Surface Mapping

Xiao-Ming Fu

Outlines

- Definition
- Application
- Algorithms
  - Common base domain
  - Parameterization-based method
  - ……

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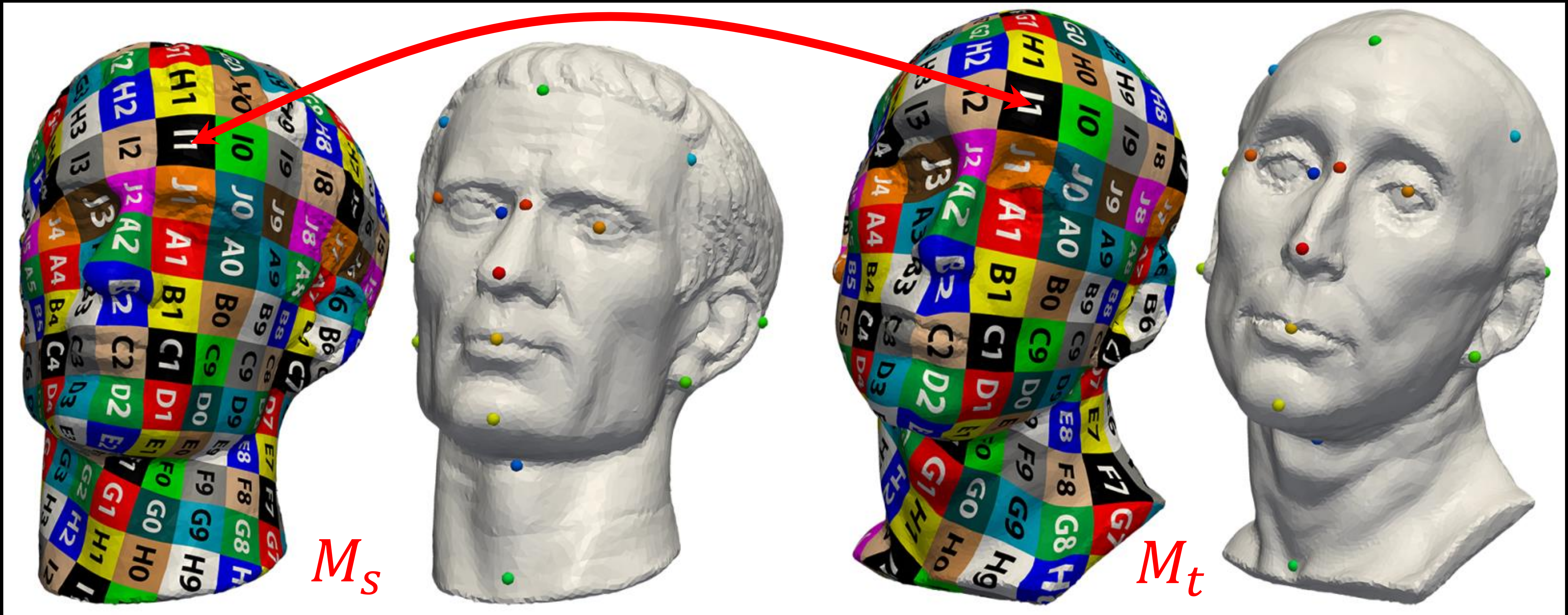
Surface Mapping

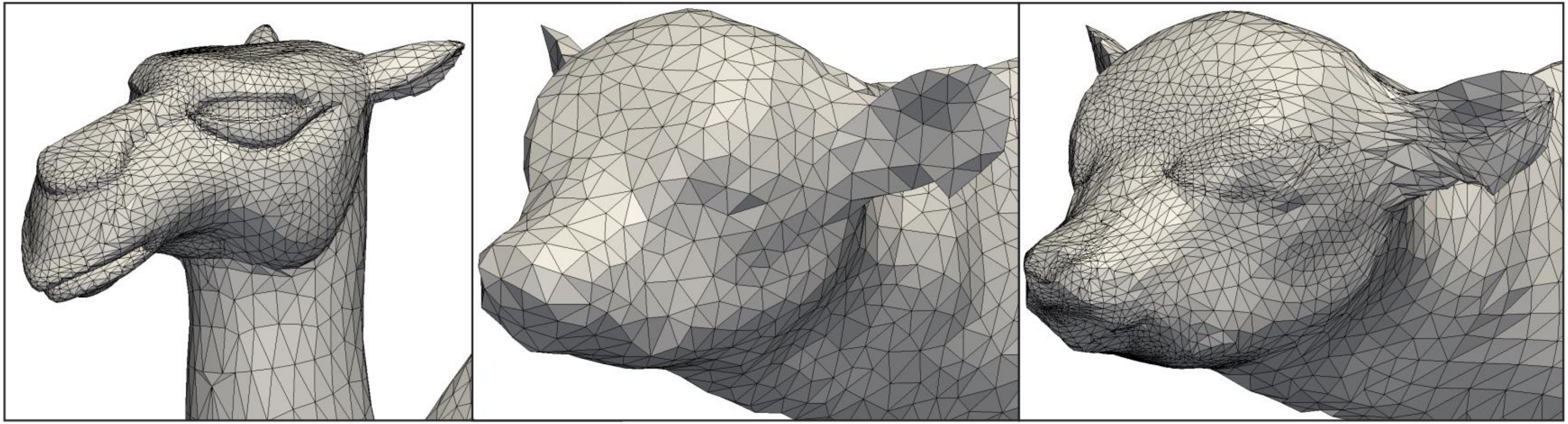
Inter-surface mapping, Cross parameterization

- A one-to-one mapping f between the two surfaces M_s and M_t



Compatible meshes

- Meshes with identical connectivity



M_s $\quad$ M_t $\quad$ $\widehat{M_t} = f(M_s) \approx M_t$

Applications

- Morphing
- Attribute transfer
- ……

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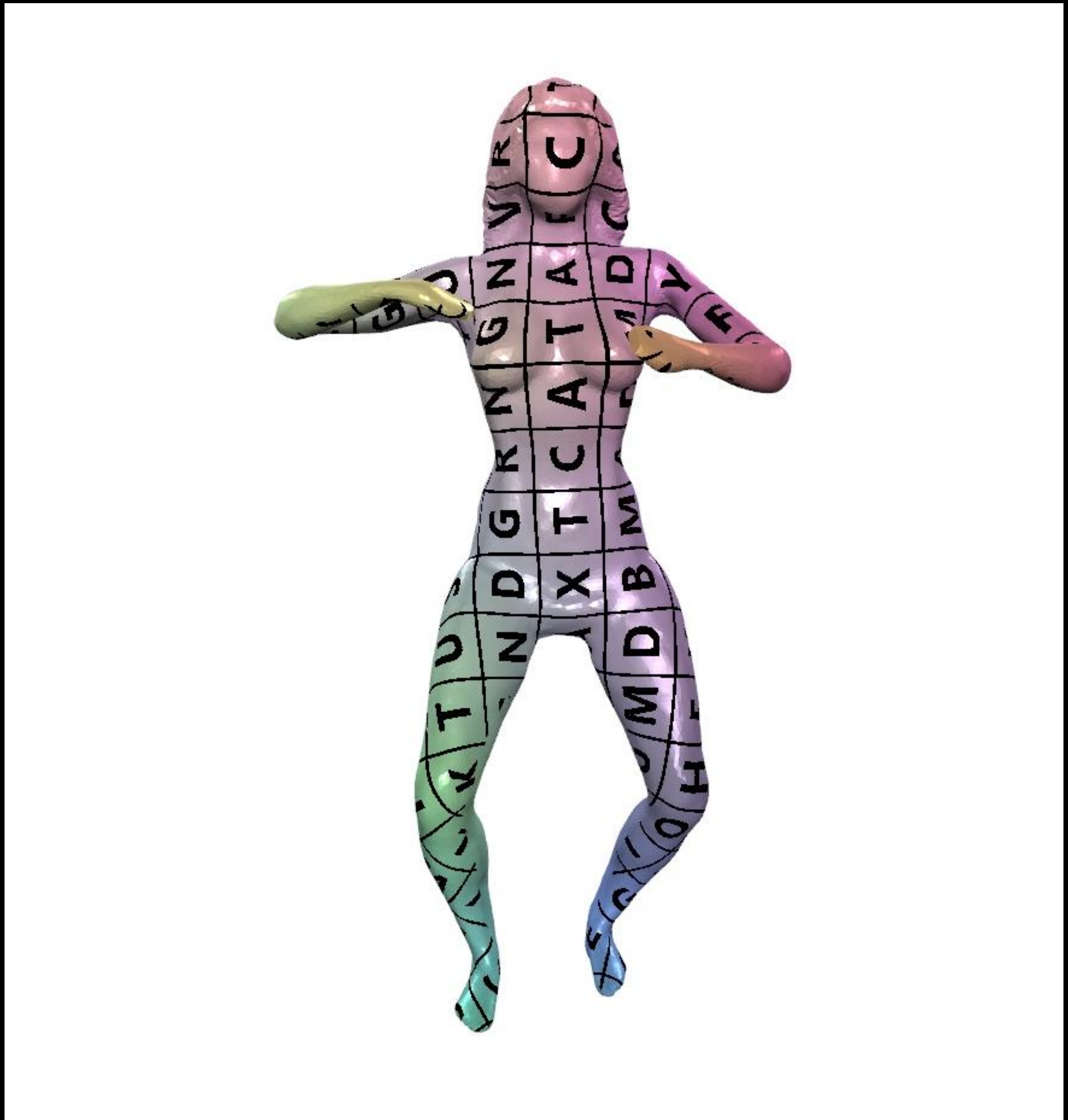
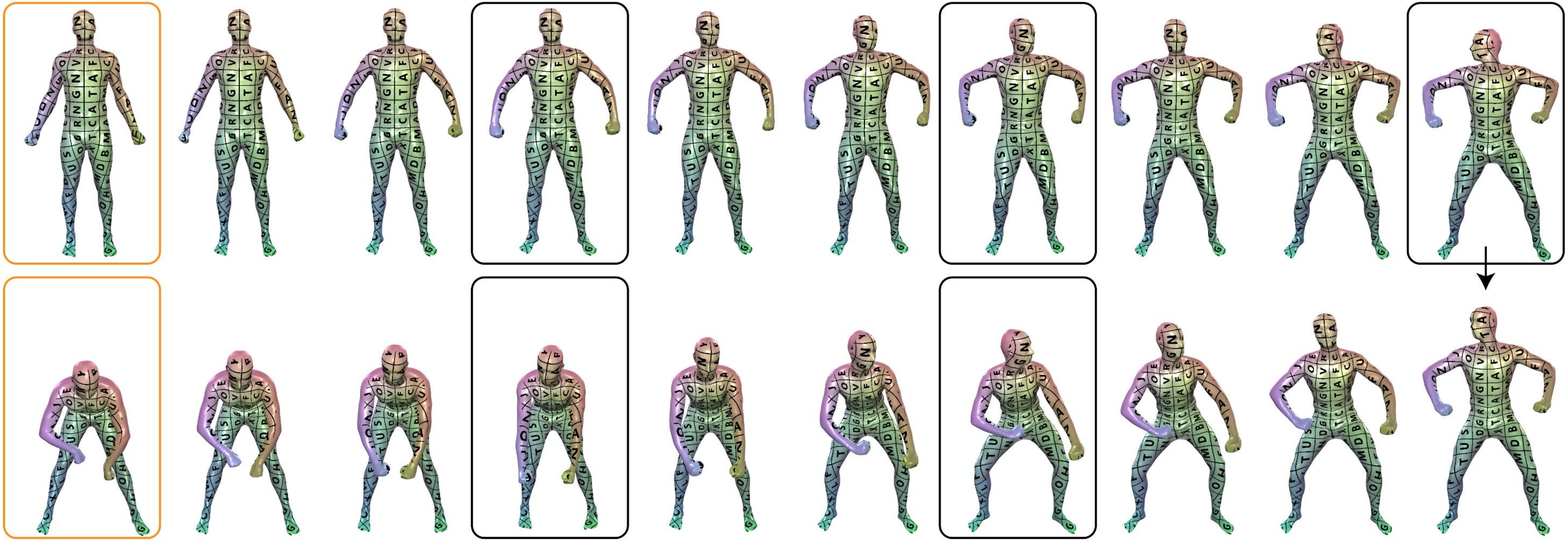
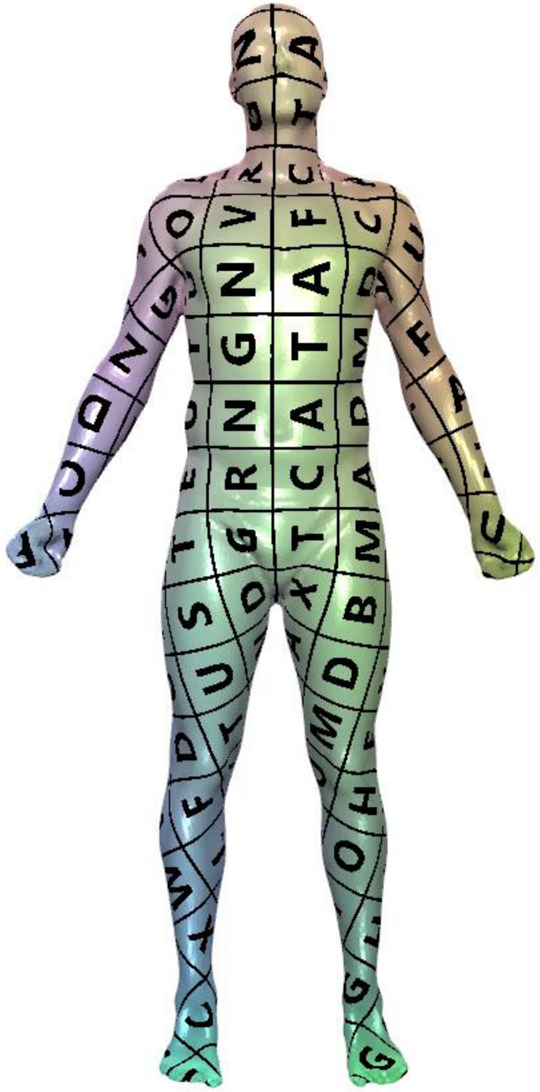
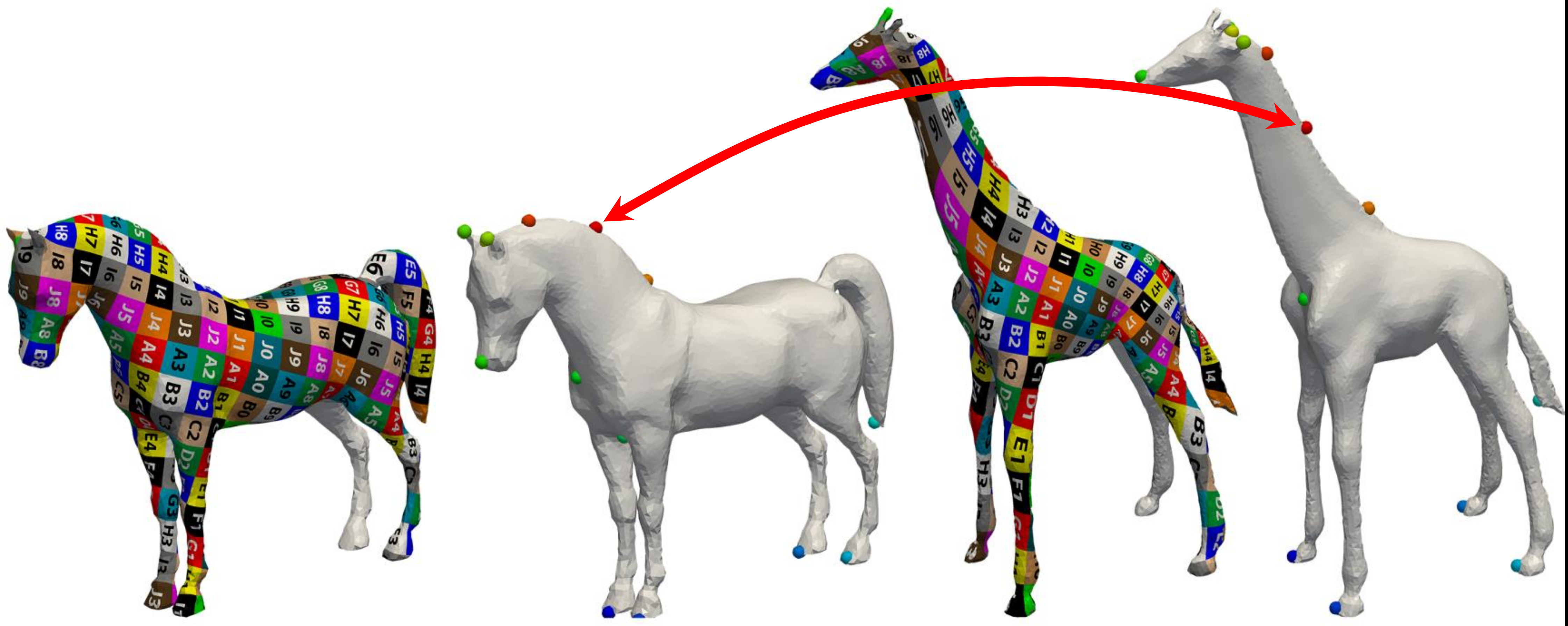






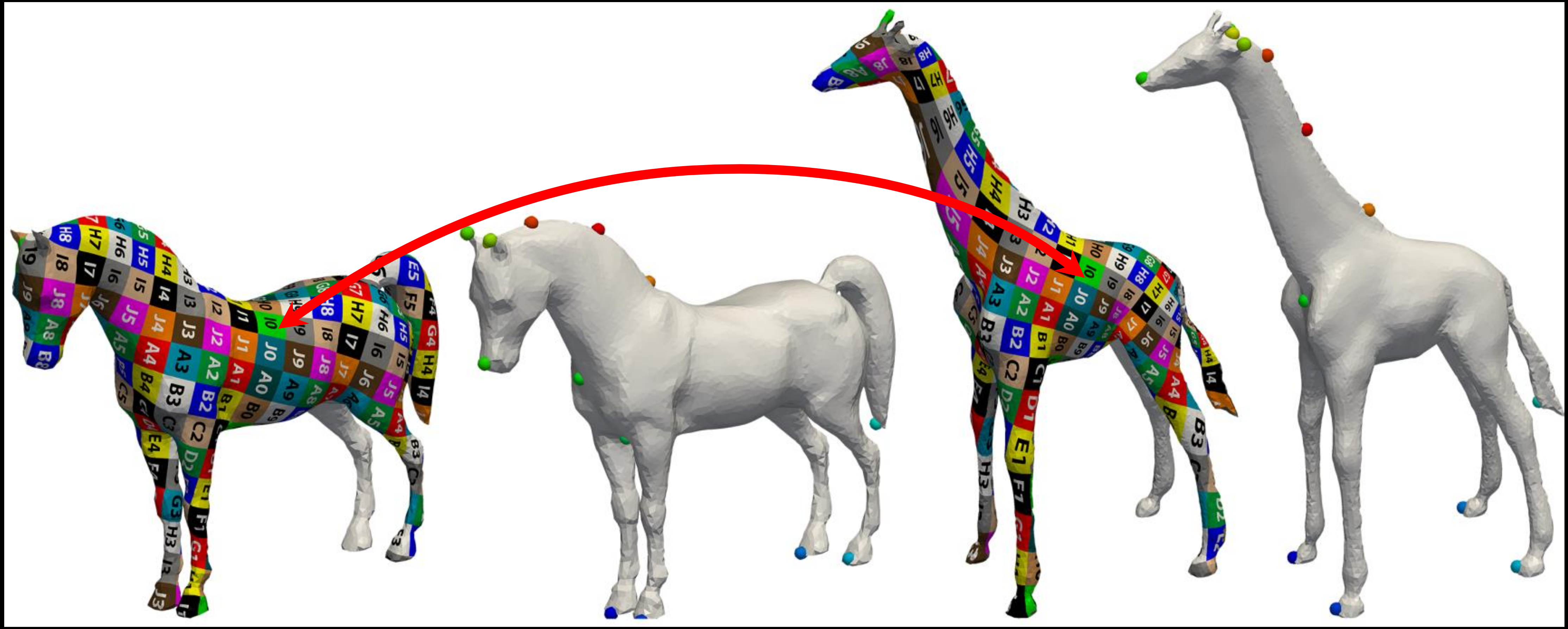
Inputs

- Two (n) models and some corresponding landmarks



Goal

- Bijection and low distortion



Outlines

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  - Common base domain
    - Cross-Parameterization and Compatible Remeshing of 3D Models
  - Parameterization-based method
  - ……

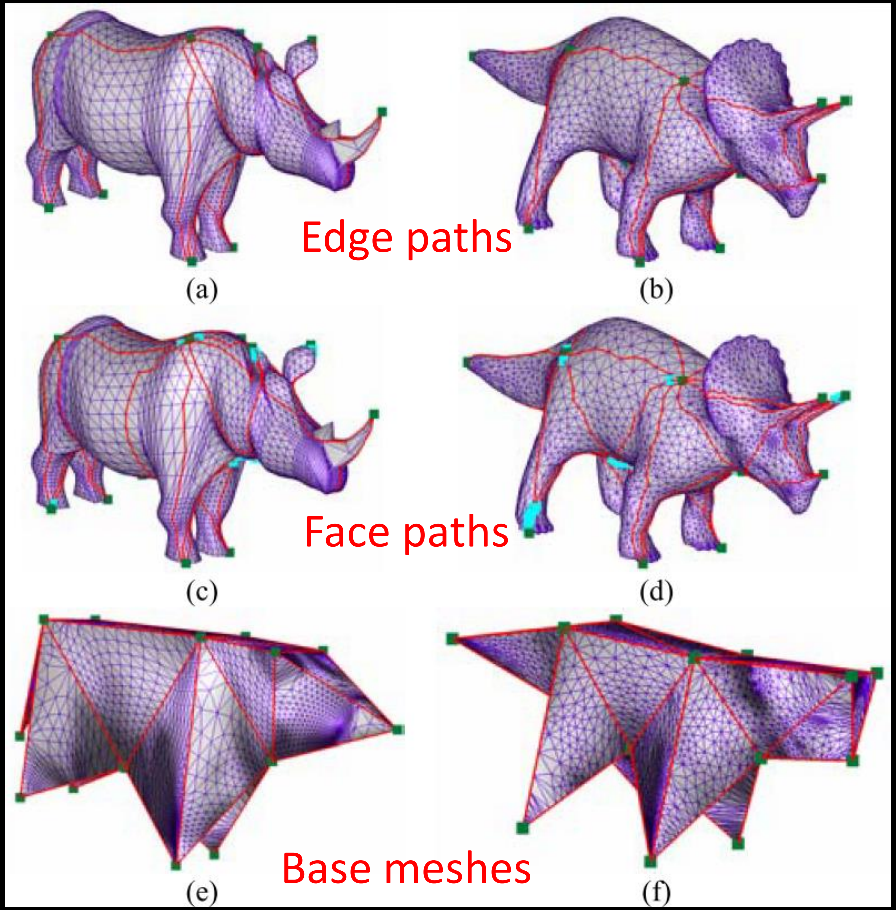
Algorithm stages

- Construct a common base domain
  - Topologically identical triangular layouts of the two meshes.
- Compute a low distortion cross-parameterization
  - Each patch is mapped to the corresponding base mesh triangle.
- Compatibly remeshes the input models using the parameterization

Common base domain

Topologically identical triangular layouts

- Incrementally adding pairs of matching edge paths between feature vertices.



Algorithm *PathMatch*

$M_s' = Ms$

$M_t' = M_t$

Compute the shortest paths s^{ij} for each pair of vertices in V_s

Compute the shortest paths t^{ij} for each pair of vertices in V_t

$ST = \varnothing$

foreach s^{ij}

$ST \leftarrow \langle s^{ij}, t^{ij} \rangle$ /* pairs of matching paths */

while $ST \neq \varnothing$

$\langle s,t \rangle = ST.RemoveShortest()$

if *NonBlocking*(s,t)

Add s to P_s; Add t to P_t

Remove all interior vertices of s from M_s'

Remove all interior vertices of t from M_t'

Update(ST, s, t)

end

end

end

a pair of paths with the smallest length sum

Cross-Parameterization

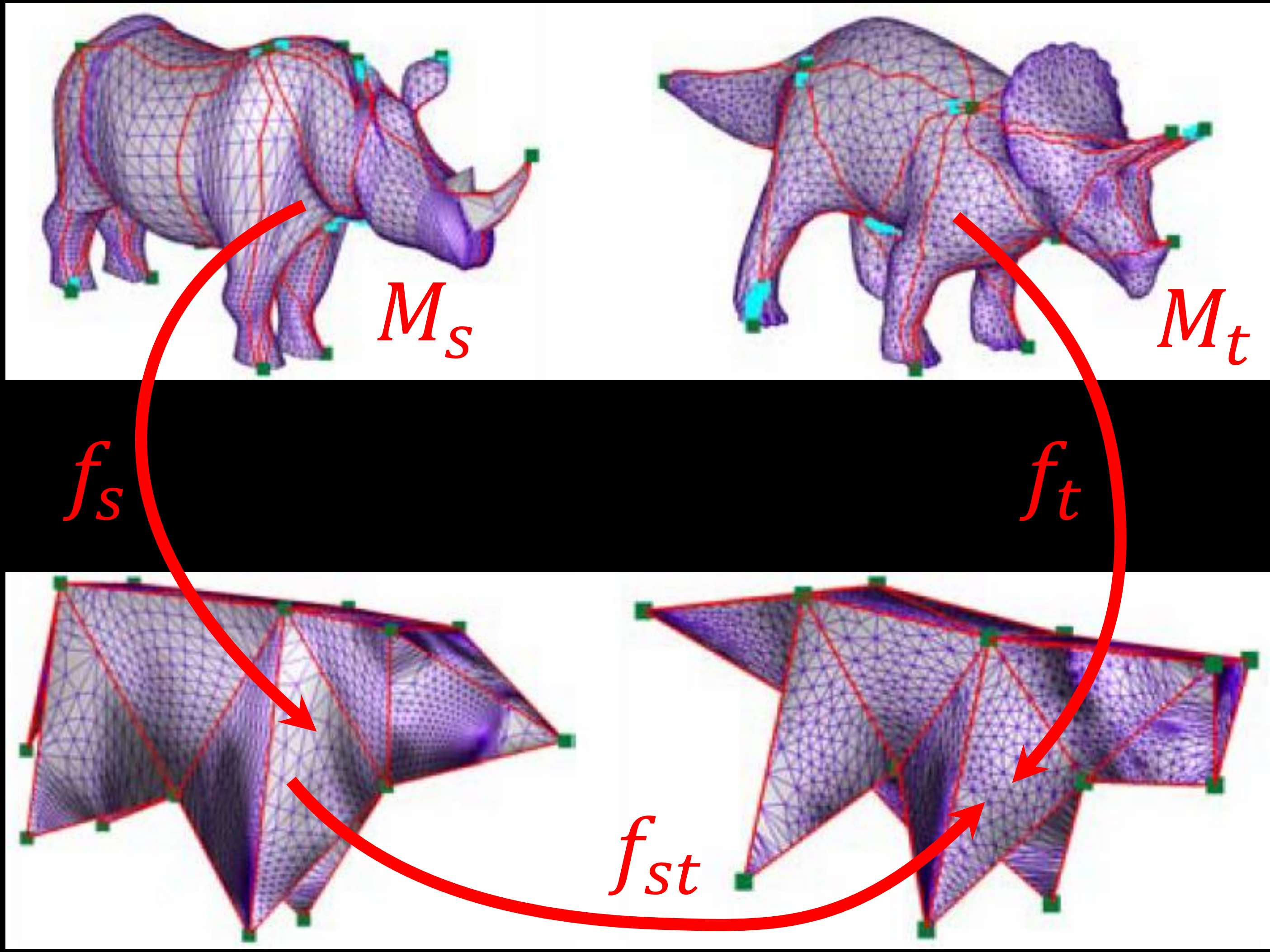
- Tutte's embedding:

  Given a triangulated surface homeomorphic to a disk, if the (u, v) coordinates at the boundary vertices lie on a convex polygon in order, and if the coordinates of the internal vertices are a convex combination of their neighbors, then the (u, v) coordinates form a valid parameterization (without self-intersections, bijective).

- Each patch is a triangle, i.e., it is a convex boundary.
  - Bijection guarantee.

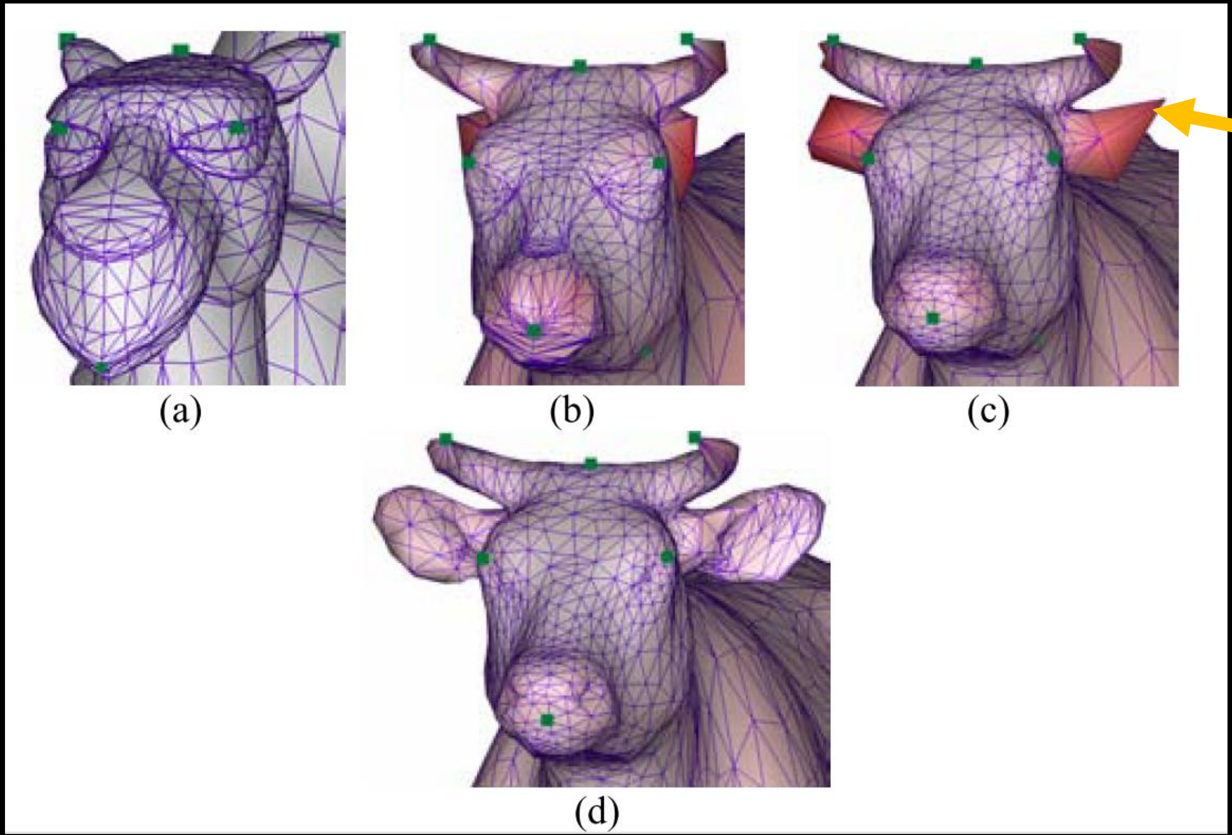
Cross-Parameterization

- $f = f_t^{-1} \circ f_{st} \circ f_s$



Compatible Remeshing

- First remeshes the target model with the connectivity of the source mesh
- Perform smoothing and refinement



High approximation error

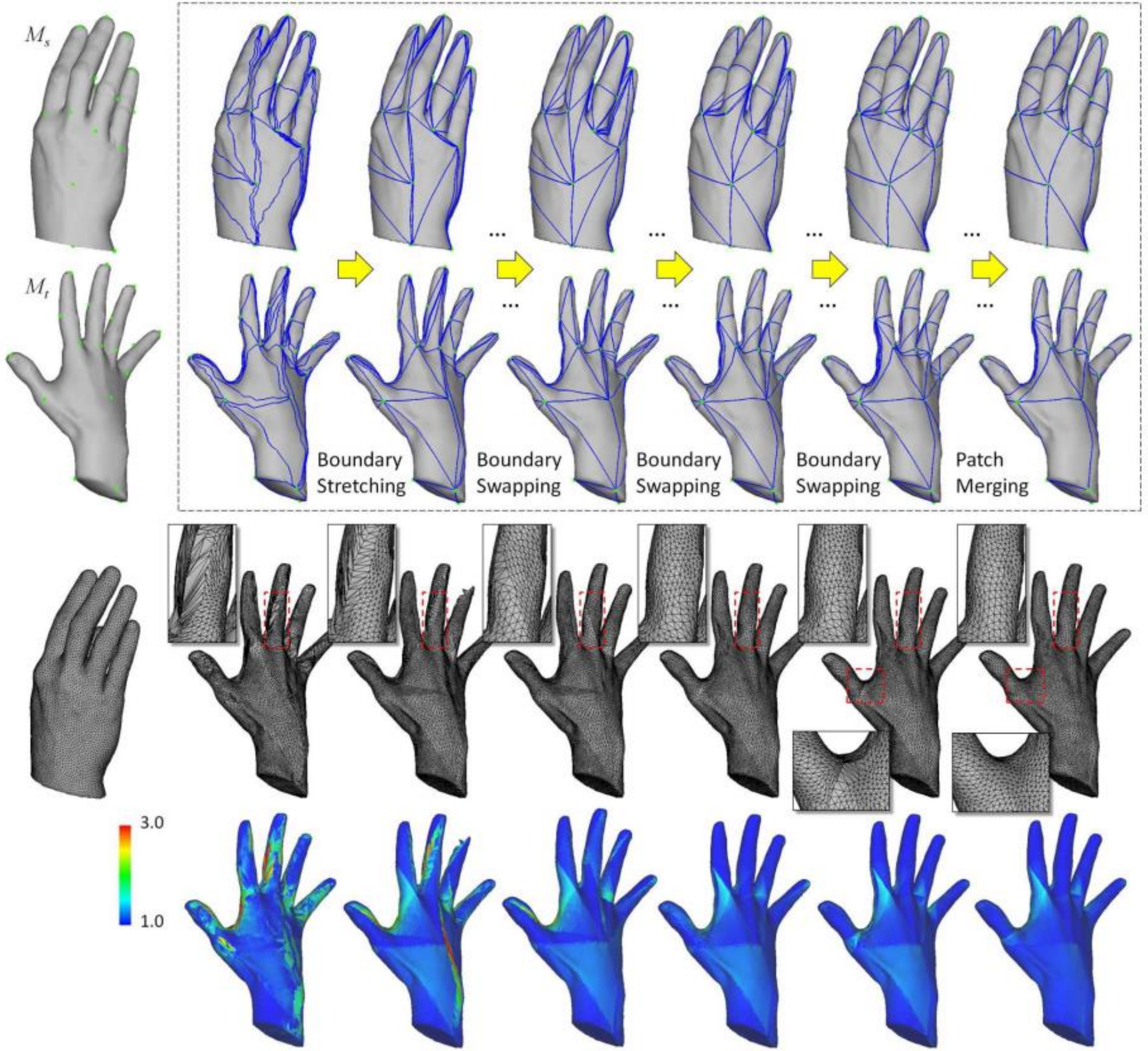
(b) Initial projection
(c) After smoothing
(d) Smoothing and refinement

Disadvantages

- The construction of common base domain is non-trivial.
- The distortion of surface mappings is not optimized directly.

Efficient Optimization of Common Base Domains for Cross Parameterization *2012*

- Initial Base Domain Construction (previous method)
- Boundary Stretching
  - curve stretching operator is to convert a curve into a geodesic curve locally
- Boundary Swapping
  - Similar to edge flip
- Patch Merging
  - helps reduce the distortion


M_s
M_t
Boundary Stretching
Boundary Swapping
Boundary Swapping
Boundary Swapping
Patch Merging
3.0
1.0

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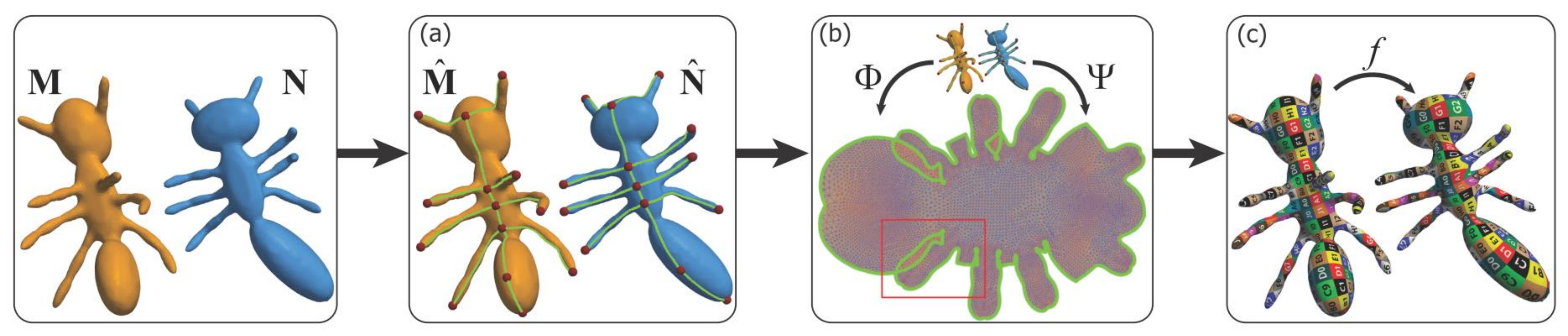
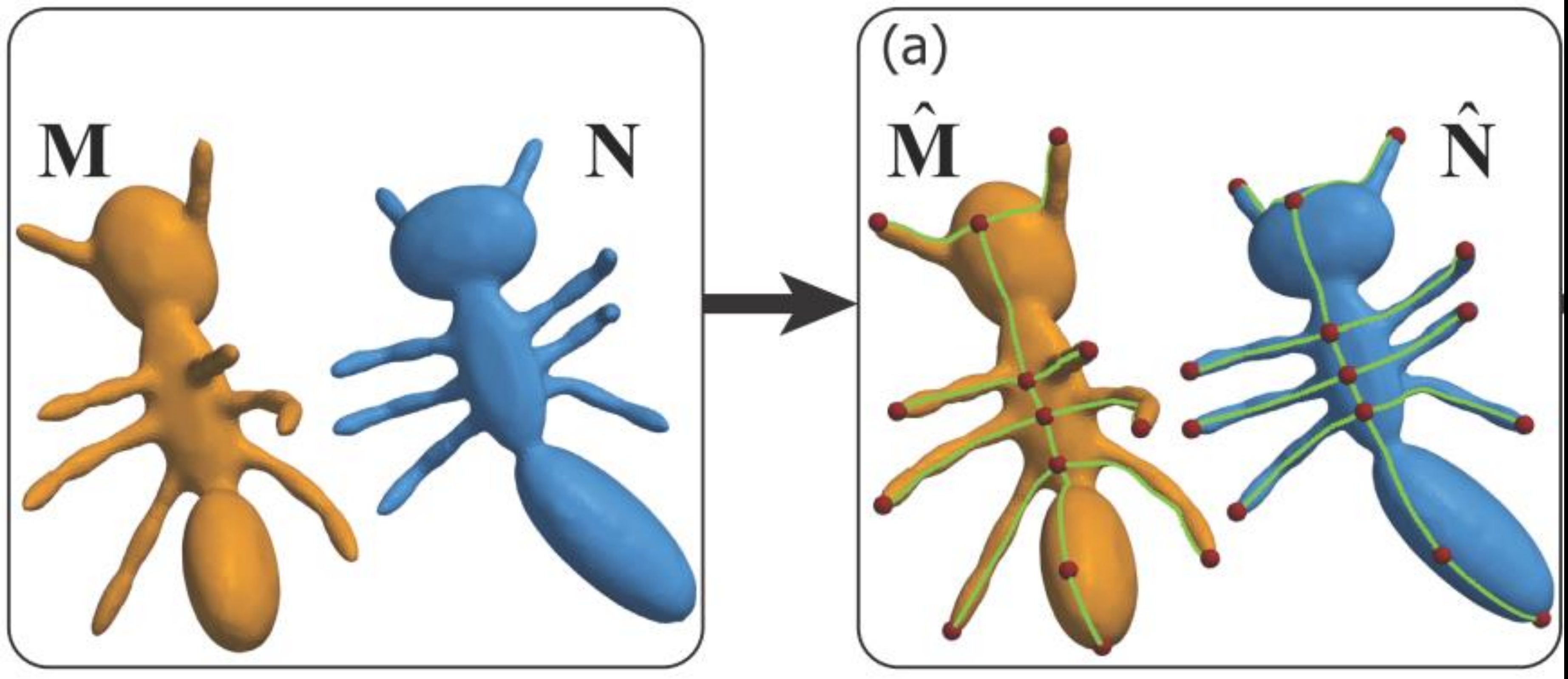
Algorithm steps

- (a) Cutting to disk topology.
- (b) Computing the joint flattenings Φ, Ψ.
- (c) Bijection Lifting.



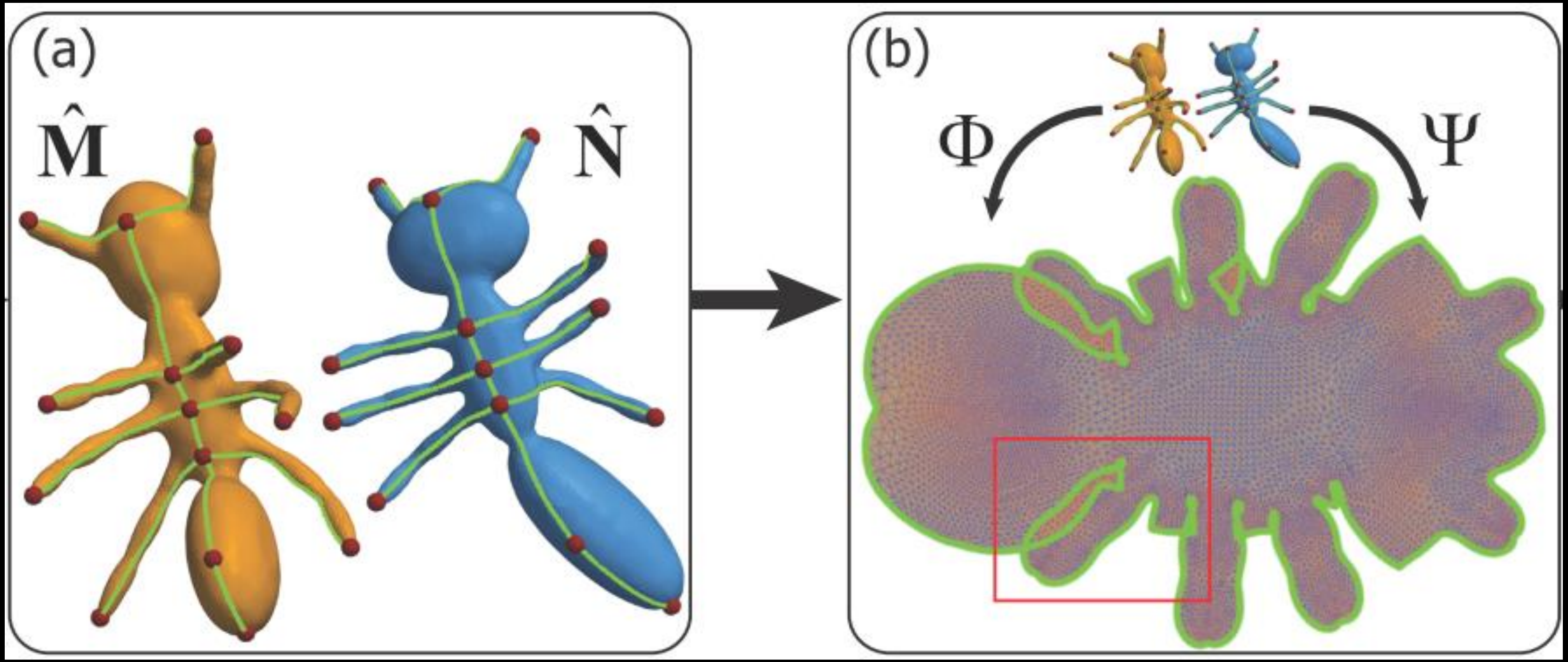
Cutting paths

- Bijective correspondence
  - Shortest path
  - Minimal spanning tree



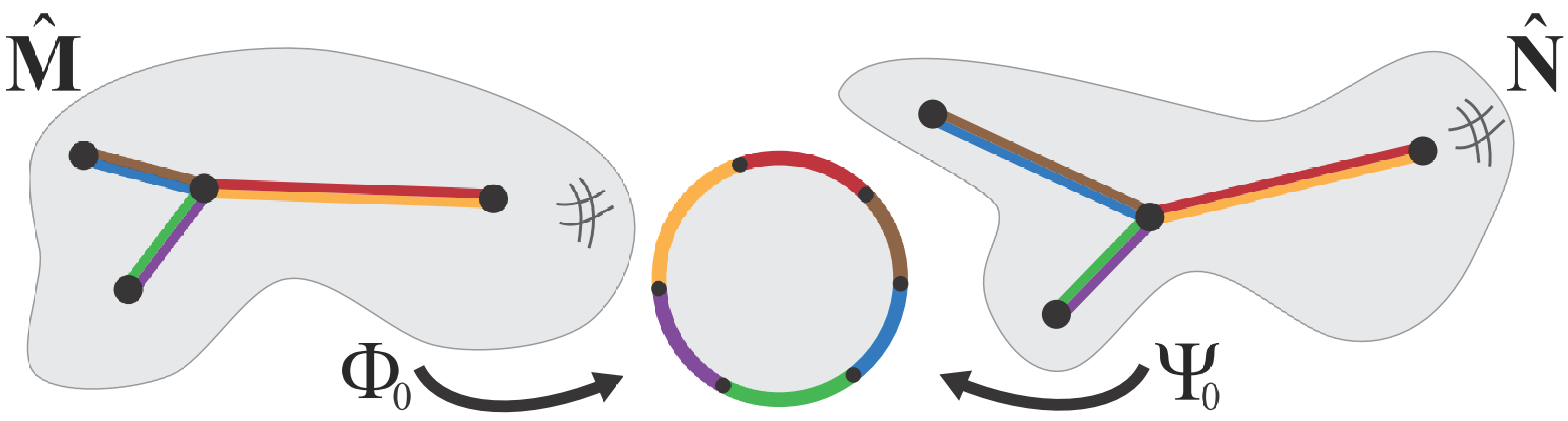
Computing Φ, Ψ

- Constraint
  - **Common boundary condition**
  - Locally injective
- Solvers:
  - Former methods



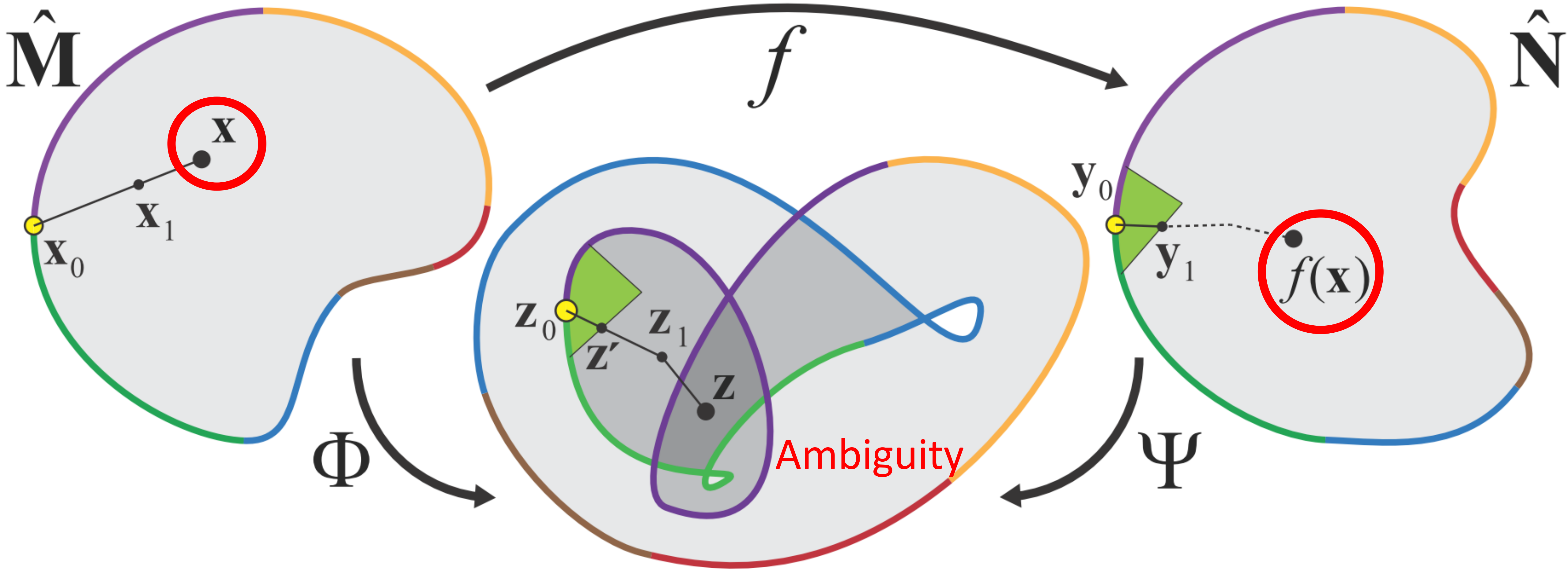
Bijection Lifting

- Bijective parameterizations



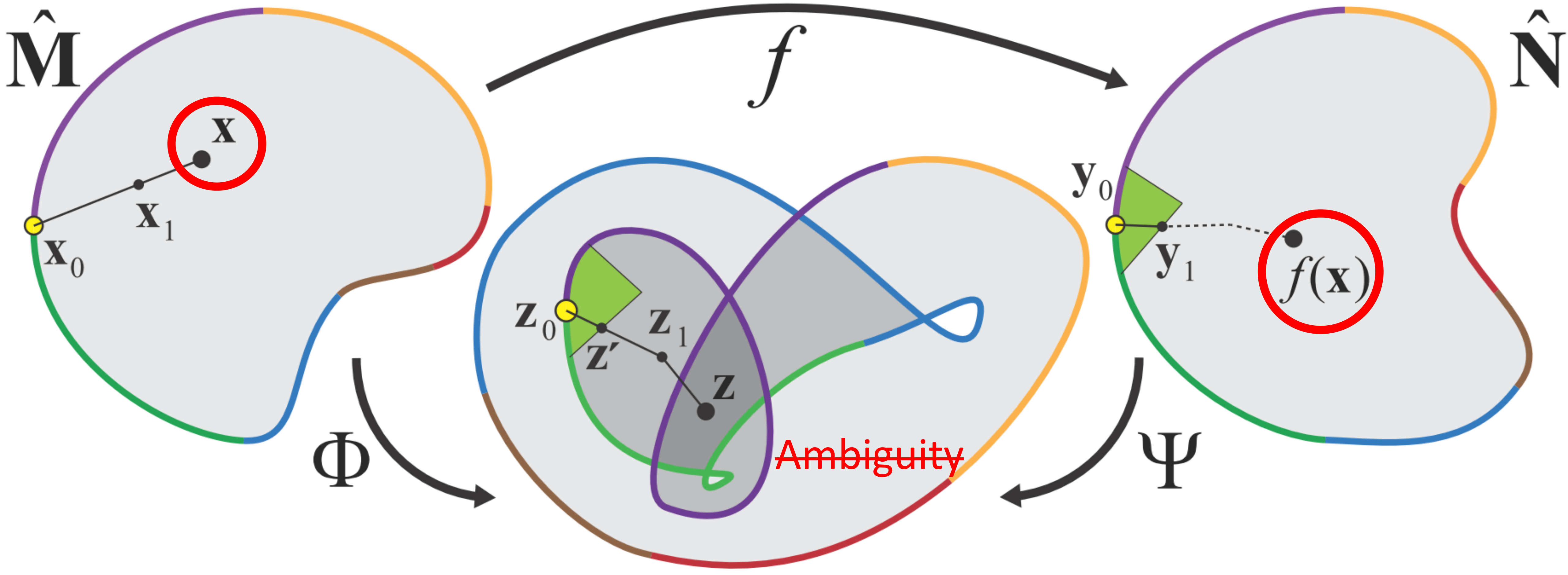
Bijection Lifting

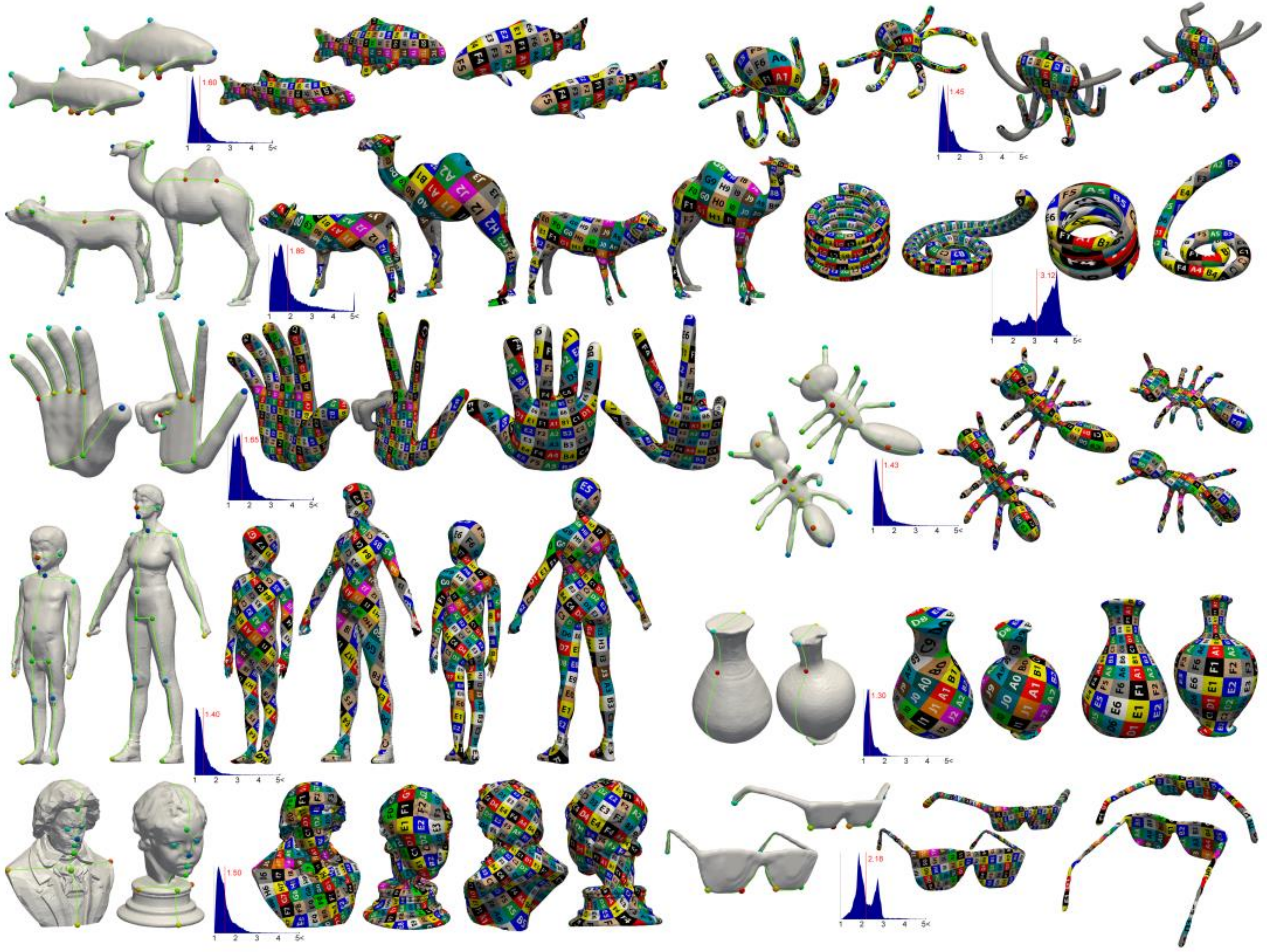
- Only locally injective constrains



Bijection Lifting

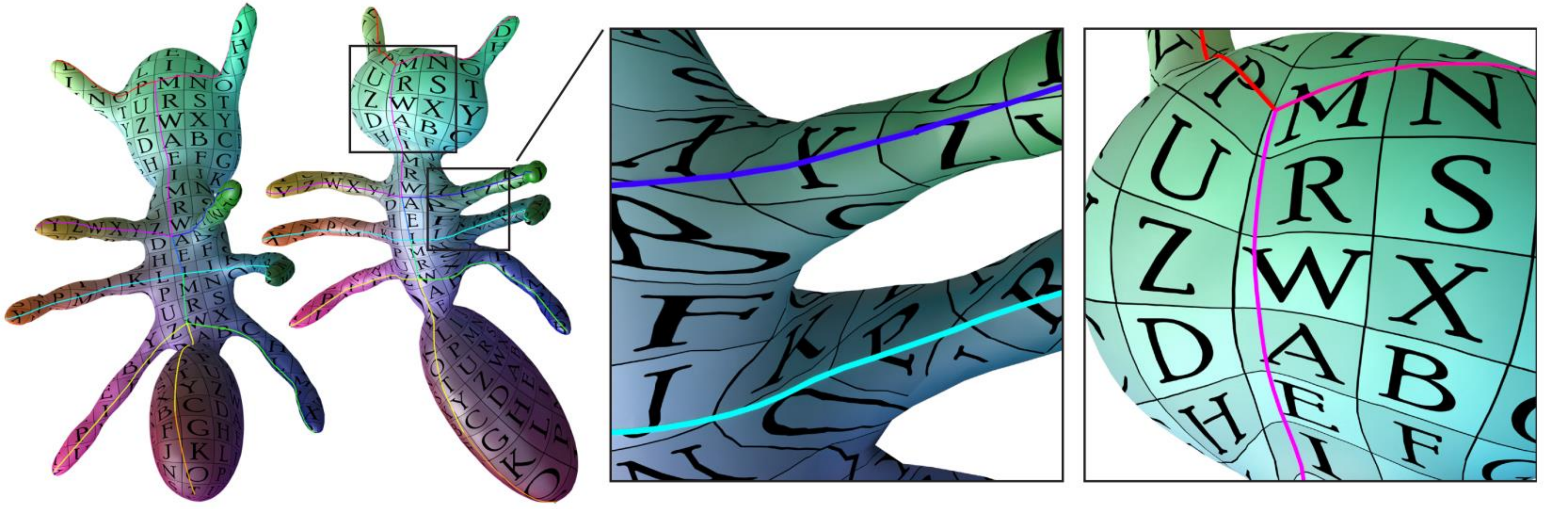
- Only locally injective constrains




1.60
1.45
1.86
3.12
1.65
1.43
1.40
1.30
1.50
2.18
1 2 3 4 5<

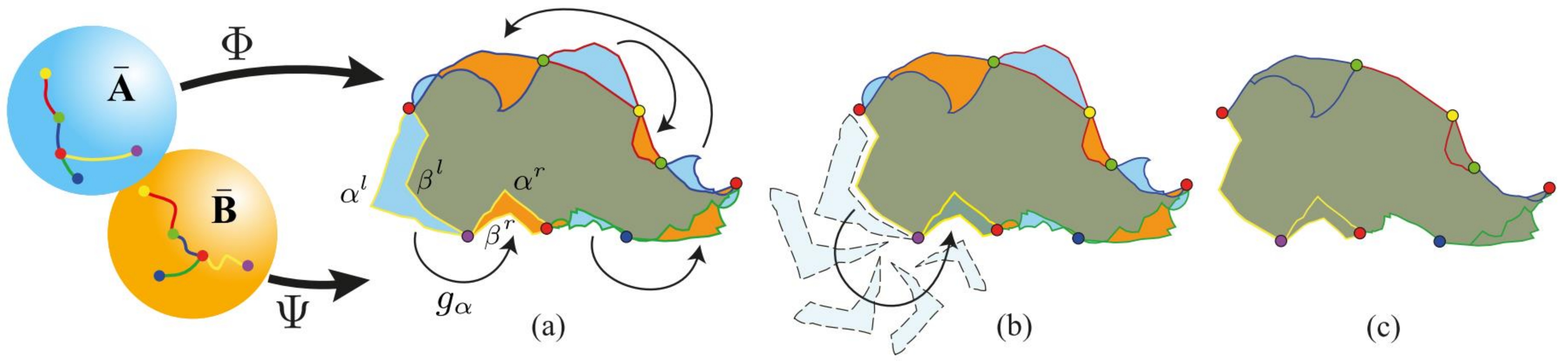
Disadvantages

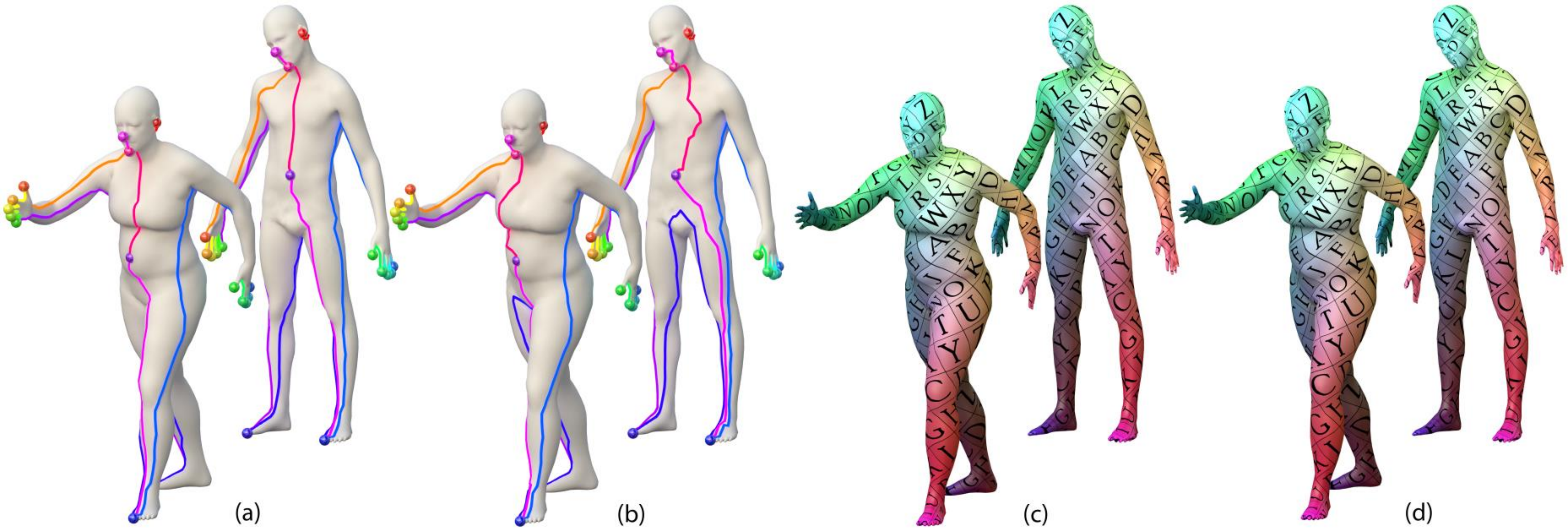
- Cut-dependent



Seamless Surface Mappings

SIGGRAPH 2015





Cut-independent

More methods

- Inter-Surface Mapping, 2004
- Functional Maps: A Flexible Representation of Maps Between Shapes, 2012
- Hyperbolic Orbifold Tutte Embeddings, 2016
- Variance-Minimizing Transport Plans for Inter-surface Mapping, 2017
- ……