# Homework 1
# Background[1]

## CMU 10-601: Machine Learning (Spring 2019)

# START HERE: Instructions

- **Collaboration policy:** Collaboration on solving the homework is allowed, after you have thought about the problems on your own. It is also OK to get clarification (but not solutions) from books or online resources, again after you have thought about the problems on your own. There are two requirements: first, cite your collaborators fully and completely (e.g., "Jane explained to me what is asked in Question 2.1"). Second, write your solution *independently*: close the book and all of your notes, and send collaborators out of the room, so that the solution comes from you only. See the Academic Integrity Section on the course site for more information: http://www.cs.cmu.edu/~mgormley/courses/10601/about.html#7-academic-integrity-policies

- **Late Submission Policy:** See the late submission policy here: http://www.cs.cmu.edu/~mgormley/courses/10601/about.html#6-general-policies

- **Submitting your work:**

  - **Autolab:** You will submit your code for programming questions on the homework to Autolab (https://autolab.andrew.cmu.edu/). After uploading your code, our grading scripts will autograde your assignment by running your program on a virtual machine (VM). The software installed on the VM is identical to that on linux.andrew.cmu.edu, so you should check that your code runs correctly there. If developing locally, check that the version number of the programming language environment and versions of permitted libraries match those on linux.andrew.cmu.edu. (Octave users: Please make sure you do not use any Matlab-specific libraries in your code that might make it fail against our tests.) You have a **total of 10 Autolab submissions.** Use them wisely. In order to not waste Autolab submissions, we recommend debugging your implementation on your local machine (or the linux servers) and making sure your code is running correctly first before any Autolab submission. The above is true for future assignments, but this one allows **unlimited submissions.**

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[1]Compiled on Wednesday 16th January, 2019 at 20:30

- **Gradescope:** For written problems such as short answer, multiple choice, derivations, proofs, or plots, we will be using Gradescope (https://gradescope.com/). Please use the provided template. Submissions can be handwritten onto the template, but should be labeled and clearly legible. If your writing is not legible, you will not be awarded marks. Alternatively, submissions can be written in LaTeX. Regrade requests can be made, however this gives the TA the opportunity to regrade your entire paper, meaning if additional mistakes are found then points will be deducted. Each derivation/proof should be completed on a separate page. For short answer questions you **should not** include your work in your solution. If you include your work in your solutions, your assignment may not be graded correctly by our AI assisted grader. For this assignment only, if you answer at least 90% of the written questions correctly, you get full marks on the Gradescope portion of this assignment. For this assignment only, **we will offer two rounds of grading**. The first round of grading will happen immediately following the due date specified above. We will then release your grades to you and if you got less than 90% on the written questions, you will be allowed to submit once again by a second due date. The exact due date for the second round will be announced after we release the first round grades.

- **Materials:** Download from autolab the tar file ("Download handout"). The tar file will contain all the data that you will need in order to complete this assignment.

For multiple choice or select all that apply questions, shade in the box or circle in the template document corresponding to the correct answer(s) for each of the questions. For LaTeX users, use ■ and ● for shaded boxes and circles, and don't change anything else.

# 1 Hello, Autolab! [32 Points]

## 1.1 Introduction

This homework is neither representative the standard difficulty of programming assignments for this course nor is it designed to test your ability to program. **In this homework you have to choose Python, Octave, Java, or C++ as your programming language. Submitting code for more than one language may result in undefined behavior..**

The goal of this assignement is to ensure that you:

1. Have a way to edit and test your code (i.e. a text editor and compiler/interpreter)

2. Are familiar with submitting to Autolab

3. Are familiar with file I/O and standard output in the language of your choice

**Warning:** This handout assumes that you are using a unix command prompt (with `zsh, bash, csh` or similar). All of the command prompts lines listed in this handout will work on the linux.andrew.cmu.edu machines. You may need to use other commands or methods if you are working locally - especially if you are using Windows.

## 1.2 Reading from a file [22pts]

In `reverse.{py|m|java|cpp}`, implement a program that reads in the lines of a file, then writes them in reverse order to an output file. Specifically, your program should take two command line arguments: the name of the input file and the name of the output file. It should read the lines of the input file and write them to the output file from last to first, separated by "\n". You should assume that the input file has unix-style line breaks. (Windows uses "\r\n" to indicate a new line. Unix uses only "\n".)

For example, if the file `input.txt` contained the stream

    #pineapples\n#pinstripes\n#pinwheelofdoom\n#pinsir\n

which is commonly displayed as

    #pineapples
    #pinstripes
    #pinwheelofdoom
    #pinsir

depending on your language of choice, one of the following:

- `python reverse.py input.txt output.txt`

- `octave -qH reverse.m input.txt output.txt`

- `javac reverse.java; java reverse input.txt output.txt`

- g++ reverse.cpp; ./a.out input.txt output.txt

should write the following to output.txt

    #pinsir\n#pinwheelofdoom\n#pinstripes\n#pineapples\n

which is displayed as

    #pinsir
    #pinwheelofdoom
    #pinstripes
    #pineapples

You may assume that the contents of the input file will fit in memory for any reasonable machine and the contents of the file will be ASCII-encoded and that all inputs end with a '\n'. You will be provided with two example files `example.txt` and `sentences.txt` with which you can test your code. However, do not assume that if your code works on just those files than it will receive full points. We will grade your code on different, hidden test cases. Attempts to directly determine the specific contents of these tests constitute violation of the course policy.

Note to Octave users: Please be sure that `reverse.m` is a *script* that gets its arguments from the command line rather than a *function*.

## 1.3   Test Code on `linux.andrew.cmu.edu` Machines

Before submitting to Autolab on this and **every future assignment**, you should check that it behaves correctly when run on the `linux.andrew.cmu.edu` machines, since they mirror the software installed on the Autolab virtual machines (VMs). These instructions assume you are working on a unix based operating system (e.g. Linux, Mac OS) - if you are using Windows you can install cygwin (https://www.cygwin.com/) which provides a unix-like environment. (Of course, you are also welcome to develop your code directly on these same servers.)

Follow the three stpes below. Here we assume your code and the `example.txt` file are located in a subdirectory `./reverse/`.

1. Copy your code to the linux server:

        rsync -a ./reverse/ <Your Andrew ID>@linux.andrew.cmu.edu:~/reverse/

2. Log into the linux server:

        ssh <Your Andrew ID>@linux.andrew.cmu.edu

3. Change directories to where you just copied the code:

        cd  ~/reverse/

4. Run the code with one of the below:

- `python reverse.py example.txt output.txt`

- `octave -qH reverse.m example.txt output.txt`

- `javac reverse.java; java reverse example.txt output.txt`

- `g++ -g -std=c++11 reverse.cpp; ./a.out example.txt output.txt`

5. Check that it was properly reversed:

   `cat output.txt`

## 1.4  Command Line Arguments

In this and future programming assignments, we will use command line arguments to run your programs with different parameters. Below, we provide some simple examples for how to do this in each of the programming languages you can use in the course.

Python:

```python
from __future__ import print_function
import sys

if __name__ == '__main__':
    infile = sys.argv[1]
    outfile = sys.argv[2]
    print("The input file is: %s" % (infile))
    print("The output file is: %s" % (output))
```

Java:

```java
public class myclass {
   public static void main(String[] args) {
        String infile = args[0];
        String outfile = args[1];
        System.out.println("The input file is: " + infile);
        System.out.println("The output file is: " + outfile);
   }
}
```

C++:

```cpp
#include<iostream>
#include <string>

using namespace std;

int main(int argc, char **argv){
```

```
    if (argc >= 3) {
        string infile = string(argv[1]);
        string outfile = string(argv[2]);
        cout << "The input file is: " << infile << endl;
        cout << "The output file is: " << outfile << endl;
    }
    return 0;
}
```

Octave:

```
args=argv();
infile=args{1};
outfile=args{2};
disp(horzcat("The intput file is: ", infile))
disp(horzcat("The output file is: ", outfile))
```

## 1.5   Autolab Submission [10pts]

You must submit a .tar file named `reverse.tar` containing `reverse.{py|m|java|cpp}`.
You can create that file by running:

```
tar -cvf reverse.tar reverse.{py|m|java|cpp}
```

from the directory containing your code.

Some additional tips: **DO NOT** compress your files; you are just creating a tarball. Do not
use tar `-czvf`. **DO NOT** put the above files in a folder and then tar the folder. Autolab
is case sensitive, so observe that all your files should be named in **lowercase**. You must
submit this file to the corresponding homework link on Autolab.

Note: For this assignment, you may make arbitrarily many submissions to Autolab before
the deadline, but only your last submission will be graded.

> **Python3 Users:** Please include a blank file called python3.txt (case-sensitive) in your tar
> submission and we will execute your submitted program using Python 3 instead of Python
> 2.7. If the file is not present, we will default to running your code with Python 2.7.

# Instructions for Specific Problem Types

For "Select One" questions, please fill in the appropriate bubble completely:

**Select One:** Who taught this course?

● Matt Gormley

○ Marie Curie

○ Noam Chomsky

If you need to change your answer, you may cross out the previous answer and bubble in the new answer:

**Select One:** Who taught this course?

● Matt Gormley

○ Marie Curie
✖ Noam Chomsky

For "Select all that apply" questions, please fill in all appropriate squares completely:

**Select all that apply:** Which are scientists?

■ Stephen Hawking

■ Albert Einstein

■ Isaac Newton

☐ I don't know

Again, if you need to change your answer, you may cross out the previous answer(s) and bubble in the new answer(s):

**Select all that apply:** Which are scientists?

■ Stephen Hawking

■ Albert Einstein

■ Isaac Newton
✖ I don't know

For questions where you must fill in a blank, please make sure your final answer is fully included in the given space. You may cross out answers or parts of answers, but the final answer must still be within the given space.

**Fill in the blank:** What is the course number?

| 10-601 | 10-7601 |
|---|---|

# 2    Prerequisite Practice[68 Points]

In this section, you will work through a number of problems covering probability, statistics, calculus, linear algebra, geometry, and computer science.

## 2.1    Probability and Statistics [30pts]

**Use the following data to answer questions 1-4**. Consider data created by flipping a coin five times $S = [1, 1, 0, 1, 1]$ , where 1 denotes that the coin turned up heads and 0 denotes that it turned up tails.

1. **[2pt]** The sample mean for this data is:

   **Select one:**

   ○ 1

   ○ $\frac{3}{5}$

   ○ $\frac{1}{5}$

   ⊙ $\frac{4}{5}$

2. **[2pt]** The (uncorrected) sample variance for this data is:

   **Select one:**

   ○ $\frac{1}{25}$

   ○ $\frac{2}{25}$

   ⊙ $\frac{4}{25}$

   ○ $\frac{8}{25}$

   $$\frac{4 \times \left(\frac{1}{5}\right)^2 + \left(\frac{4}{5}\right)^2}{5} \qquad \frac{16}{25} + \frac{4}{25} = \frac{20}{25}$$

3. **[2pt]** With reference to the previous question, what is the probability of observing any combination of this data (4 heads and 1 tails), assuming it was generated by flipping a coin X with an unequal probability of heads (1) and tails (0), where now the distribution is $P(X = 1) = 0.75$, $P(X = 0) = 0.25$?

   **Select one:**

   ⊙ $\frac{405}{1024}$

   ○ $\frac{1}{32}$

   ○ $\frac{324}{1024}$

   ○ $\frac{81}{1024}$

   $$\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{1}{4} = \frac{81}{1024} \times 5$$

4. **[2pt]** Note that the probability of this data sample would be greater if the value of $P(X = 1)$ was not 0.5, but instead some other value. What is the value of $P(X = 1)$ that maximizes the probability of the sample S? Provide your answer as a fraction.

$$\boxed{\frac{4}{5}}$$

$P = X^4(1-X)$ $\qquad$ $P' = 4X^3(1-X) - X^4$
$\qquad\qquad\qquad\qquad = 4X^3 - 5X^4$
$\qquad\qquad\qquad\qquad \approx X^3(4 - 5X) = 0 \qquad X = \frac{4}{5}$

5. **[2pt]** State true or false. For events A and B,

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

**Select one:**

    ⊘ True

    ◯ False

6. **[2pt]** State true or false. For events A and B,

$$P(A_1 \cap A_2 \cap A_3) = P(A_3 | A_2 \cap A_1)P(A_2|A_1)P(A_1)$$

**Select one:**

    ◯ True

    ⊘ False

**Use the following information to answer questions 7-8.** Whether your car is wet in the morning (W) is dependent on whether it rained last night (R) or not, however other factors may have lead to your car being wet. The following are probabilities of such events:

$$P(R) = 0.4$$
$$P(W|R) = 0.8$$
$$P(W|\neg R) = 0.2$$

$0.6$

$0.32 + 0.12$

7. **[2pt]** What is the probability of $P(\neg R)$? (Here $\neg R$ reads: no rain last night)

**Select one:**

- ⃝ 0.1
- ⃝ 0.4
- ⃝ 0.9
- ⊘ 0.6

8. **[2pt]** Using the same probabilities as the previous question, what is the probability that your car is wet in the morning?

**Select one:**

- ⃝ 0.64
- ⃝ 0.56
- ⊘ 0.44
- ⊘ 0.4

$P(W|\neg \quad (\neg R \quad +P \quad \neg R) \cdot P(R)$

$0. \quad \sim \quad .2 \ast 0.6$

$\quad \quad + .1 \quad = \quad .44$

**Use the following information to answer questions 9-10.** Consider the following joint probability table where both X and Y are binary variables:

| X | Y | Probability | |
|---|---|---|---|
| 0 | 0 | 0.1 | ✓ |
| 0 | 1 | 0.2 | |
| 1 | 0 | 0.4 | ✓ |
| 1 | 1 | 0.3 | |

9. **[2pt]** Using the same table, what is $P(X = 1 | Y = 1)$?

**Select one:**

- ⃝ $\frac{2}{3}$
- ⃝ $\frac{3}{7}$
- ⃝ $\frac{4}{5}$
- ⊘ $\frac{3}{5}$

$0.3 / 0.5 \quad = 0.6$

10. **[2pt]** What is $P(Y = 0)$?

   **Select one:**

   ○ 0.2

   ○ 0.6

   ○ 0.5

   ○ 0.3

**Use the following information to answer questions 11-13**. Let X be a random variable and the expected value of X is $E[X] = 1$ and the variance of X is $Var[X] = 1$.

11. **[2pt]** What is $E[6X]$?

   **Select one:**

   ○ 1

   ○ 3

   ○ 6

   ○ 36

12. **[2pt]** What is $Var[3X]$?

   **Select one:**

   ○ 1

   ○ 3

   ○ 6

   ○ 9

13. **[2pt]** What is $Var[2X + 3]$?

   **Select one:**

   ○ 3

   ○ 4

   ○ 5

   ○ 7

14. **[2pt]** What is the mean, variance and entropy of a Bernoulli (p) random variable?

**Select one:**

- ⊙ $p, p(1-p), -(1-p)\log(1-p) - p\log(p)$
- ◯ $p(1-p), p, -(1-p)\log(1-p) - p\log(p)$
- ◯ $p, p(1-p), \log(1-p) - p\log(p)$
- ◯ The entropy of a Bernoulli variable is not defined.

15. **[2pt]** Please match the probability density function of the random variable X to its corresponding distribution name.

(a) $\text{prob(X=x)} = \frac{1}{\sqrt{(2\pi)^d|\Sigma|}} \exp(-\frac{1}{2}(x-\mu)^T \Sigma^{-1}(x-\mu))$

(b) $\text{prob(X=x)} = \lambda e^{-\lambda x}$ when $x \geq 0$; 0 otherwise

(c) $\text{prob(X=x)} = \binom{n}{x} p^x (1-p)^{n-x}$

(d) $\text{prob(X=x)} = \frac{1}{b-a}$ when $a \leq x \leq b$; 0 otherwise

(e) $\text{prob(X=x)} = p^x (1-p)^{1-x}$

Multivariate Gaussian: | A |

Exponential: | b |

Uniform: | d |

Bernoulli: | e |

Binomial: | c |

## 2.2 Calculus [8pts]

1. **[2pt]** Find the derivative of y with respect to x, where $y = 2x^4 - x^3 + 5x - 1$

   **Select one:**
   - ⊗ $8x^3 - 3x^2 + 5$
   - ○ $8x^4 - 3x^3 + 5x$
   - ○ $6x^3 - 2x^2$
   - ○ $16x^3 - x^2 + 5$

   *[handwritten: $8x^3 - 3x^2 + 5$]*

2. **[2pt]** Evaluate the derivative of y with respect to x, where $y = x^3 - \frac{4}{x^2}$ at x = 1.

   [boxed answer: 11]

   *[handwritten: $x^{-2}$, $y' = 3x^2 - 4(-2)x^{-3} = 3 + 8$]*

3. **[2pt]** Find the partial derivative of $y$ with respect to $x$, where $y = 3x^2 \sin(z)e^{-x}$

   **Select one:**
   - ○ $3x \sin(z)e^{-x}(2 + x)$
   - ○ $-6x \sin(z)e^{-x}$
   - ⊗ $3x \sin(z)e^{-x}(2 - x)$ ✓
   - ○ $6x \cos(z)e^{-x}$ ✗

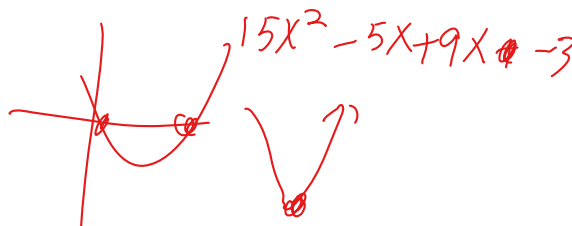   *[handwritten: $3x^2 e^{-x}$, $6x e^{-x} + (-1)e^{-x}3x^2$, $3x e^{-x}(2-x)$]*

4. **[2pt]** For the function $f(x) = 5x^3 + 2x^2 - 3x$ the value $x = \frac{1}{3}$ sets the derivative to be 0. What can you say about $f(x)$ at the point $\frac{1}{3}$:

   **Select one:**
   - ⊗ a minimum
   - ○ a maximum
   - ○ a minimum or a maximum
   - ○ None of the above

   *[handwritten: $f(x)' = 15x^2 + 4x - 3$, $= (3x-1)(5x+3)$, $x = \frac{1}{3}, -\frac{3}{5}$, $15x^2 - 5x + 9x - 3$]*

## 2.3 Vectors and Matrices [10pts]

1. **[2pt]** Consider the matrix X and the vectors y and z below: $\mathbf{X}=\begin{bmatrix} 1 & 4 \\ 2 & 6 \end{bmatrix}$, $\mathbf{y}=\begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $\mathbf{z}=\begin{bmatrix} 2 \\ 3 \end{bmatrix}$.
   What is the inner product of the vectors **y** and **z**? (this is also sometimes called the dot product)

   **Select one:**

   ○ $\begin{bmatrix} 4 & 6 \\ 2 & 3 \end{bmatrix}$    $y^{T}z$

   ○ 9

   ○ $\begin{bmatrix} 4 \\ 3 \end{bmatrix}$

   ○ 7

2. **[2pt]** Using the same values for **X**, **y**, and **z** as above, what is the product of **Xy**?

   **Select one:**

   ○ $\begin{bmatrix} 10 \\ 2 \end{bmatrix}$    $\begin{bmatrix} 1 & 4 \\ 2 & 6 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

   ○ $\begin{bmatrix} 6 \\ 10 \end{bmatrix}$    $\begin{bmatrix} 6 \\ 10 \end{bmatrix}$

   ○ $\begin{bmatrix} 7 \\ 11 \end{bmatrix}$

   ○ $\begin{bmatrix} 14 \\ 22 \end{bmatrix}$

3. **[2pt]** For the matrices $\mathbf{A} = \begin{bmatrix} 2 & 1 & 4 \\ -3 & 2 & 0 \\ 1 & 3 & -2 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 3 & 4 & 5 \\ 3 & -1 & 3 \\ 1 & 3 & -2 \end{bmatrix}$ What is the product

   **AB**?

   **Select one:**

   ○ $\begin{bmatrix} 13 & 19 & 5 \\ -3 & -14 & -9 \\ 4 & -4 & 18 \end{bmatrix}$

   ○ $\begin{bmatrix} 13 & 19 & 28 \\ 19 & 9 & -7 \\ -10 & -2 & 13 \end{bmatrix}$

   ○ $\begin{bmatrix} 20 & -20 & -28 \\ 3 & -14 & 9 \\ 3 & 2 & 13 \end{bmatrix}$

   ◉ $\begin{bmatrix} 13 & 19 & 5 \\ -3 & -14 & -9 \\ 10 & -5 & 18 \end{bmatrix}$

4. **[2pt]** True or False, The matrix **A** from the previous question has an inverse?

   **Select one:**

   ◉ True

   ○ False

5. **[2pt]** Given matrix $\mathbf{X} = \begin{bmatrix} 3 & 4 & 2 \\ 1 & 6 & 2 \\ 1 & 4 & 4 \end{bmatrix}$ and the column vector $\mathbf{y} = \begin{bmatrix} -6 \\ 1 \\ 1 \end{bmatrix}$, what is the eigenvalue of **X** associated with **y**?

   **Select one:**

   ○ **y** is not an eigenvector

   ○ -3

   ○ 2

   ○ 1.5

## 2.4 Geometry [6pts]

1. **[2pt]** What relationship does the vector $w$ share with the line $\mathbf{w}^\mathbf{T}\mathbf{x} + b = 0$? (assume $\mathbf{x}$ and $\mathbf{w}$ are both two dimensional column vectors, and $\mathbf{w}^T$ indicates the transpose of the column vector $\mathbf{w}$.)

   **Select one:**

   ○ parallel

   ○ orthogonal

   ○ depends on the value of b

2. **[2pt]** With reference to the above question, select the statement which best explains why $\mathbf{w}$ and $\mathbf{w}^\mathbf{T}\mathbf{x} + b = 0$ share the above relationship.
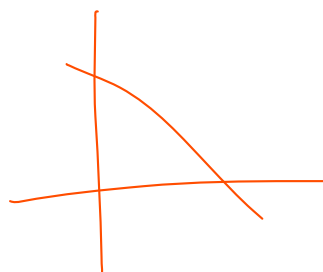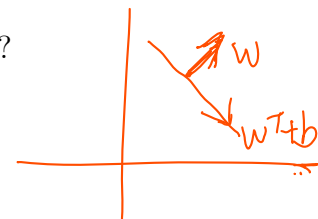
   **Select one:**

   ○ The inner product $\mathbf{w}^\mathbf{T}(\mathbf{x}' - \mathbf{x}'')$, where $\mathbf{x}'$ and $\mathbf{x}''$ are two points on the line $\mathbf{w}^\mathbf{T}\mathbf{x} + b = 0$, is 0

   ○ The inner product $\mathbf{w}^\mathbf{T}(\mathbf{x}' - \mathbf{x}'')$, where $\mathbf{x}'$ and $\mathbf{x}''$ are two points on the line $\mathbf{w}^\mathbf{T}\mathbf{x} + b = 0$, is ~~1~~

   ○ The inner product $\mathbf{w}^\mathbf{T}(\mathbf{x}' - \mathbf{x}'')$, where $\mathbf{x}'$ and $\mathbf{x}''$ are two points on the line $\mathbf{w}^\mathbf{T}\mathbf{x} + b = 0$, is $b' - b''$

3. **[2pt]** What is the distance from the origin to the line $\mathbf{w}^\mathbf{T}\mathbf{x} + b = 0$?

   (In the following answers, $\lambda$ is some constant)

   **Select one:**

   ○ $\frac{|b|}{||\mathbf{w}||}$

   ○ $\frac{|b|}{\mathbf{w}^\mathbf{T}\mathbf{w}}\mathbf{W}$

   ○ $\frac{2\lambda}{\mathbf{w}b}$

   ○ $\frac{||\mathbf{w}||}{|b|}$

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## 2.5 CS Foundations [14pts]

1. **[2pt]** If $f(n) = \ln(n)$ and $g(n) = \log_3(n)$ which of the following are true?

   **Select one:**

   ○ $f(n) \in O(g(n))$

   ○ $g(n) \in O(f(n))$

   ○ Both

   ○ Neither

2. **[2pt]** If $f(n) = n^{10}$ and $g(n) = 10^n$ which of the following are true?

   **Select one:**

   ○ $f(n) \in O(g(n))$

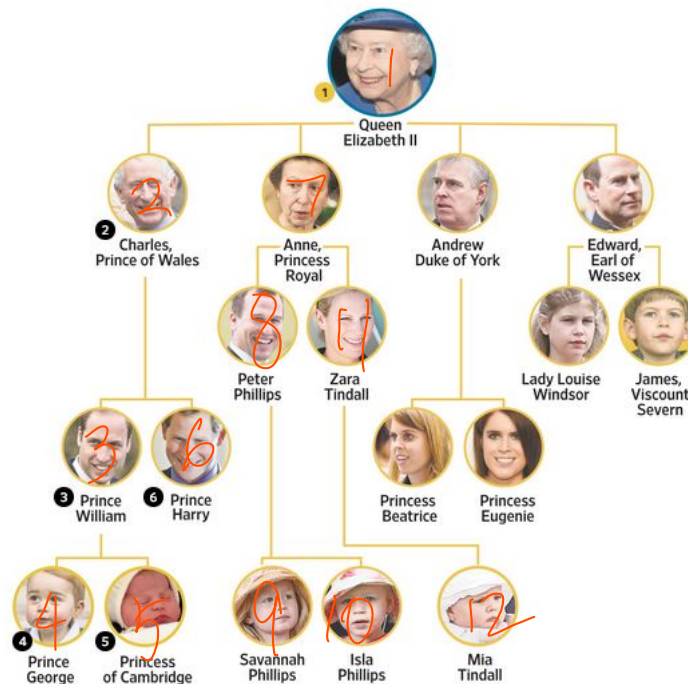   ○ $g(n) \in O(f(n))$

   ○ Both

   ○ Neither



Figure 1: Britian's Royal Family

3. **[2pt]** Using the tree shown in Figure 1, how many nodes would depth-first-search visit in finding Mia Tindall (including her node)? Assuming we search left-to-right and top-down.

   **Select one:**

   - ○ 3
   - ○ 12 *(circled)*
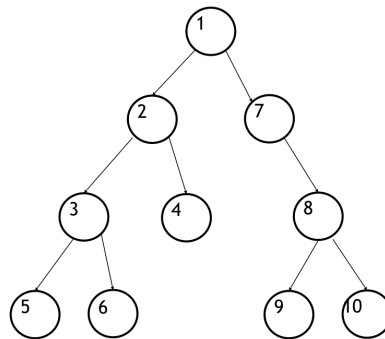   - ○ 15
   - ○ 18



Figure 2: A Binary Tree with indexed nodes

4. **[2pt]** Figure 2 is a Binary Tree with indexed nodes. Assume root node is node 1. What is the node-visit order of **DFS** and **BFS** of the above Binary Tree?

   A depth-first search (DFS) traversal of a binary tree starts with visiting the root node, and recursively searches down the left subtree (i.e., the tree rooted at the left node) before going to search the right subtree (i.e., the tree rooted at the right node) until the traversal is done.
   Note: Alternatively, we can also look right subtree before left subtree too, for the question please consider left to right order!

   A breadth-first search (BFS) traversal of a binary tree visits every node (assuming a left-to-right order) on a level (with the same distance to the root) before going to a lower level until the traversal is done.

   The node-visit order of DFS is:

   1 2 3 5 6 4 7 8 9 10

   The node-visit order of BFS is:

   1 2 7 3 4 8 5 6 9 10

18

5. **[2pt]** Fill in the blanks in the pseudo code for key search using recursive depth-first search (DFS) traversal.

```
class TreeNode:
    def __init__(self, val):
        self.val = val
        self.leftNode = None
        self.rightNode = None

# (a) the left/right node is denoted as
#     node.leftNode/node.rightNode
# (b) left/right node are of type TreeNode
# (c) the value of the node is denoted as node.val
# (d) recursive DFS to search for the node
#     with value key in a binary tree
# (e) the left node is assumed to be searched
#     before the right node

def find_val(node, key):
    if node is None:
        return None

    if (1) node.val == key :
        return node

    else:
        result = (2) find_val(key, node.LeftNode)

        if result is None:
            result = (3) find_val(key, node.

        return (4) return result
```

**Consider the following information to answer questions 6-7:**
Given the functions of computing a fibonacci number:

```python
def fib_1(n):
    if n == 0 or n == 1:
        return 1
    return fib_1(n - 1) + fib_1(n - 2)


d = {}
d[0] = 1
d[1] = 1
def fib_2(n):
    if n in d.keys():
        return d[n]
    d[n] = fib_2(n - 1) + fib_2(n - 2)
    return d[n]
```

6. [**2pt**] Which of the following is the tightest upper bound on the time complexity of computing `fib_1(n)`?

   **Select one:**

   ○ $O(n)$

   ○ $O(n \log n)$

   ○ $O(2^n)$

   ○ $O(n!)$

7. [**2pt**] Which of the following is the tightest upper bound on the time complexity of computing `fib_2(n)`?

   **Select one:**

   ○ $O(n)$

   ○ $O(n \log n)$

   ○ $O(2^n)$

   ○ $O(n!)$

**Collaboration Questions** Please answer the following:

1. Did you receive any help whatsoever from anyone in solving this assignment?
   Yes / No.

   - If you answered 'yes', give full details: _____
   - (e.g. "Jane Doe explained to me what is asked in Question 3.4")

2. Did you give any help whatsoever to anyone in solving this assignment?
   Yes / No.

   - If you answered 'yes', give full details: _____
   - (e.g. "I pointed Joe Smith to section 2.3 since he didn't know how to proceed with Question 2")

3. Did you find or come across code that implements any part of this assignment ?
   Yes / No. (See below policy on "found code")

   - If you answered 'yes', give full details: _____
   - (book & page, URL & location within the page, etc.).