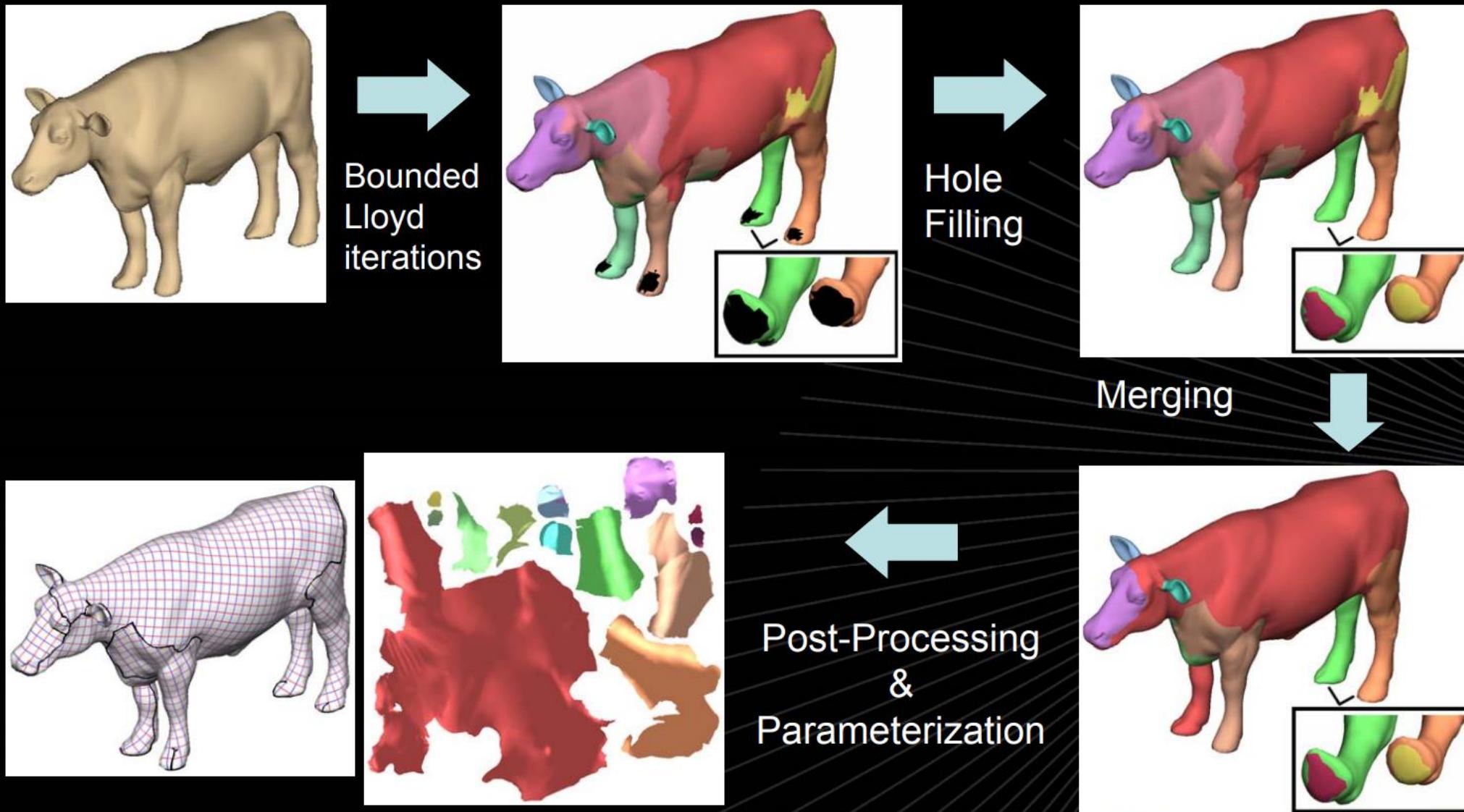
- Cost function

$$Cost(C, t) = A_t F(C, t)^\alpha C(C, t)^\beta$$

# Segmentation method

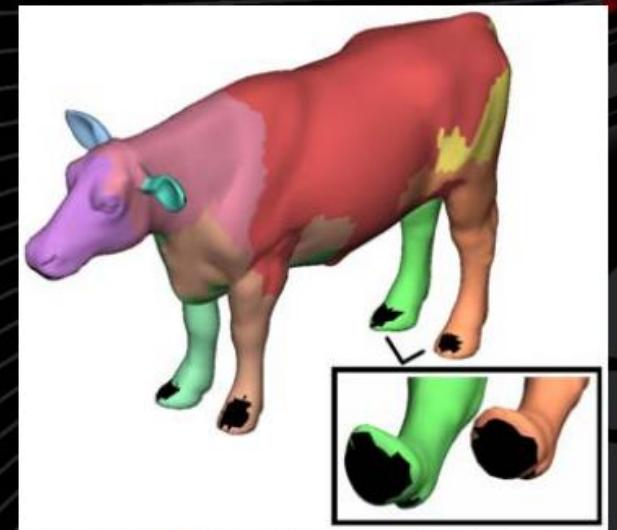
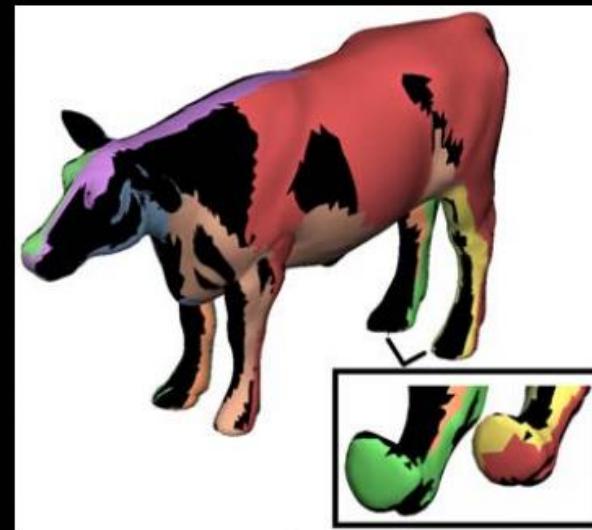
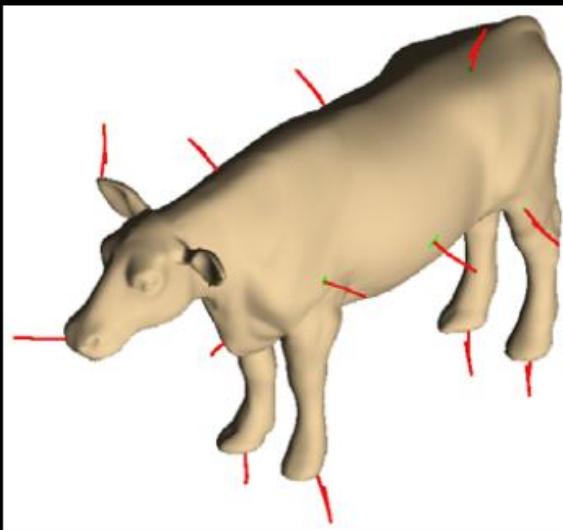
- Lloyd algorithm
  - 1. Select random triangles to act as seeds
  - 2. Grow charts around seeds using a greedy approach
  - 3. Find new proxy for each chart
  - 4. Repeat from step 2 until convergence
- K-means
- CVT

# Algorithm overview



# Bounded Lloyd iterations

- Initialization
  - Random / Furthest point seeds
  - Compute initial proxy
- Bounded Growing/Reseeding iterations
- Termination



# Bounded Lloyd iterations – Growing

- Use greedy approach
  - Prioritize by  $\text{Cost}(C, t)$
- Bound Fitting Error
  - Guarantee (nearly) developable charts
  - $F(C, t) < F_{max}$

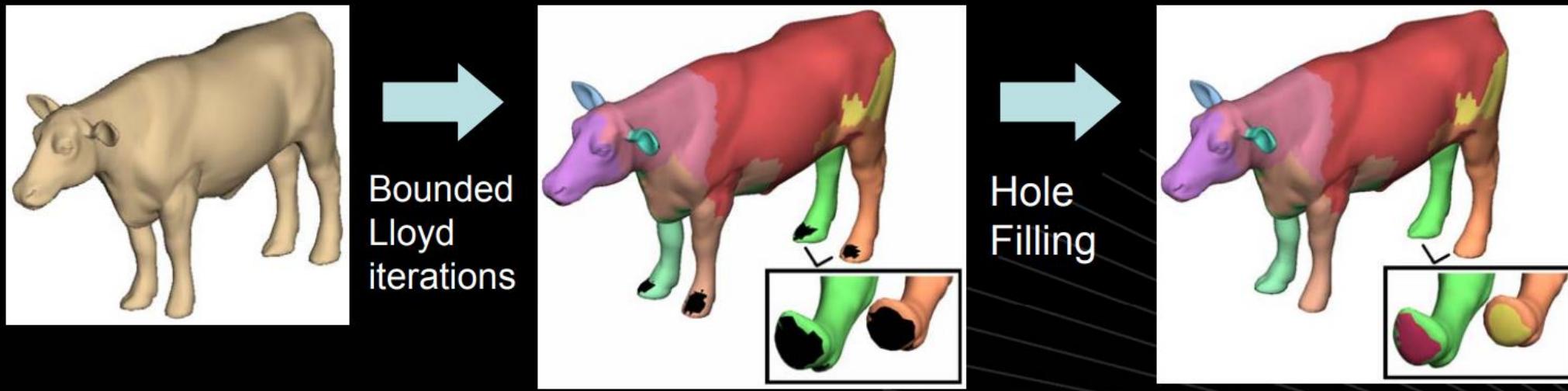
# Bound Lloyd iterations – Reseeding

- Find new proxy

$$\min_{N_c, \theta_c} \frac{1}{A_c} \sum_{t \in C} A_t F(C, t) \text{ s.t. } \|N_c\| = 1$$

- Find new seed
  - Minimal Fitting Error
  - Close to center of chart
  - To find such seeds, we examine the first  $k$  triangles in the chart with minimal fitting error ( $k = 10$  in all our examples), and then select the one closest to the center of the chart.

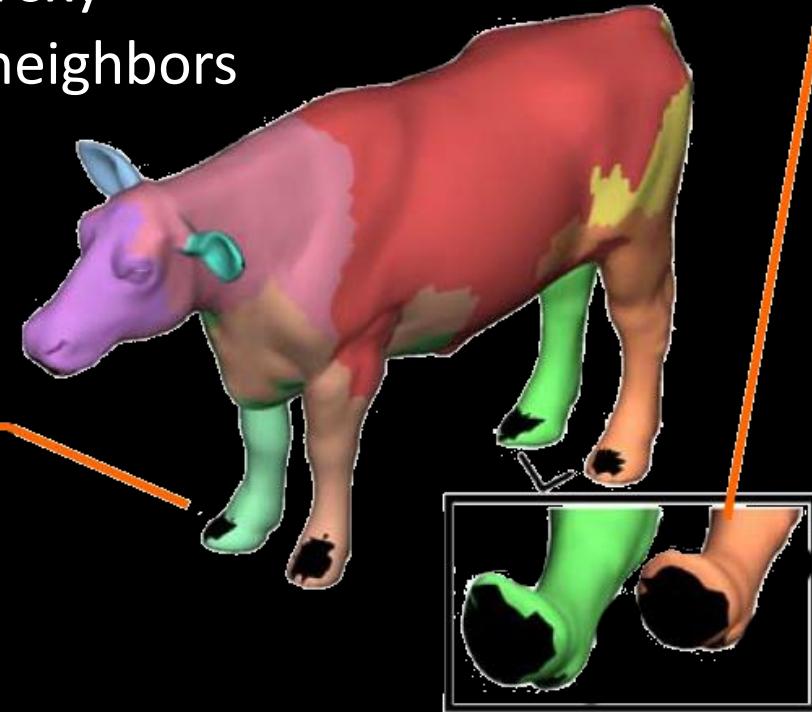
# Algorithm overview



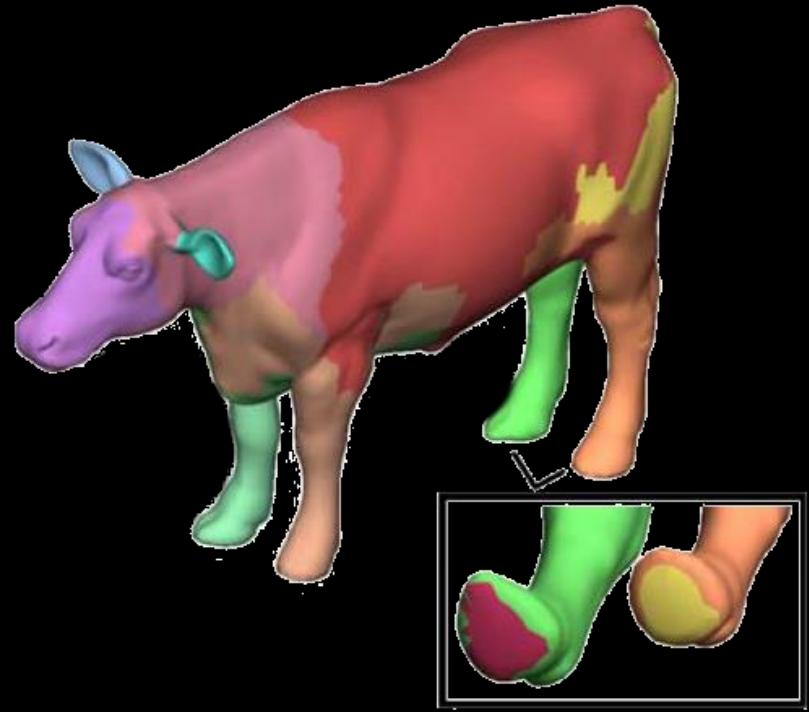
# Hole filling

- Bound on Fitting Error
  - Unclassified triangles
- Fill holes
  - Large holes → New proxy
  - Small holes → Grow neighbors

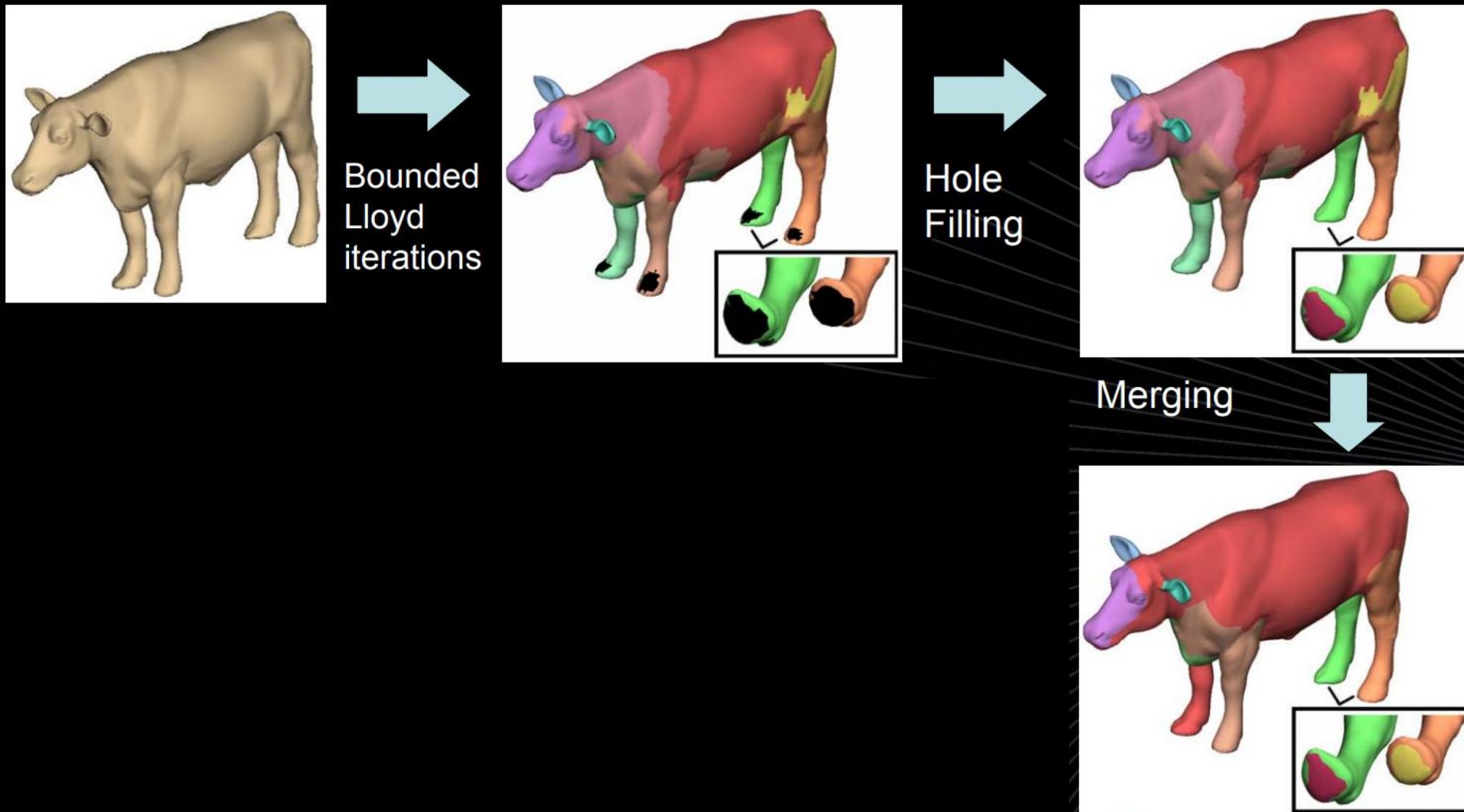
Small



Large

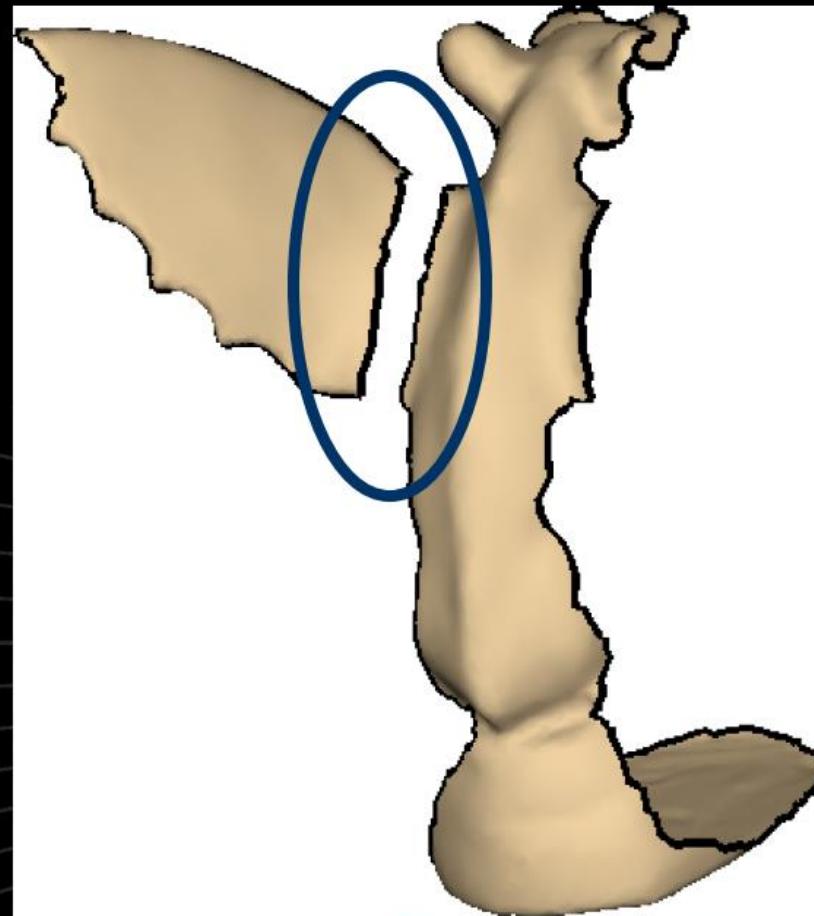


# Algorithm overview

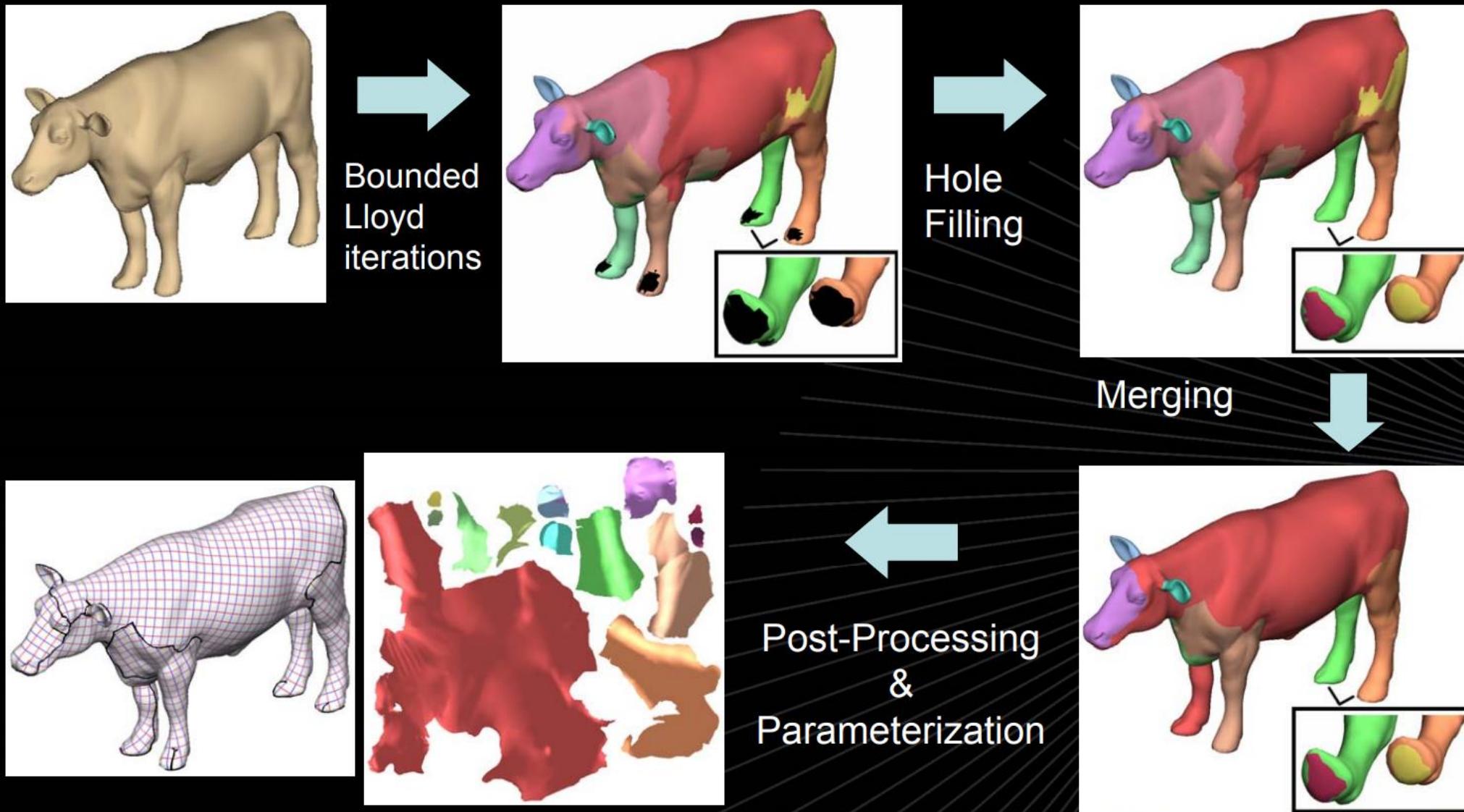


# Merging

- Broaden set of captured developable surfaces
- Reduce number of charts



# Algorithm overview



# Post processing

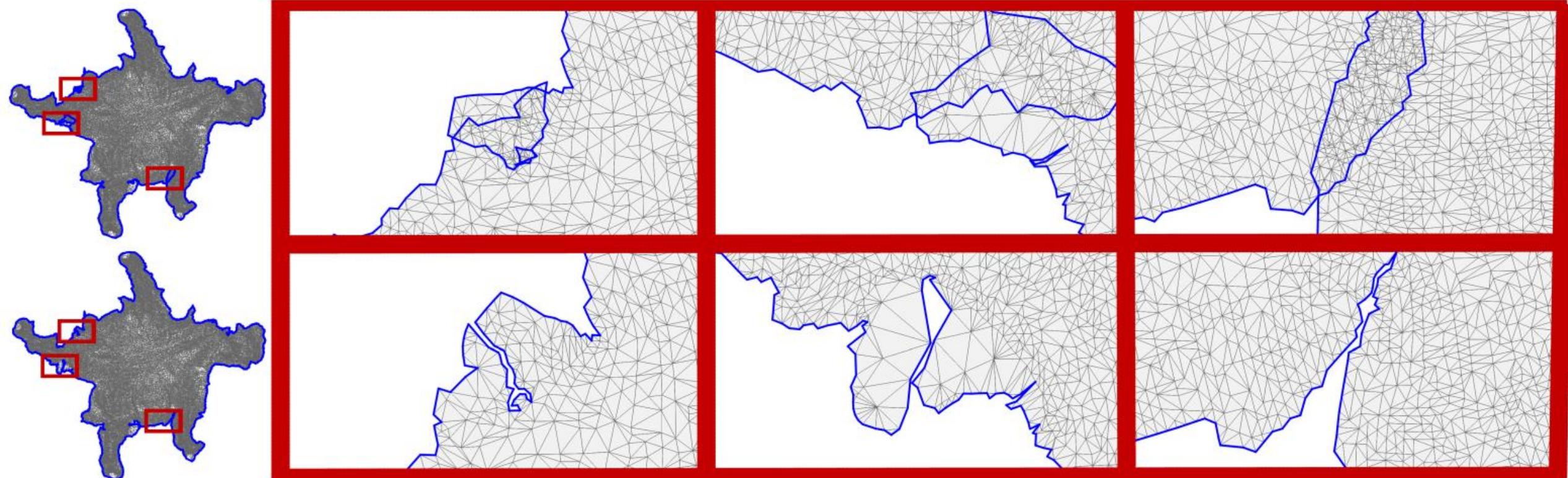
- Straighten boundaries
- Darts/Gussets relax stress
  - Add seams toward high error regions
- Verify disc topology
- Parameterization

# Outlines

- Definition
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# Bijection

- Preserving orientations
- No intersections of boundary



# Barrier

Bijective Parameterization with Free Boundaries, SIGGRAPH 2015

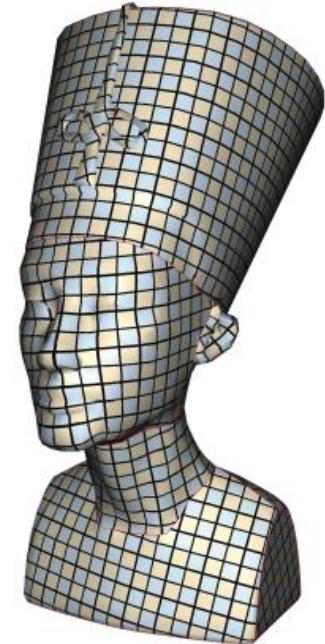
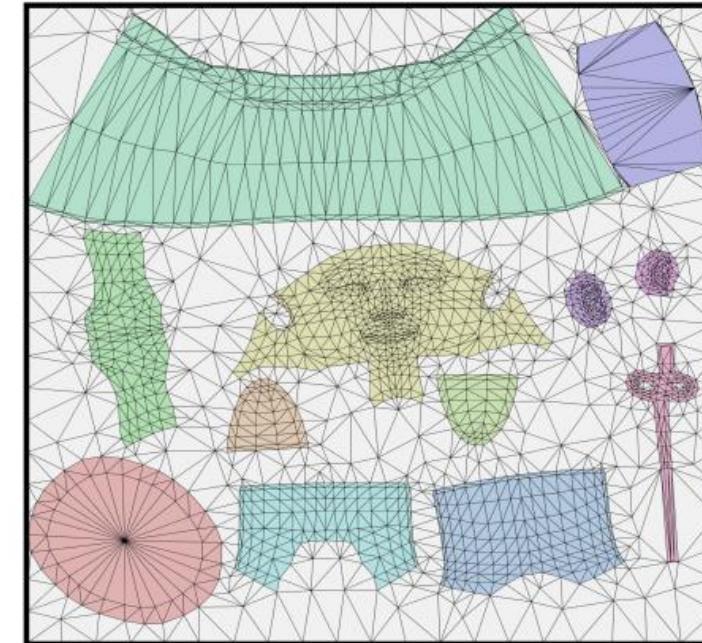
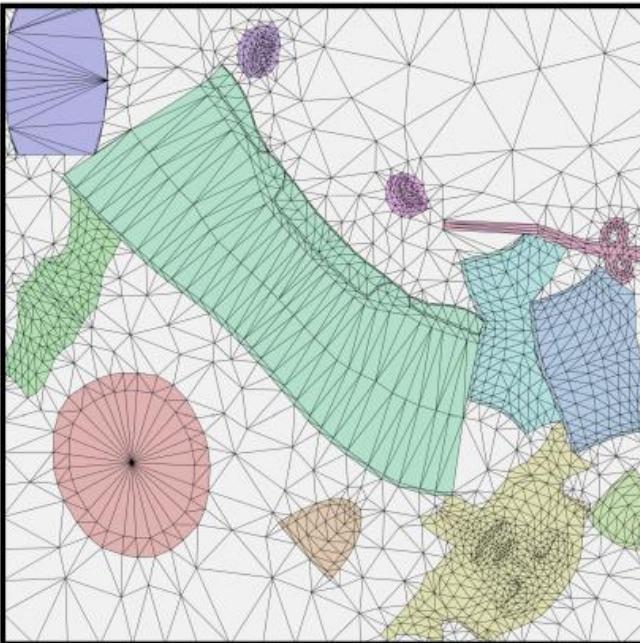
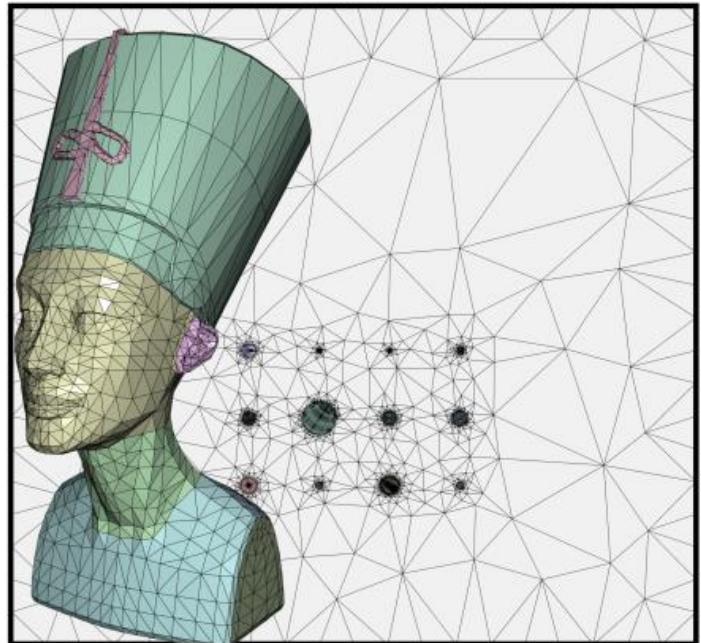
- For each boundary edge with vertices  $U_1, U_2$ , we associate a barrier function:

$$\max\left(0, \frac{\varepsilon}{dist(U_1, U_2, U_i)} - 1\right)^2$$

where,  $dist(U_1, U_2, U_i)$  measures the distance from a boundary point  $U_{i \neq 1,2}$  to the edge  $(U_1, U_2)$ .

# Scaffold

Simplicial Complex Augmentation Framework for Bijective Maps, SIGGRAPH Asia 2017



# Outlines

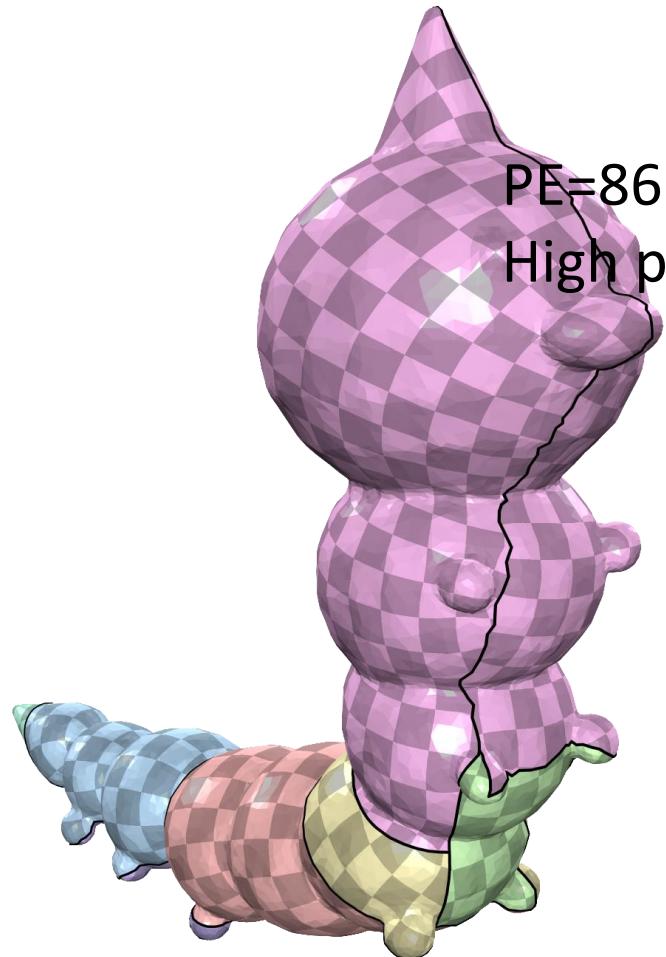
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# Atlas Refinement with Bounded Packing Efficiency

Hao-Yu Liu, Xiao-Ming Fu, Chunyang Ye, Shuangming Chai, Ligang Liu

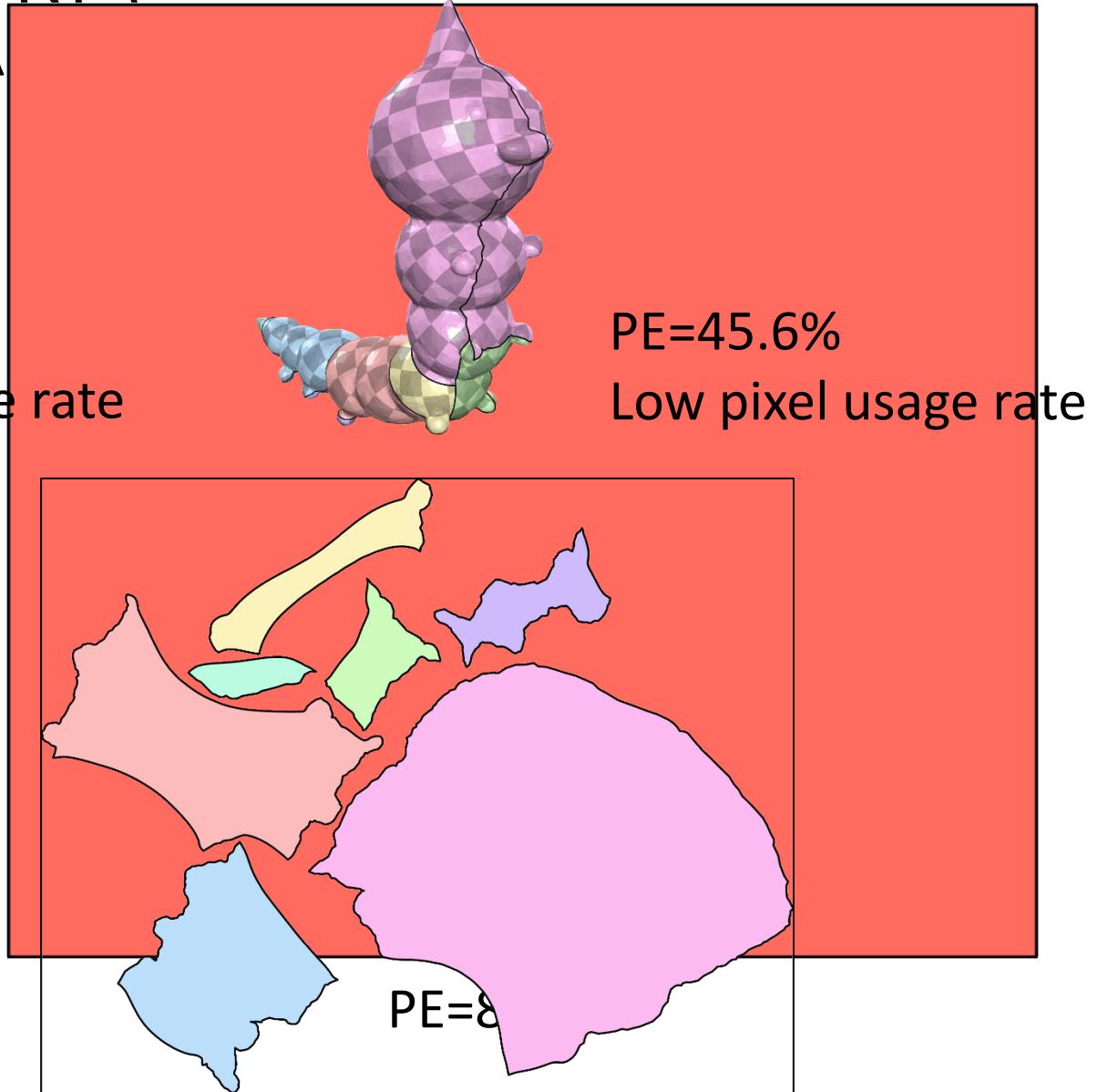
ACM Transactions on Graphics (SIGGRAPH) 38(4), 2019.

# Packing Efficiency (PE)



PE=86.1%

High pixel usage rate



# Packing Efficiency (PE)

**Maximizing atlas packing efficiency is NP-hard!**

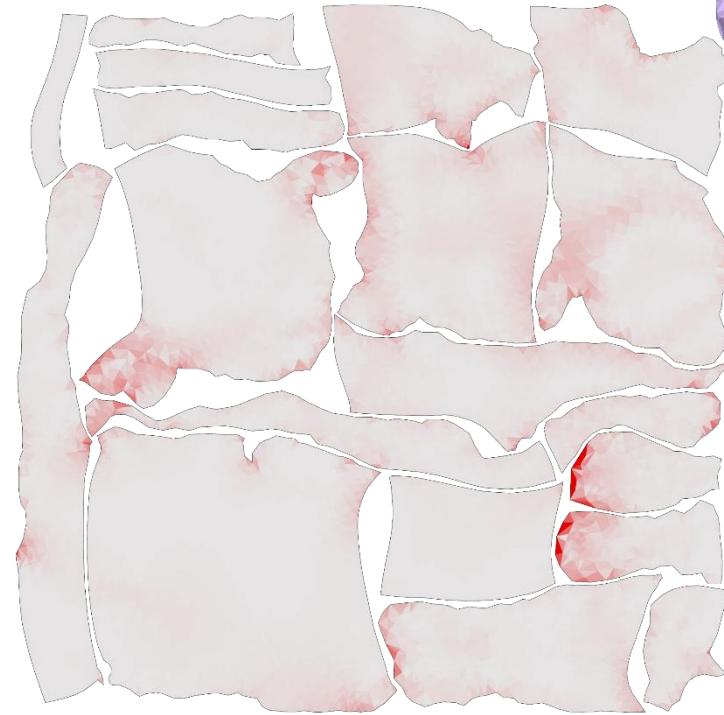
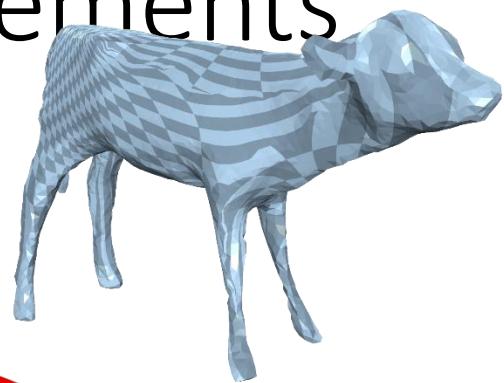
[Garey and Johnson 1979; Milenkovic 1999]

# Other Requirements

- Low distortion



High Distortion



Low Distortion

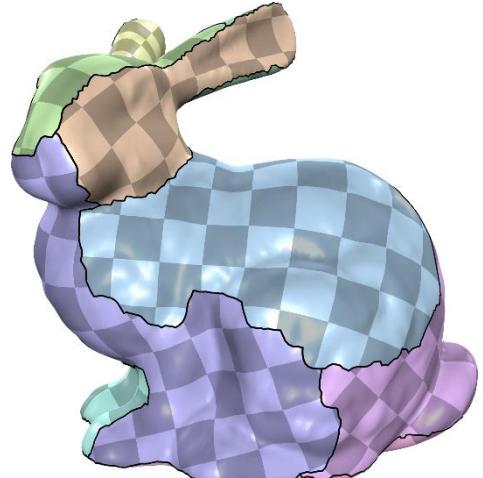


# Other Requirements

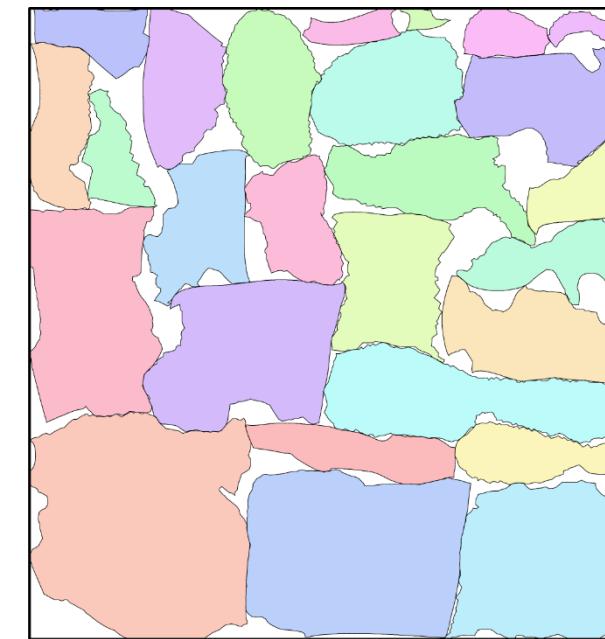
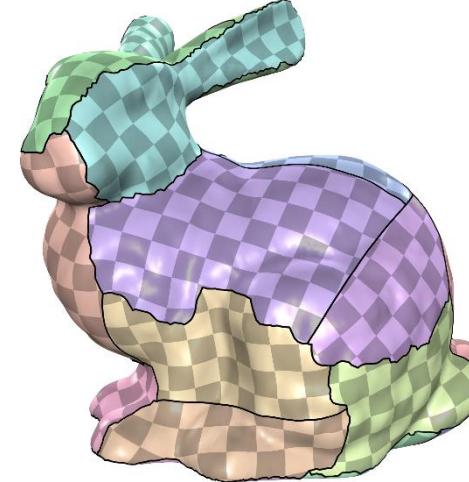
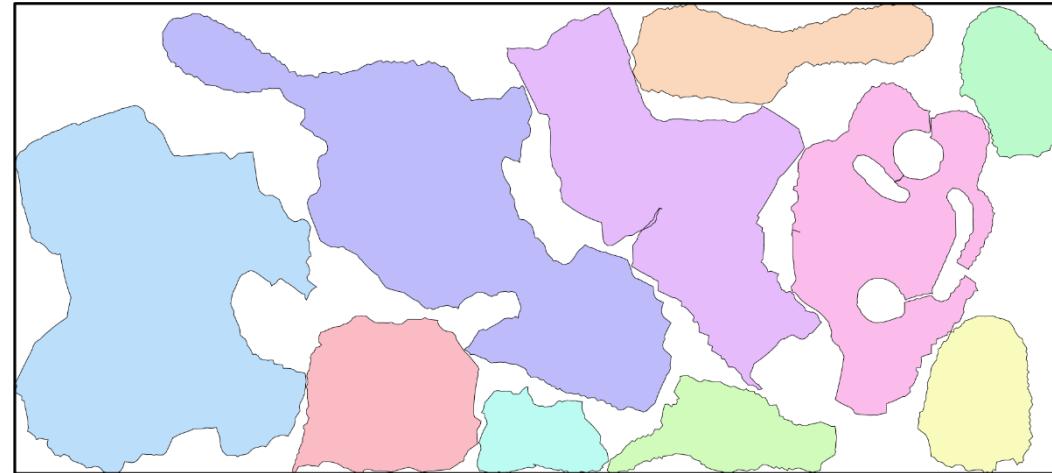
- Low distortion
  - [Golla et al. 2018; Liu et al. 2018; Shtengel et al. 2017; Zhu et al. 2018]
- Consistent orientation
  - [Floater 2003; Tutte 1963; Claici et al. 2017; Hormann and Greiner 2000; Rabinovich et al. 2017; Schüller et al. 2013]
- Overlap free
  - [Jiang et al. 2017; Smith and Schaefer 2015]
- Low boundary length
  - [Li et al. 2018; Poranne et al. 2017; Sorkine et al. 2002]

**These methods do not consider PE!**

# Atlas Refinement



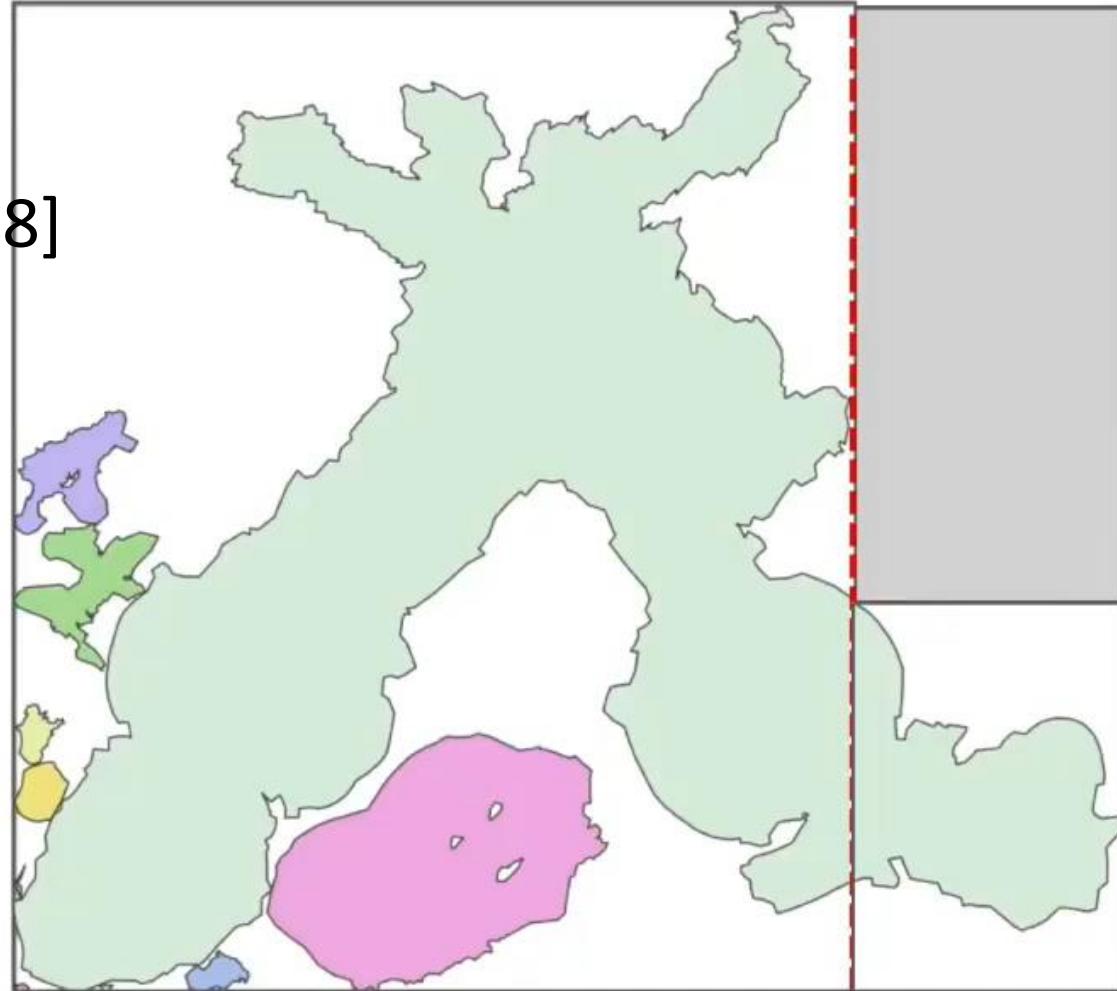
Input



No overlap  
High PE

# Previous Work

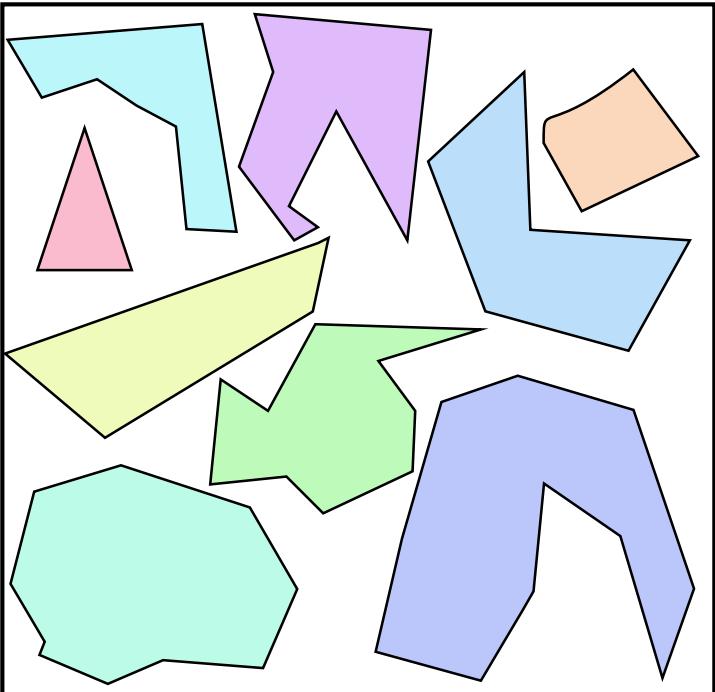
Box Cutter [Limper et al. 2018]



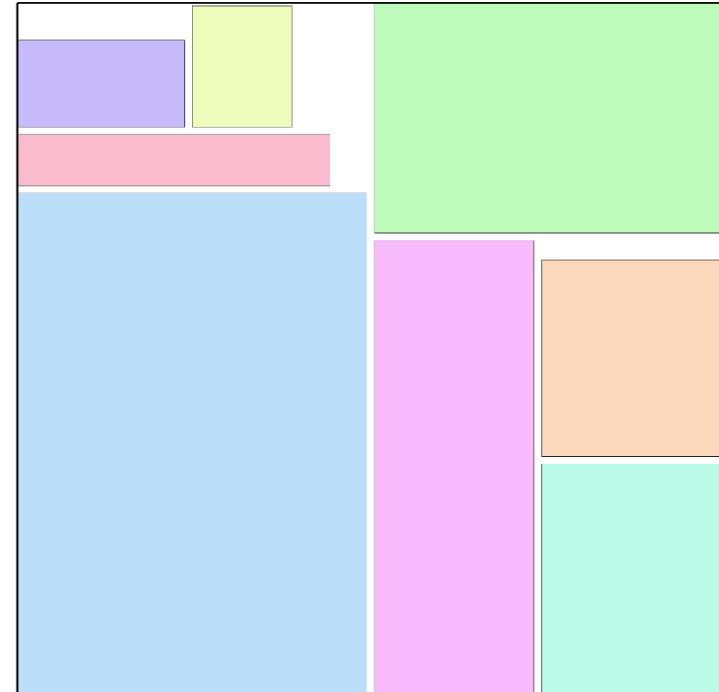
**No guarantee for a high PE result!**

# Motivation

# Packing Problems

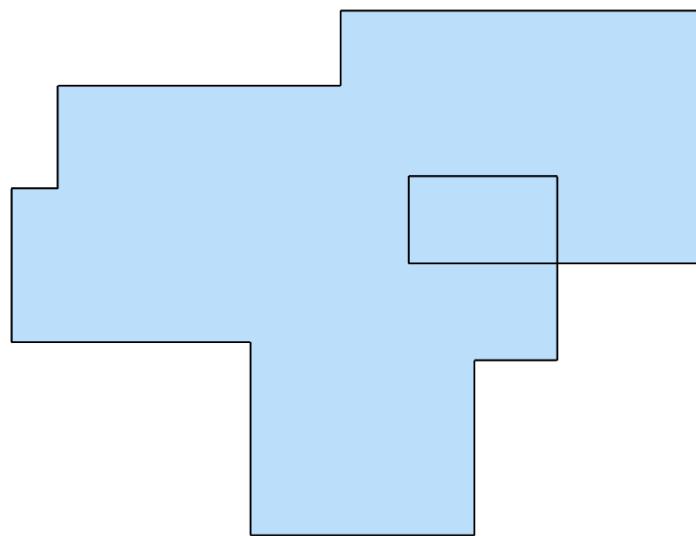


Irregular shapes  
Hard to achieve high PE

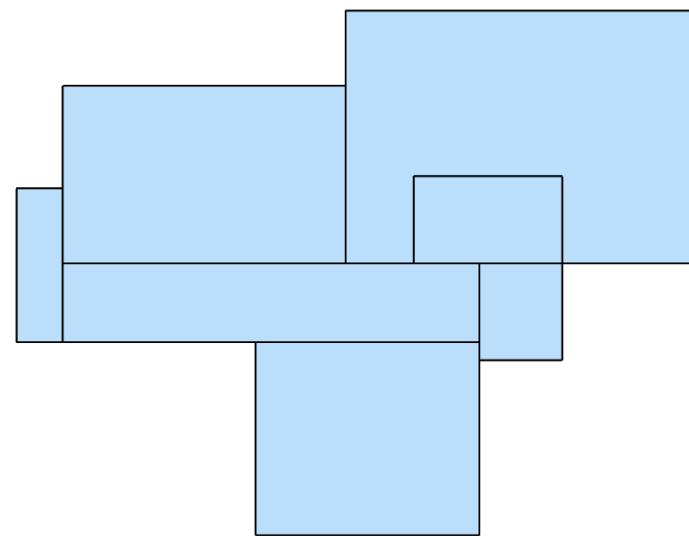


Rectangles  
Simple to achieve high PE  
Widely used in practice

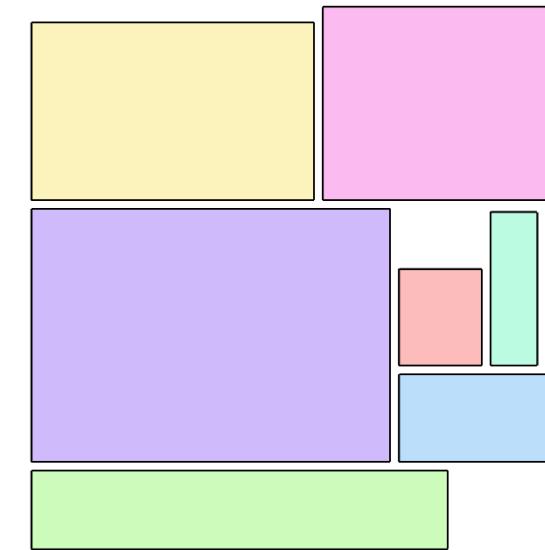
# Axis-Aligned Structure



Axis-aligned structure

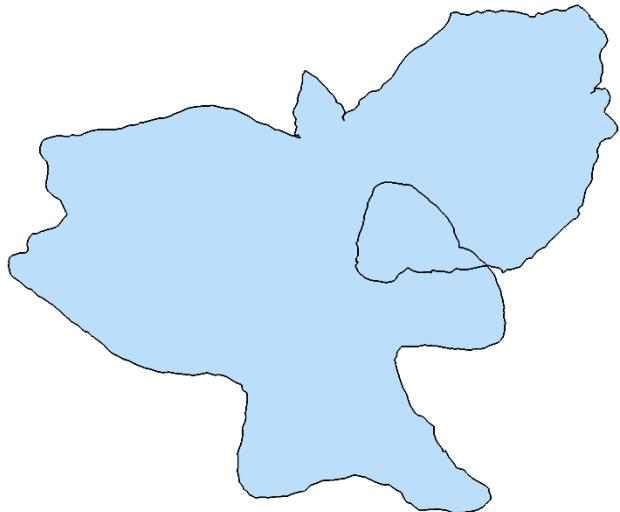


Rectangle decomposition

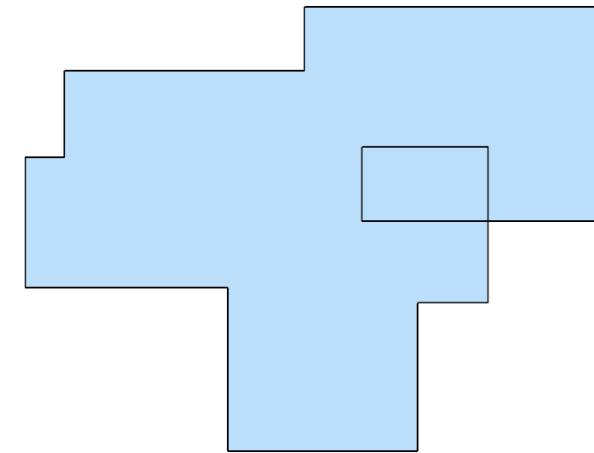


High PE (87.6%)!

# General Cases

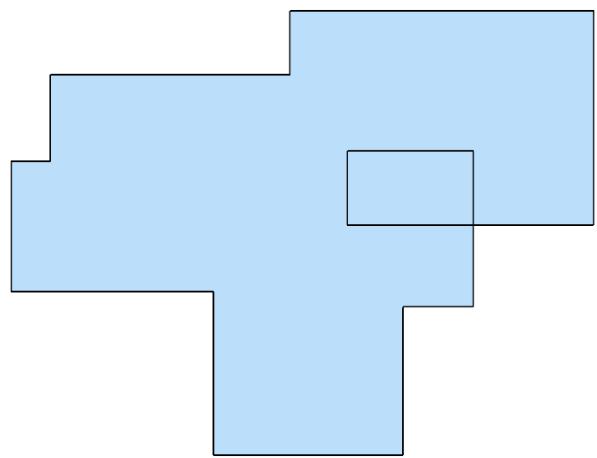


Not axis-aligned

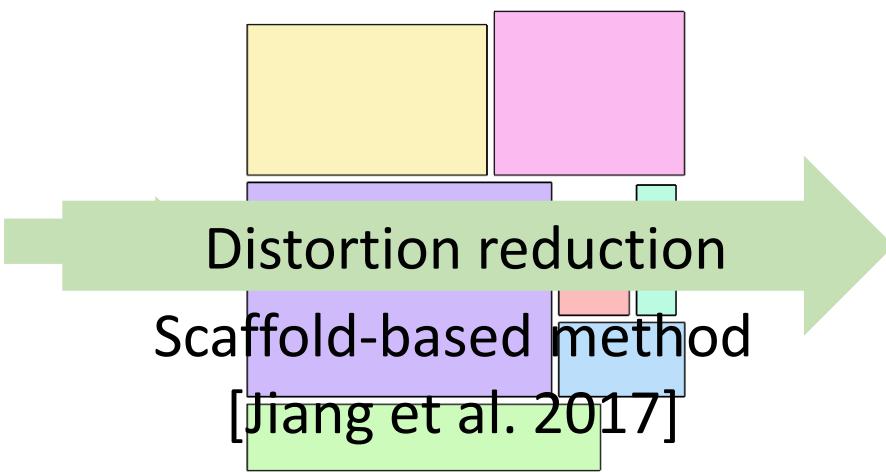


Axis-aligned  
Higher distortion

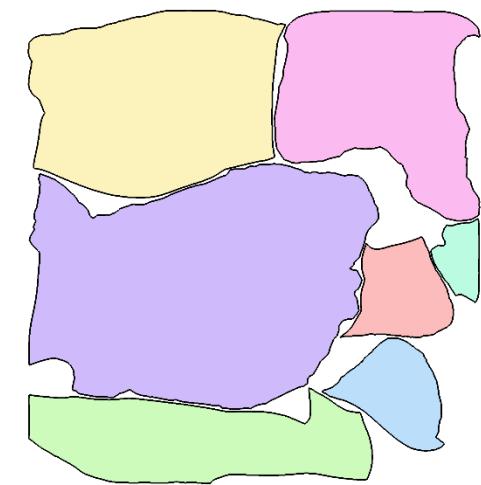
# Distortion Reduction



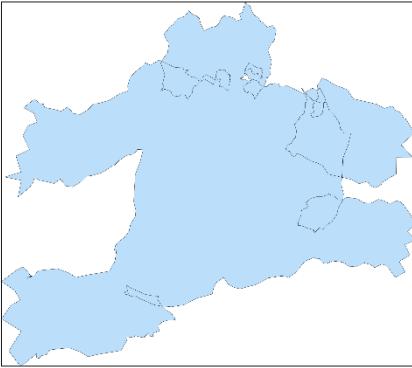
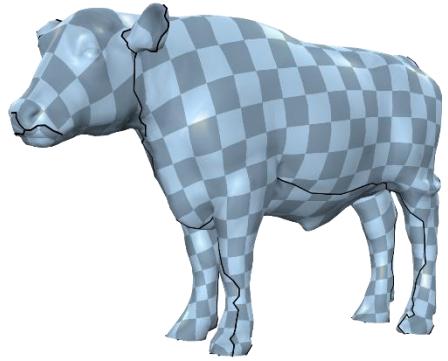
Axis-aligned  
High distortion



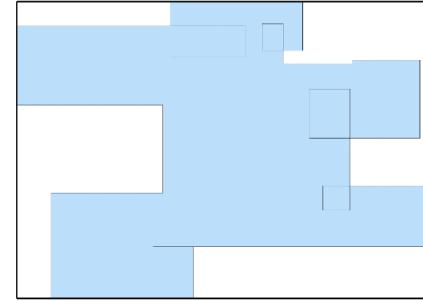
No overlap & High PE  
High distortion



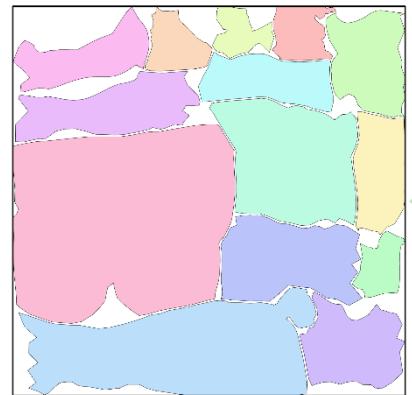
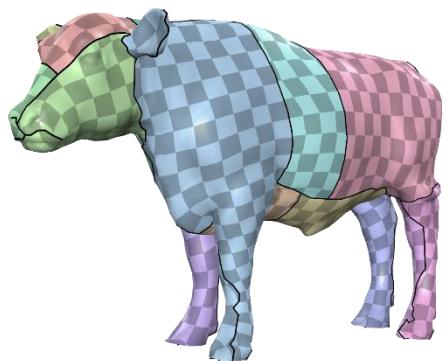
No overlap & High PE  
Low distortion  
Bounded PE



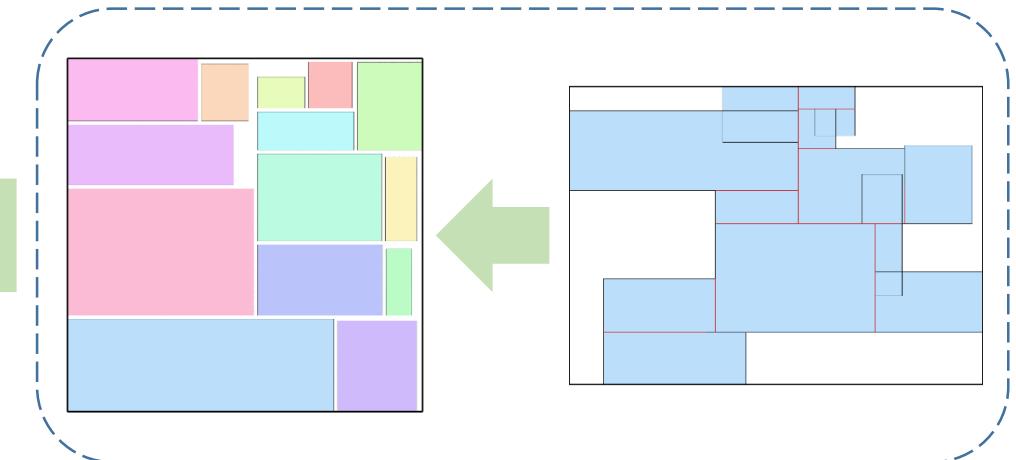
Axis-aligned deformation



Rectangle  
decomposition  
and packing

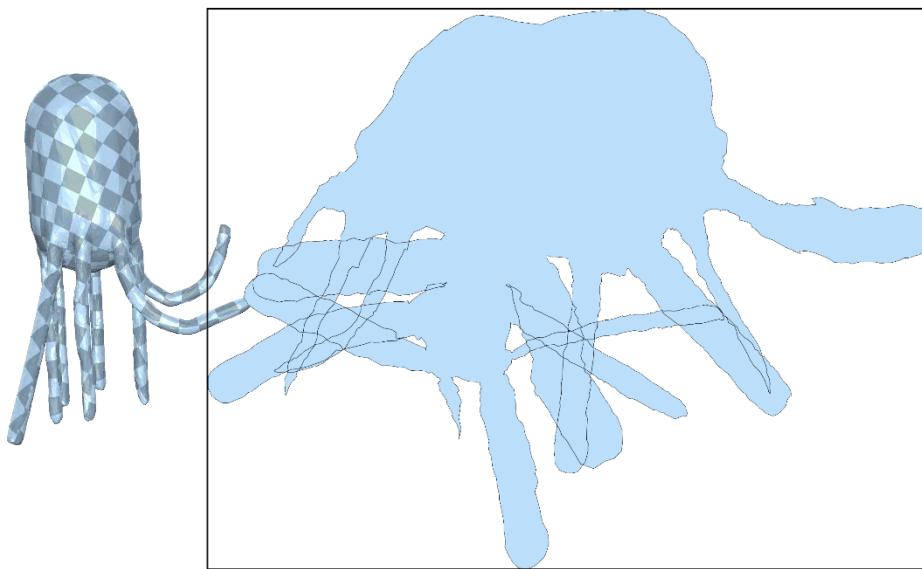


Distortion reduction



# Axis-Aligned Deformation

- Input

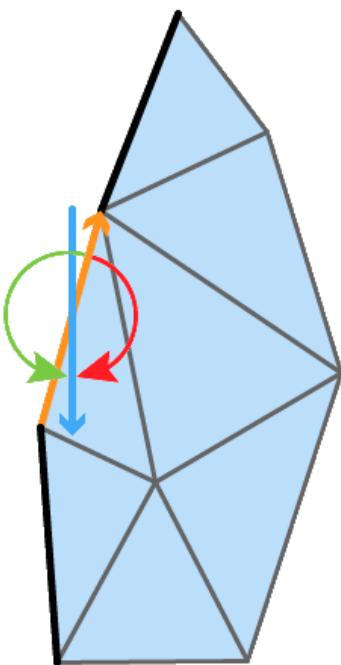


Single chart  
With overlap



10 charts  
Without overlap

# Axis-Aligned Deformation

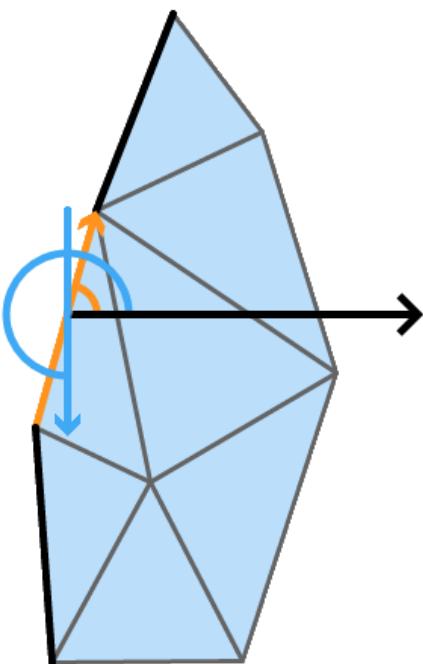


Direction vector  
Ambiguous rotating directions

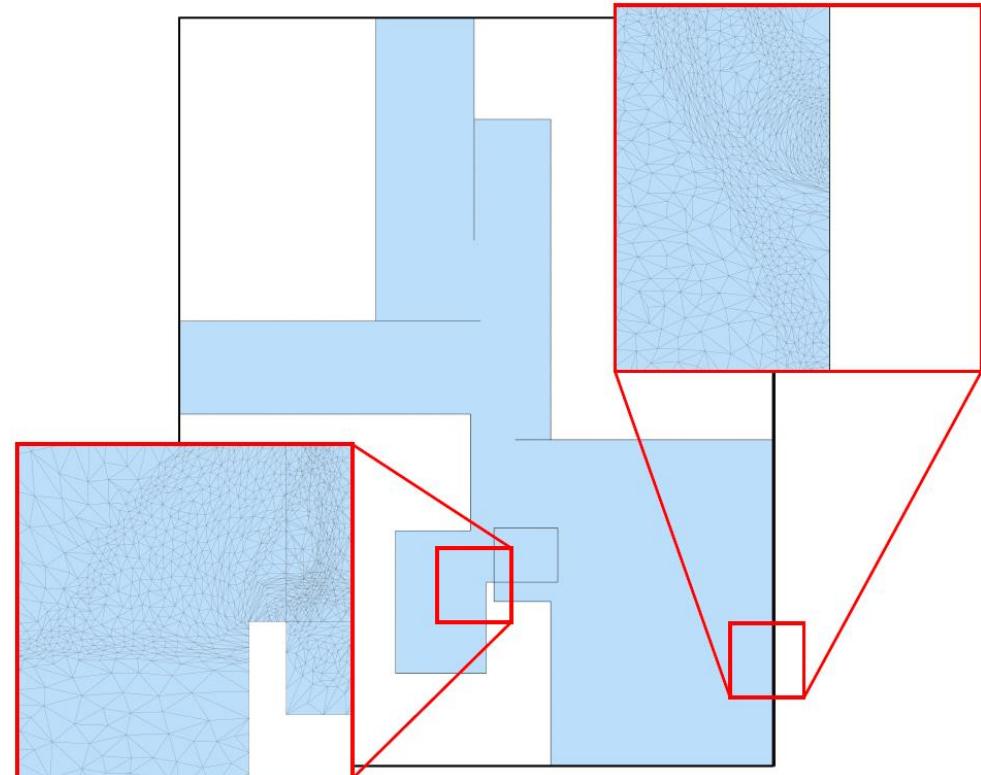


Fail!

# Axis-Aligned Deformation

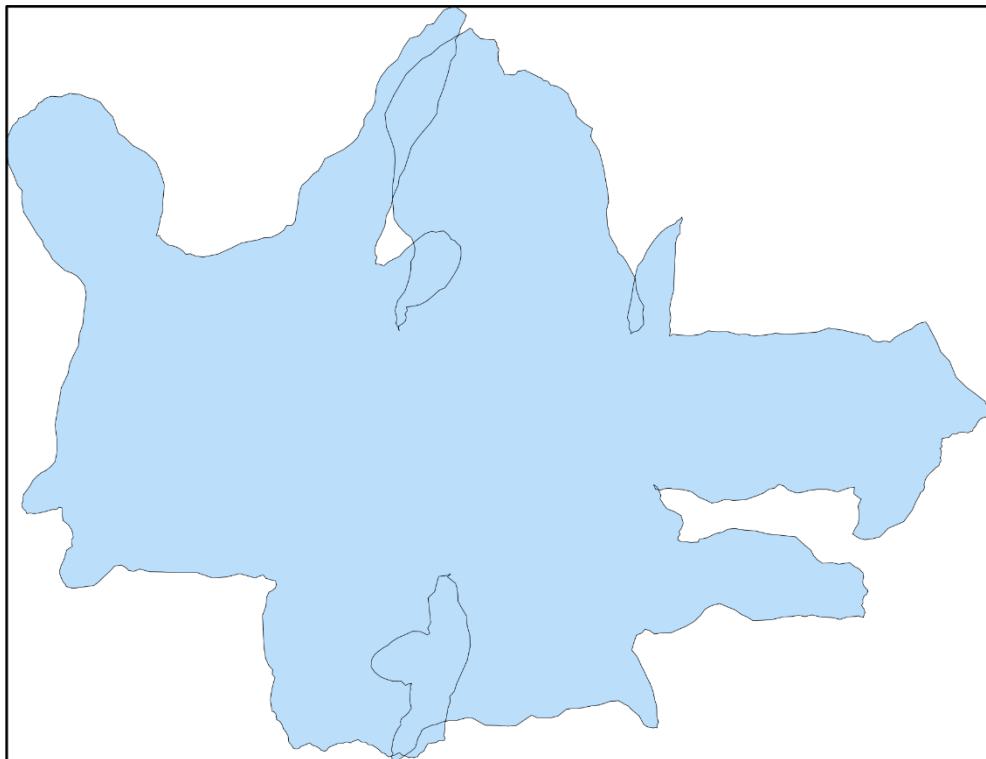


Polar angle  
Clear rotating direction

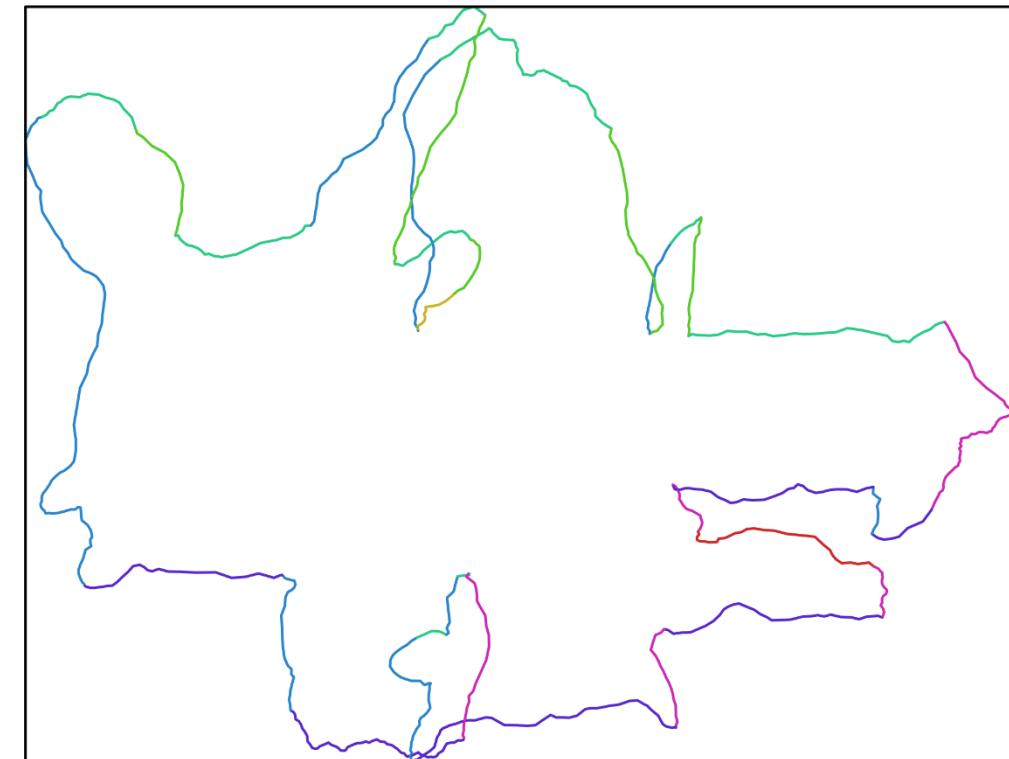


Success!

# Axis-Aligned Deformation



Input



Target polar angle

# Axis-Aligned Deformation

- Energy of boundary alignment

$$E_{\text{edge}}(\mathbf{b}_i) = \frac{1}{2}(1 - \gamma) \left( \theta_i - \frac{\pi}{2} \Theta_i \right)^2 + \frac{1}{2} \gamma \left( \frac{l_i}{l_i^0} - 1 \right)^2$$

Rotate polar angle                      Keep length

$$E_{\text{align}}(\mathbf{c}) = \sum_{i=1}^{N_b} \frac{l_i^0}{l^0} E_{\text{edge}}(\mathbf{b}_i)$$

# Axis-Aligned Deformation

- Energy of isometric distortion(symmetric Dirichlet)

$$E_d(c) = \frac{1}{4} \sum_{f_i \in F^c} \frac{\text{Area}(f_i)}{\text{Area}(M^c)} (\|J_i\|_F^2 + \|J_i^{-1}\|_F^2)$$

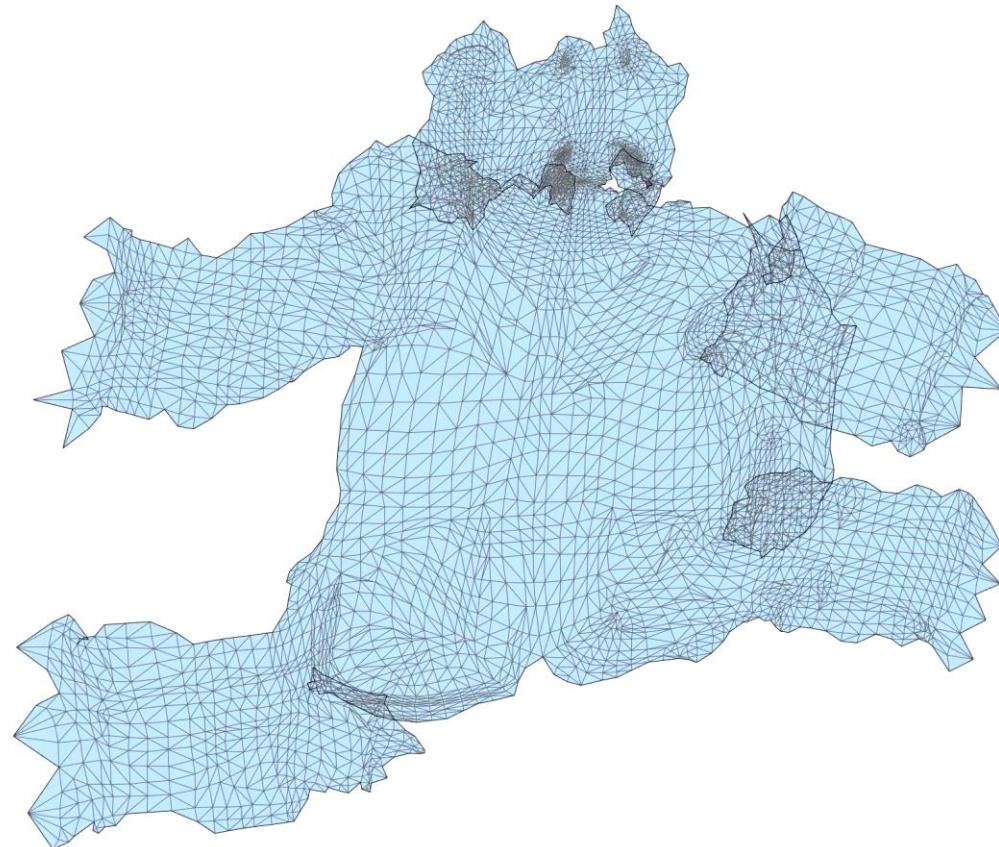
Keep low distortion and orientation consistency.

# Axis-Aligned Deformation

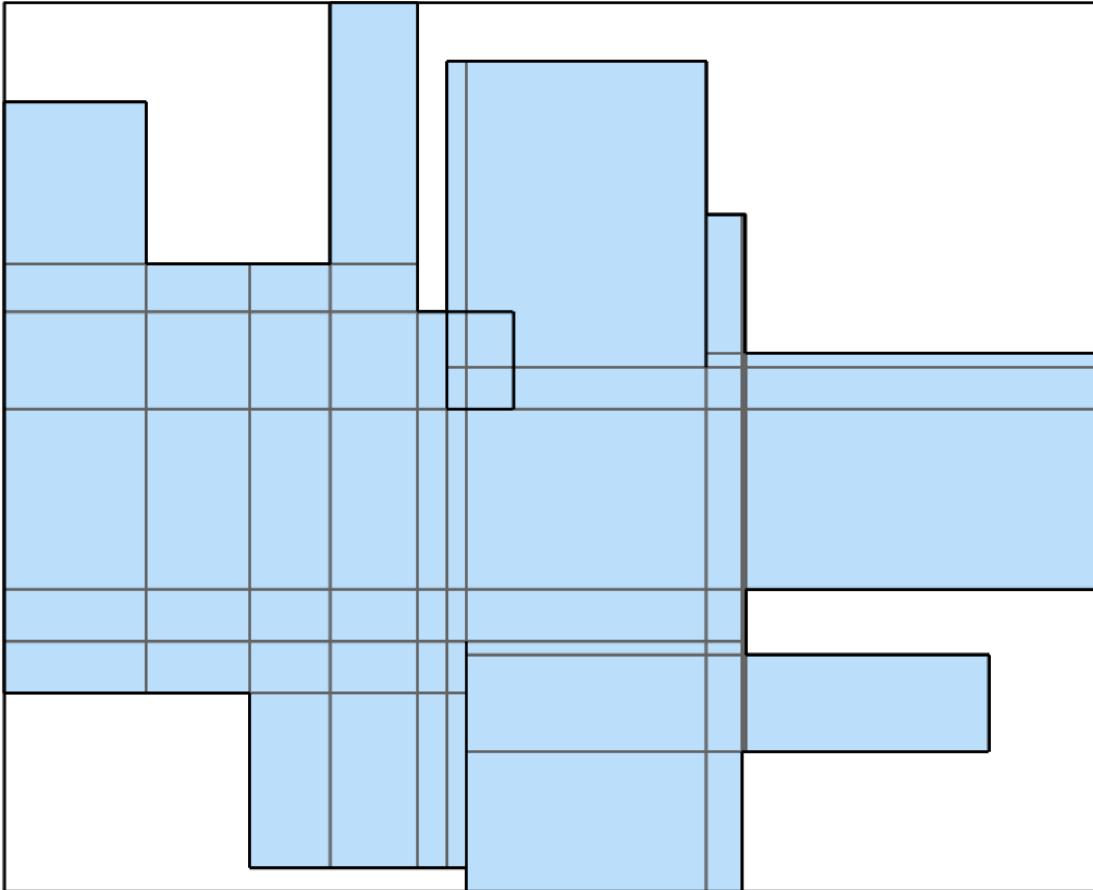
0.2X Playback

$$\min_c \quad E_d(c) + \lambda E_{\text{align}}(c)$$

$$\text{s.t.} \quad \det J_i > 0, \forall i$$



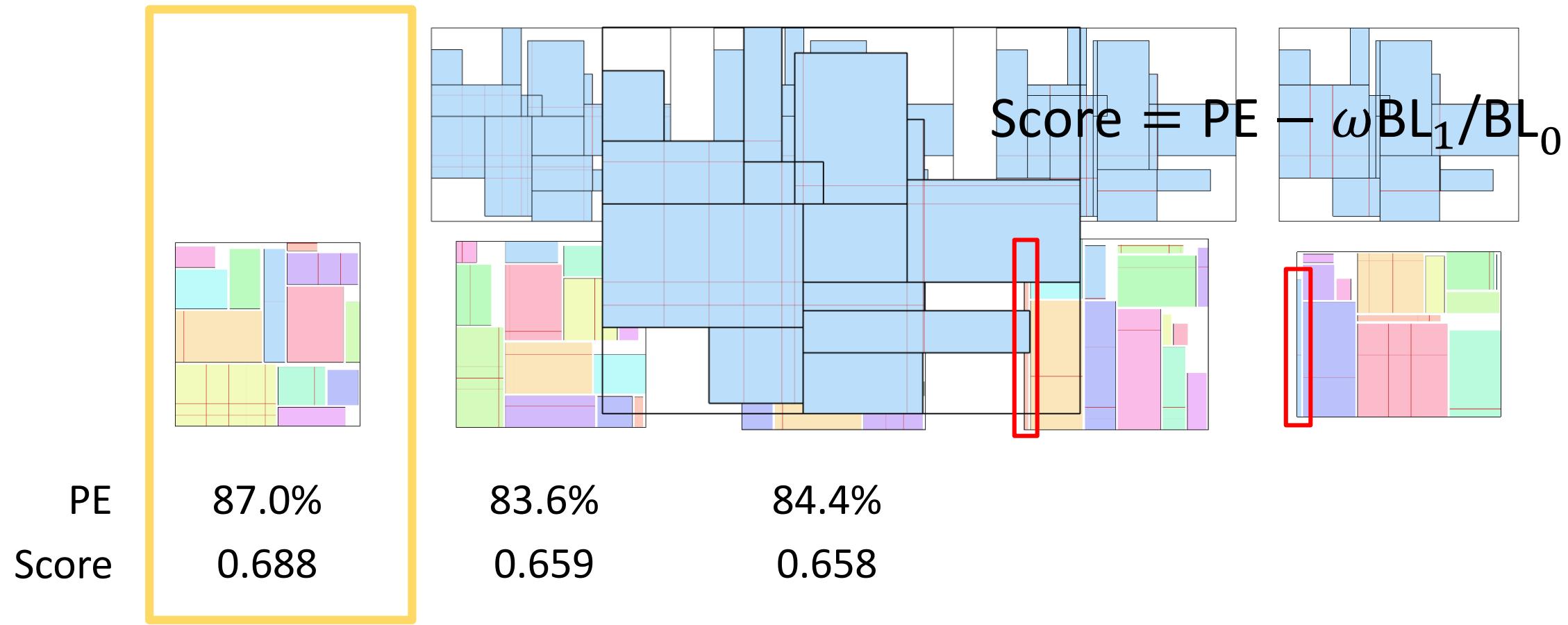
# Rectangle Decomposition and Packing



The faces are all rectangles.  
But the number is too many.

# Rectangle Decomposition and Packing

- Motorcycle graph algorithm



# Distortion Reduction

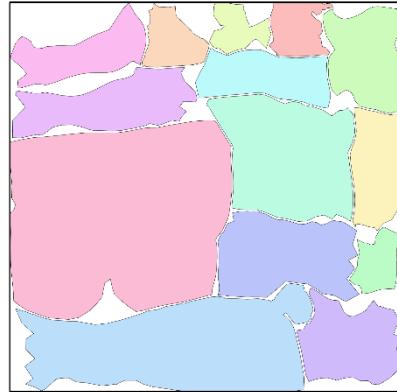
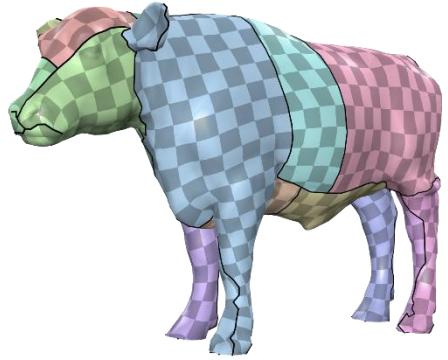
$$\min_C E_{\text{reduction}} = E_d(C) + E_{\text{PE}}(C)$$

s.t.  $\Phi$  is bijective

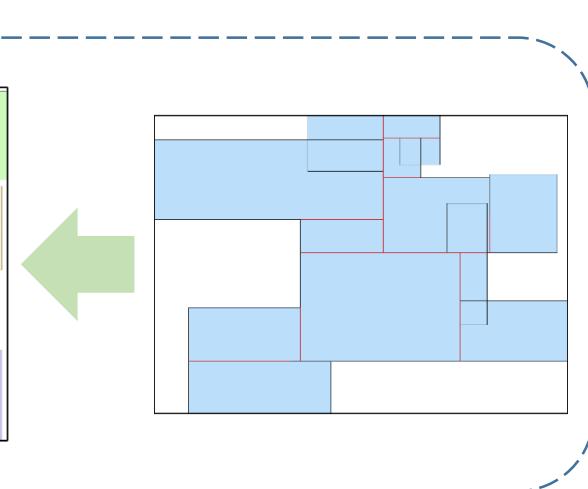
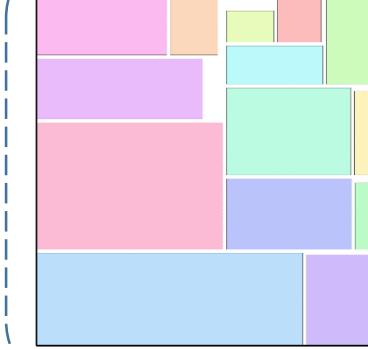
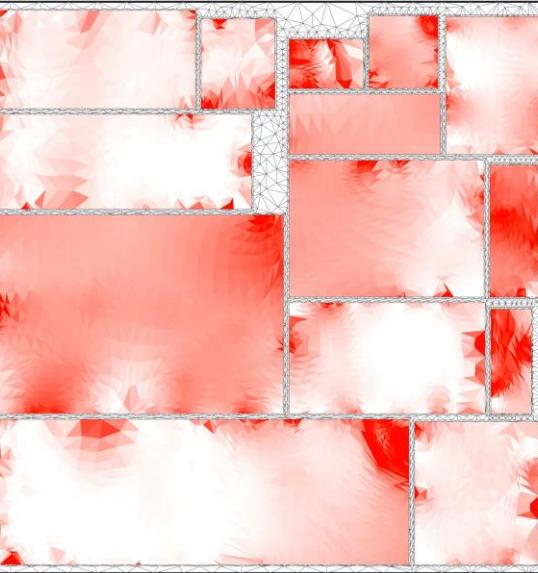
Isometric energy

Barrier function of PE bound

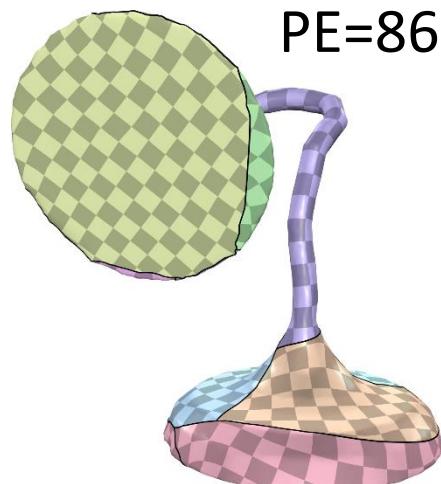
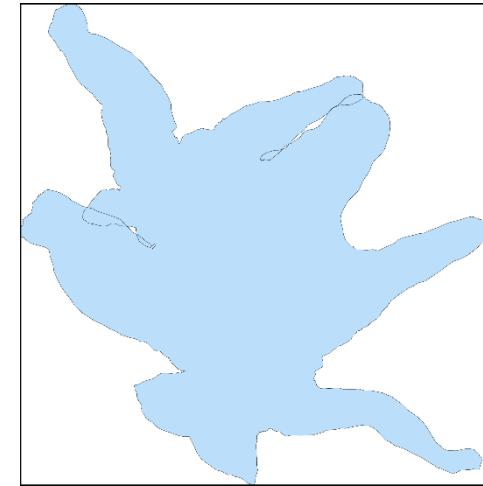
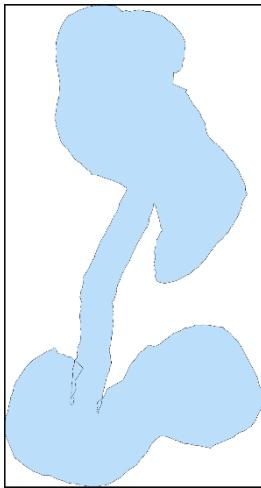
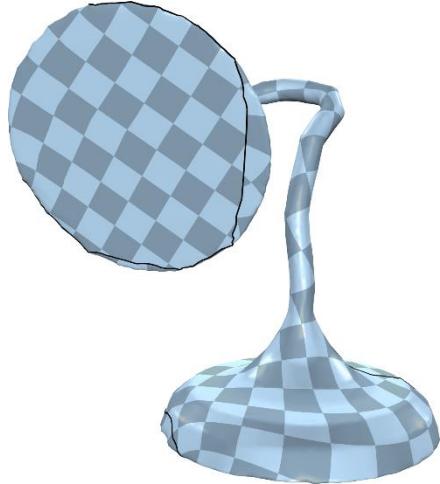
Scaffold-based method  
[Jiang et al. 2017]



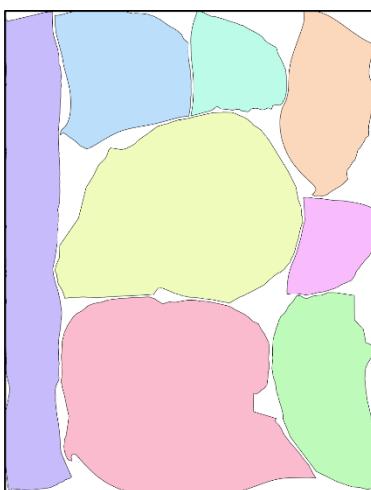
Distortion reduction



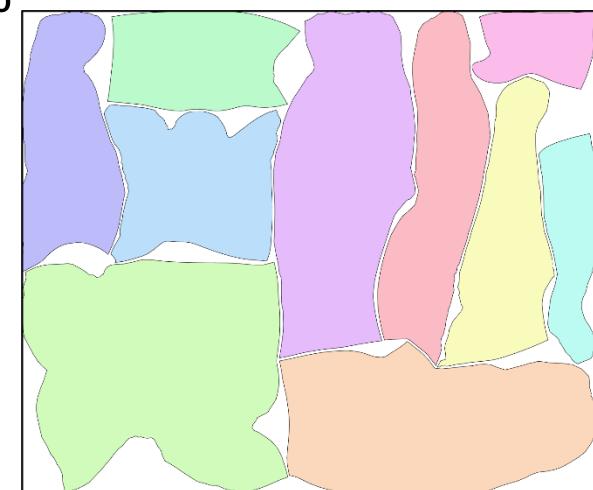
# Benchmark (5,588)



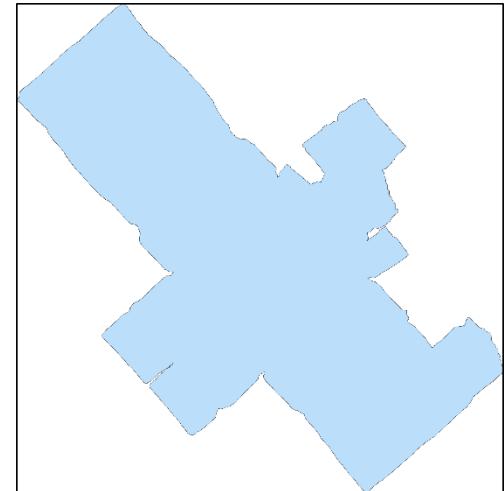
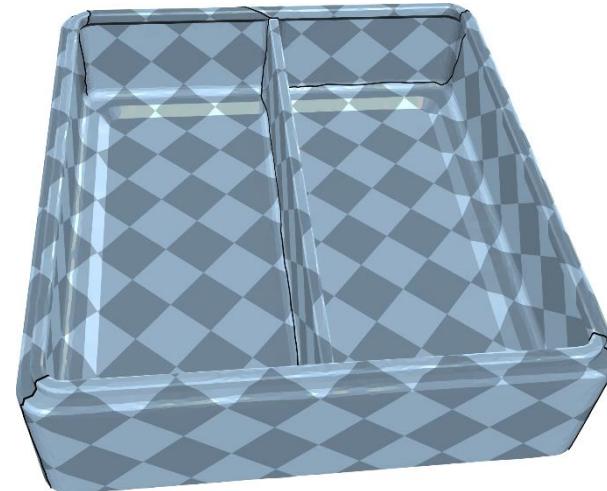
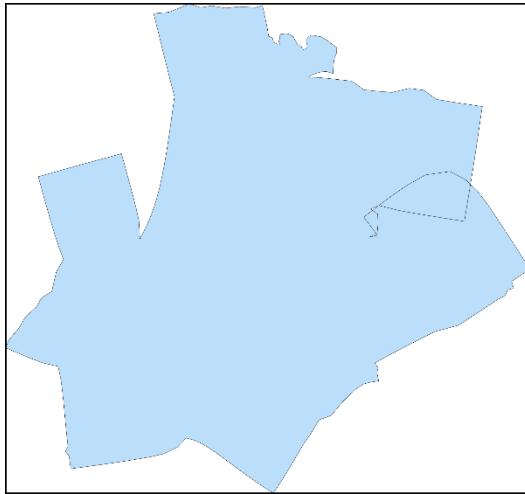
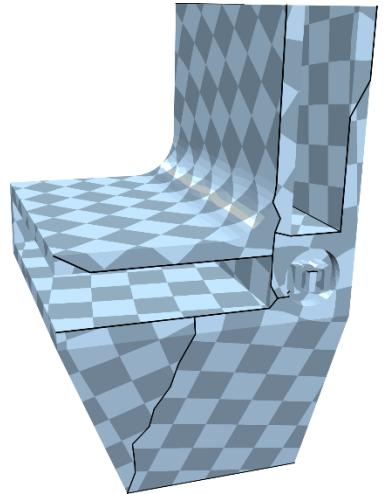
PE=86.2%



PE=86.7%

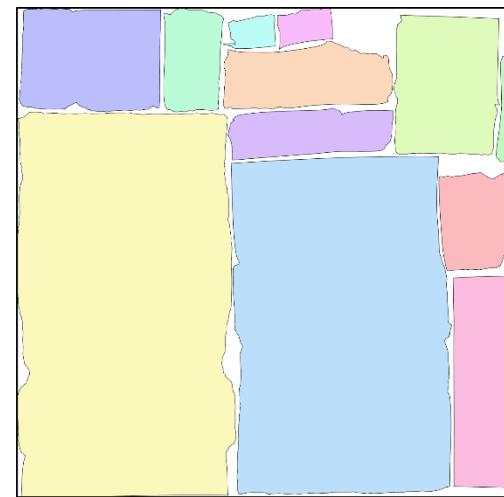
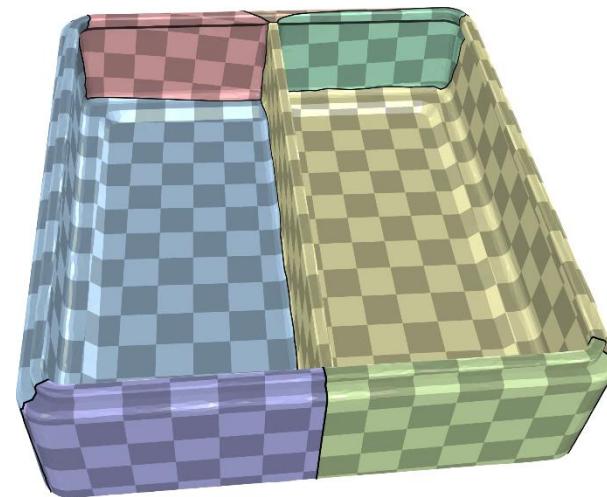
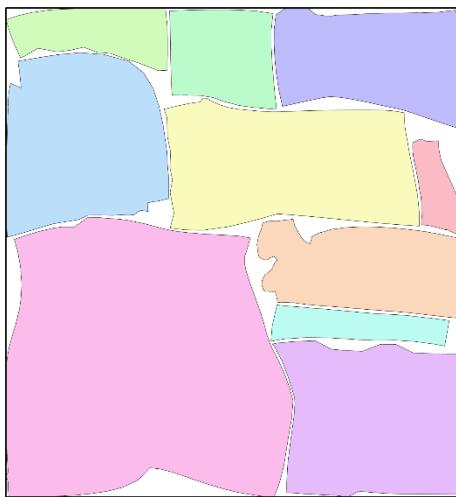
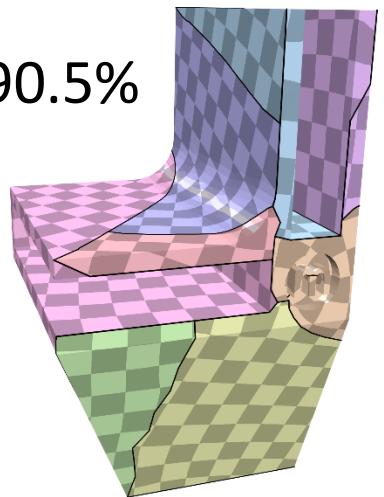


# Benchmark (5,588)



PE=91.0%

PE=90.5%



# PolyAtlas: Atlas Refinement with Bounded Packing Efficiency

*Submitted to ACM SIGGRAPH 2019*

ID: 339