Homework 2
ISyE 6420: Fall 2024

Question 1

Let $y_i \mid \theta \stackrel{\text{iid}}{\sim}$ Uniform $(-\theta, \theta)$, for $i = 1, \ldots, n$. Assume the prior distribution for θ to be Pareto(a, b), where $p(\theta) = ba^b/\theta^{b+1}$ for $\theta \geq a$ and 0 otherwise. Find the posterior distribution of θ.

Question 2

The data set `neurondiffs.dat|xlsx` comes from the lab of Dr Steve Potter at the Department of Biomedical Engineering, Georgia Tech. It consists of 988 time intervals between successive firings in a cell culture of neurons. The firing times are defined as time instances when a neuron sends a signal to another linked neuron (a spike). The cells, from the cortex of an embryonic rat brain, were cultured for 18 days on multielectrode arrays. The measurements were taken while the culture was stimulated at the rate of 1 Hz. It was postulated that firing times form a Poisson process; thus the interspike intervals provided in the data set should have an exponential distribution.

(a) Check the histogram of interspike intervals and discuss its resemblance to the exponential density. What is the MLE for exponential rate parameter λ.

(b) Given the exponential model for interspike intervals T_is, find the posterior distribution of rate parameter λ when the prior for λ is gamma $\mathcal{G}a(18, 20)$. What is the Bayes estimator for λ. Find also the posterior variance of λ.

(c) If the model for T_is is parametrized by a scale parameter μ $(= 1/\lambda)$, find the posterior mean of μ if the prior on μ is inverse-gamma $\mathcal{IG}(18, 20)$.[1]

Question 3

A lifetime X (in years) of a particular device is modeled by a Weibull distribution

$$f(x|\nu, \theta) = \nu\theta x^{\nu-1} \exp\left\{-\theta x^{\nu}\right\}, \; x \geq 0,$$

with shape parameter $\nu = 3$ and unknown rate parameter θ. The lifetimes of $X_1 = 3, X_2 = 4$, and $X_3 = 2$ are observed. Assume that an expert familiar with this type of devices suggested an exponential prior on θ with rate parameter $\lambda = \frac{5}{2}$.

[1] Random variable X has inverse gamma $\mathcal{IG}a(a, b)$ distribution, if $1/X$ has gamma $\mathcal{G}a(a, b)$ distribution. Here a and b are shape and rate parameters, respectively. The density of inverse gamma $\mathcal{IG}a(a, b)$ distribution is

$$f(x|a, b) = \frac{b^a}{\Gamma(a)x^{a+1}} \exp\left\{-\frac{b}{x}\right\}.$$

(a) For the prior suggested by the expert, find the posterior distribution of θ.

(b) What are the posterior mean and variance? No need to integrate if you recognize to which family of distributions the posterior belongs.