十八世纪与现代的交响：链式法则、贝叶斯网络和休谟——概率、相关和因果

The Symphony of the Eighteenth Century and the Modern Times: Chain Rule, Bayesian Networks, and Hume - Probability, Correlation, and Causation

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让我们从一个动机开始：**下雨（Rain, R）**和**堵车（Traffic, T）**两个事件之间的关系到底是什么?

Let's start with a motivation: What is the relationship between the two events **rain (Rain, R)** and **traffic jam (Traffic, T)** ?

如果像我一样，你也观察到每当下雨就总是堵车，正要不假思索地回答“因果关系！”，那么你一定是一个观察敏锐的司机，但缺少一些深思熟虑。它们之间的关系果真如此显然吗?

If, like me, you've observed that traffic jams always occur when it rains, and are about to respond without a second thought, "Cause and effect!", then you must be an observant driver, but lack some forethought. Is the relationship really so obvious?

首先，让我们先定义一些符号： First, let's define some notation:

- $P(T)$：交通堵塞（Traffic）的概率 $P(T)$: Probability of traffic jam
- $P(R)$：下雨（Rain）的概率 $P(R)$: Probability of Rain
- $P(T|R)$： 下雨时交通堵塞的条件概率 $P(T|R)$: Conditional probability of traffic jam when it rains
- $P(R|T)$： 交通堵塞时下雨的条件概率 $P(R|T)$: Conditional probability of rain when there is a traffic jam
- $P(R,T)$： 交通堵塞和下雨同时发生的概率 $P(R,T)$: The probability of traffic jam and rain occurring at the same time

如果我们着眼于$P(T,R)$，根据**链式法则（Chain Rule）**我们可以得到两种分解：

If we focus on $P(T,R)$, we can get two decompositions according to **the chain rule** :

$$P(T,R) = P(T)\cdot P(R|T)$$

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然而，根据 **Judea Pearl（1936 — ）**提出的**贝叶斯网络（Bayesian network）**，它们却分别对应两种相反的方向建模：

However, according to **the Bayesian network** proposed by **Judea Pearl (1936-**), they model two opposite directions:

$$P(T,R) = P(T)\cdot P(R|T) \implies T \to R$$

$$P(T,R) = P(R)\cdot P(T|R) \implies R \to T$$

这两种建模在数学上都能表示同一个联合概率分布 P(T,R) 。

Both of these modeling approaches can mathematically represent the same joint probability distribution P(T,R).

这个结果暴露了一个令人不安的事实：仅凭 **观测数据**，我们无法确定因果的方向。无论是 T->R 还是 R->T，贝叶斯网络都能与数据兼容。

This result reveals a disturbing fact: Only **Observational data** , we cannot determine the direction of causality. Whether it is T->R or R->T, the Bayesian network is compatible with the data.

然而，在现实中，这两种因果关系显然并非等价。下雨导致交通堵塞符合常识，而交通堵塞导致下雨则显得荒诞不经。这一矛盾揭示：**观测数据** 在描述相关性时游刃有余，但在因果性面前却显得力不从心。

However, in reality, these two causal relationships are obviously not equivalent. It is common sense that rain causes traffic jams, but it is absurd that traffic jams cause rain. This contradiction reveals that **observational data** It is adept at describing correlation, but struggles with causality.

大卫·休谟（David Hume，1711—1776）在《人性论》中对因果关系提出了深刻的怀疑，他写道：

David Hume (1711-1776) expressed profound skepticism about causality in his Treatise of Human Nature, writing:

> "We have no other notion of cause and effect, but that of certain objects, which have been always conjoin'd together." 我们对因果关系的理解，仅限于某些物体总是连结在一起。
>
> "We have no other notion of cause and effect, but that of certain objects, which have been always conjoin'd together." Our understanding of causality is limited to the fact that certain objects are always connected together.

休谟认为，因果关系并非逻辑上的必然，而是基于经验的习惯性联想。我们无法感知某种神秘的“因果力”，只能观察到事件的先后顺序和频繁共现。在下雨与堵车的例子中，休谟大概会这样讲：我们之所以认为下雨导致堵车，或许只是因为二者常同时出现，而非真正理解了其中的必然联系。

Hume believed that causality is not a logical necessity, but a habitual association based on experience. We cannot perceive some mysterious "causal force", but can only observe the sequence of events and their frequent co-occurrence. In the example of rain and traffic jams, Hume would probably say this: the reason why we think that rain causes traffic jams may be just because the two often appear at the same time, rather than truly understanding the necessary connection between them.

为了回应休谟的怀疑，现代科学引入了**实验数据** or **数学模拟的实验干预数据**，**随机对照试验（RCT）**和 **do-算子（do-calculus）**分别是两个对应方法。

In response to Hume's skepticism, modern science introduced **experimental data** or **mathematically simulated experimental intervention data** , **with randomized controlled trials (RCT)** and **do-calculus** being the two corresponding methods.

随机对照试验（RCT）通过实验中的随机化干预来隔离因果效应。设想一个理想实验，实验通过人工强制造雨和人工强制避免下雨进行：

Randomized controlled trials (RCTs) experiment Imagine an ideal experiment that uses artificially forced rain and artificially forced rain avoidance to isolate causal effects.

- 实验组：强制下雨（R = 1） Experimental group: forced rain (R = 1)

- 对照组：强制不下雨（R = 0） Control group: forced no rain (R = 0)

通过比较两组的交通堵塞概率，我们可以计算平均因果效应（ATE）：

By comparing the traffic jam probabilities of the two groups, we can calculate the average causal effect (ATE):

ATE = P(T = 1 | R = 1) - P(T = 1 | R = 0)

若 ATE > 0，则表明下雨确实增加了堵车的概率，从而支持 R->T 的因果方向。

If ATE > 0, it means that rain does increase the probability of traffic jams, thus supporting the causal direction of R->T.

而当实验不可行时，**Judea Pearl** 提出的**do-算子**提供了一种数学模拟的实验干预的方法。假设存在混淆变量 Z（如节假日），它同时影响下雨和堵车。我们可以用后门调整公式剔除其干扰：

When experiments are not feasible, **the do-operator** proposed by **Judea Pearl** provides a Experimental intervention with mathematical simulations method. Assume that there is a confounding variable Z (such as holidays), which affects both rain and traffic jams. We can use the backdoor adjustment formula to eliminate its interference:

P(Y=y|do(X=x)) = ∑_z P(Y=y|X=x,Z=z) · P(Z=z)

通过控制 Z ，我们能够估计下雨对堵车的直接影响，而非仅仅依赖相关性。

By controlling Z, we are able to estimate the direct effect of rain on traffic jams rather than relying solely on correlations.

RCT 和 do-算子各有千秋，前者需要真实实验，后者在无法实验时可以通过数学模拟实验干预。相较于观测数据的局限，这两种方法让我们得以超越相关性，逼近因果关系。

RCT and do-operator have their own advantages. The former requires real experiments, while the latter can intervene through mathematical simulation experiments when experiments are not possible. Compared with the limitations of observational data, these two methods allow us to go beyond correlation and approach causality.

然而，即便有了实验数据或模拟干预，我们是否能彻底回应休谟的怀疑?

However, even with experimental data or simulation interventions, can we fully respond to Hume's skepticism?

休谟可能会部分认同实验能强化因果信念，因为它主动制造了“下雨后堵车”的模式，减少了混淆。

Hume would likely partially agree that experiments can strengthen causal beliefs because they actively create a “traffic jam after rain” pattern and reduce confusion.

但休谟可能会进一步追问：即使实验显示下雨导致堵车，我们凭什么相信这种因果关系在未来依然成立？实验只是增加了经验的频率和一致性，仍然无法证明因果关系的“必然性”——它仍然是信念。

But Hume might have asked further: Even if the experiment shows that rain causes traffic jams, why do we believe that this causal relationship will still hold true in the future? The experiment only increases the frequency and consistency of experience, and still cannot prove the "necessity" of causality - it is still a belief.

尽管如此，实验数据 or 数学模拟的实验干预数据确是对休谟的实用回应——我们承认因果不可绝对证明，但通过实验和模型，可以更可靠地预测和干预。这些方法虽无法证明因果的终极必然性，却也多少安抚了我们漂

泊的心灵，不再总是感到如风中的芦苇、水中的浮萍了。

Nevertheless, experimental data or experimental intervention data from mathematical simulations is indeed a practical response to Hume - we admit that causality cannot be absolutely proven, but through experiments and models, we can predict and intervene more reliably. Although these methods cannot prove the ultimate inevitability of causality, they have more or less soothed our wandering hearts, and we no longer always feel like reeds in the wind or duckweed in the water.