Morphing

Xiao-Ming Fu

Outlines

- Definition
- Angle, length, area, volume, and curvature
  - Example-Driven Deformations Based on Discrete Shells
- Affine transformation
  - As-Rigid-As-Possible Shape Interpolation
- Data-driven morphing
  - A Data-Driven Approach to Realistic Shape Morphing
  - Data-Driven Shape Interpolation and Morphing Editing

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Definition

- Morphing is a special effect in motion pictures and animations that changes (or morphs) one image or shape into another through a seamless transition.

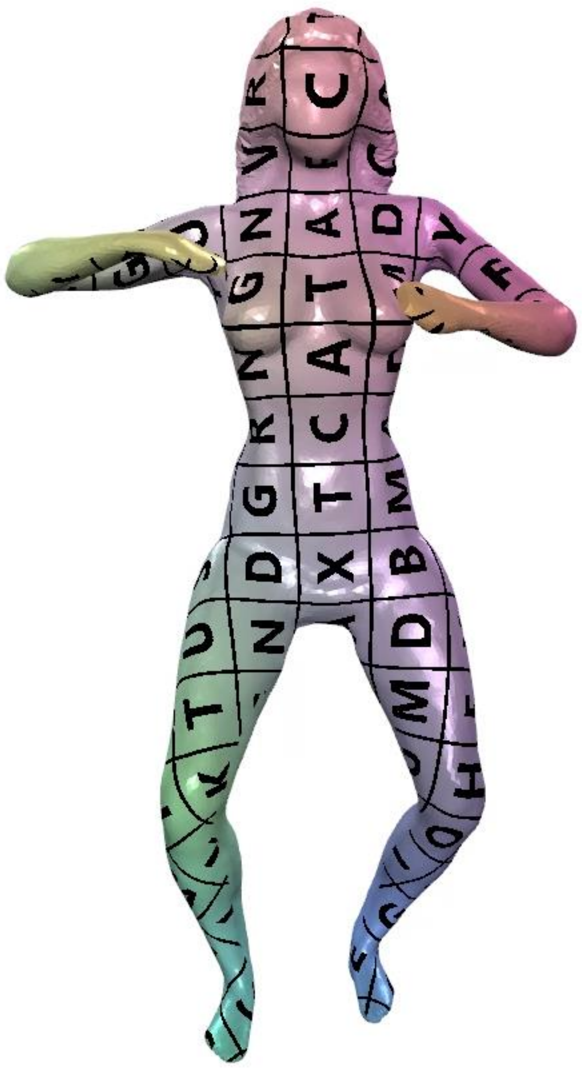
Definition

- Problem: Given M^0, M^1, and t, how to compute the shape M^t?
- $t \in [0,1]$, interpolation
- $t \notin [0,1]$, extrapolation

M^0
Source

M^t

M^1
Target



Requirements

- **Look naturally and intuitively**
- Symmetry
- Smooth vertex paths
- Bounded distortion / low distortion
- Foldover-free
- Large deformation
- ……

Some methods

- First interpolate some values/metrics, then reconstruct the shape.

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Interpolation

Angle, length, and volume

$$l_e^t = (1-t)l_e^0 + tl_e^1$$
$$\theta_e^t = (1-t)\theta_e^0 + t\theta_e^1$$
$$\mathrm{V}^t = (1-t)V^0 + tV^1$$

l_e: edge length

θ_e: dihedral angles

V: volume

$$V = \frac{1}{6}\sum_{f_{i,j,k}} (\boldsymbol{x}_i \times \boldsymbol{x}_j) \cdot \boldsymbol{x}_k$$

Reconstruction

- A mesh with prescribed edge lengths and dihedral angles does not exist.

$$E_l = \frac{1}{2}\sum_e (l_e - l_e^t)^2$$

$$E_a = \frac{1}{2}\sum_e (\theta_e - \theta_e^t)^2$$

$$E_v = \frac{1}{2}(V - V_e^t)^2$$

$$E = \lambda E_l + \mu E_b + \nu E_v$$

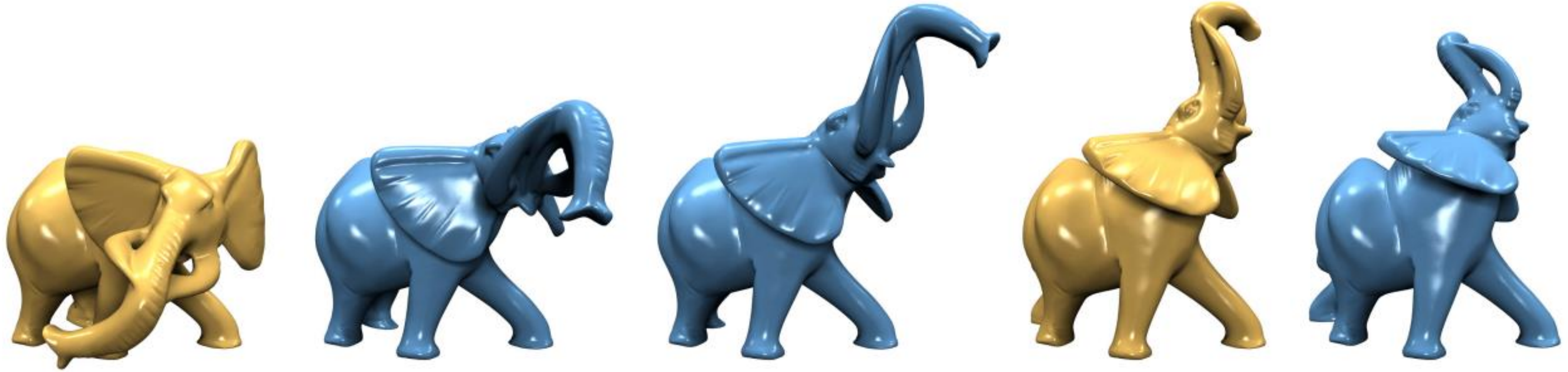

Figure 5: *Interpolation and extrapolation of the yellow example poses. The blending weights are* 0, 0.35, 0.65, 1.0, *and* 1.25.

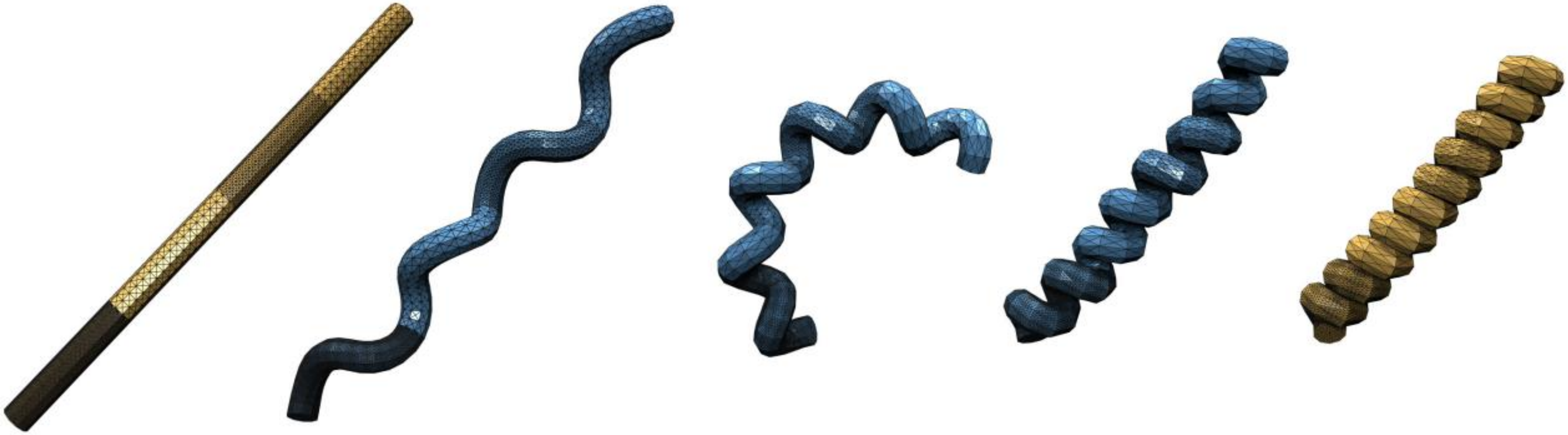

Figure 6: *Interpolation of an adaptively meshed and strongly twisted helix with blending weights* 0, 0.25, 0.5, 0.75, 1.0.

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Interpolation

- How to define $A(t)$ reasonably?
- Simplest solution:

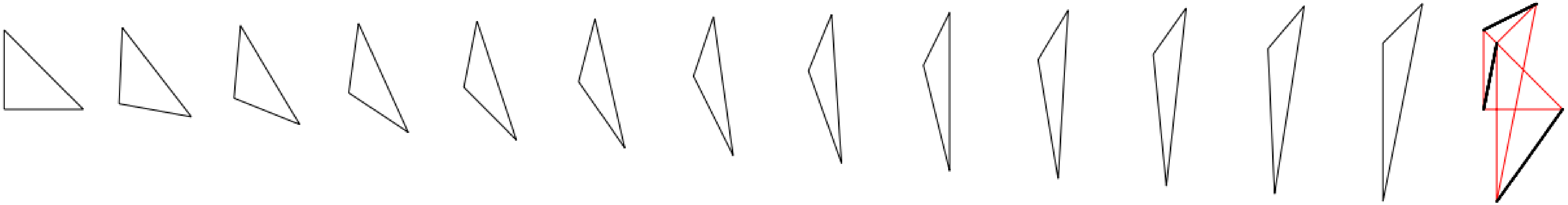
$$A(t) = (1-t)I + tA$$

- More elaborate approaches:
  - Singular value decomposition

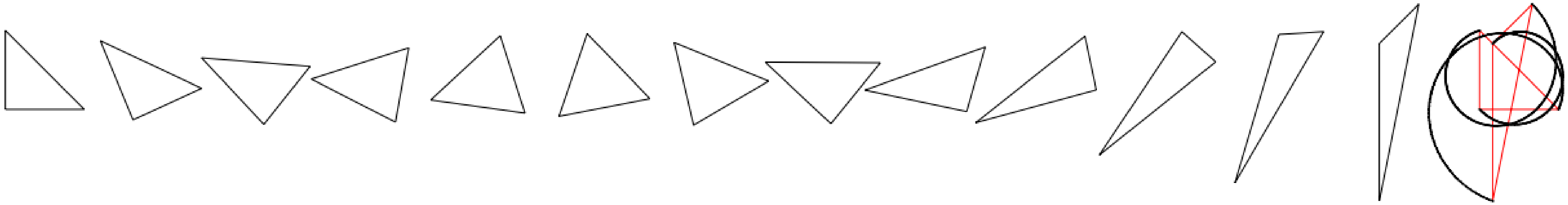
$$A = U\Sigma \mathrm{V}^{\mathrm{T}}$$
$$A(t) = U(t)\big((1-t)I + t\Sigma\big)V^T(t)$$

  - Polar decomposition

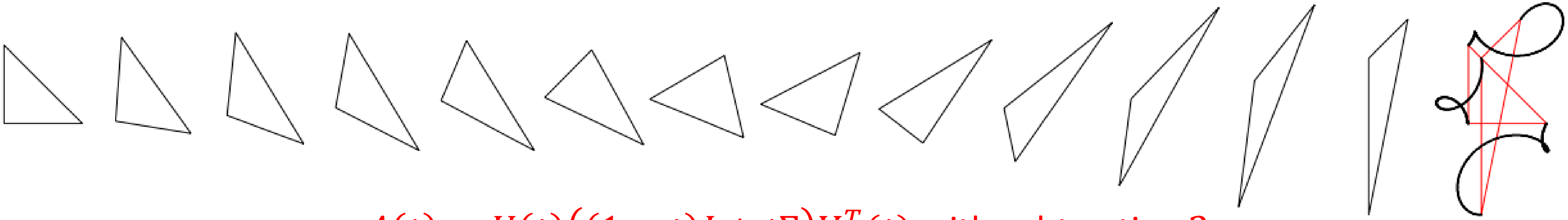
$$A = U\Sigma \mathrm{V}^{\mathrm{T}} = UV^T V\Sigma V^T = RS$$
$$A(t) = R(t)\big((1-t)I + tS\big)$$



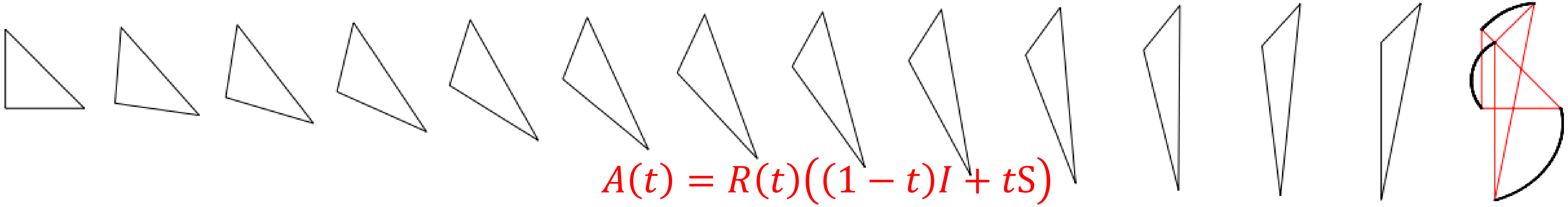
$A(t) = (1-t)I + tA$



$A(t) = U(t)\big((1-t)I + t\Sigma\big)V^T(t)$



$A(t) = U(t)\big((1-t)I + t\Sigma\big)V^T(t)$ with subtracting 2π



$A(t) = R(t)\big((1-t)I + tS\big)$

Reconstruction

- Least squares:

$$E = \sum_{f} \|J - A(t)\|_F^2$$

Figure 12: Morph between photographs of an elephant and a giraffe.

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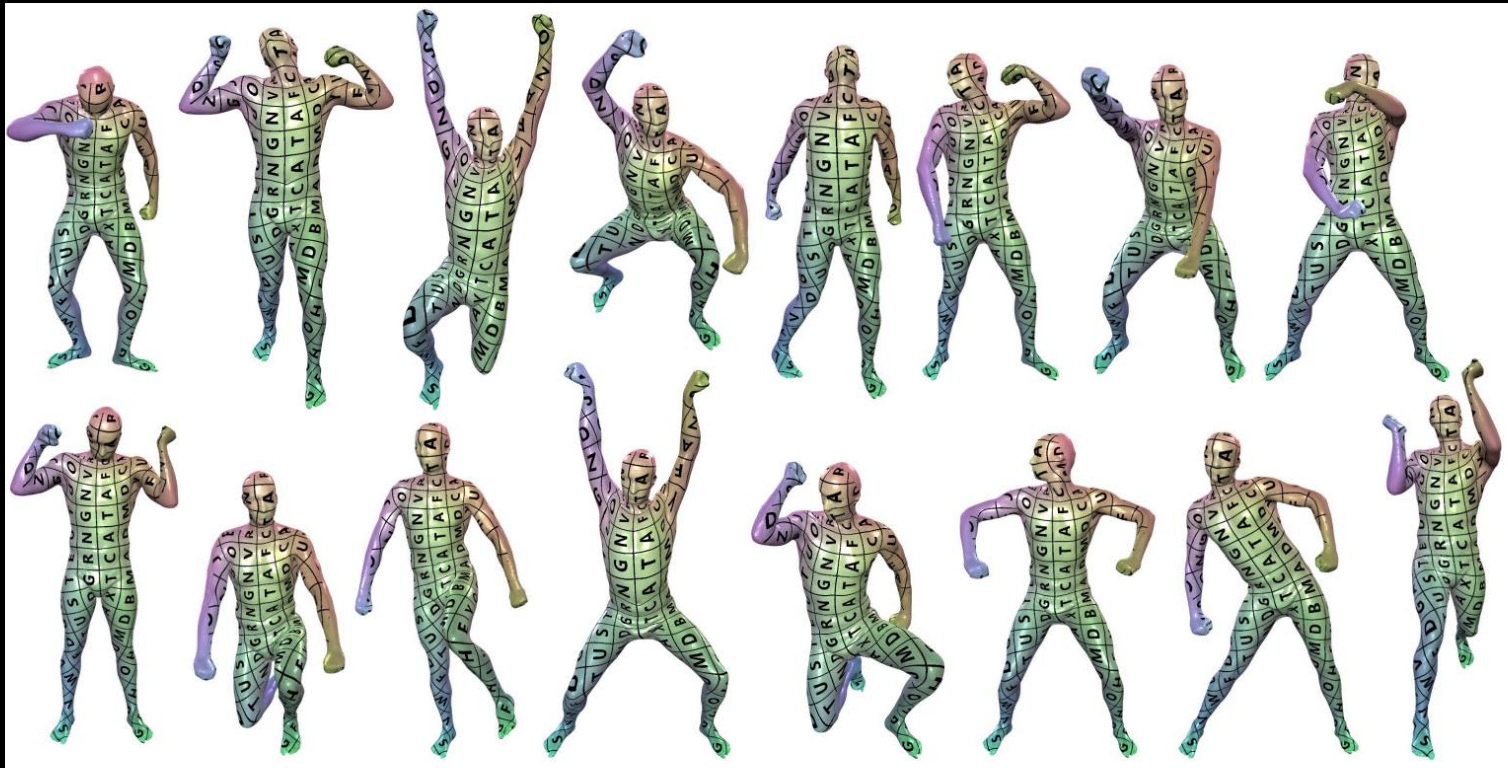
Data-driven approach

- Problem:
  - Input: a database with various models belonging to the same category and containing identical connectivity
  - Given source and target models, how to utilize the database to generate the morphing?



Two stages

- Offline stage
  - Analyze the model database to form **local shape spaces** that better characterize the plausible distribution of models in the category.
- Online stage
  - When the source and target models are given, we **find reference models** in the local shape spaces and **use them to guide the as-rigid-as-possible shape morphing**.

More details

- Offline stage
  - Define distance between pairs of models
- Online stage
  - Find a minimal distance path connecting the source and target models
  - In-between reference models, do as-rigid-as-possible shape interpolation.

Distance Measure

$$\bar{d}(M_i, M_j) = \sqrt{\frac{\sum_{k=1}^{n} \left\| v_k^i - v_k^j \right\|^2}{n}}$$

v_k^i: the k^{th} vertex of the i^{th} model (M_i).

n: the vertex number of the model

Pre-alignment: align models in a database using rigid transforms with the known correspondences.

Morphing

- Path Optimization
  - Shortest path (see more complex algorithm in the paper)
- Interpolation

$$E = \sum_{k=1}^{N_R} w_k(t) E_k$$

N_R The number of models on the generated path.

$$E_k = \sum_{i=1}^{n} \left(\sum_{j \in \Omega(i)} w_{ij} \left\| (\hat{v}^i - \hat{v}^j) - R_k^i (v_k^i - v_k^j) \right\|^2 + \gamma \left\| \hat{v}^i - v_k^i \right\|^2 \right)$$

$w_k(t)$: $\exp(-\varepsilon |t - t_k|)$ where $t_k = \frac{k-1}{N_R - 1}$

Solver: Local/global

Point set registration

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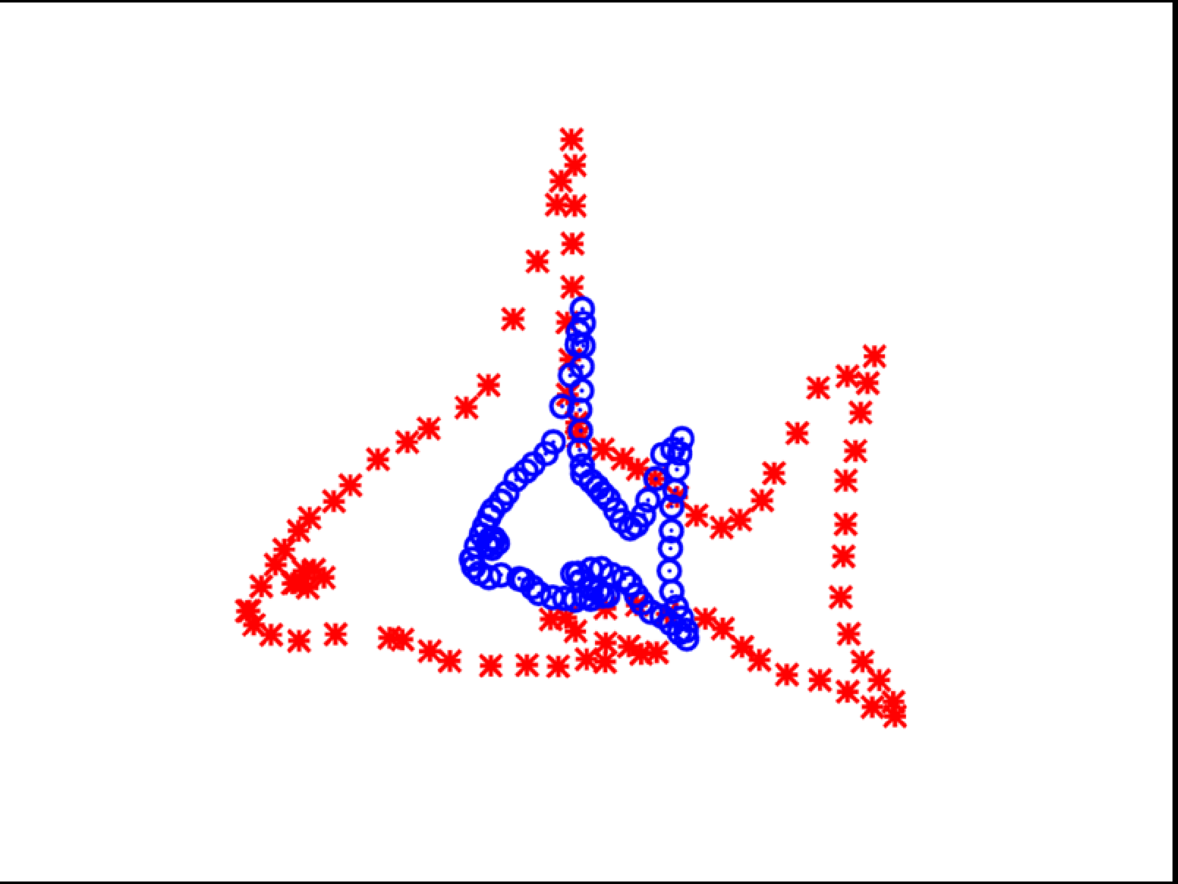
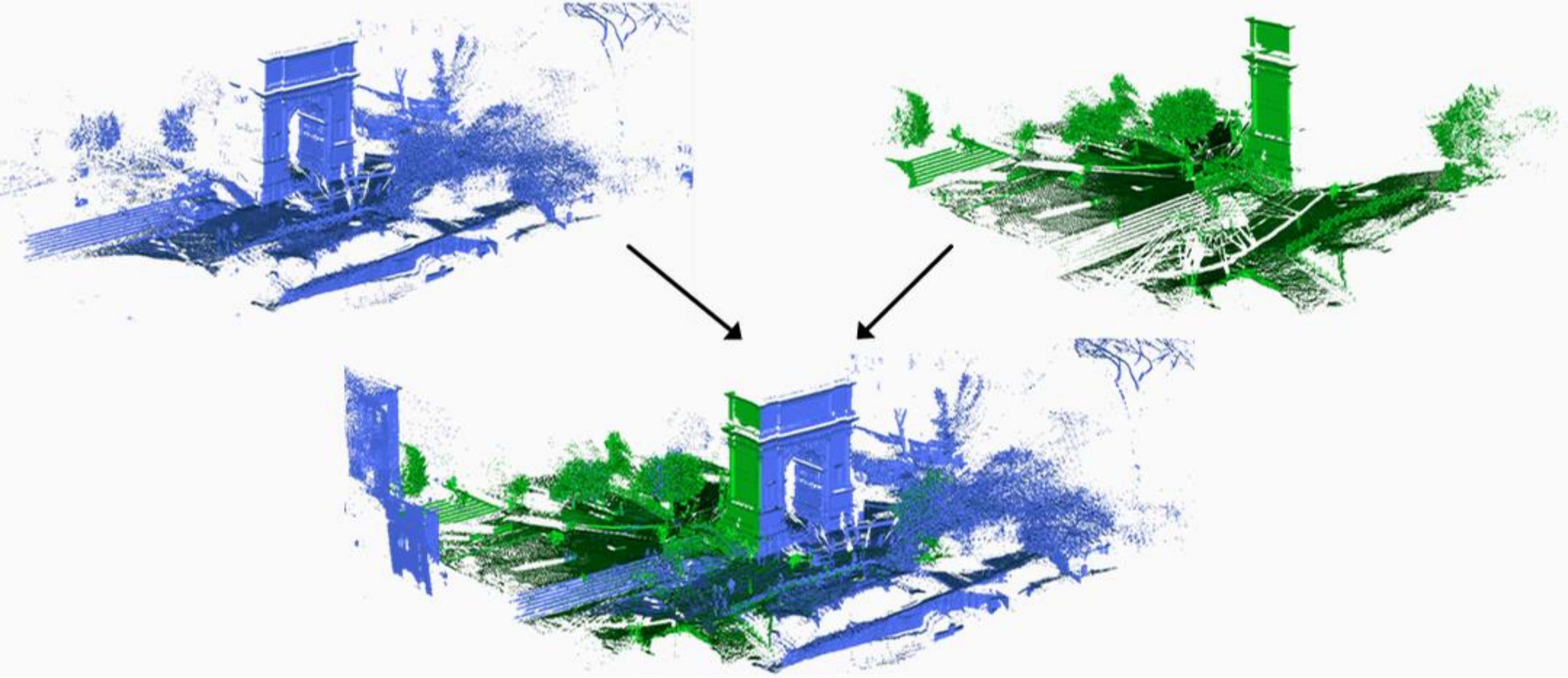
Point set registration

- The process of finding a spatial transformation that aligns two point sets.



- The purpose of finding such a transformation includes merging multiple data sets into a globally consistent model, and mapping a new measurement to a known data set to identify features or to estimate its pose.



Problem

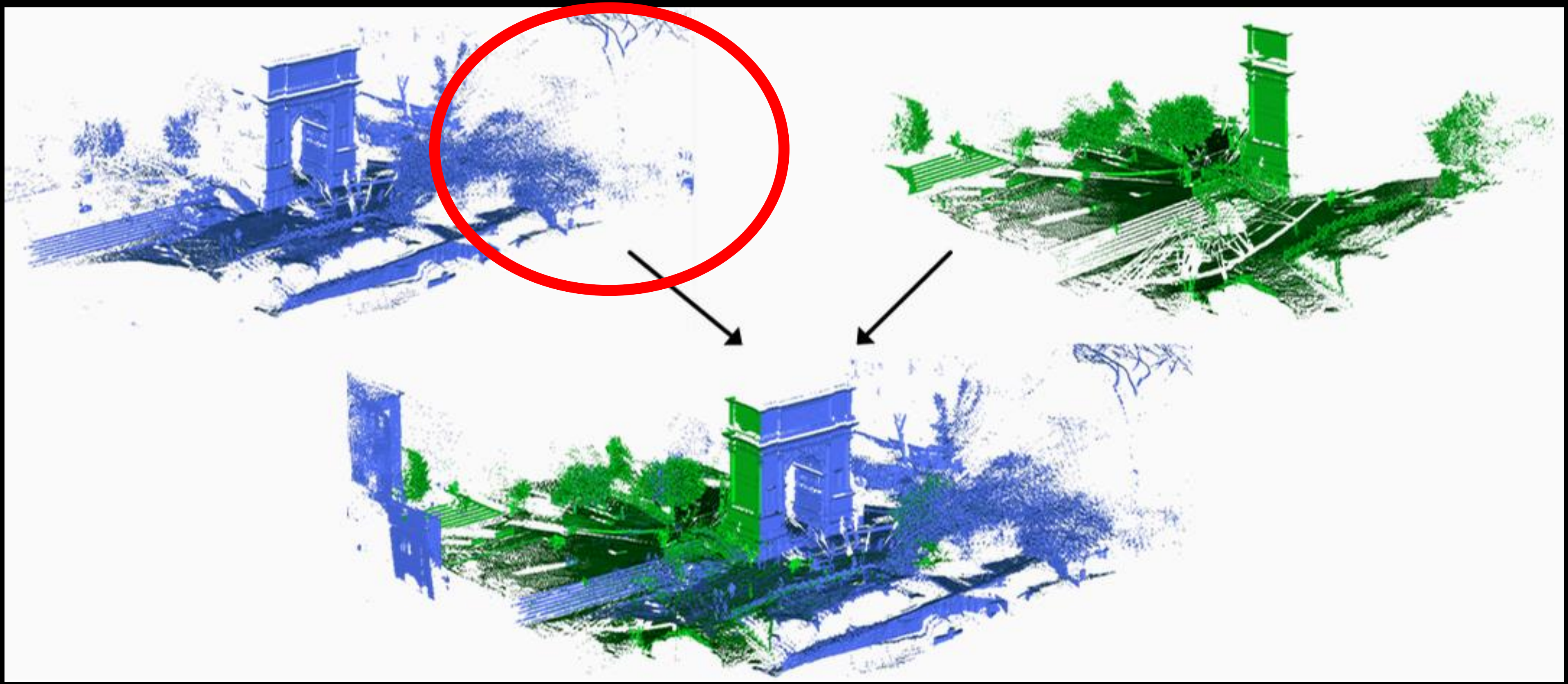
- Input: two finite size point sets $\{P, Q\}$, which contain M and N points.
- Output: a transformation to be applied to the moving "model" point set P such that the difference between P and the static "scene" set Q is minimized.

- The mapping may consist of a rigid or non-rigid transformation.
  - Rigid registration: translation and rotation
  - Non-rigid registration: affine transformations or any nonlinear transformation

For example: Spline

Challenges

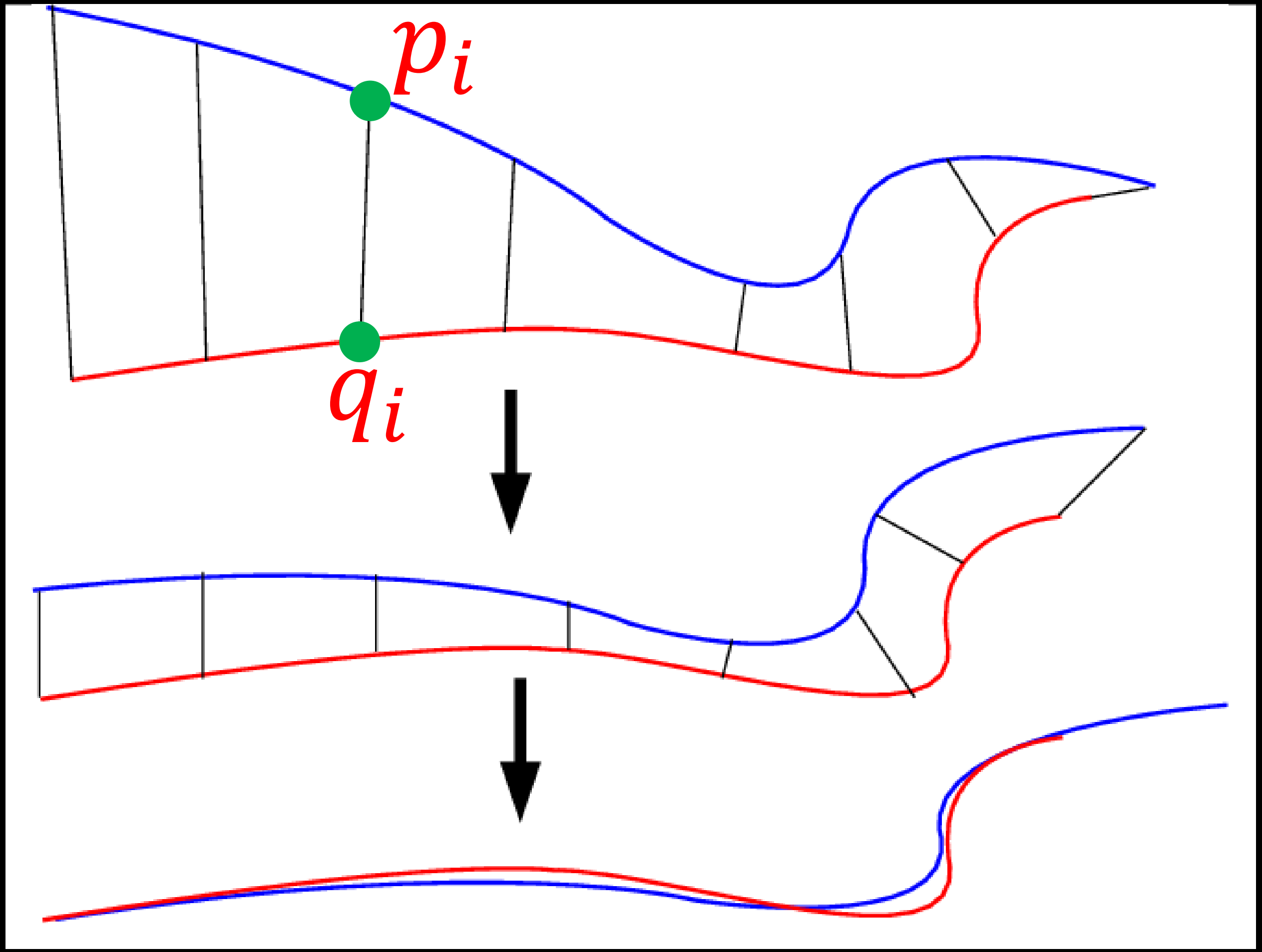
- No correspondences.
- Noisy point cloud.



Iterative closest point (ICP)

https://en.wikipedia.org/wiki/Iterative_closest_point

- 1. $\forall p_i \in P$, match the closest point in Q, denoted as q_i
- 2. Estimate the rigid transformation that aligns the corresponding points as much as possible.
- 3. Iterate above two steps.



Estimation of rigid transformation

- Error:

$$E(P,Q) = \sum_{(p_i,q_i)} \|p_i - q_i\|_2^2$$

Compute rotation R and translation t:

$$E(P,Q) = \sum_{(p_i,q_i)} \|Rp_i + t - q_i\|_2^2$$

Analytical solution

- Define $\mu_p = \frac{1}{n}\sum_{i=1}^{n} p_i$, $\mu_q = \frac{1}{n}\sum_{i=1}^{n} q_i$

$$E(P,Q) = \sum_{i=1}^{n} \|Rp_i + t - q_i\|^2$$

$$= \sum_{i=1}^{n} \left\|Rp_i + t - q_i + R\mu_p - \mu_q - R\mu_p + \mu_q\right\|^2$$

$$= \sum_{i=1}^{n} \left\|R(p_i - \mu_p) - (q_i - \mu_q) + t + R\mu_p - \mu_q\right\|^2$$

Analytical solution

$$E(P,Q) = \sum_{i=1}^{n} \left\| R(p_i - \mu_p) - (q_i - \mu_q) \right\|^2 + \left\| t + R\mu_p - \mu_q \right\|^2 + 2\left(t + R\mu_p - \mu_q\right)^T \left(R(p_i - \mu_p) - (q_i - \mu_q)\right)$$

Since:

$$\sum_{i=1}^{n} 2\left(t + R\mu_p - \mu_q\right)^T \left(R(p_i - \mu_p) - (q_i - \mu_q)\right)$$

$$= 2\left(t + R\mu_p - \mu_q\right)^T \sum_{i=1}^{n} \left(R(p_i - \mu_p) - (q_i - \mu_q)\right)$$

$$= 2\left(t + R\mu_p - \mu_q\right)^T \left(\sum_{i=1}^{n} R(p_i - \mu_p) - \sum_{i=1}^{n} (q_i - \mu_q)\right) = 0$$

Analytical solution

$$E(P,Q) = \sum_{i=1}^{n} \left\| R(p_i - \mu_p) - (q_i - \mu_q) \right\|^2 + \left\| t + R\mu_p - \mu_q \right\|^2$$

No matter what R is got, set $t = -R\mu_p + \mu_q$.

Thus,

$$E(P,Q) = \sum_{i=1}^{n} \left\| R(p_i - \mu_p) - (q_i - \mu_q) \right\|^2$$

$$= \sum_{i=1}^{n} (p_i - \mu_p)^T \cancel{R^T R} (p_i - \mu_p) + \left\| (q_i - \mu_q) \right\|^2 - 2(q_i - \mu_q)^T R (p_i - \mu_p)$$

$$= \sum_{i=1}^{n} (p_i - \mu_p)^T (p_i - \mu_p) + \left\| (q_i - \mu_q) \right\|^2 - 2(q_i - \mu_q)^T R (p_i - \mu_p)$$

Analytical solution

$$\arg\min_R E(P, Q)$$

$$= \arg\min_R \sum_{i=1}^{n} \cancel{(p_i - \mu_p)^T (p_i - \mu_p) + \|(q_i - \mu_q)\|^2} - 2(q_i - \mu_q)^T R(p_i$$

Analytical solution

- If M is a positive-symmetric-definite matrix then for any orthogonal R, $tr(M) > tr(RM)$.
- Proof: Set $M = AA^T$

$$tr(RM) = tr(RAA^T) = tr(A^T RA) = \sum a_i^T (Ra_i)$$

Schwarz inequality: $a_i^T (Ra_i) \leq \sqrt{a_i^T a_i (a_i R^T Ra_i)} = a_i^T a_i = tr(M)$

Analytical solution

- Denote H $= \sum_{i=1}^{n}\left(\left(p_i - \mu_p\right)\left(q_i - \mu_q\right)^T\right) = U\Sigma V^T$.

Solve $\arg\max_R tr(2RH)$.

Set $X = VU^T$,

Then, $XH = V\Sigma V^T$

For any orthonormal matrix B,

$$tr(XH) \geq tr(BXH)$$

Thus,

$$VU^T = X = \arg\max_R tr(2RH)$$