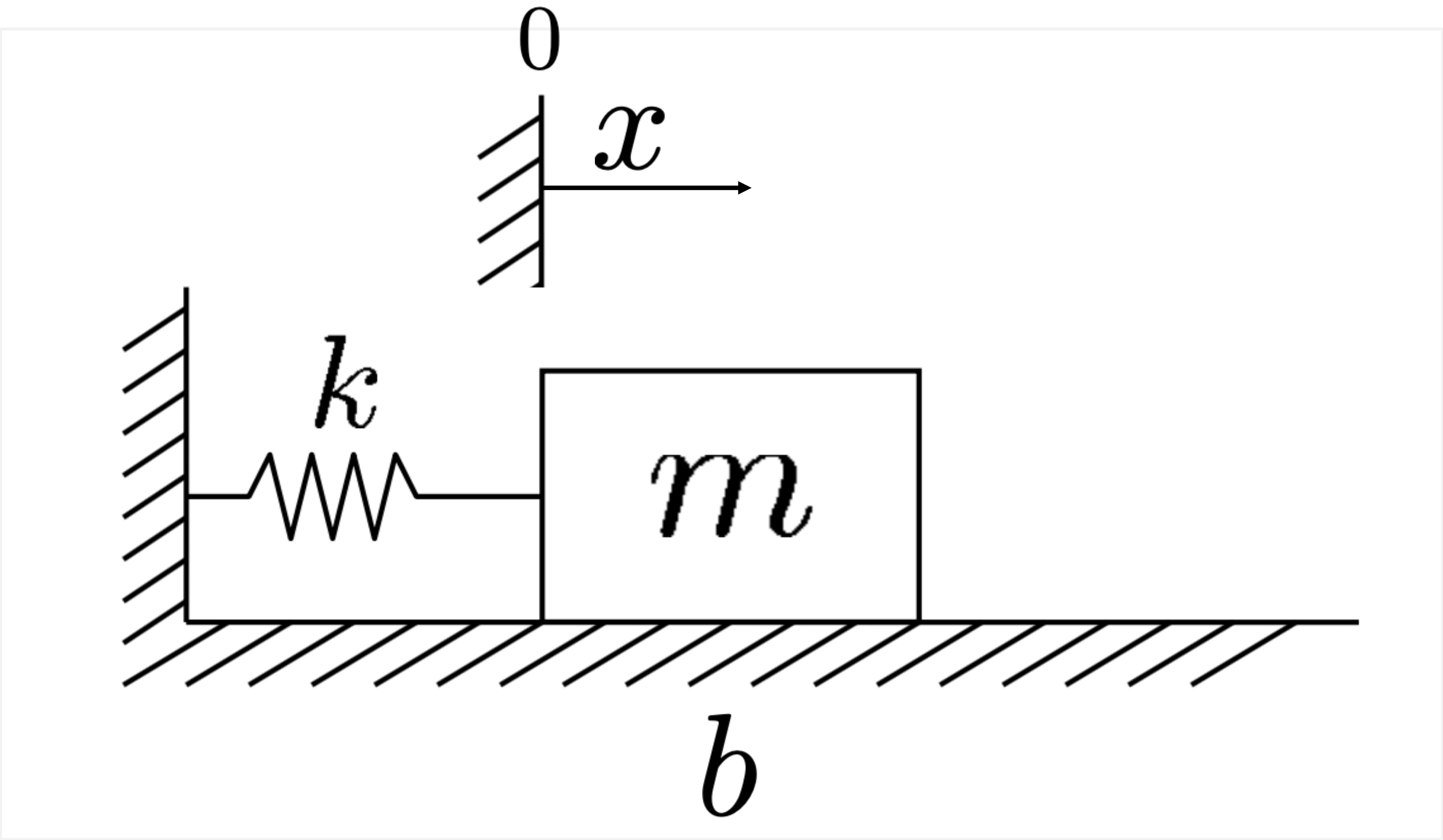
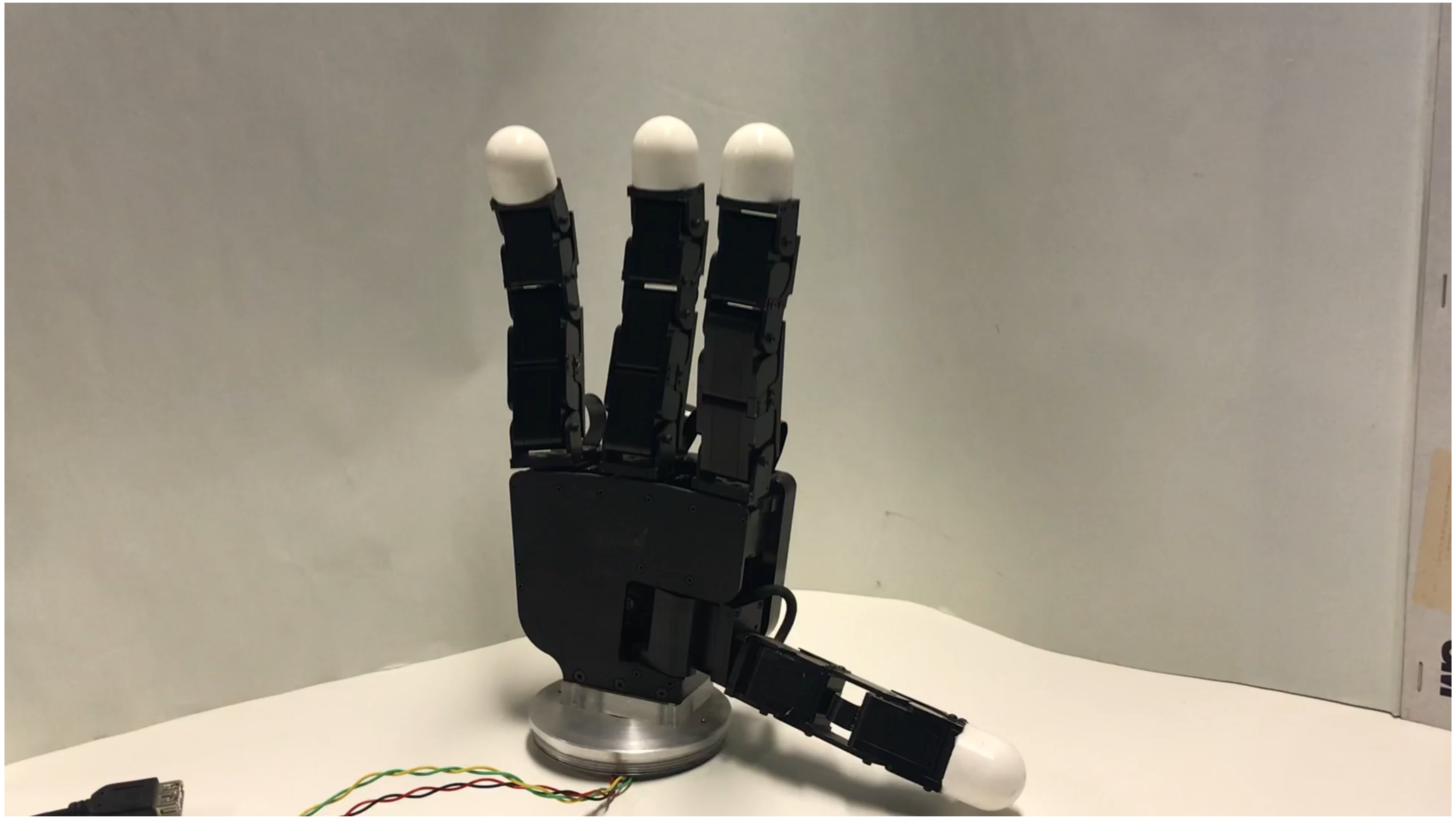



# Robotics Tutorial Assignment #3

Steffen Puhlmannn

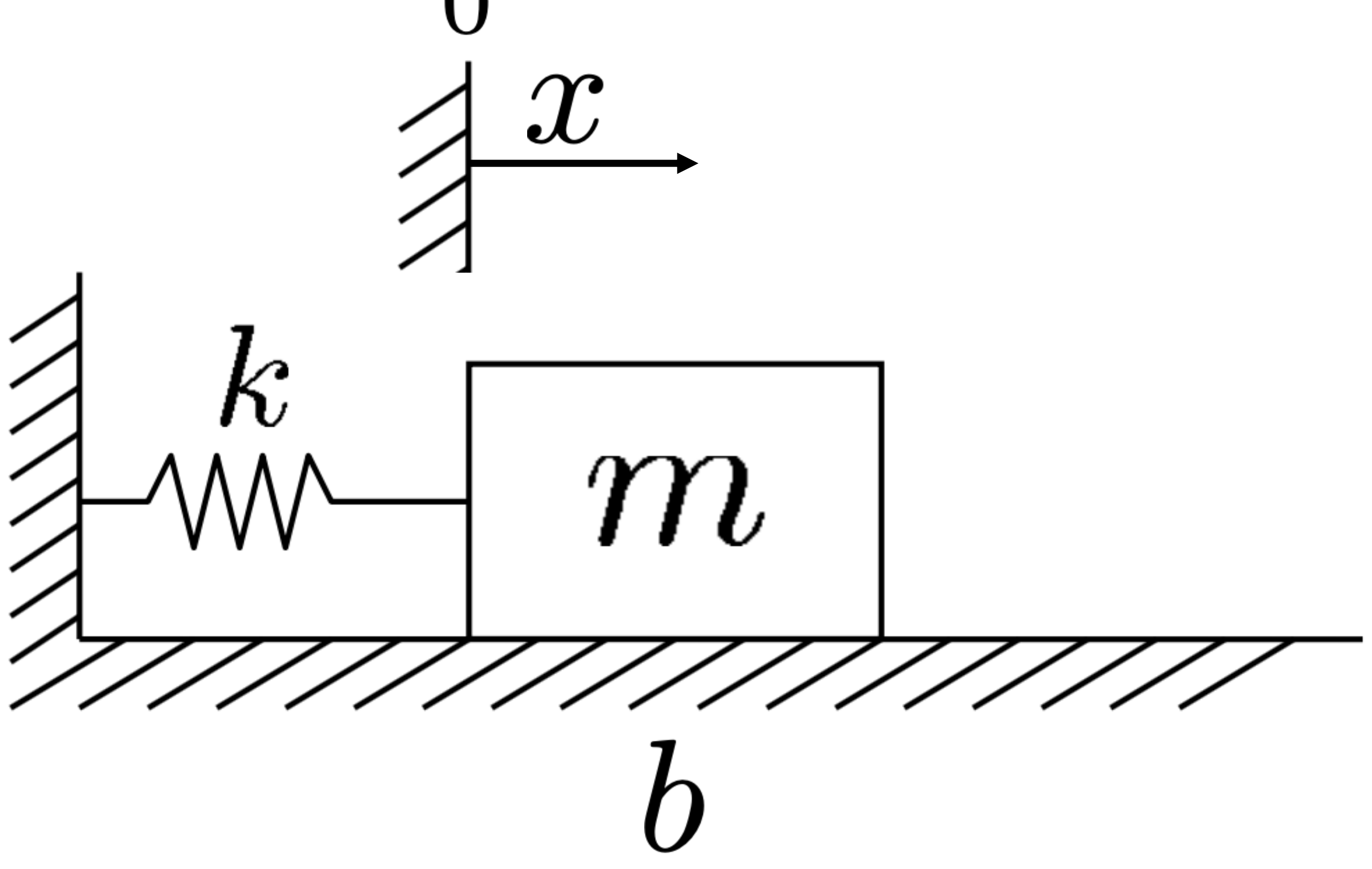
Technische Universität Berlin



Allegro Hand

# 2nd-Order Linear Systems

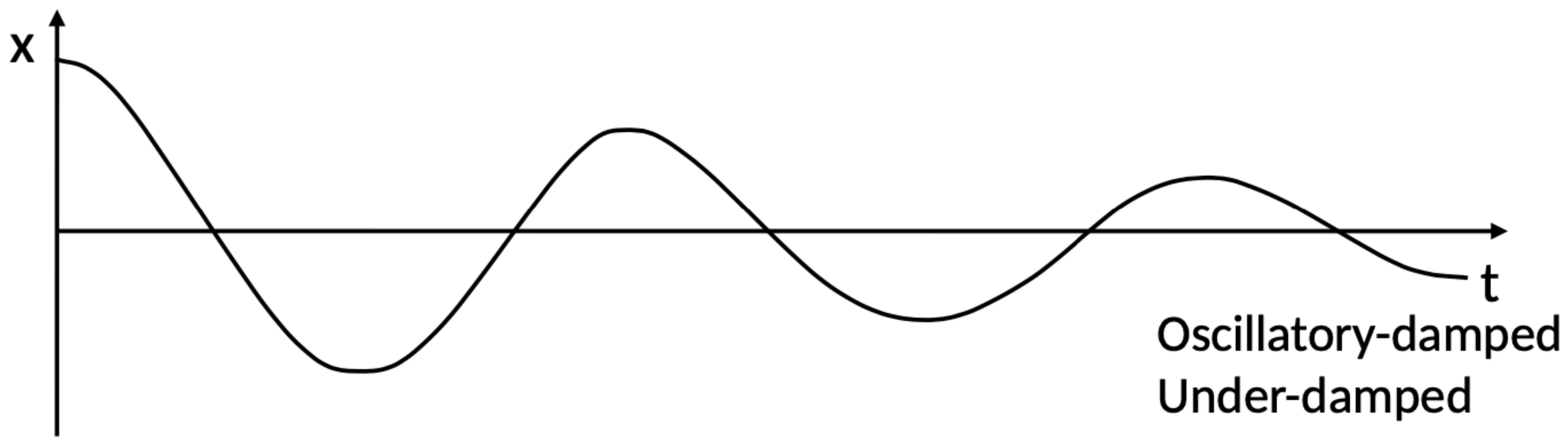
# 2nd-Order Linear Systems



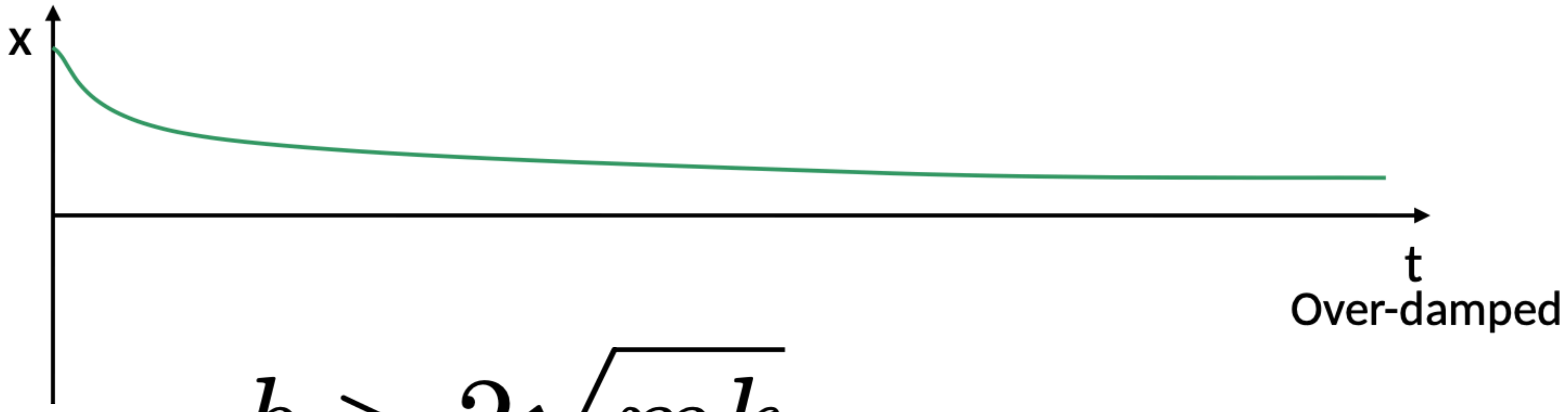

$$m\ddot{x} + b\dot{x} + kx = 0$$

# 2nd-Order Linear Systems

Possible behaviors




$$b < 2\sqrt{mk}$$

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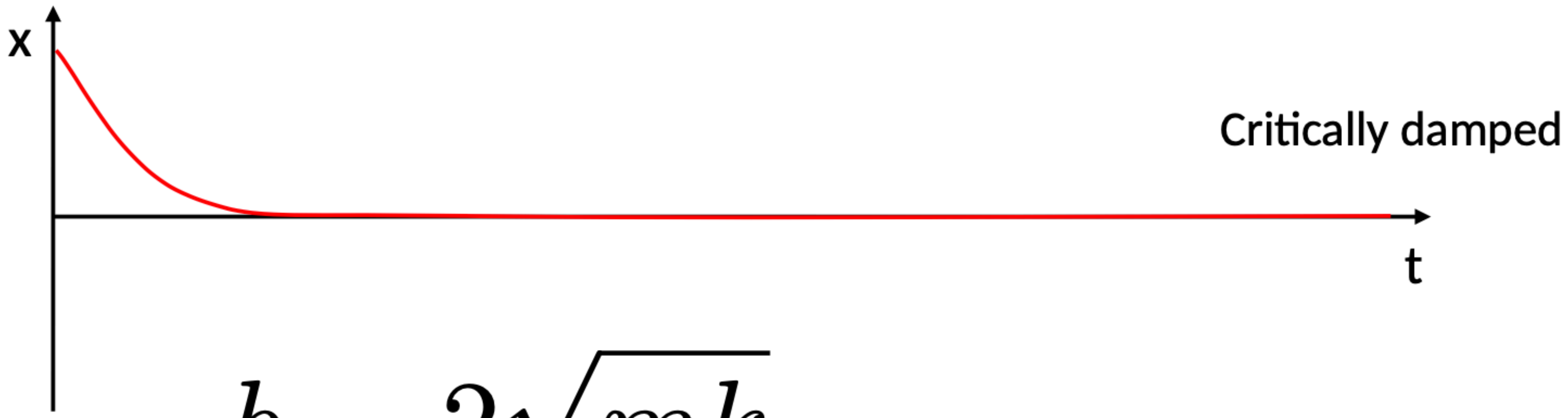


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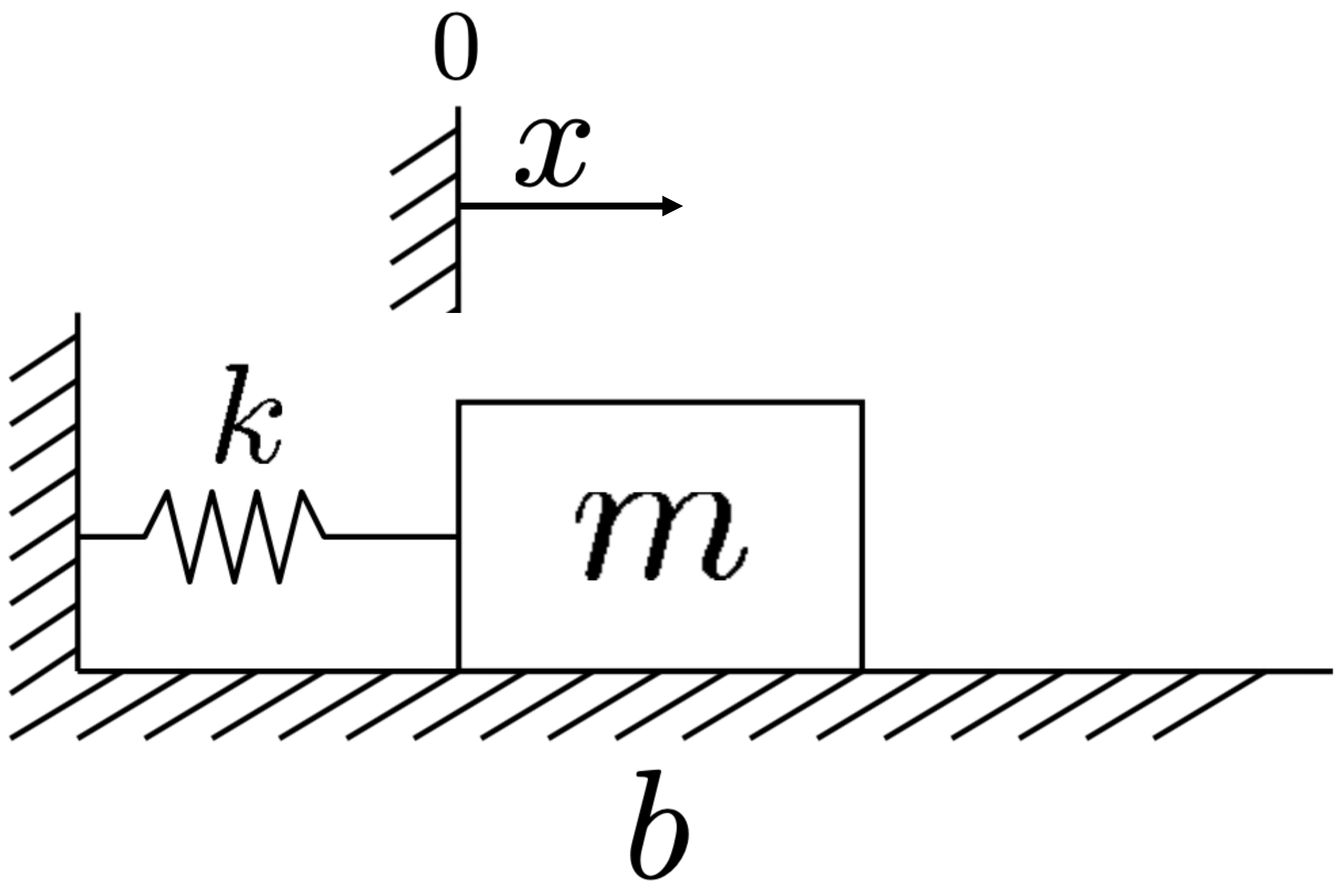
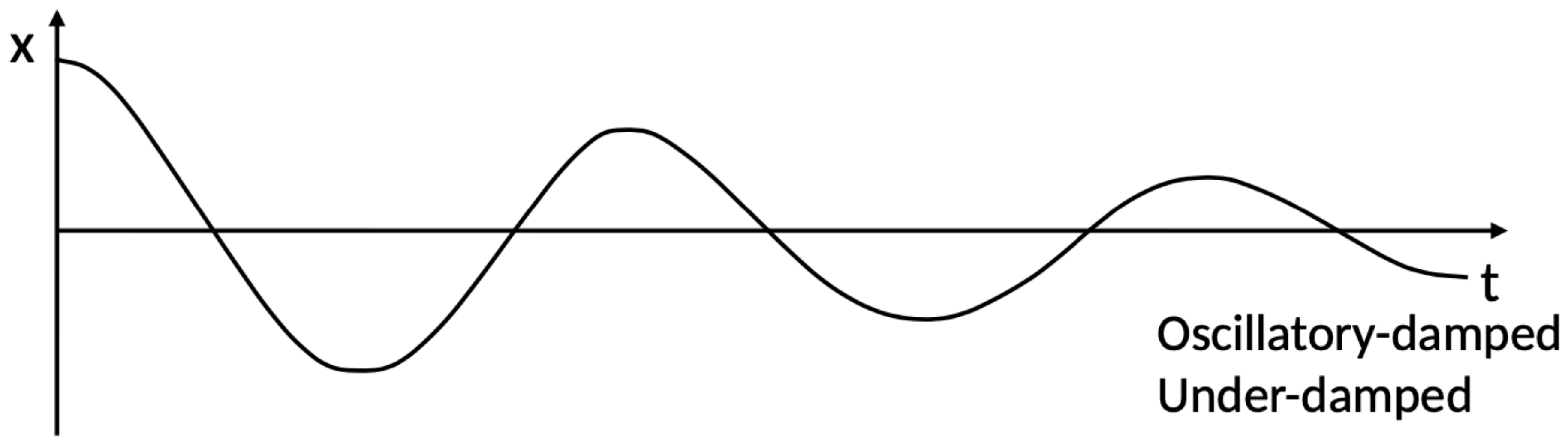


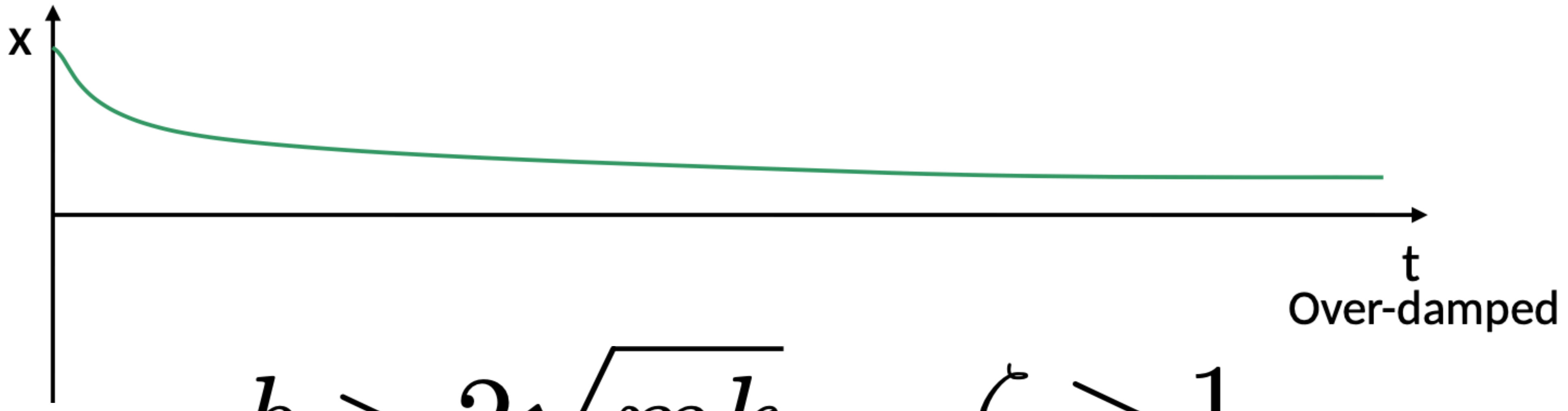

$$m\ddot{x} + b\dot{x} + kx = 0$$

# 2nd-Order Linear Systems

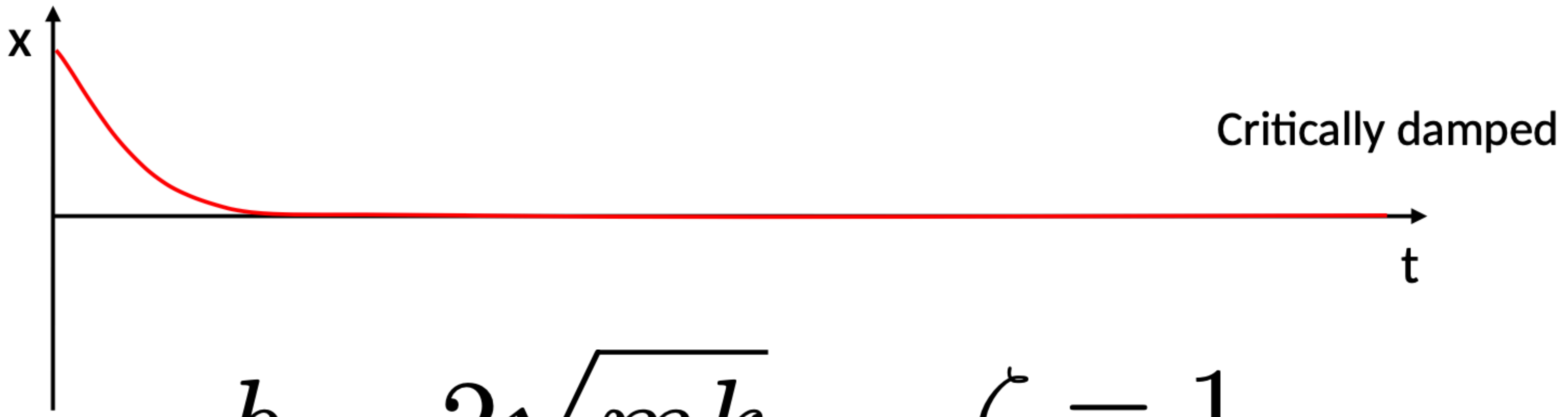
Possible behaviors




$$b < 2\sqrt{mk} \qquad \zeta < 1$$




$$b > 2\sqrt{mk} \qquad \zeta > 1$$



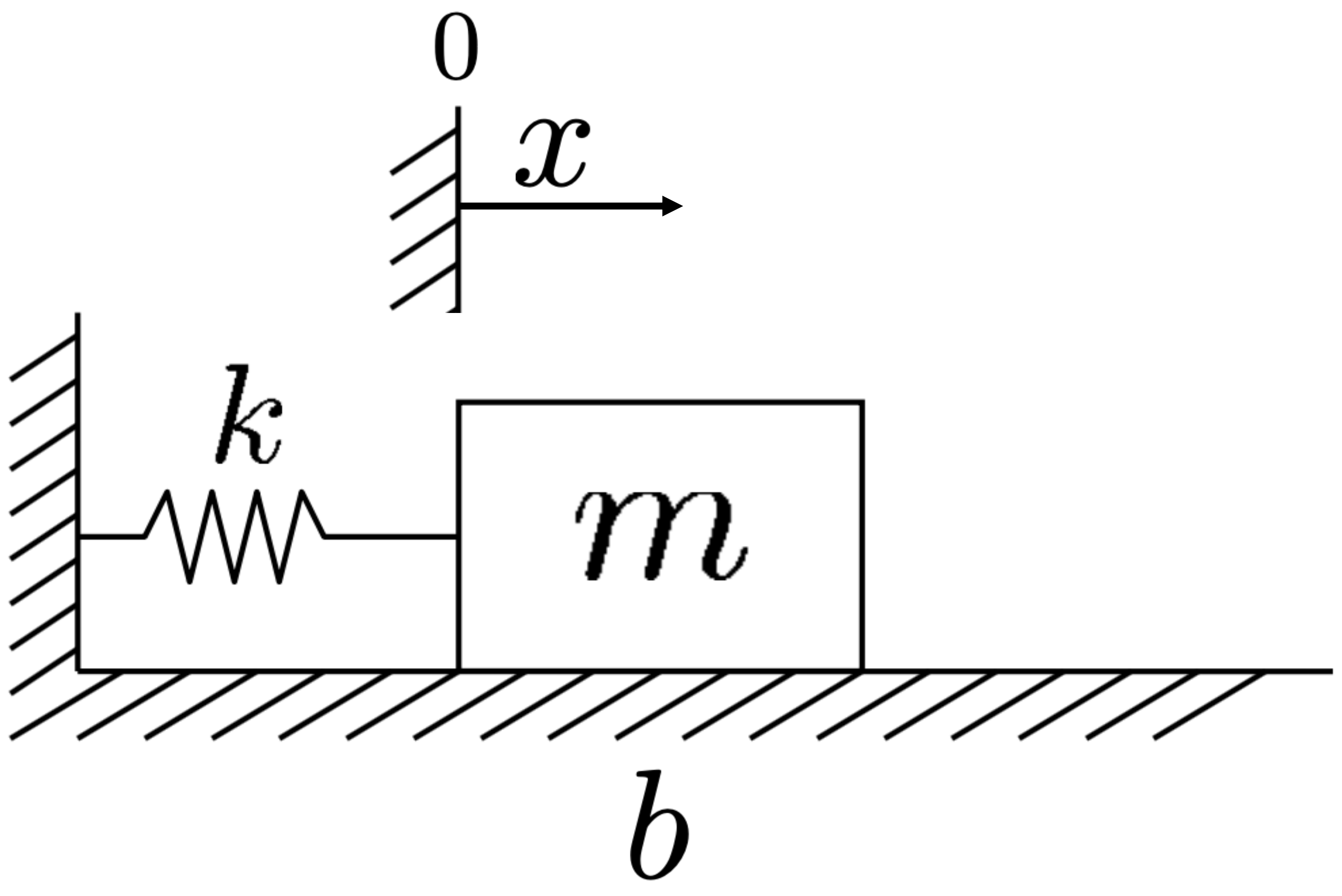

$$b = 2\sqrt{mk} \qquad \zeta = 1$$

$$\zeta = \frac{b}{2\sqrt{km}}$$

Damping ratio

$$\omega_n = \sqrt{\frac{k}{m}}$$

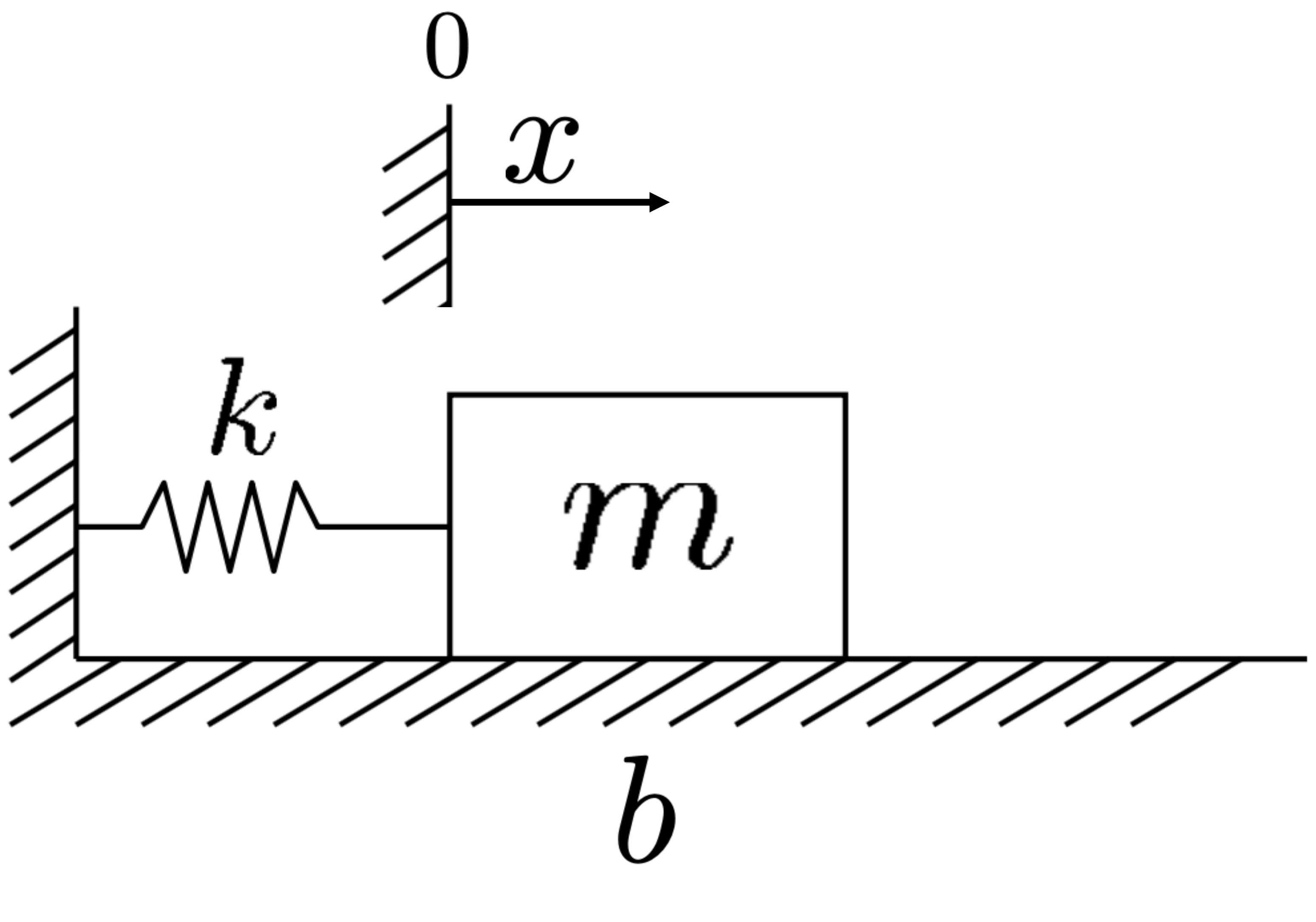
Natural frequency




$$m\ddot{x} + b\dot{x} + kx = 0$$

[redacted]

# 2nd-Order Linear Systems

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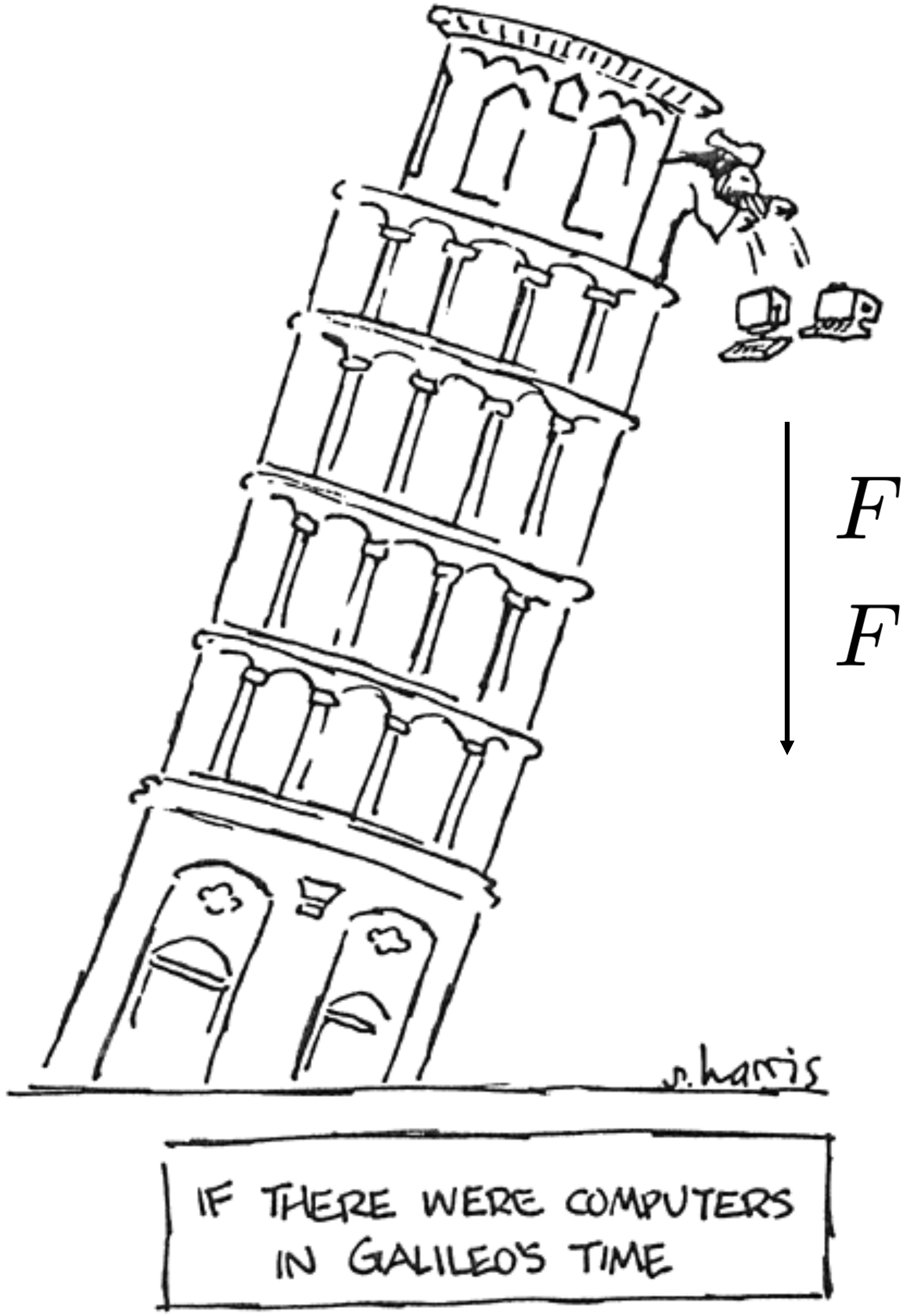
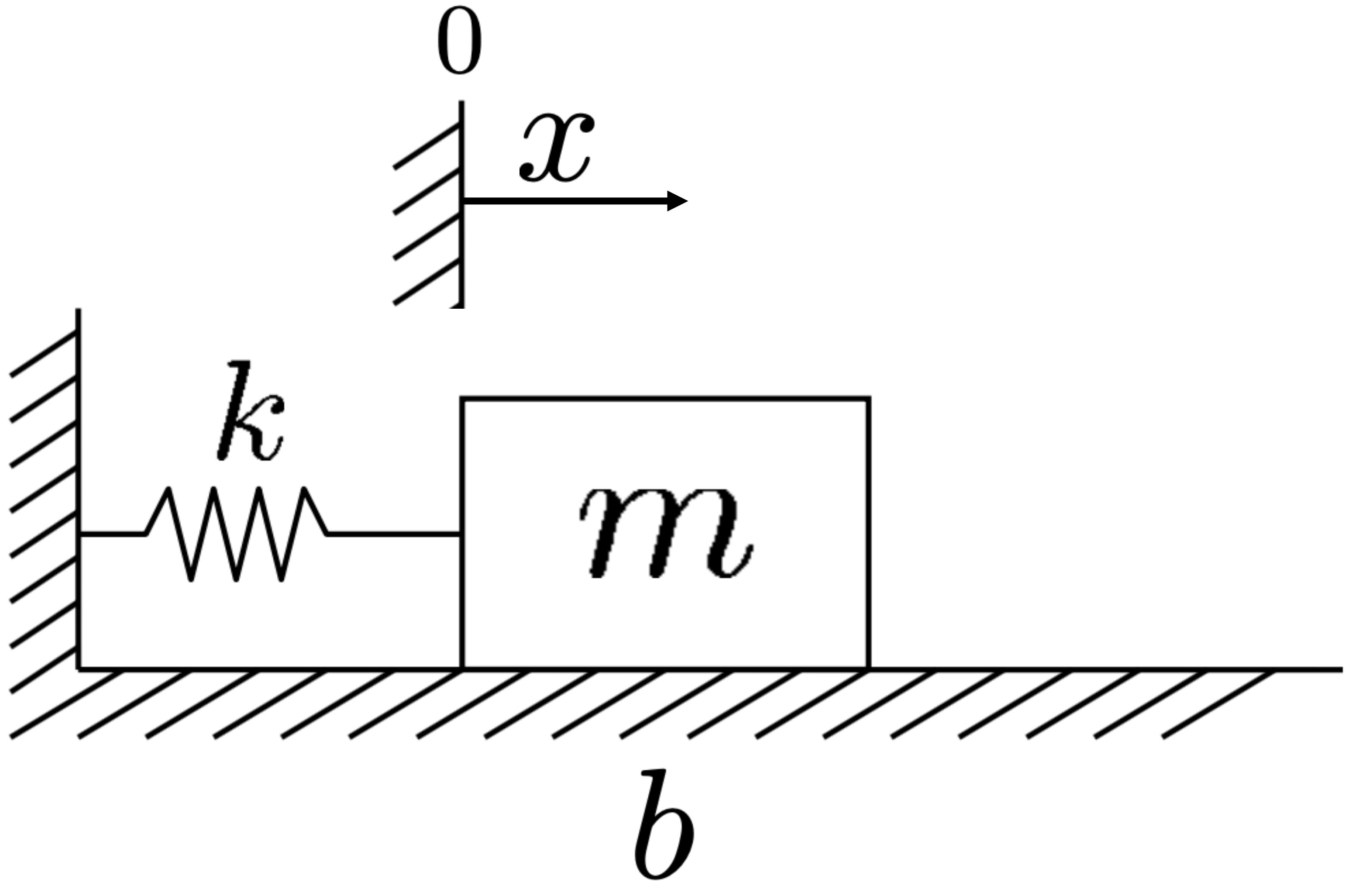
# 2nd-Order Linear Systems




$$F = mg$$

$$F = m\ddot{x}$$

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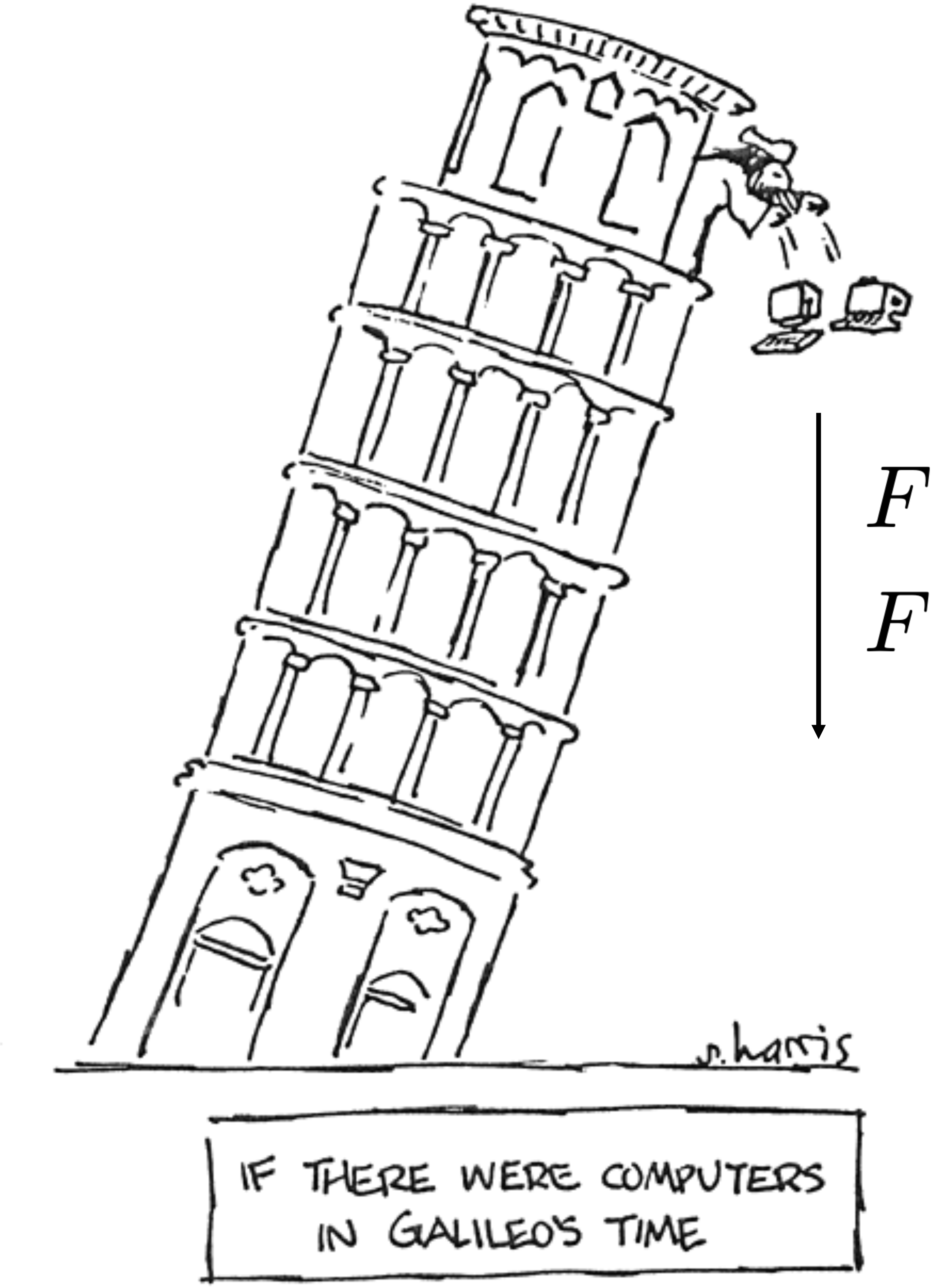
# 2nd-Order Linear Systems

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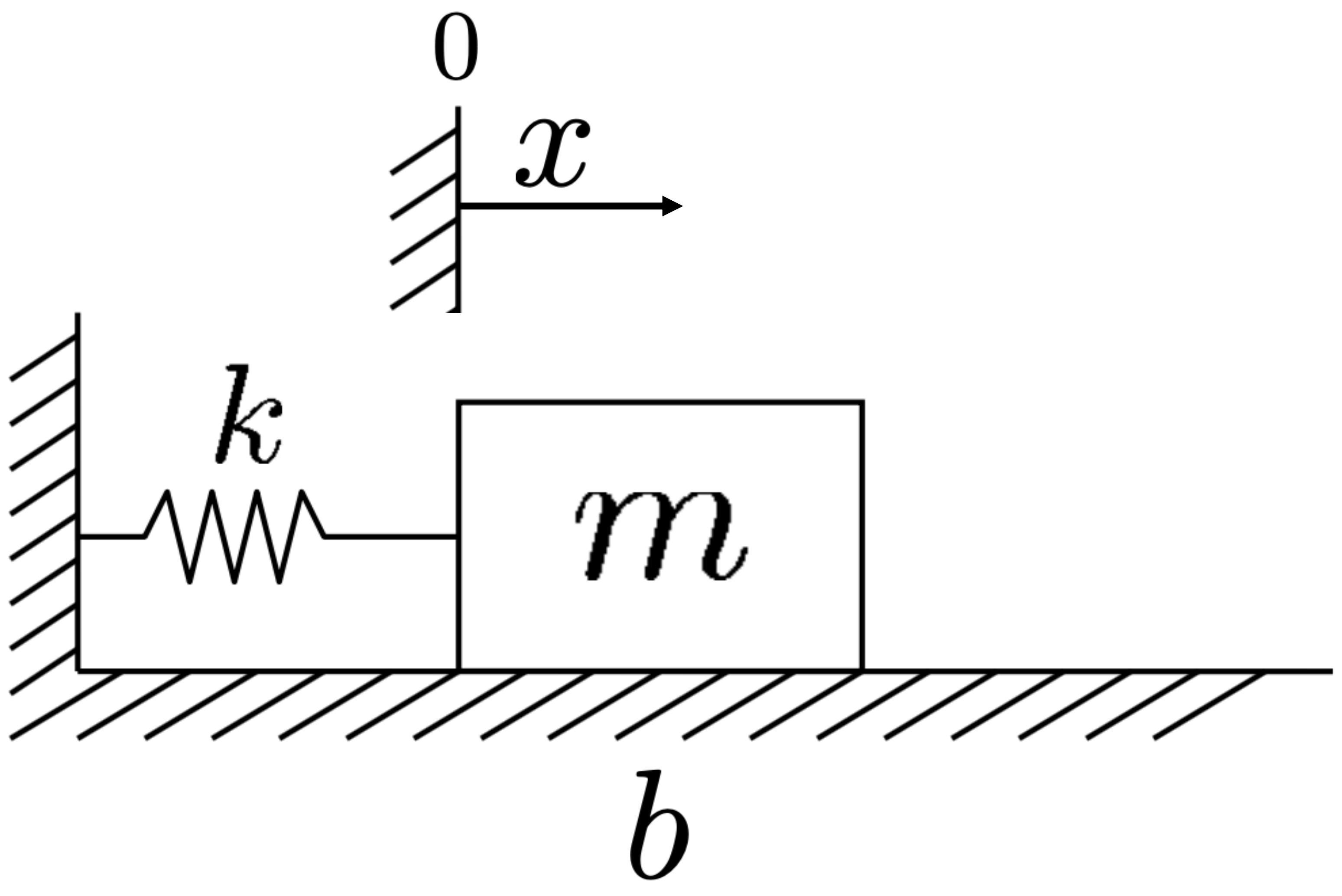
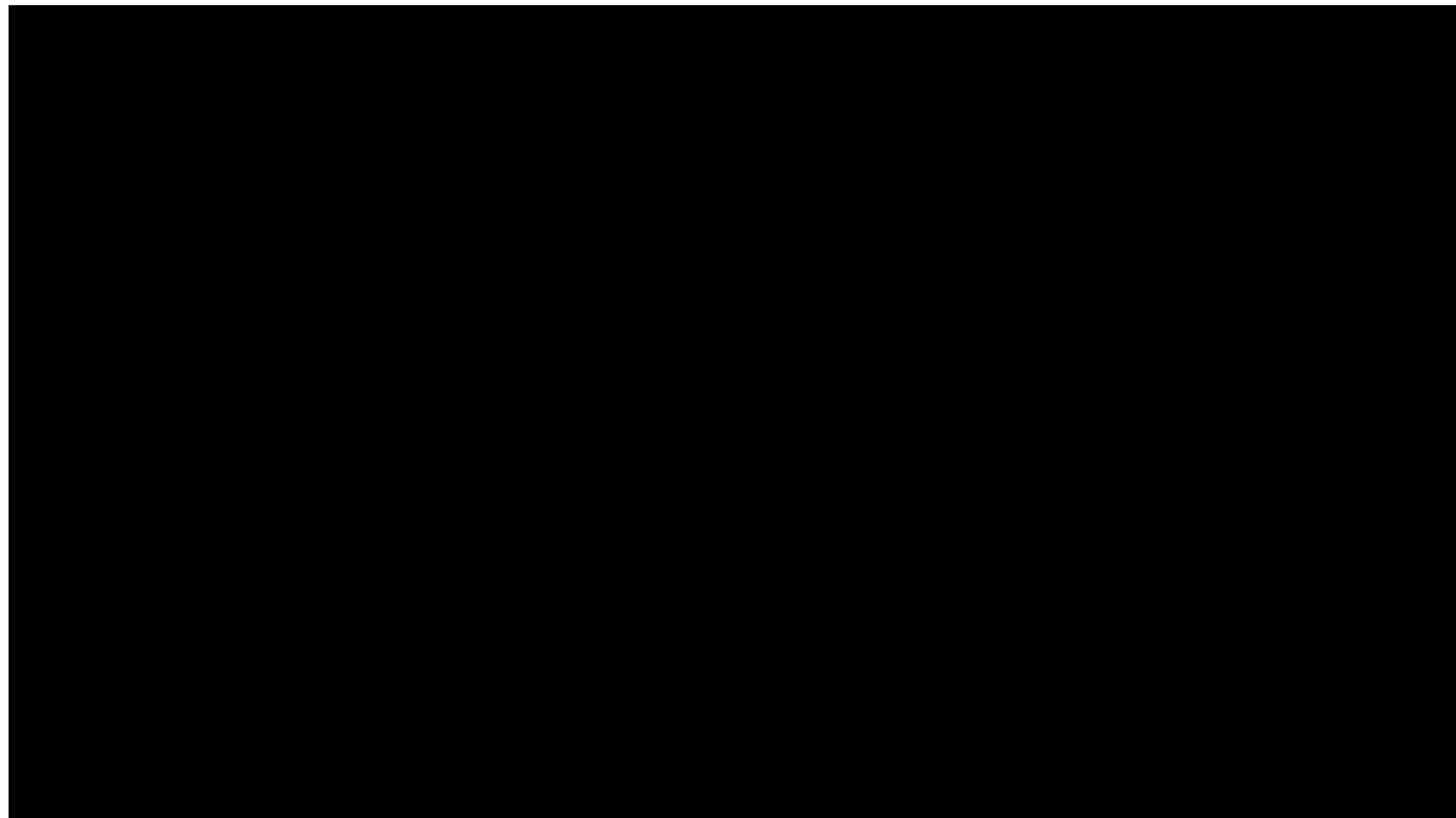



$$m\ddot{x} + b\dot{x} + kx = 0$$

reformulate

$$m\ddot{x} = -kx - b\dot{x}$$

# Control of 2nd-Order System



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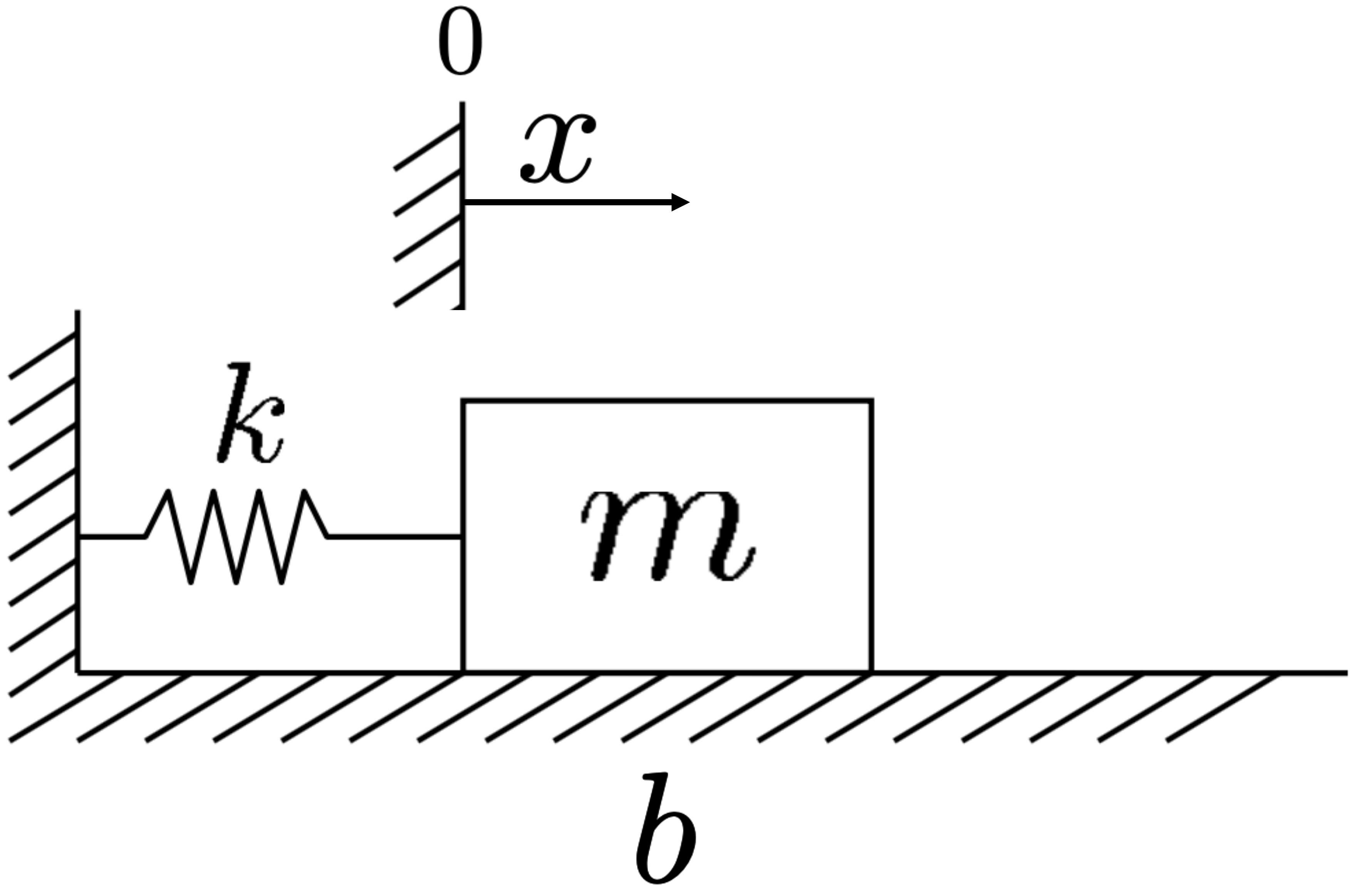


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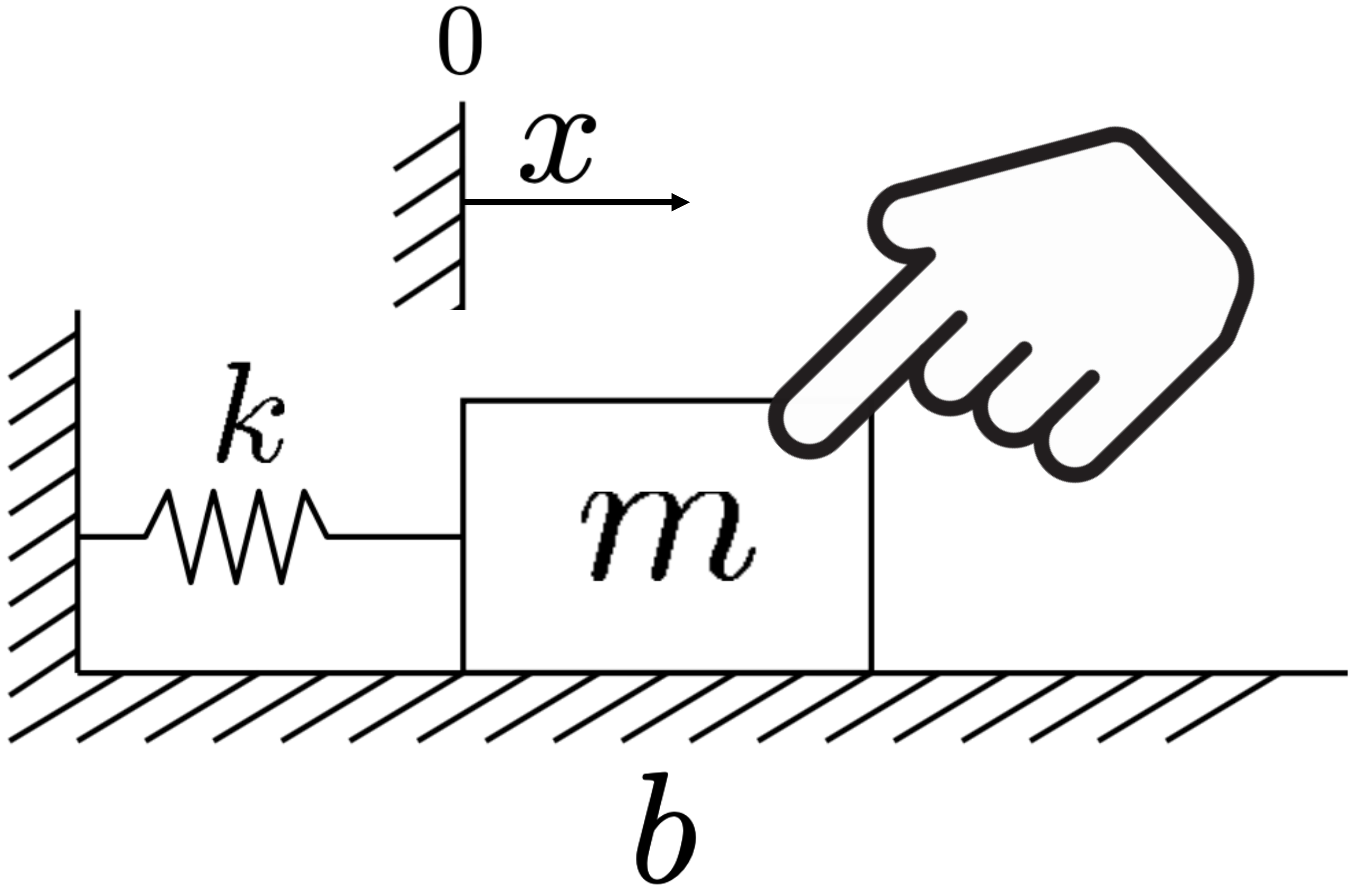


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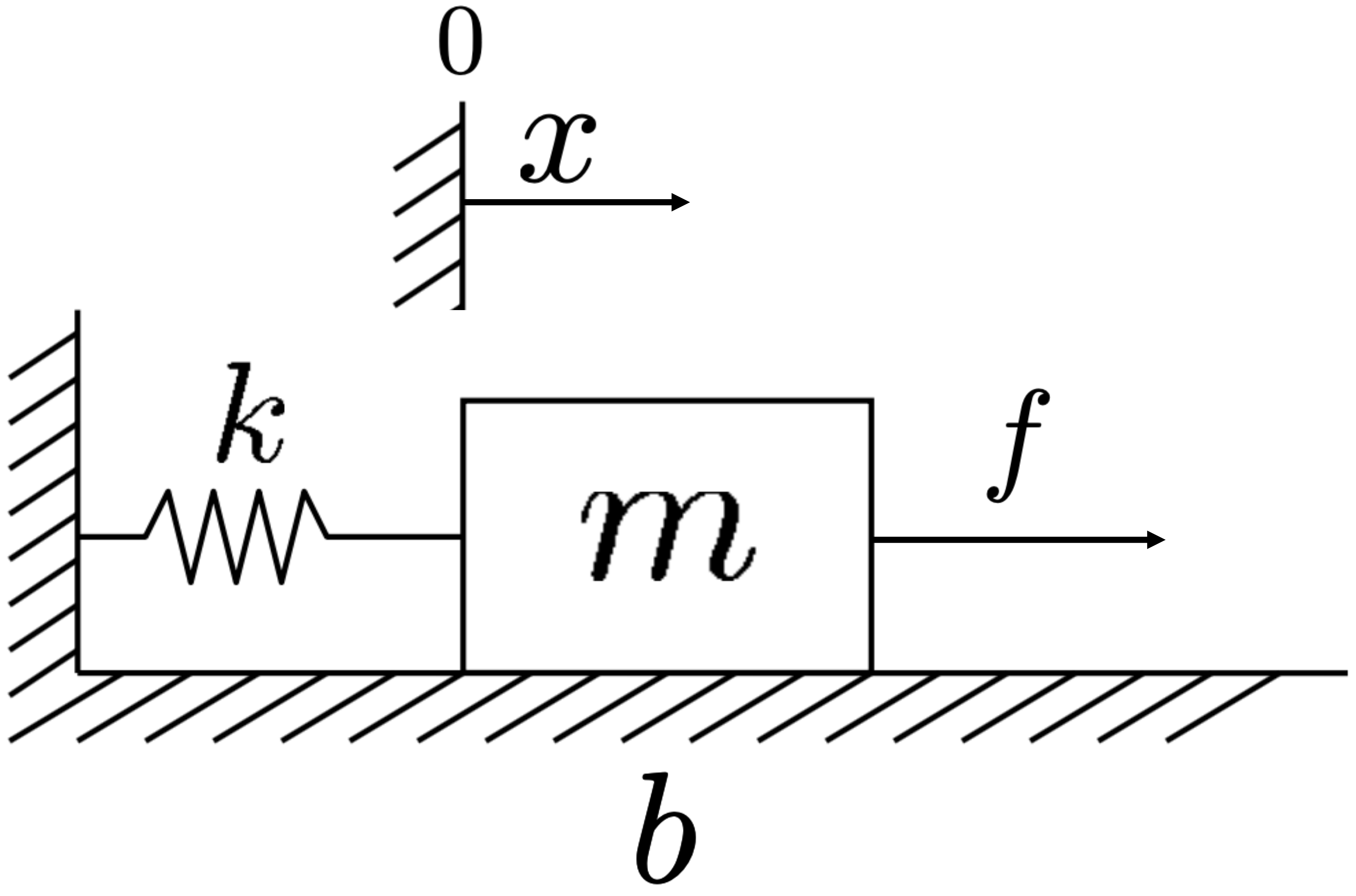


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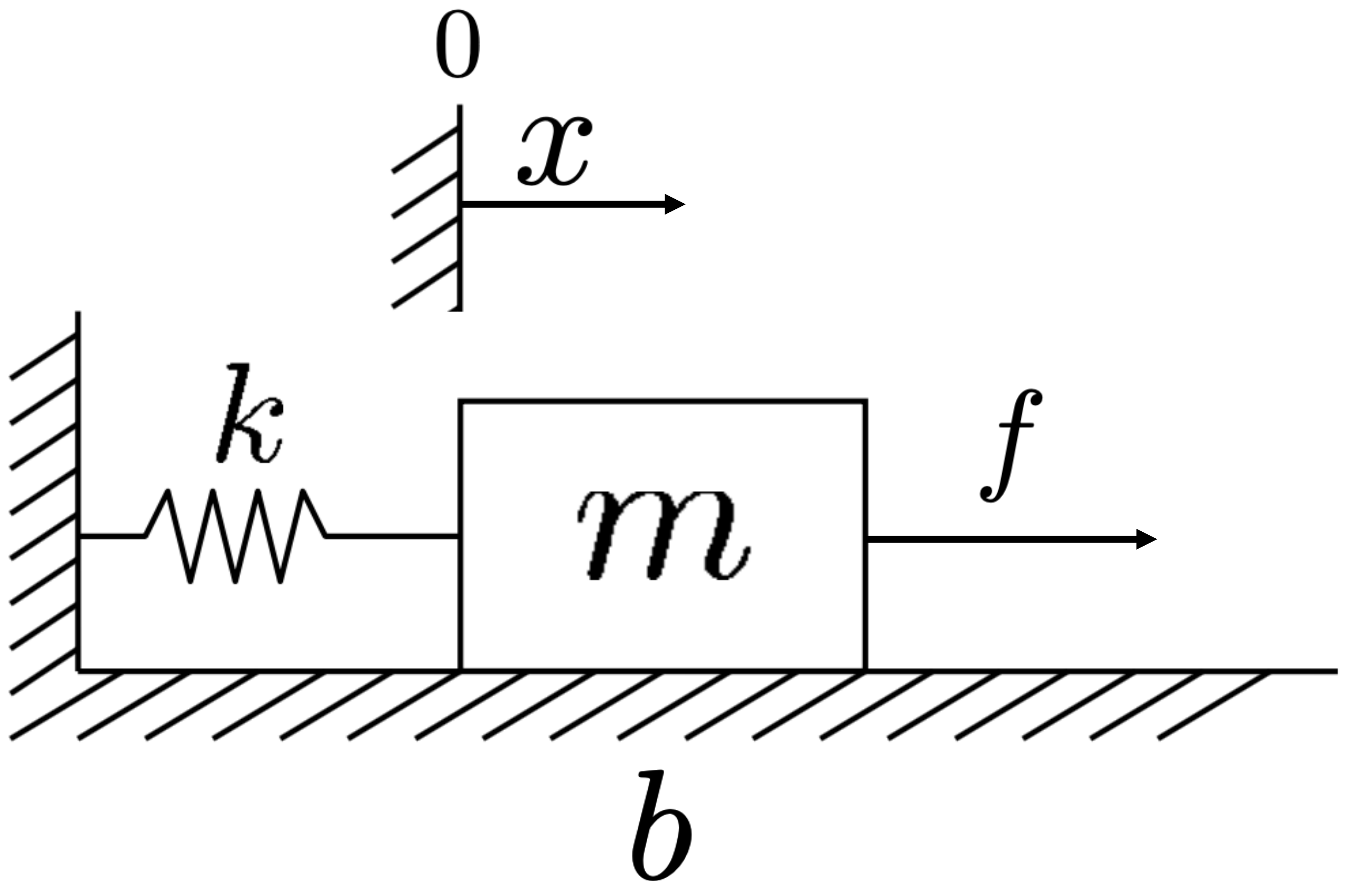


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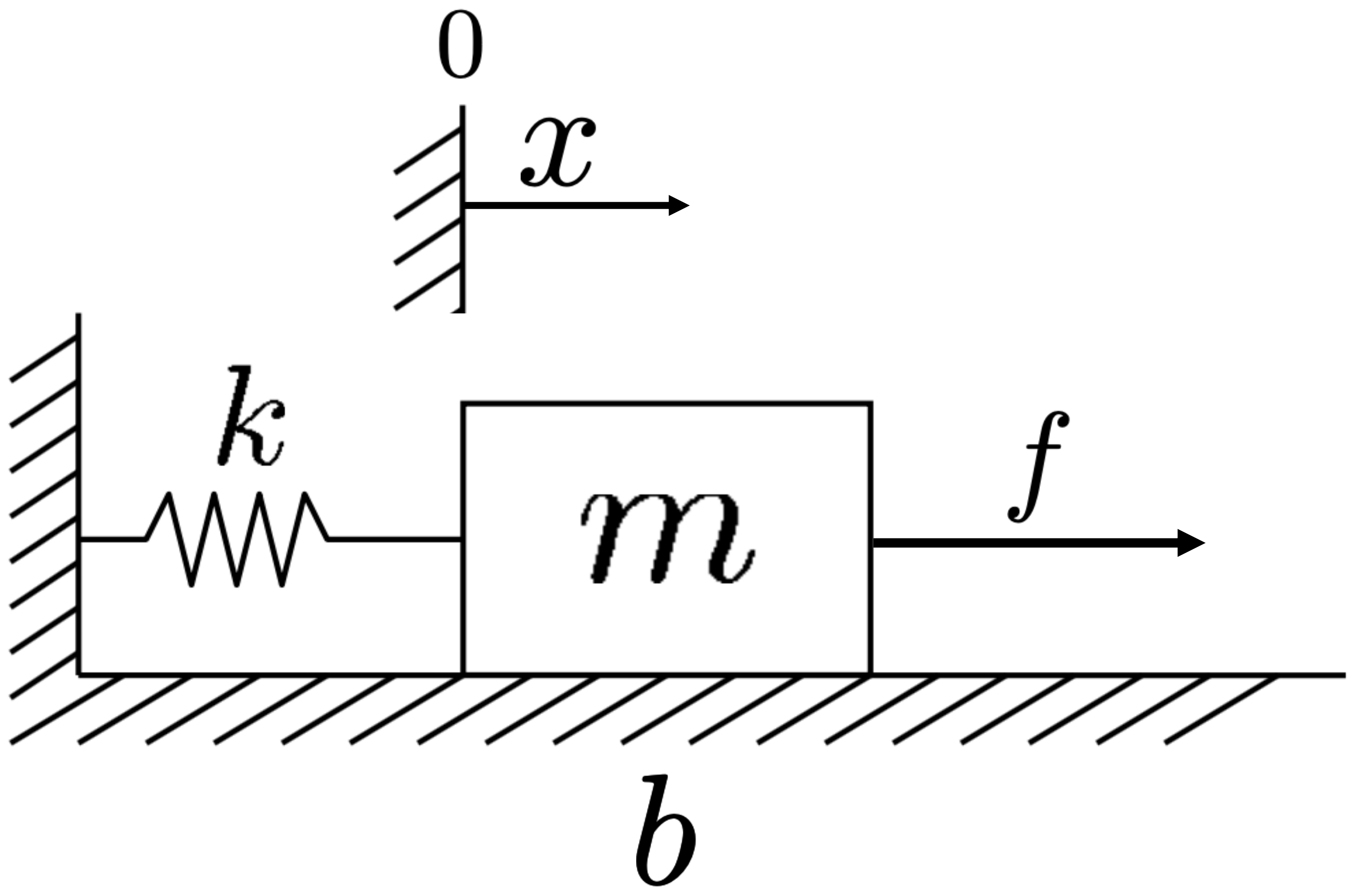


add control force

$$m\ddot{x} = -kx - b\dot{x}$$

$$m\ddot{x} = -kx - b\dot{x} + [f]$$

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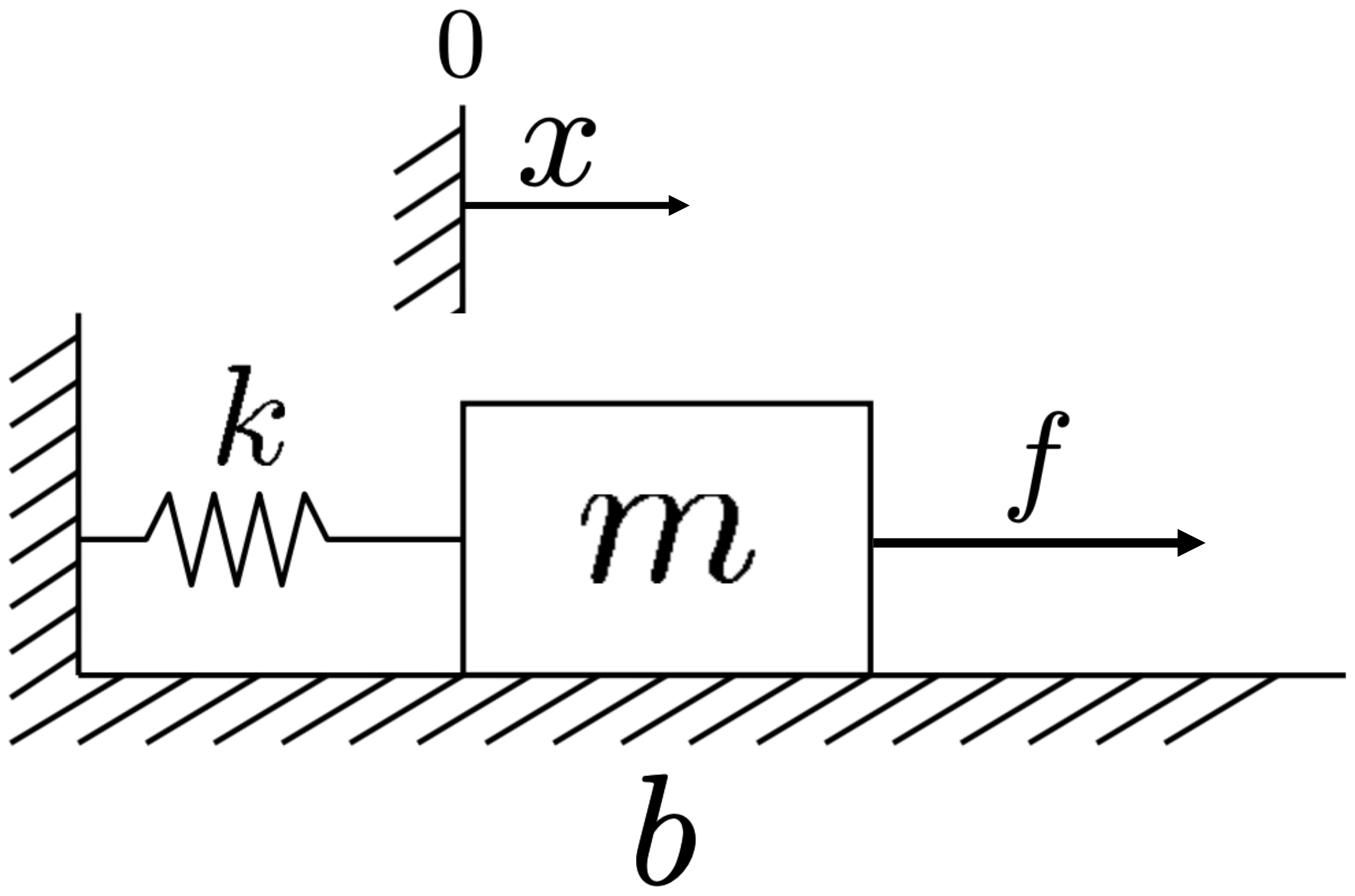


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We can define this to be whatever we want!

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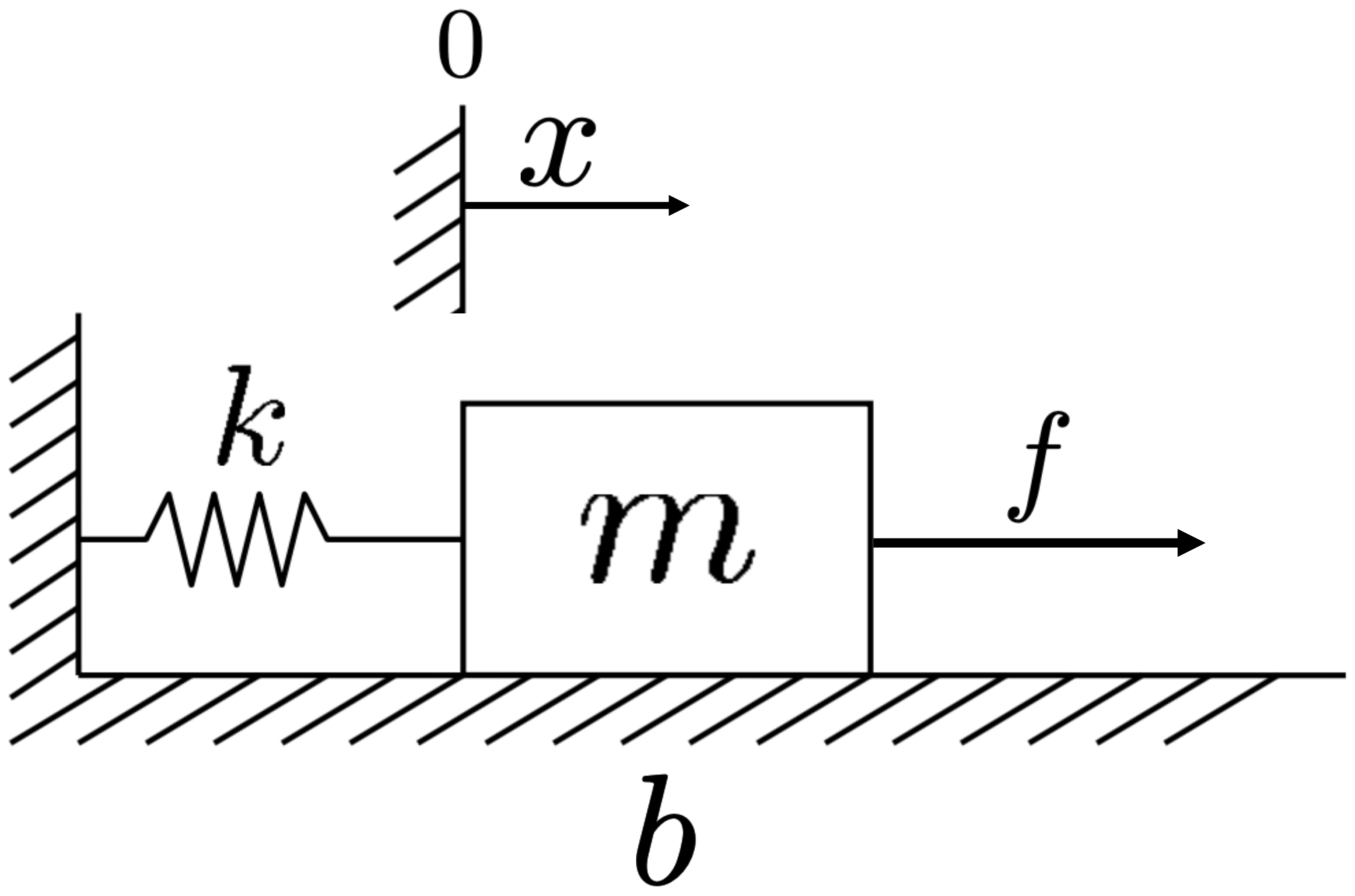


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We can define this to be whatever we want!

$f := -k_p x - k_v \dot{x}$

$$m\ddot{x} = -kx - b\dot{x} + [-k_p x - k_v \dot{x}]$$

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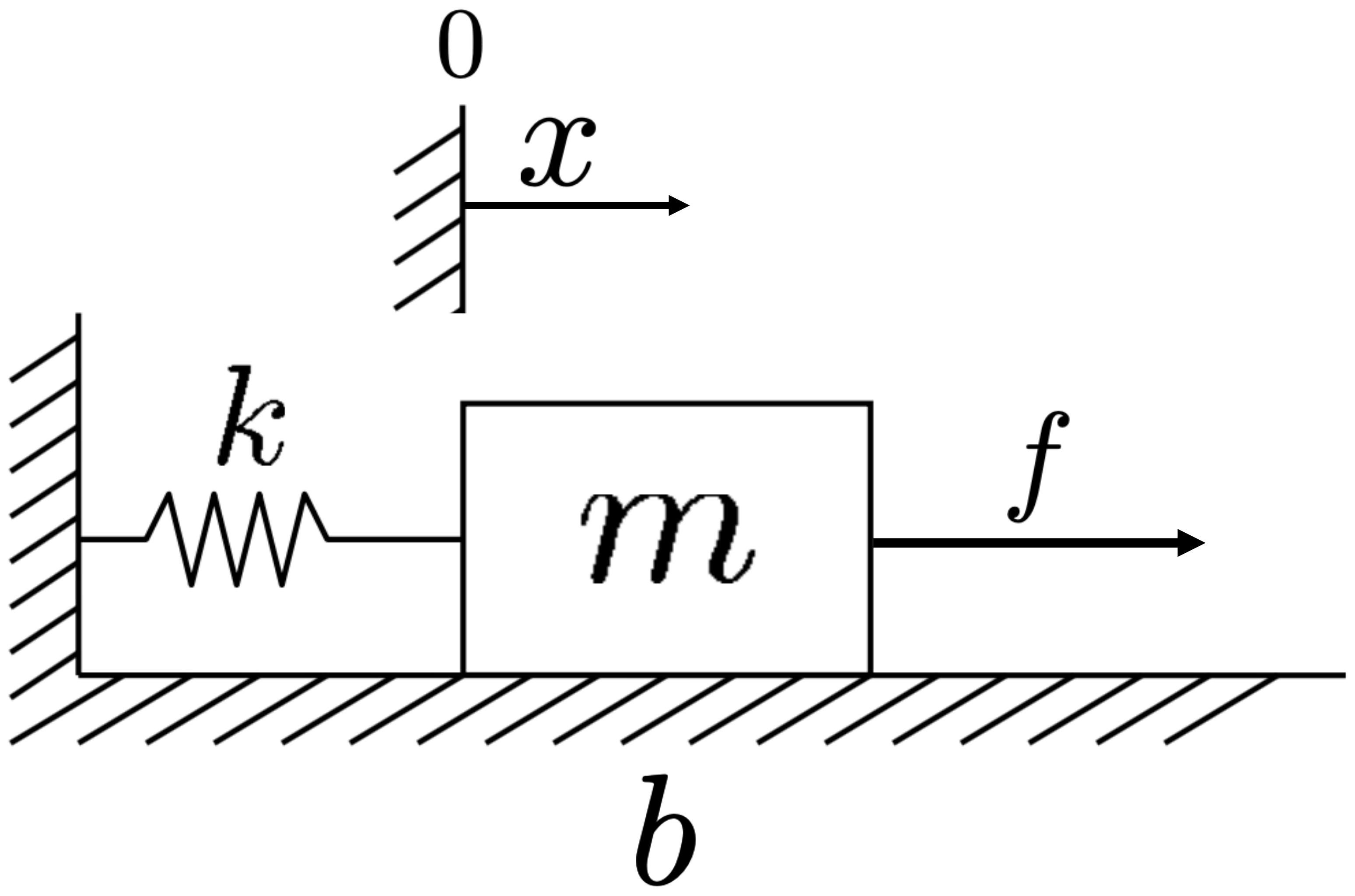


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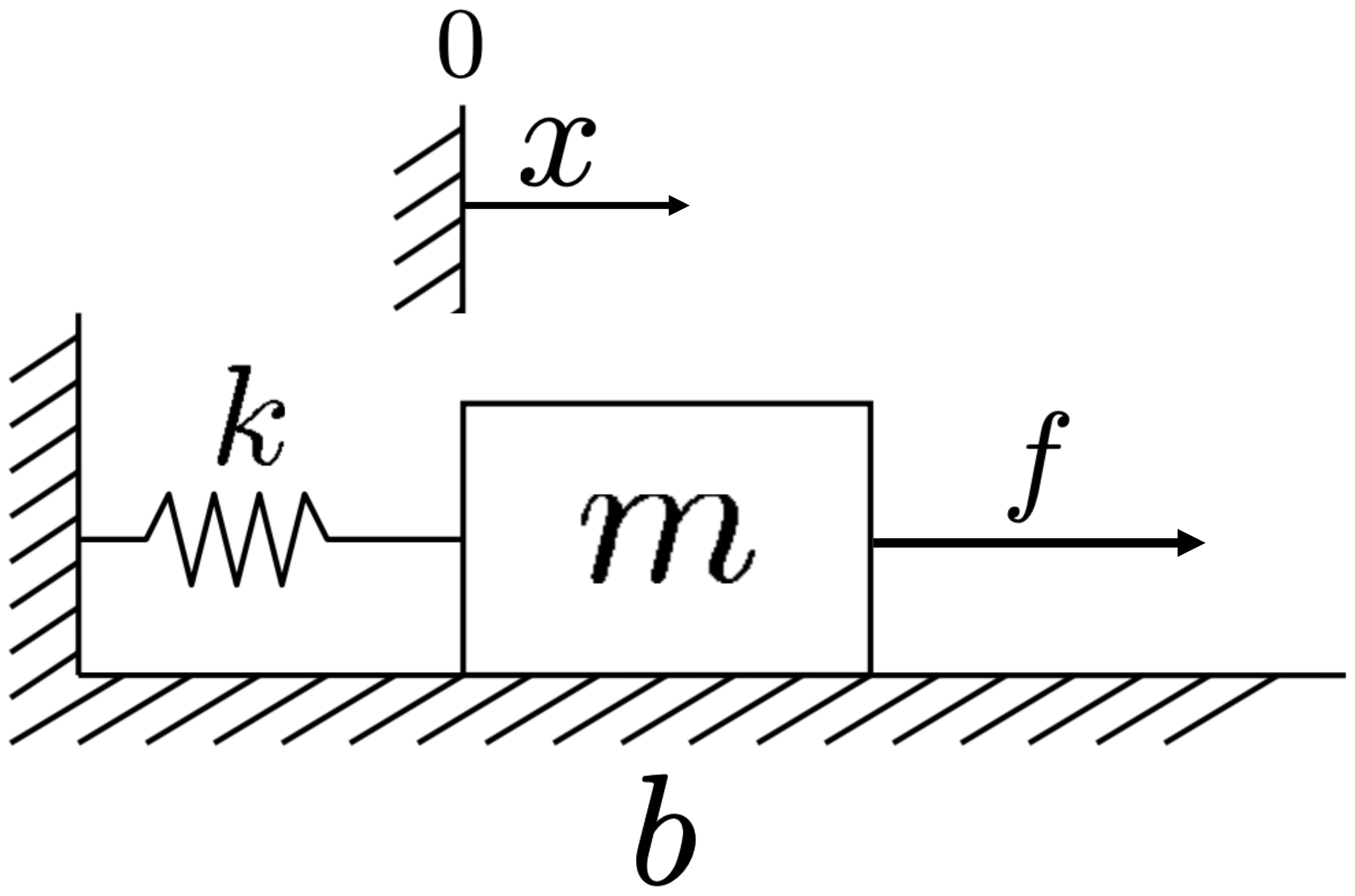
reformulate

$$m\ddot{x} = -\underbrace{(b + k_v)}_{b'}\dot{x} - \underbrace{(k + k_p)}_{k'}x$$

# Control of 2nd-Order System

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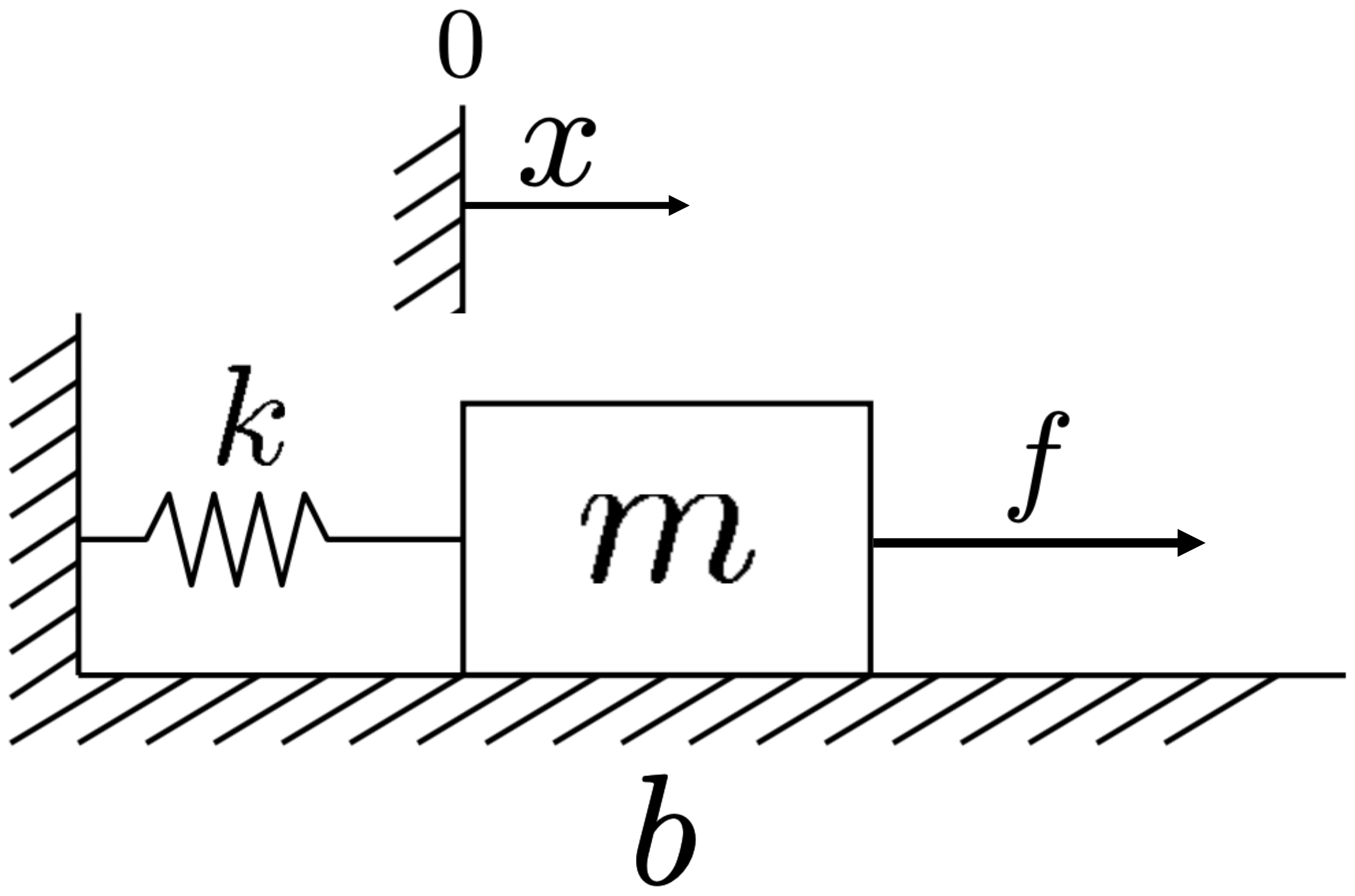
Remember:

For: $m\ddot{x} + b\dot{x} + kx = 0$

Critical damping: $b = 2\sqrt{mk}$

# Control of 2nd-Order System

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Critical damping: $b = 2\sqrt{mk}$

Critical damping:

$b' = 2\sqrt{mk'}$

# Control-Law Partitioning

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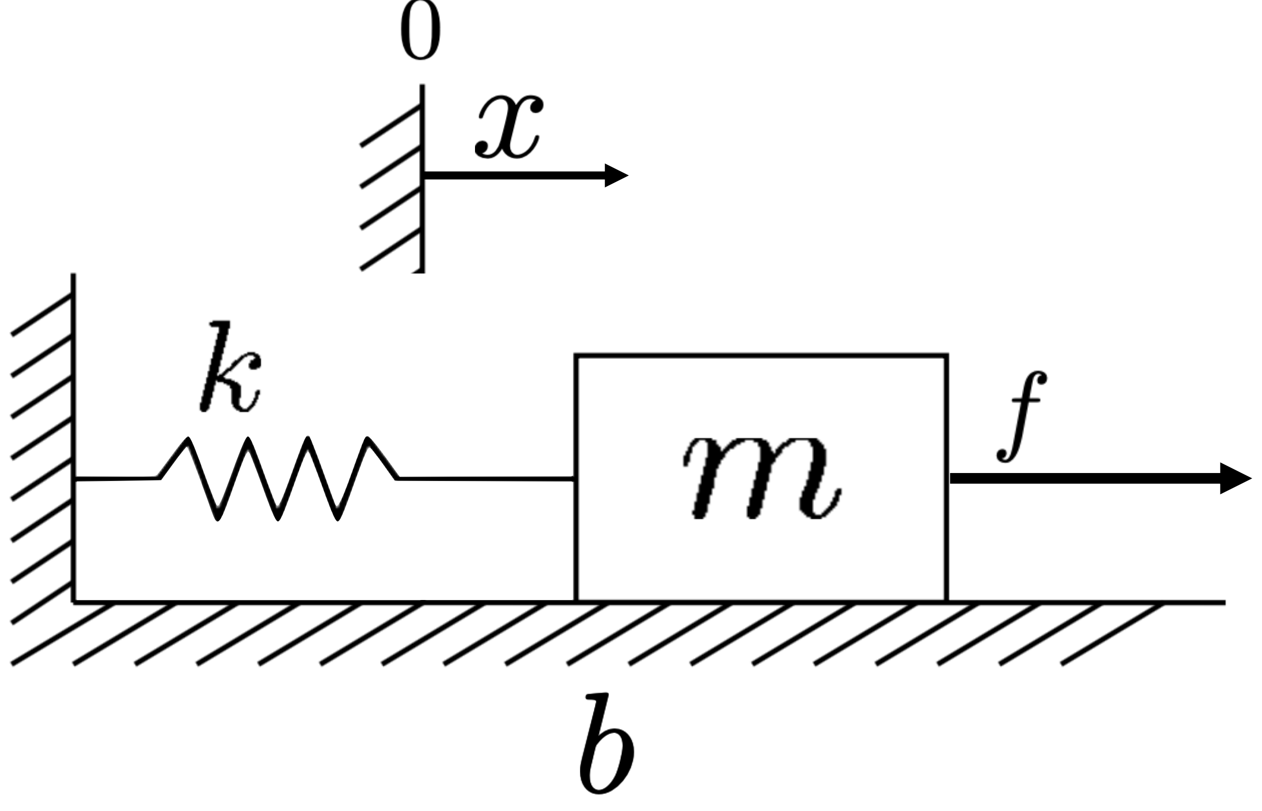


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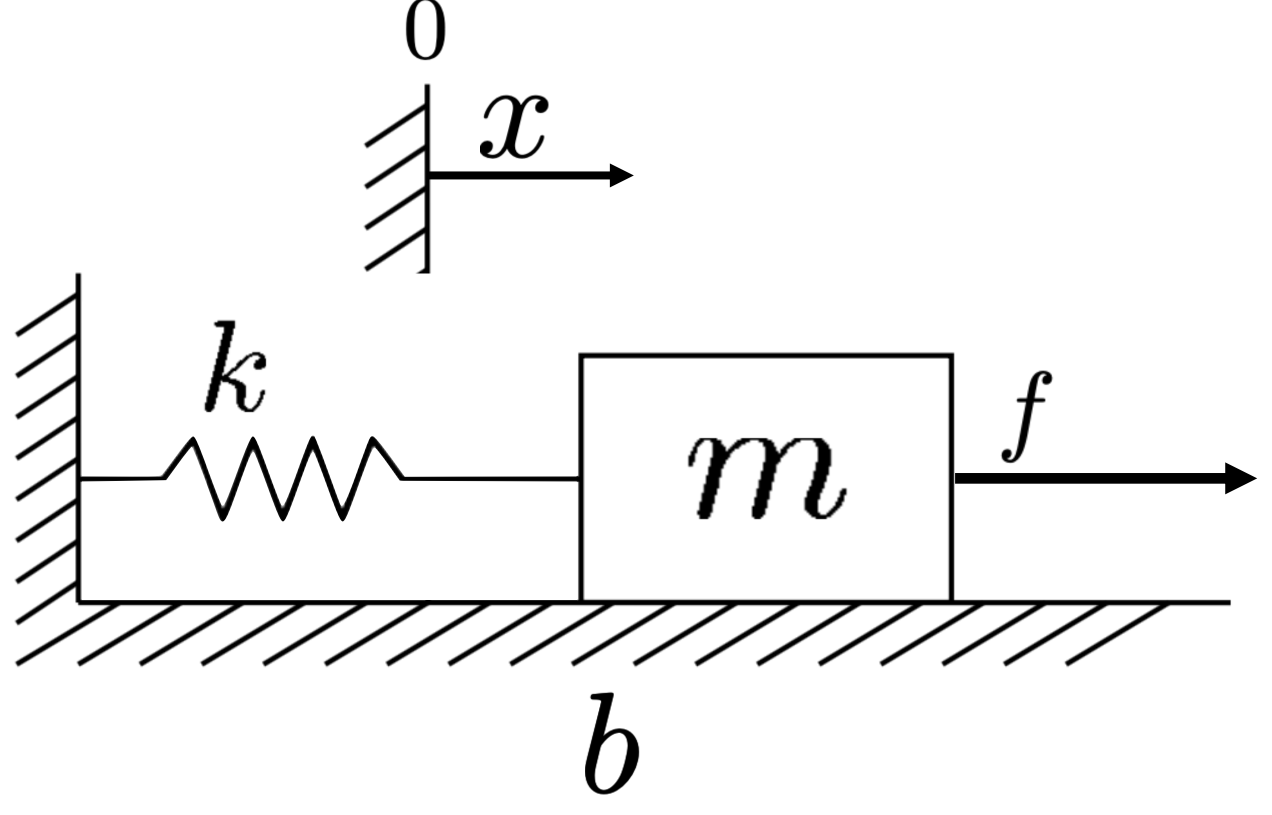


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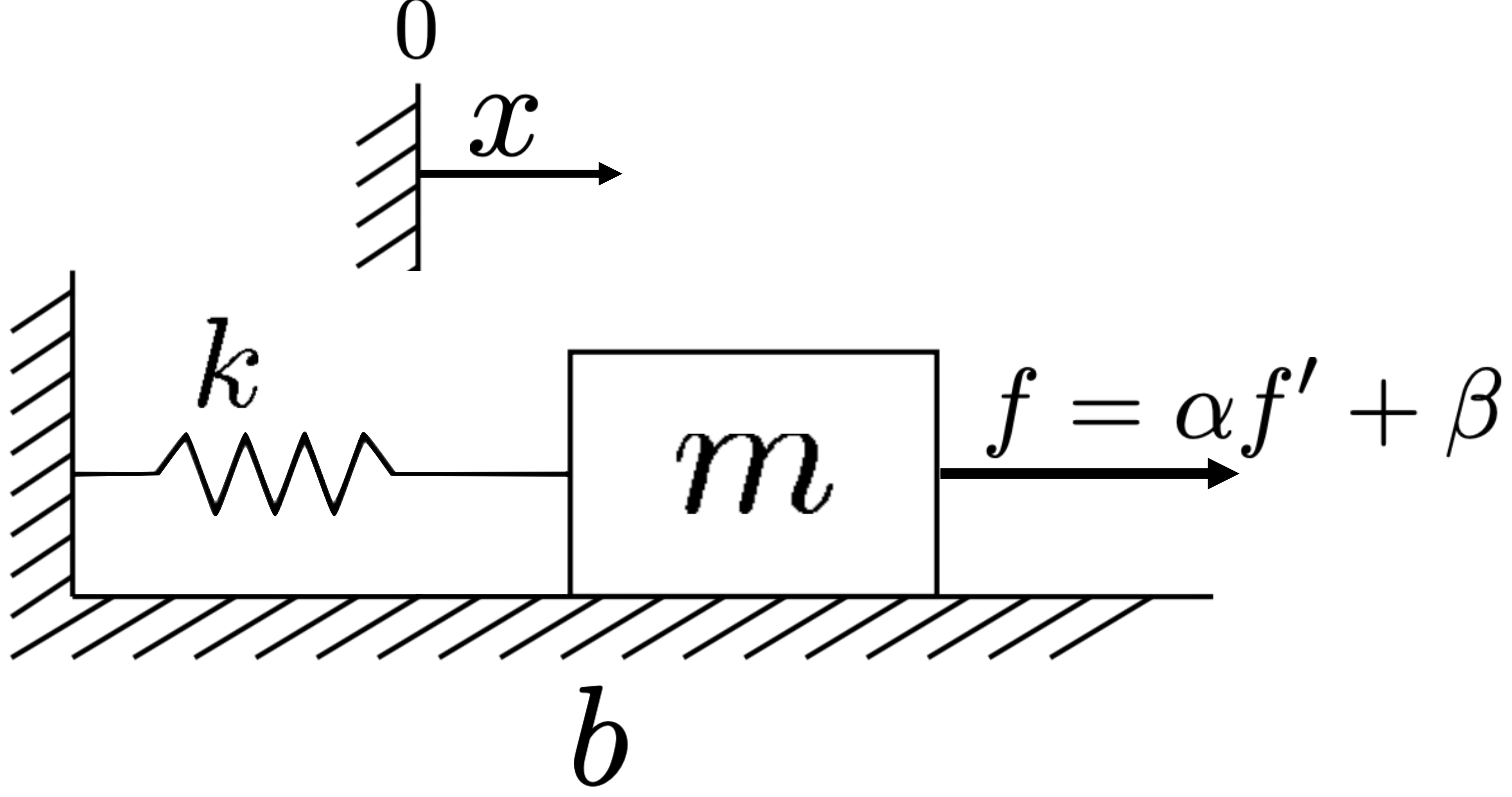


$$f := \alpha f' + \beta$$

$$m\ddot{x} = -kx - b\dot{x} + [f]$$

We can define this to be whatever we want!

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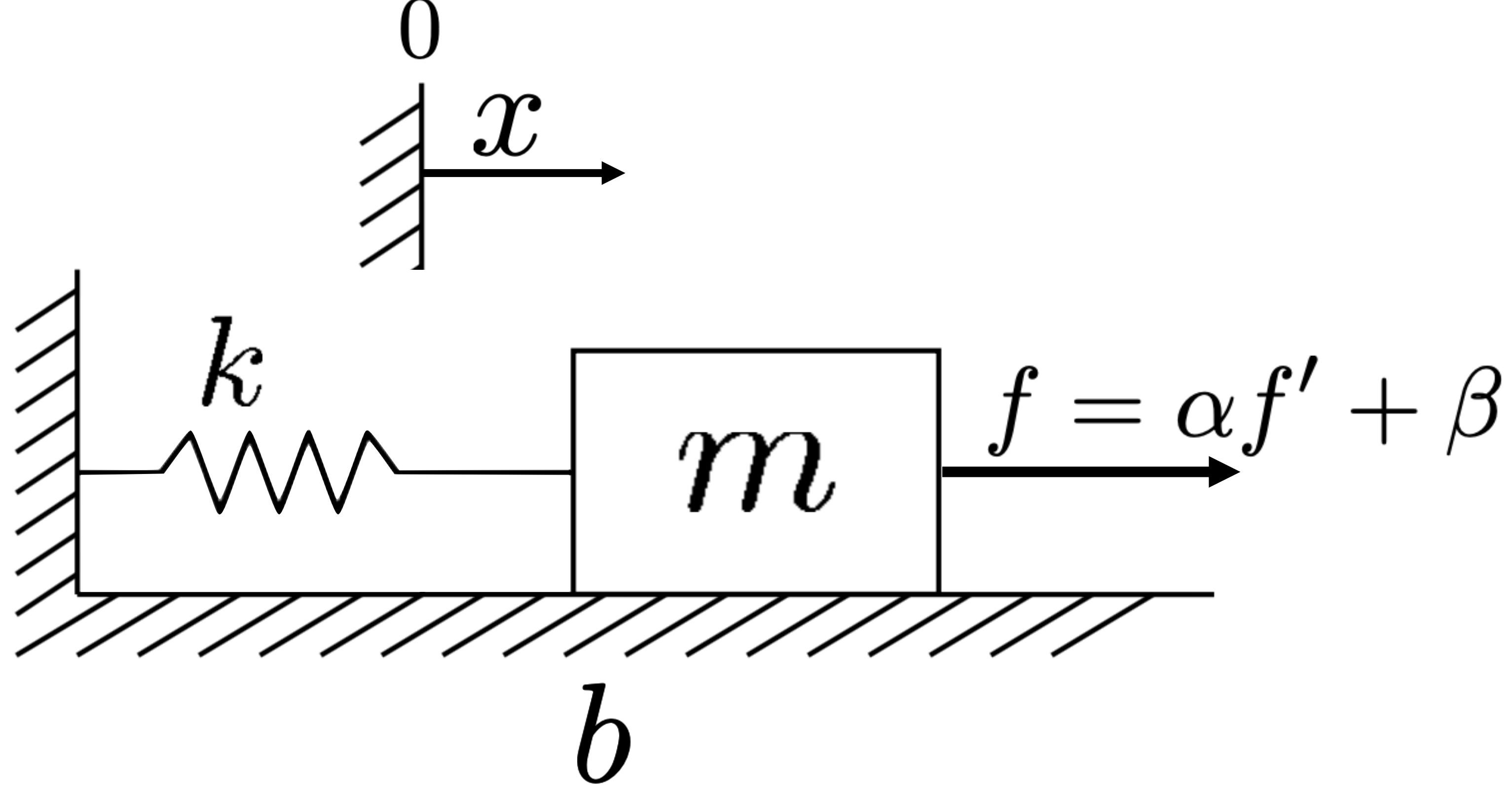


$$f := \alpha f' + \beta$$

$$\beta := b\dot{x} + kx$$
$$\alpha := m$$

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$$m\ddot{x} = -kx - b\dot{x} + [\alpha f' + \beta]$$

$$m\ddot{x} = -kx - b\dot{x} + [mf' + b\dot{x} + kx]$$

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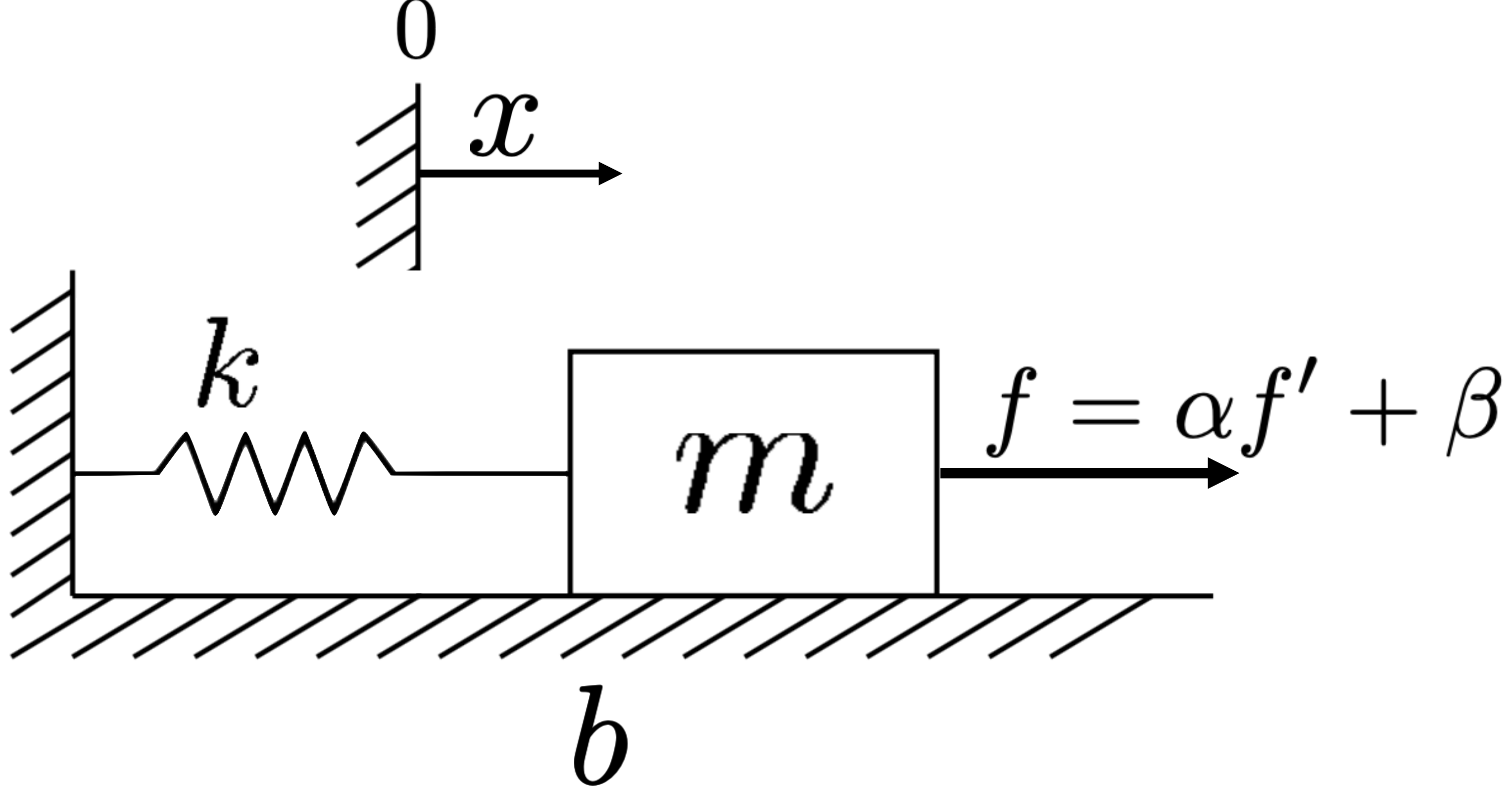


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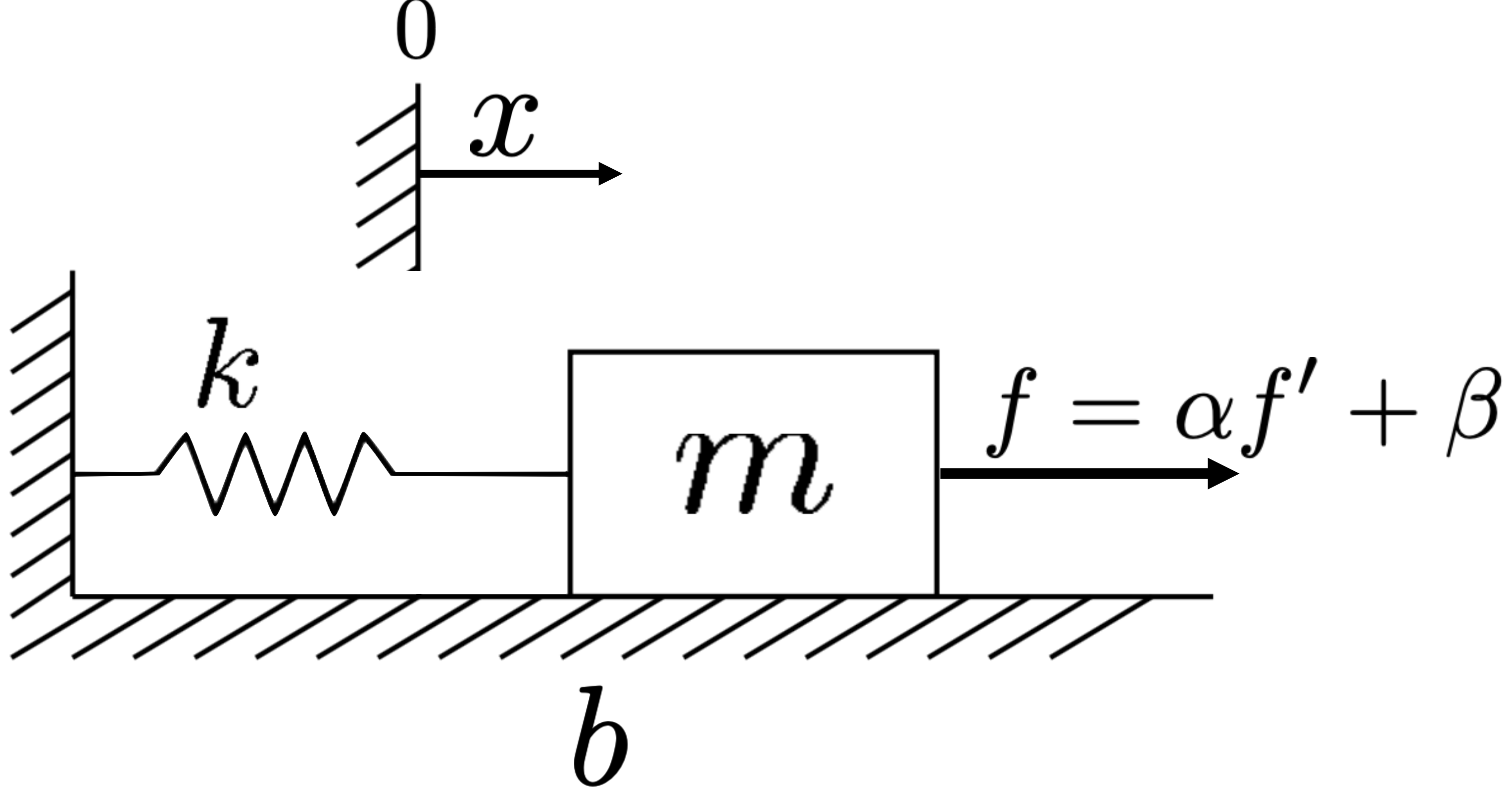


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$\beta := b\dot{x} + kx$
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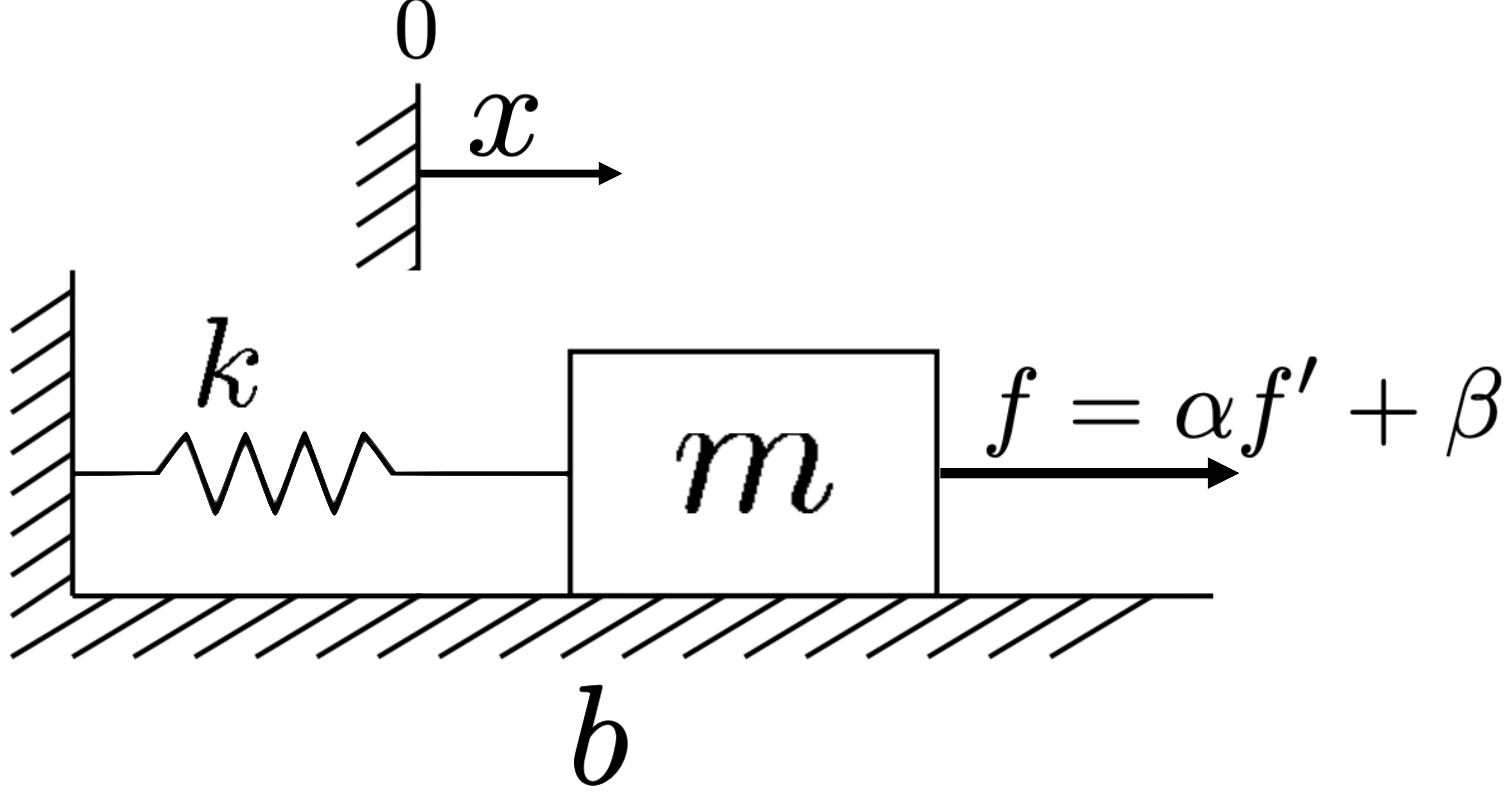
$$m\ddot{x} = -kx - b\dot{x} + [mf' + b\dot{x} + kx]$$

simplify

$$\ddot{x} = f'$$

# Control-Law Partitioning

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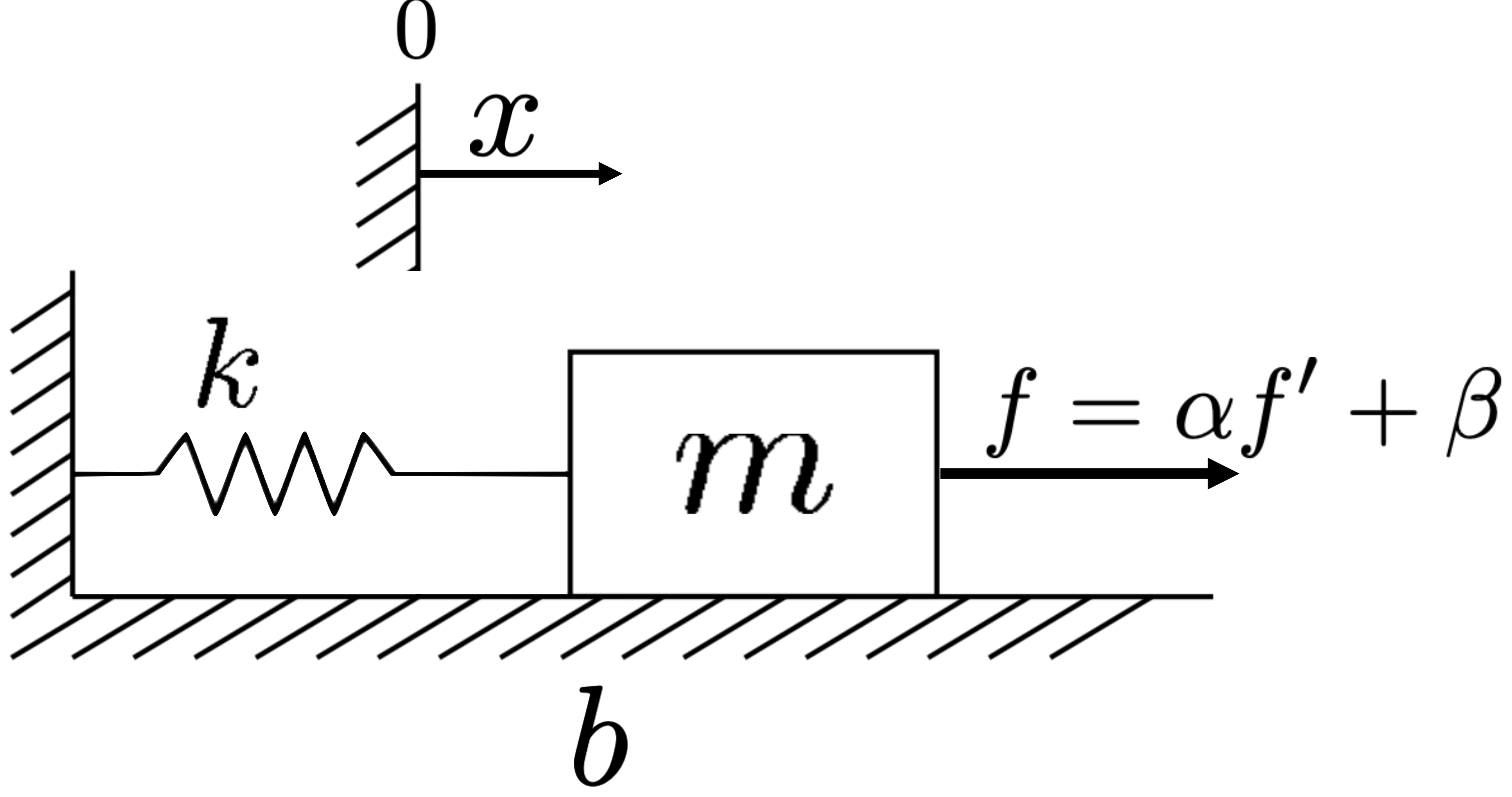


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$$m\ddot{x} = -kx - b\dot{x} + [mf' + b\dot{x} + kx]$$

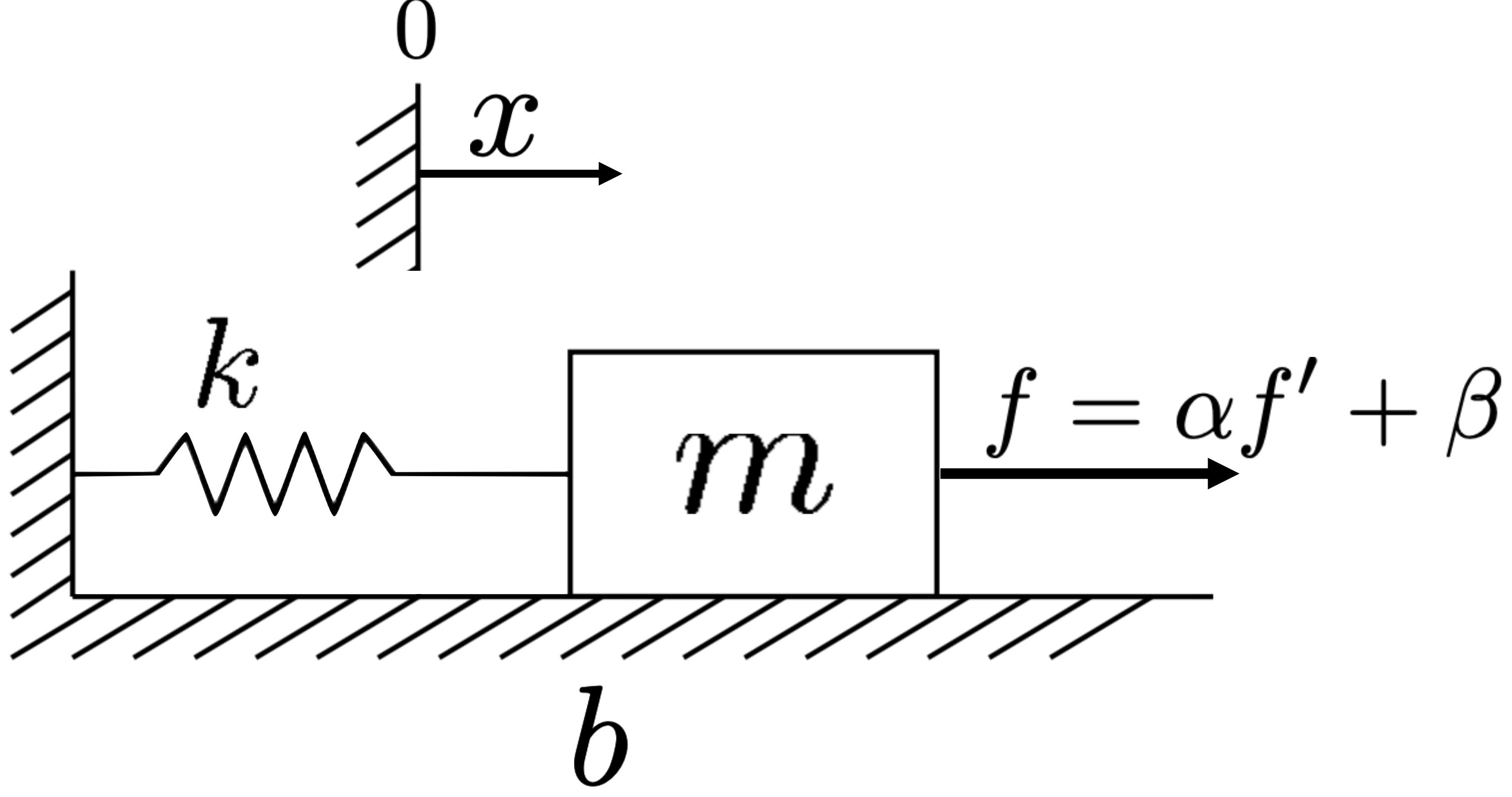
simplify

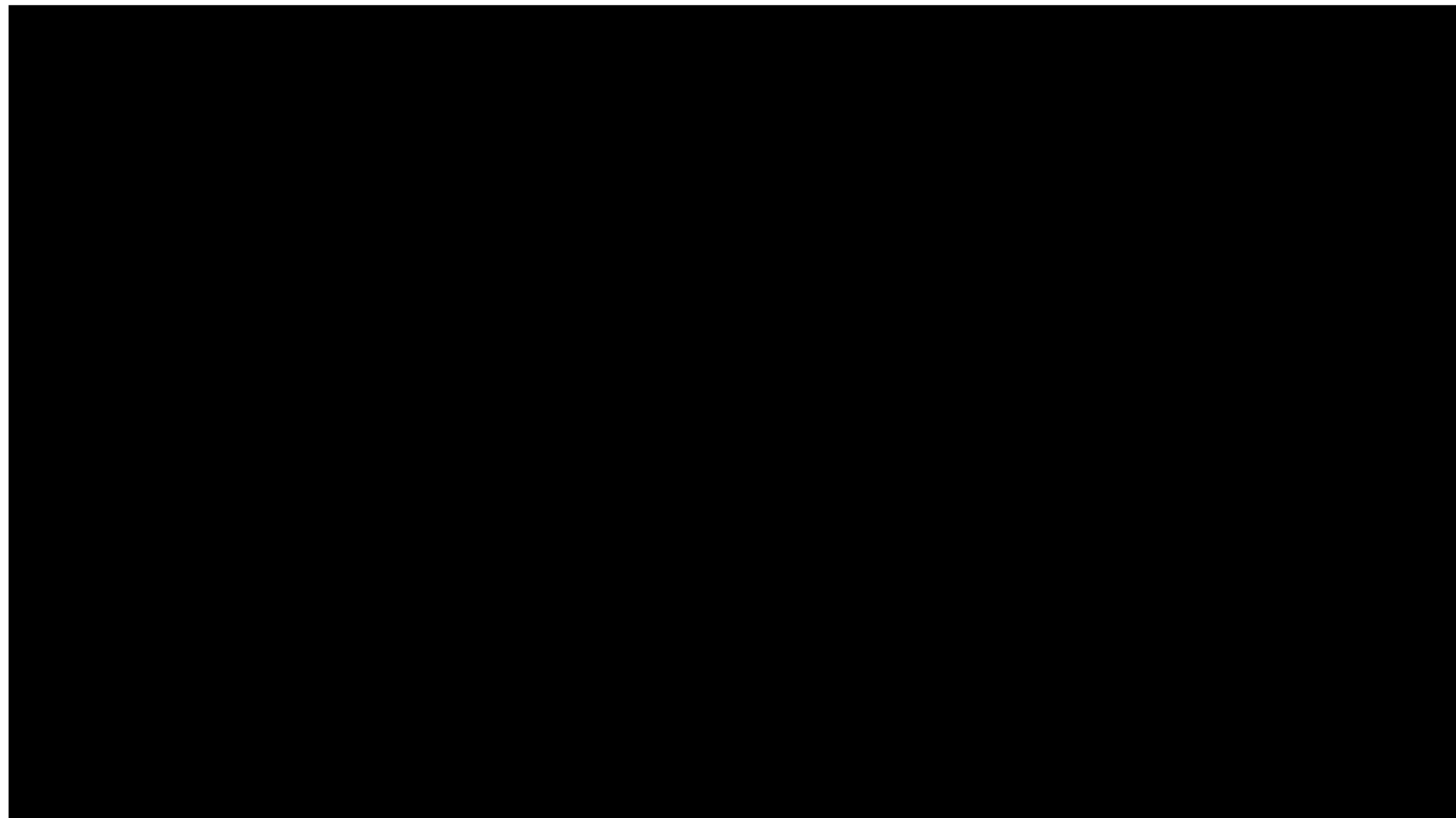
$$\ddot{x} = f'$$

unit-mass controller

$k = b = 0$
$m = 1$

# Critical Damping



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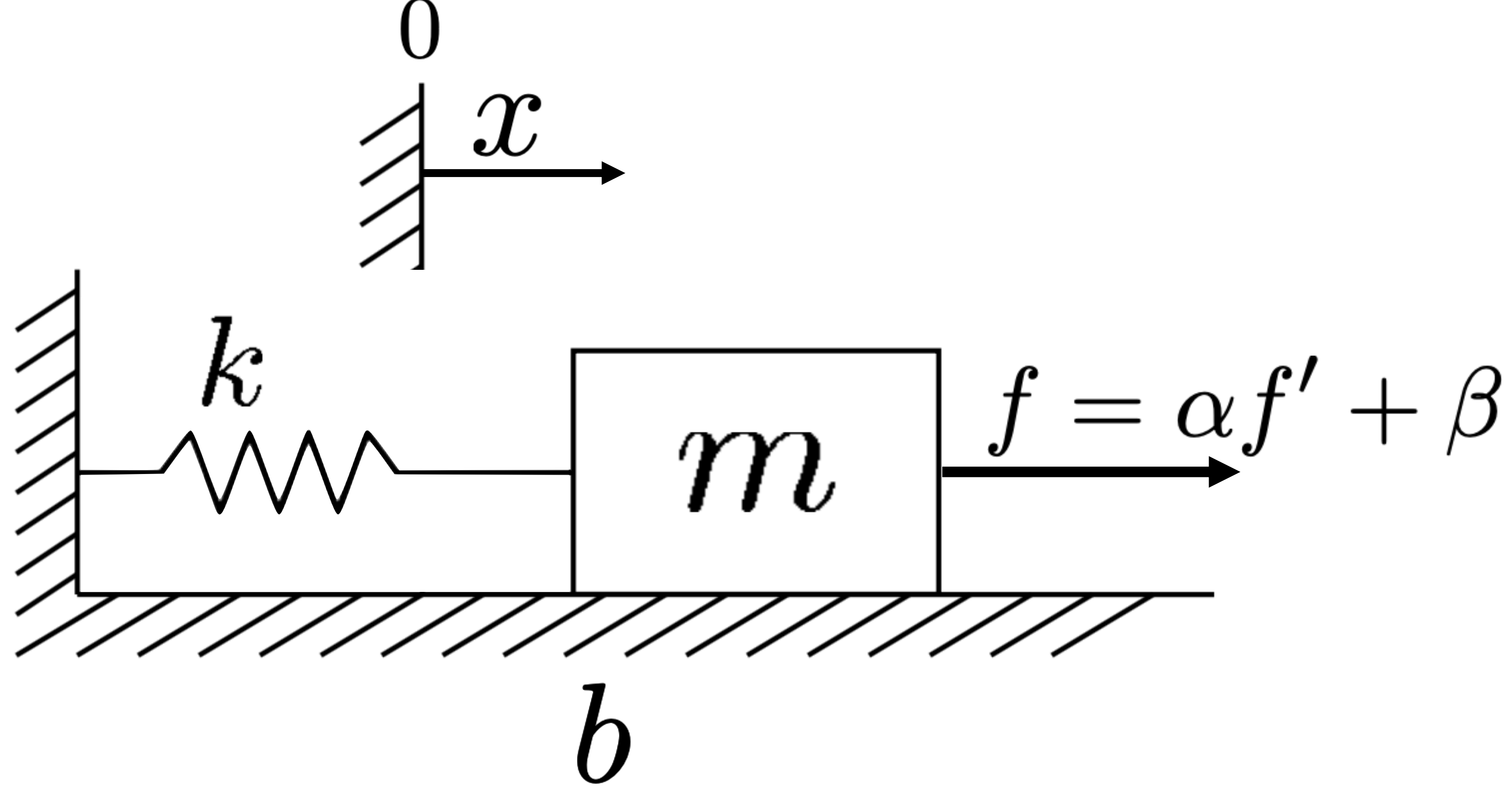


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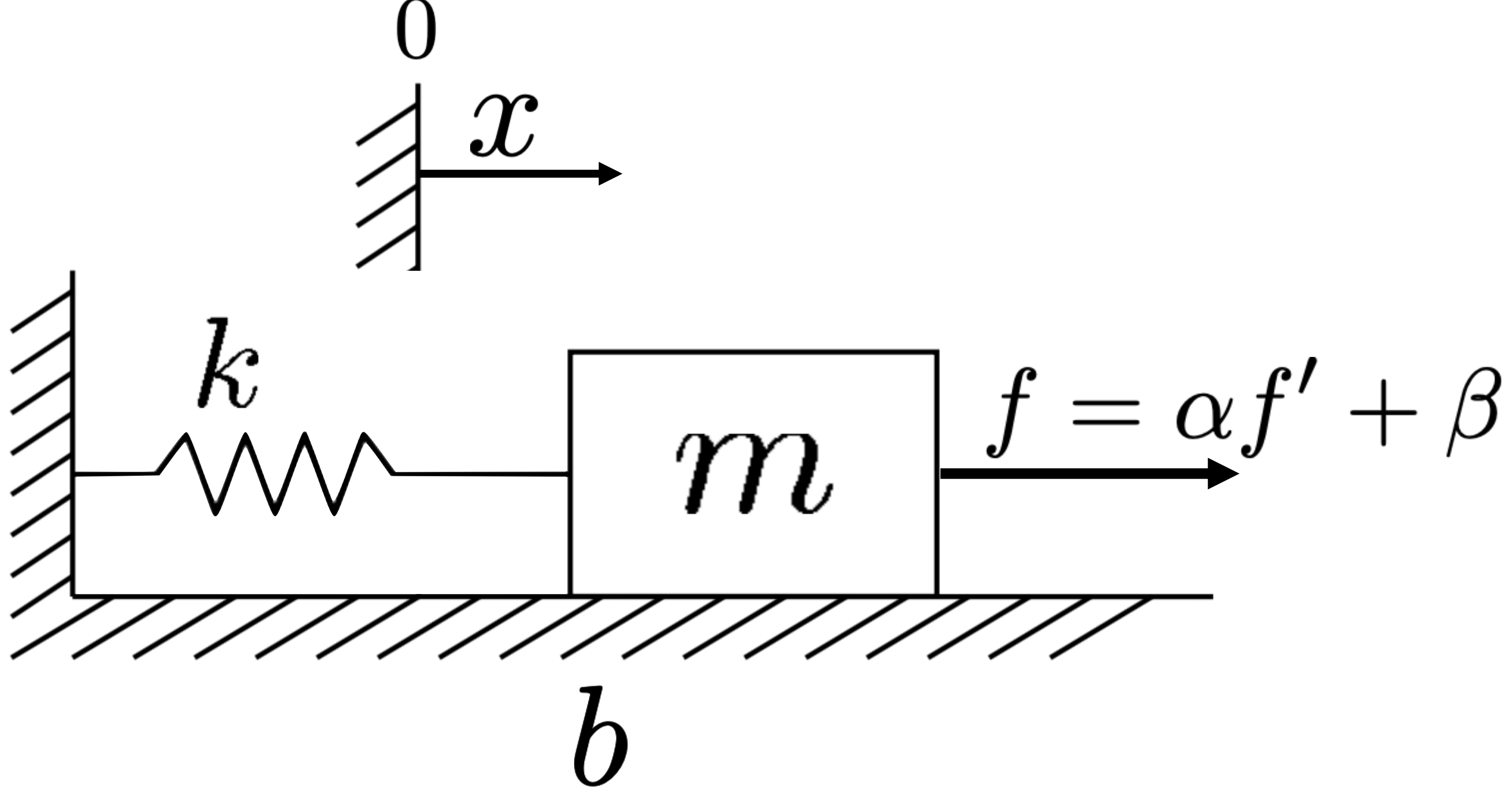


$$f' := -k_v \dot{x} - k_p x$$

$$m\ddot{x} = -kx - b\dot{x} + [mf' + b\dot{x} + kx]$$

$$m\ddot{x} = -kx - b\dot{x} + [m < -k_v \dot{x} - k_p x > + b\dot{x} + kx]$$

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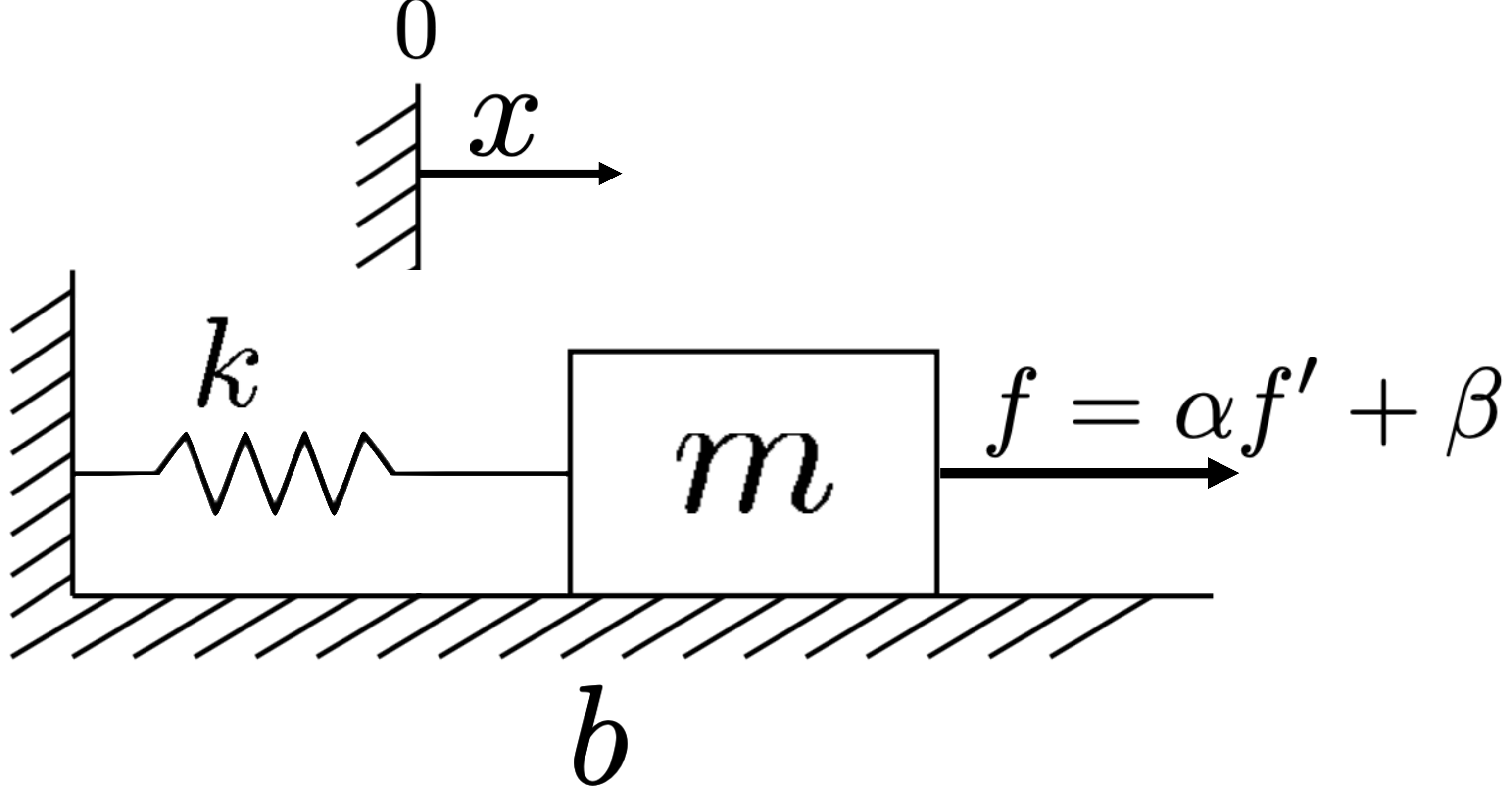


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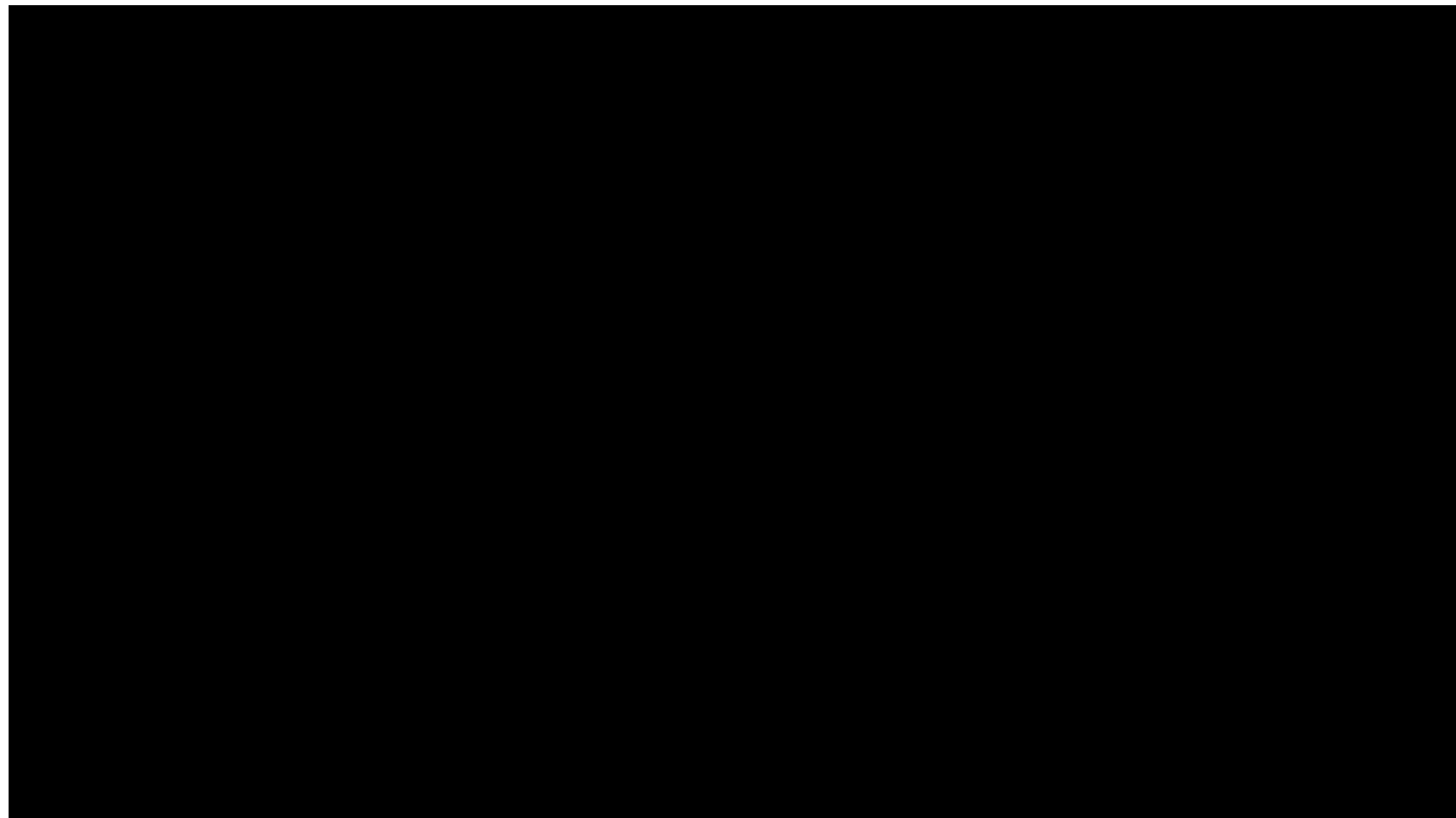

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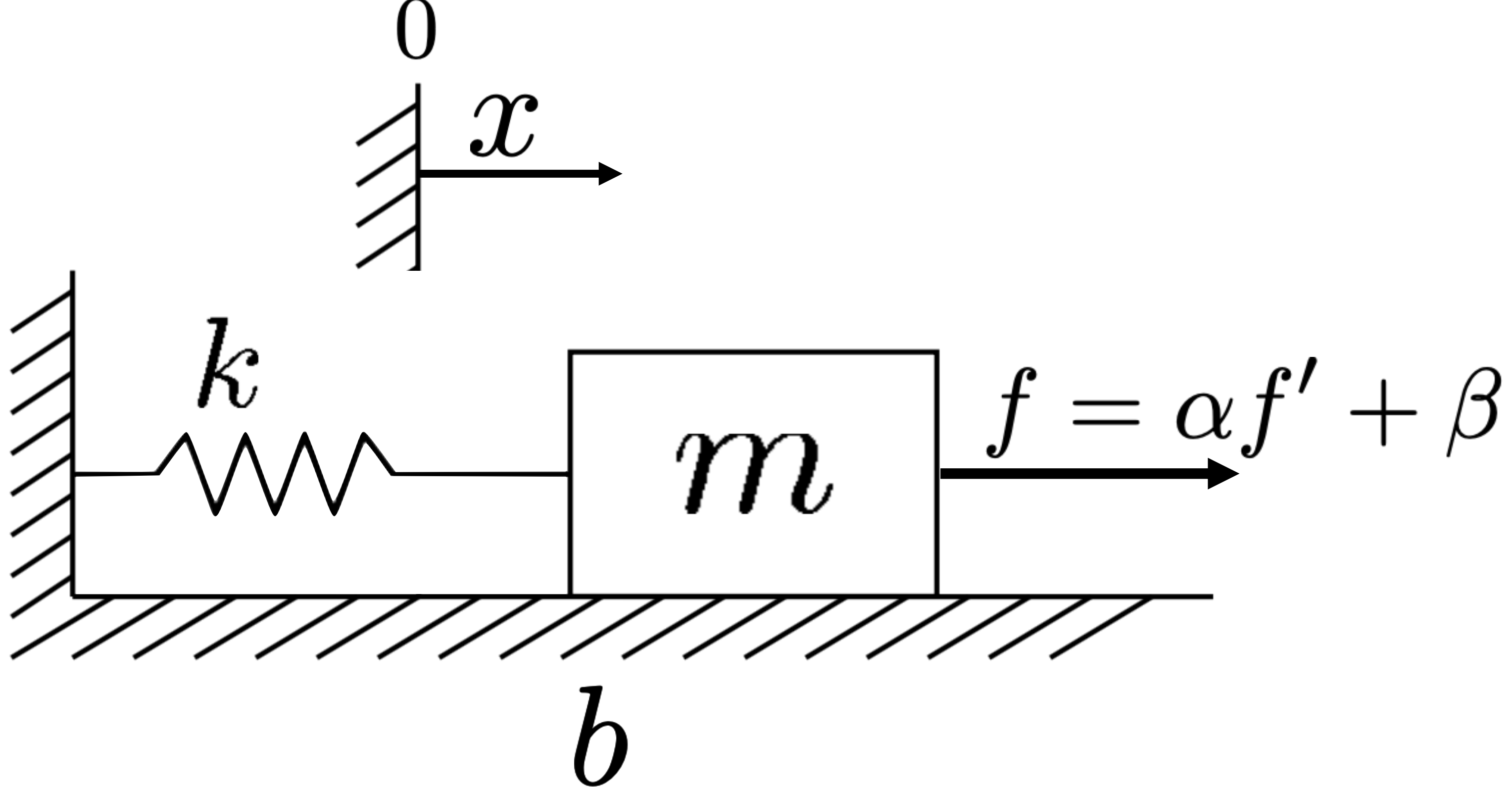


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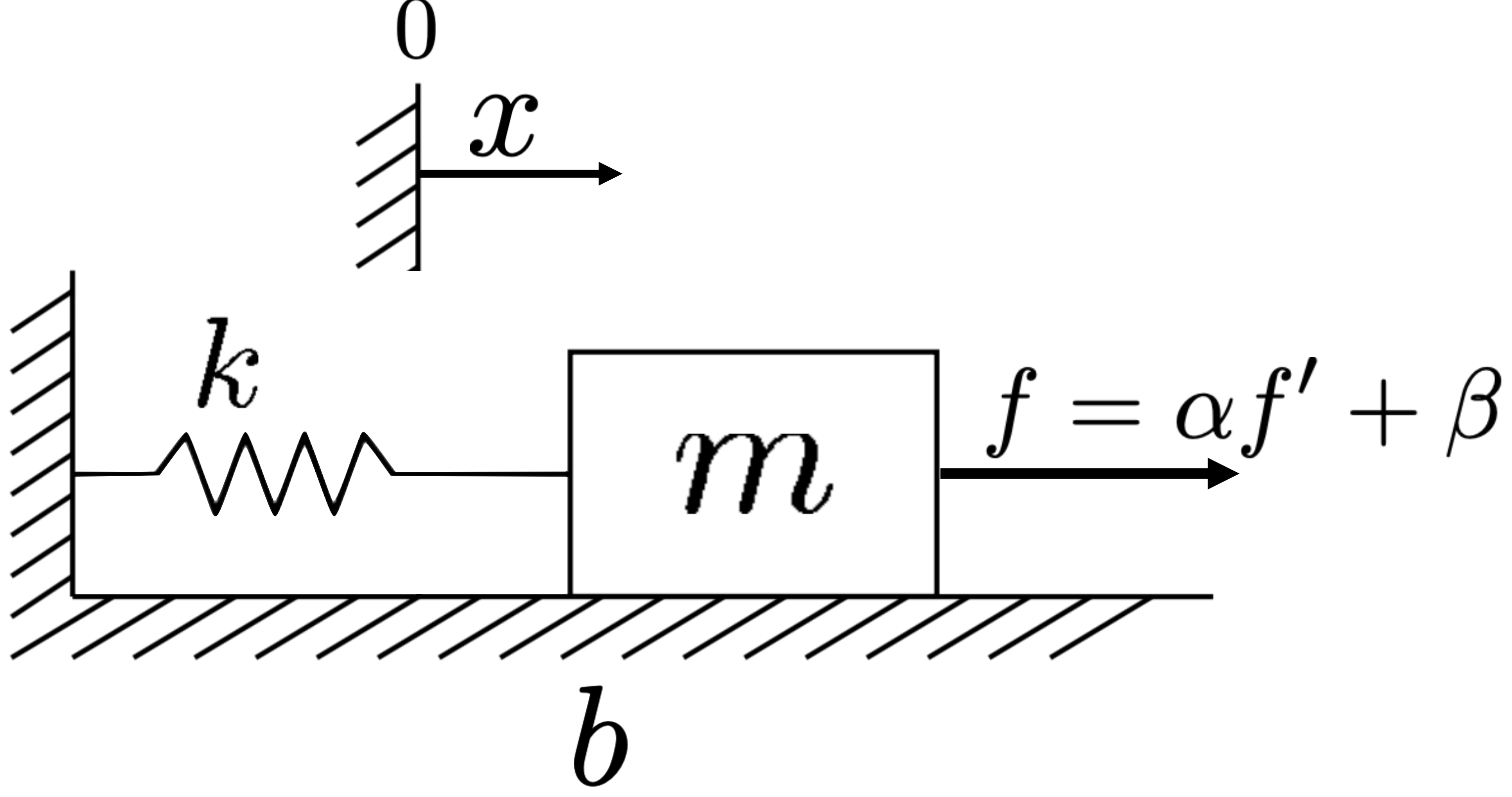
simplify

$$\ddot{x} = -k_v\dot{x} - k_p x$$

reformulate

$$\ddot{x} + k_v\dot{x} + k_p x = 0$$

# Critical Damping

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simplify

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reformulate

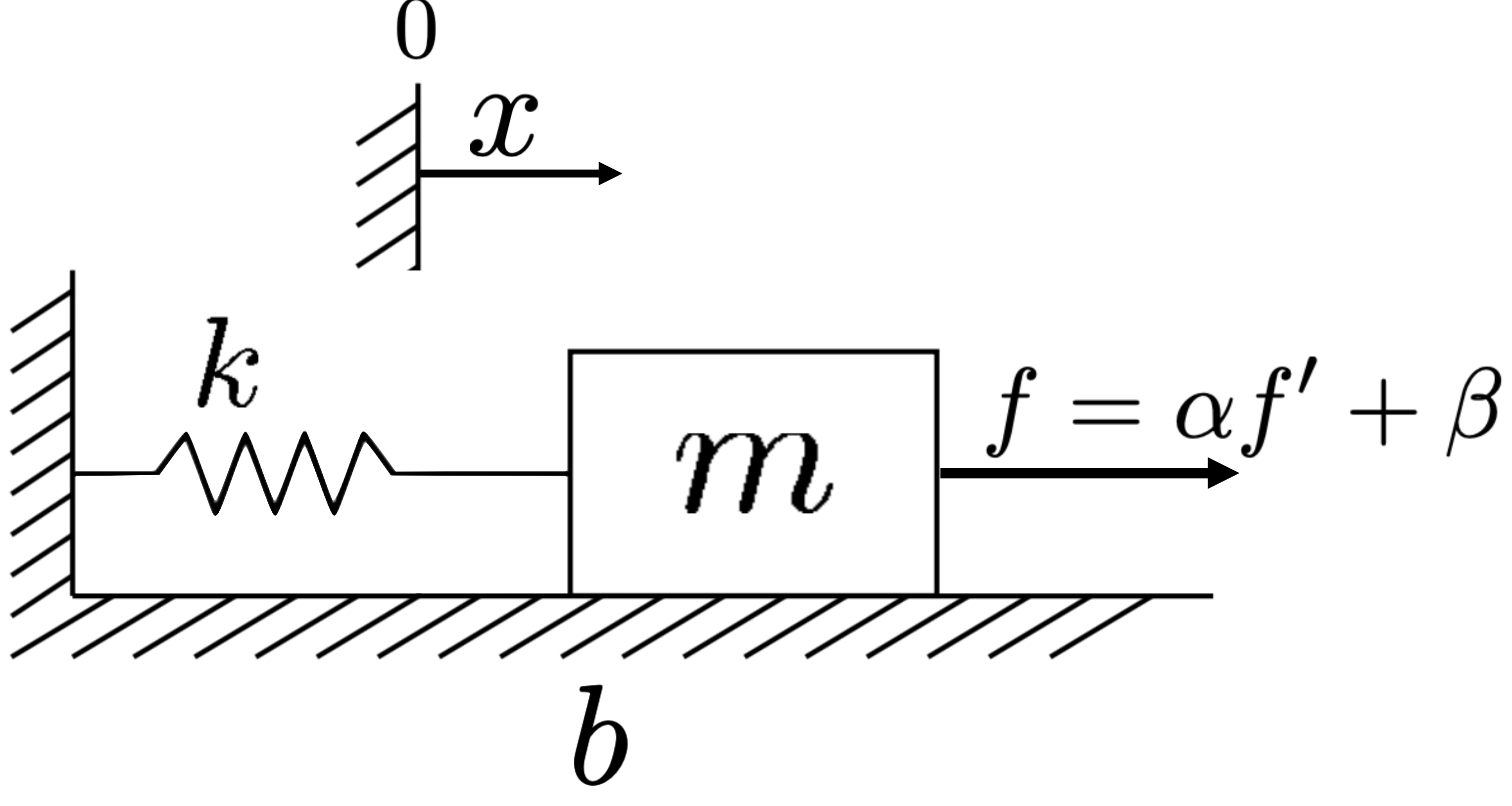
$$\ddot{x} + k_v \dot{x} + k_p x = 0$$

Remember:

For: $m\ddot{x} + b\dot{x} + kx = 0$

Critical damping: $b = 2\sqrt{mk}$

# Critical Damping

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$$m\ddot{x} = -kx - b\dot{x} + [m < -k_v \dot{x} - k_p x > +b\dot{x} + kx]$$

simplify

$$\ddot{x} = -k_v \dot{x} - k_p x$$

reformulate

$$\ddot{x} + k_v \dot{x} + k_p x = 0$$

Remember:

For: $m\ddot{x} + b\dot{x} + kx = 0$

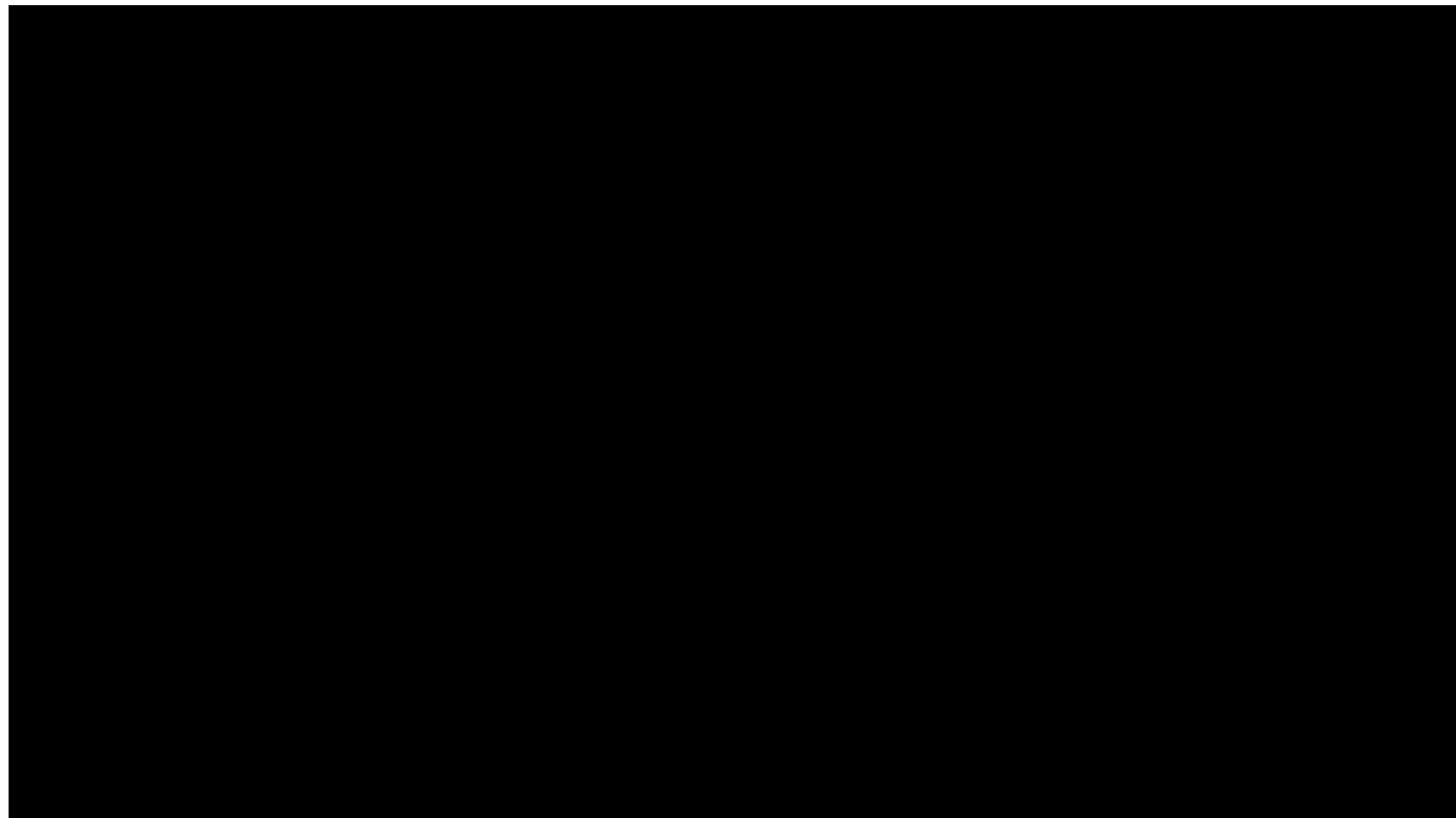
Critical damping: $b = 2\sqrt{mk}$

Outcome:

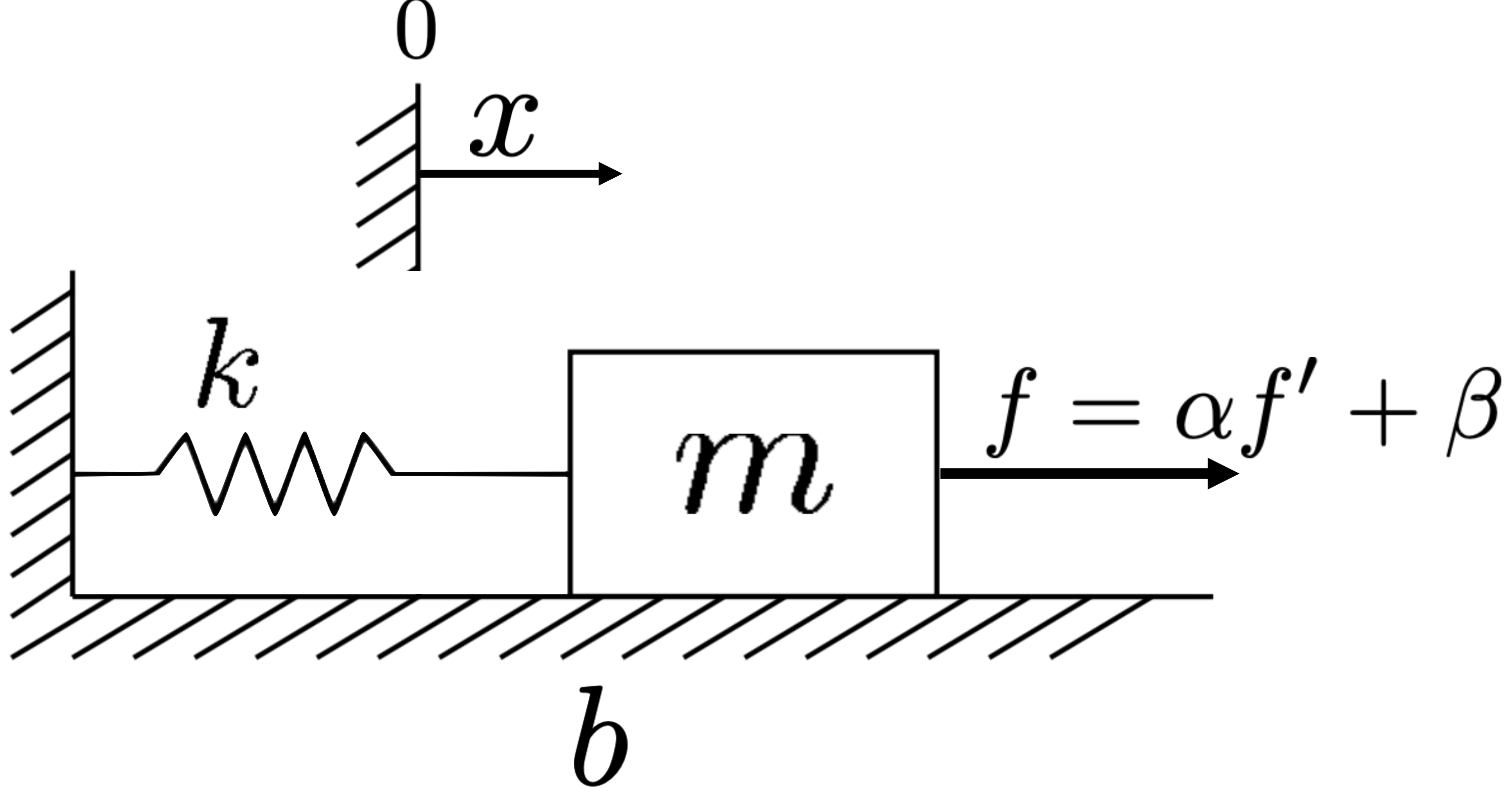
Critical damping:

$$k_v = 2\sqrt{k_p}$$

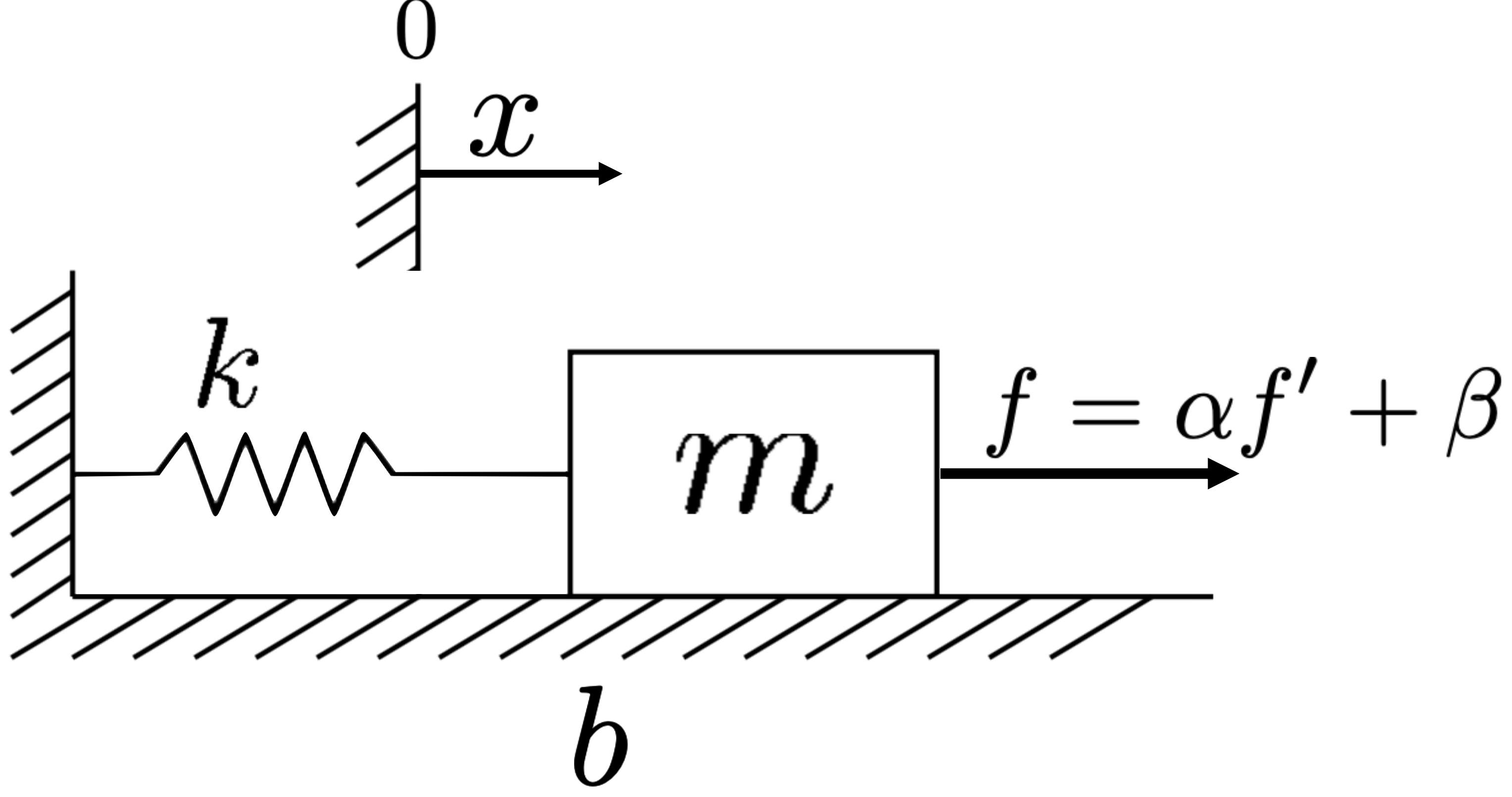
Independent of the physical system!



# Closed-loop stiffness

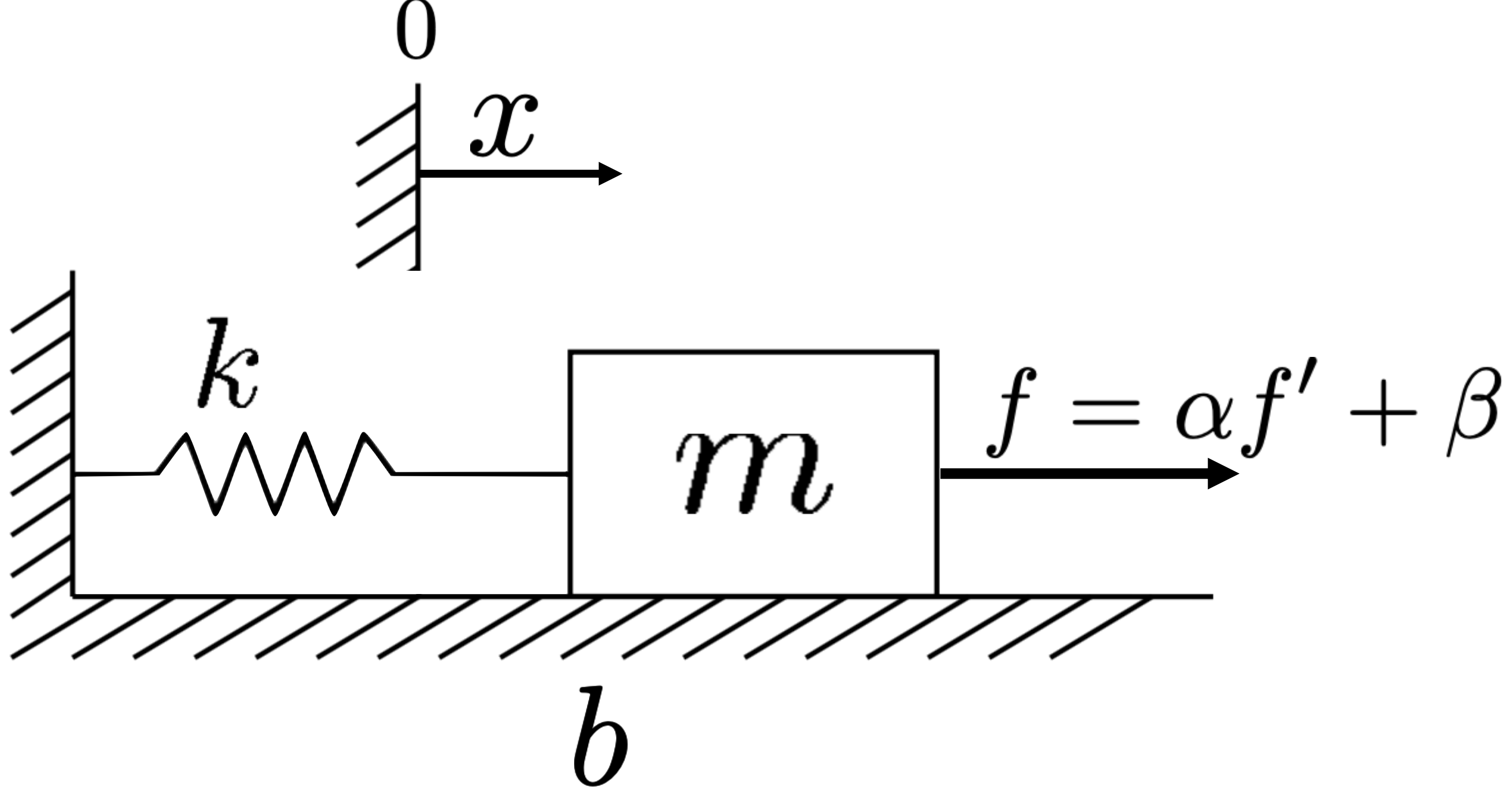


# Closed-loop stiffness




$$m\ddot{x} = -kx - b\dot{x} + [m < -k_v\dot{x} - k_p x > +b\dot{x} + kx]$$

# Closed-loop stiffness

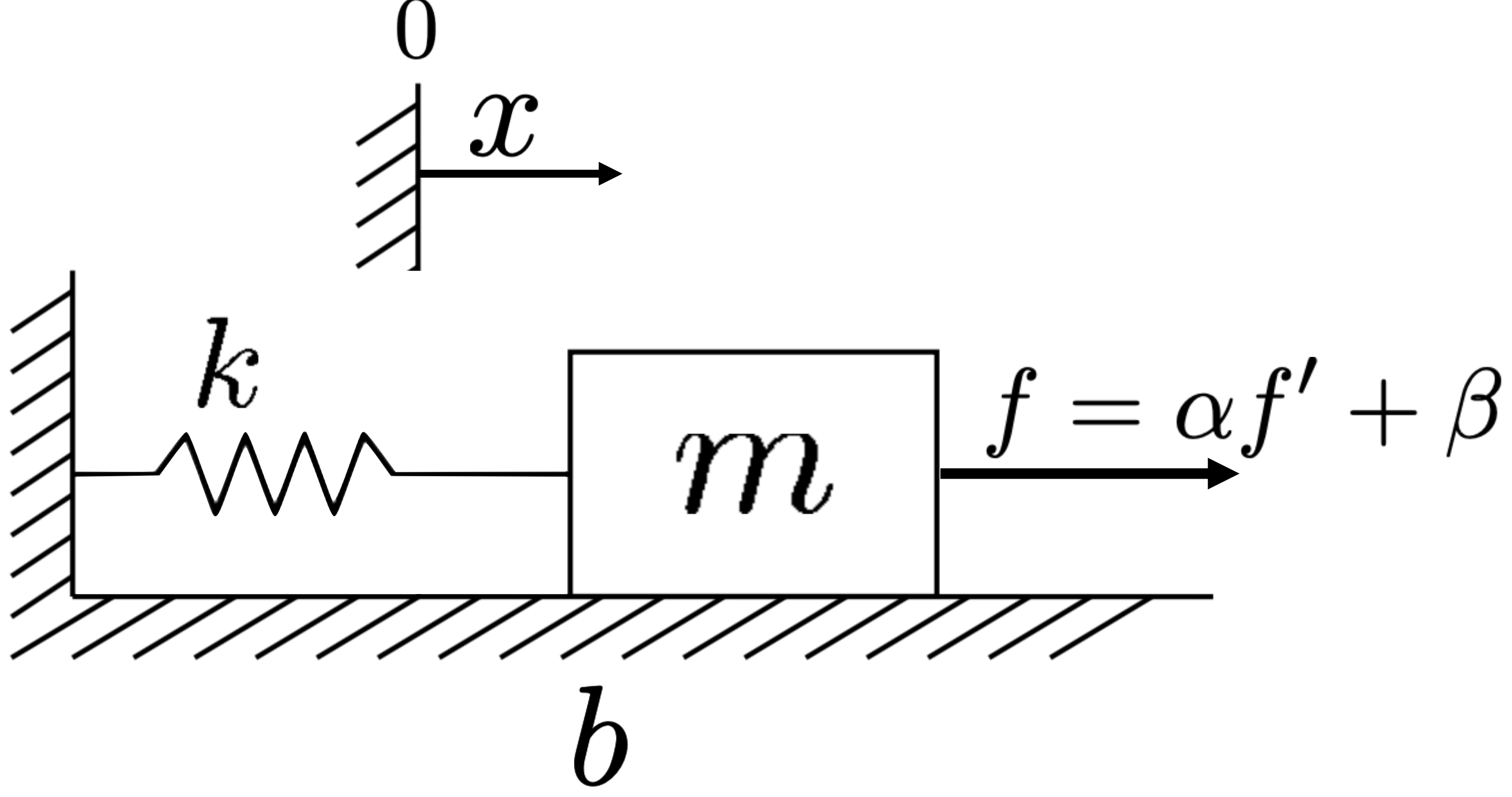



$$m\ddot{x} = -kx - b\dot{x} + [m < -k_v\dot{x} - k_p x > + b\dot{x} + kx]$$

reformulate

$$m\ddot{x} = -(b + mk_v - b)\dot{x} - (k + mk_p - k)x$$

# Closed-loop stiffness




$$m\ddot{x} = -kx - b\dot{x} + [m < -k_v\dot{x} - k_p x > + b\dot{x} + kx]$$

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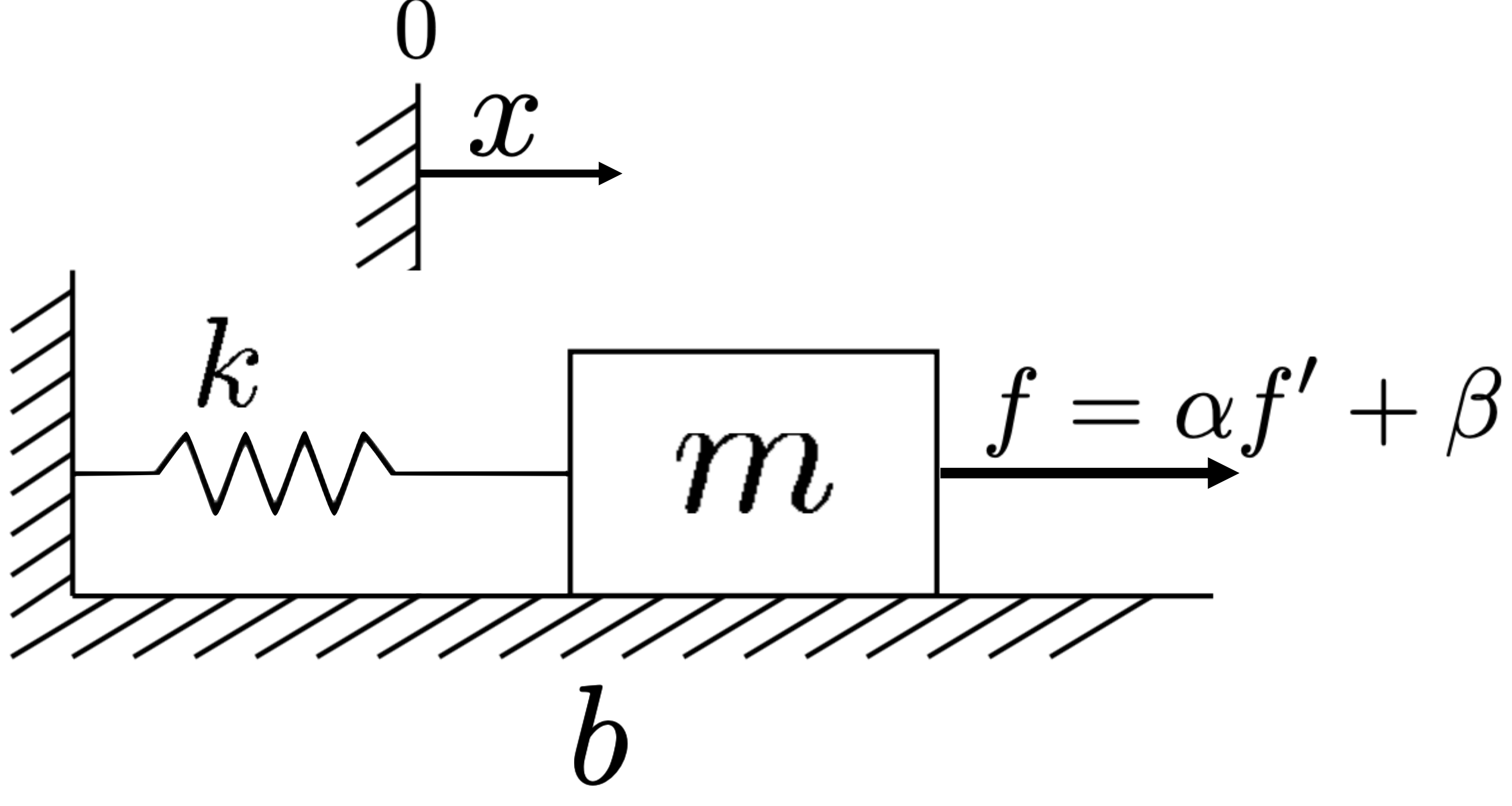
$$m\ddot{x} = -(b + mk_v - b)\dot{x} - (k + mk_p - k)x$$

simplify

$$m\ddot{x} = -\underbrace{mk_v}_{b'}\dot{x} - \underbrace{mk_p}_{k'}x$$

# Closed-loop stiffness




reformulate

simplify

$$m\ddot{x} = -kx - b\dot{x} + [m < -k_v\dot{x} - k_p x > + b\dot{x} + kx]$$

$$m\ddot{x} = -(b + mk_v - b)\dot{x} - (k + mk_p - k)x$$

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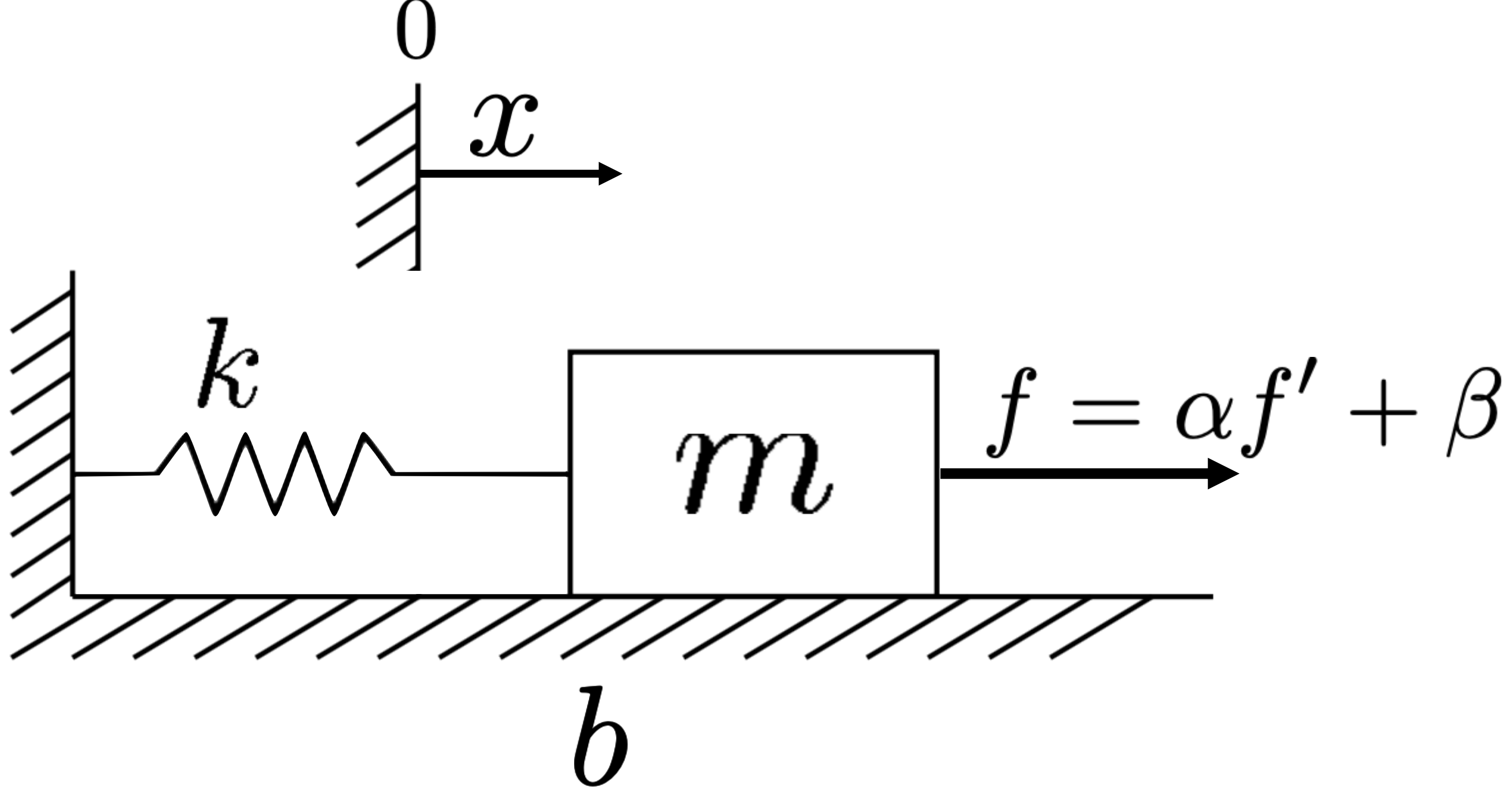
Remember:

$$m\ddot{x} = -\underbrace{(b + k_v)}_{b'}\dot{x} - \underbrace{(k + k_p)}_{k'}x$$

# Closed-loop stiffness




$$m\ddot{x} = -kx - b\dot{x} + [m < -k_v\dot{x} - k_p x > + b\dot{x} + kx]$$

reformulate

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simplify

$$m\ddot{x} = -\underbrace{mk_v}_{b'}\dot{x} - \underbrace{mk_p}_{k'}x$$

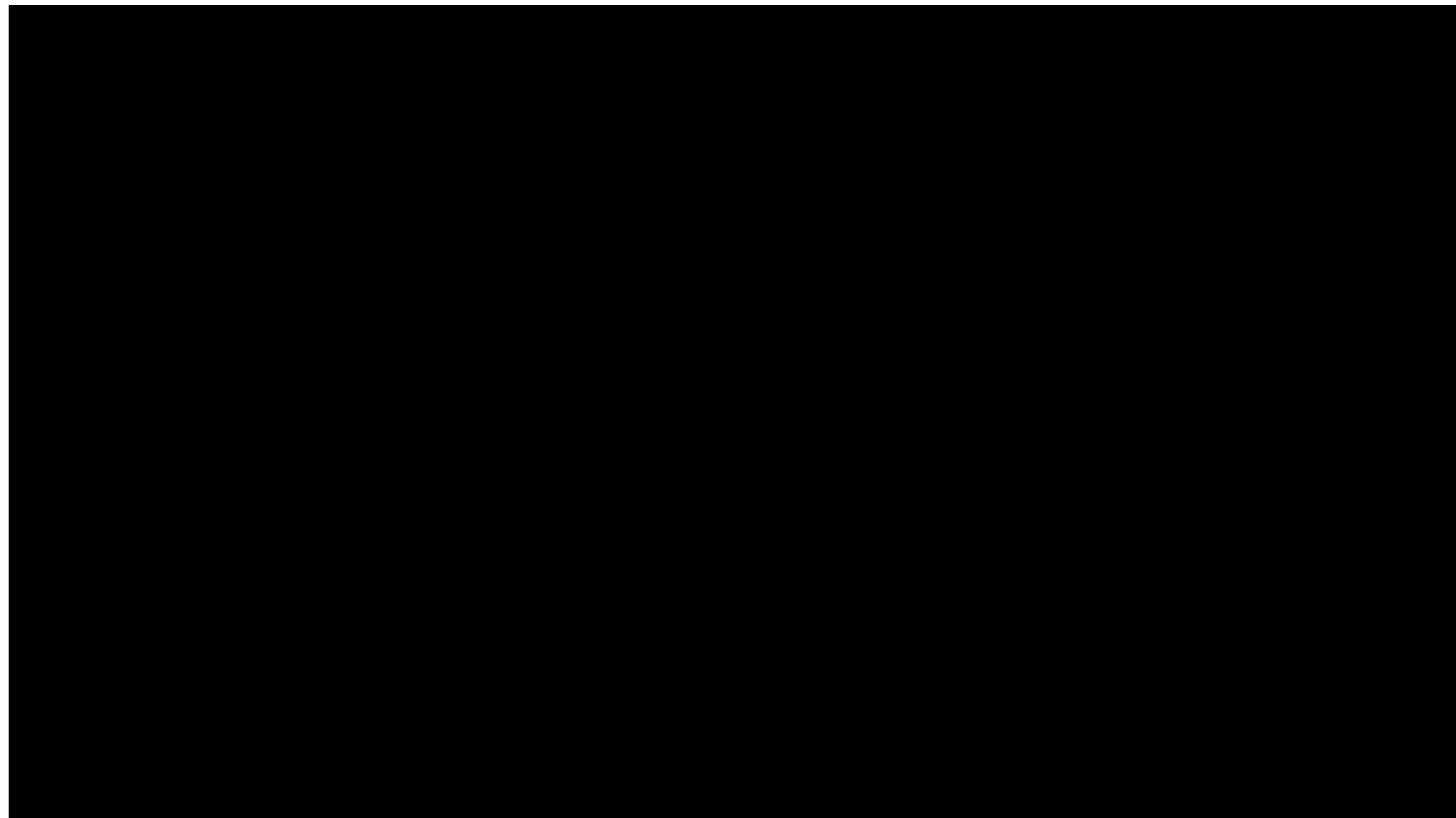
Remember:

$$m\ddot{x} = -\underbrace{(b + k_v)}_{b'}\dot{x} - \underbrace{(k + k_p)}_{k'}x$$

Outcome:

$$b' = mk_v$$
$$k' = mk_p$$

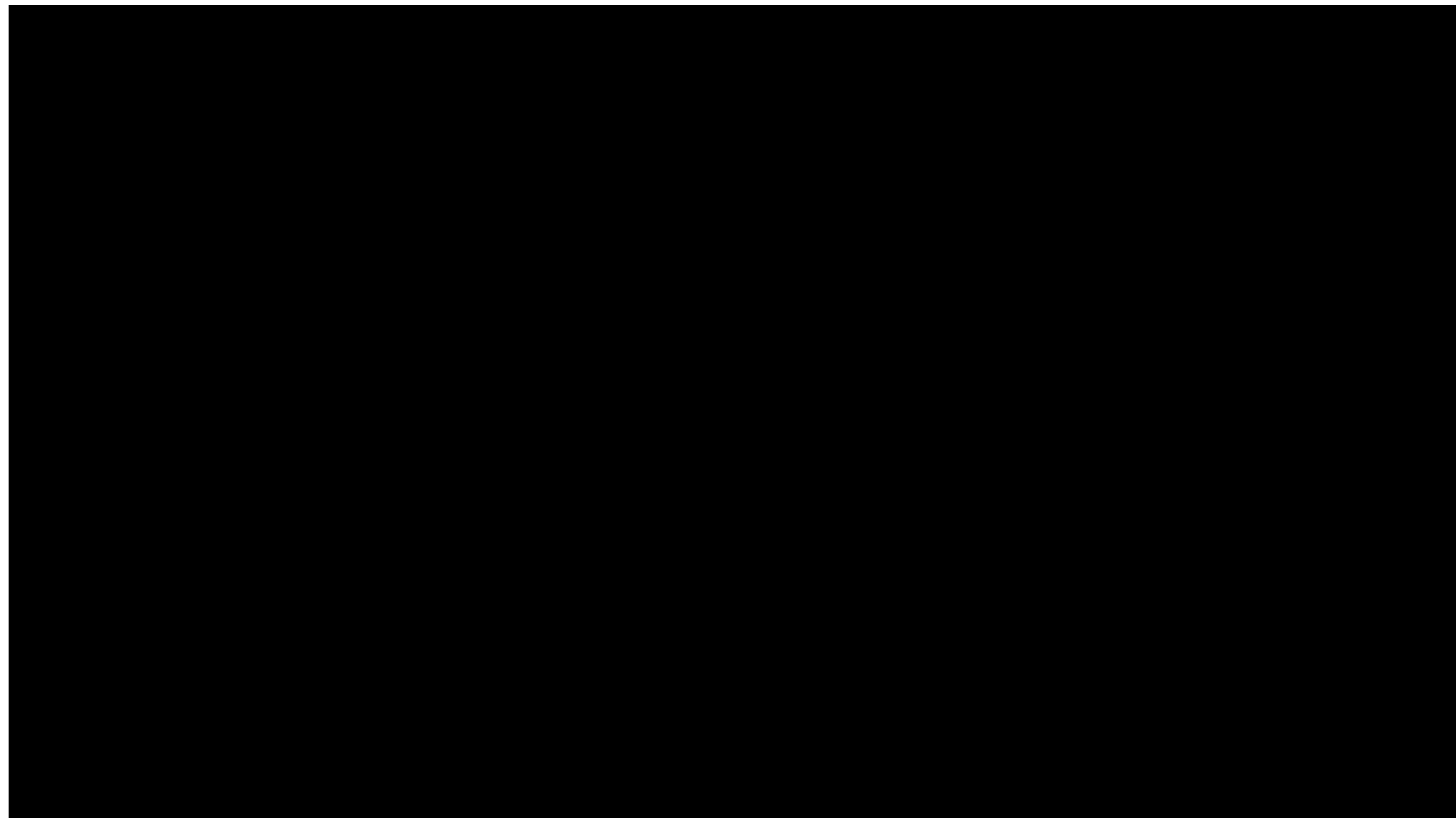
# Trajectory-Following Control



# Trajectory-Following Control

Trajectory:

$$[x_d(t), \dot{x}_d(t), \ddot{x}_d(t)]$$



# Trajectory-Following Control

Trajectory:

$$[x_d(t), \dot{x}_d(t), \ddot{x}_d(t)]$$

Error:

$$e = x_d - x$$
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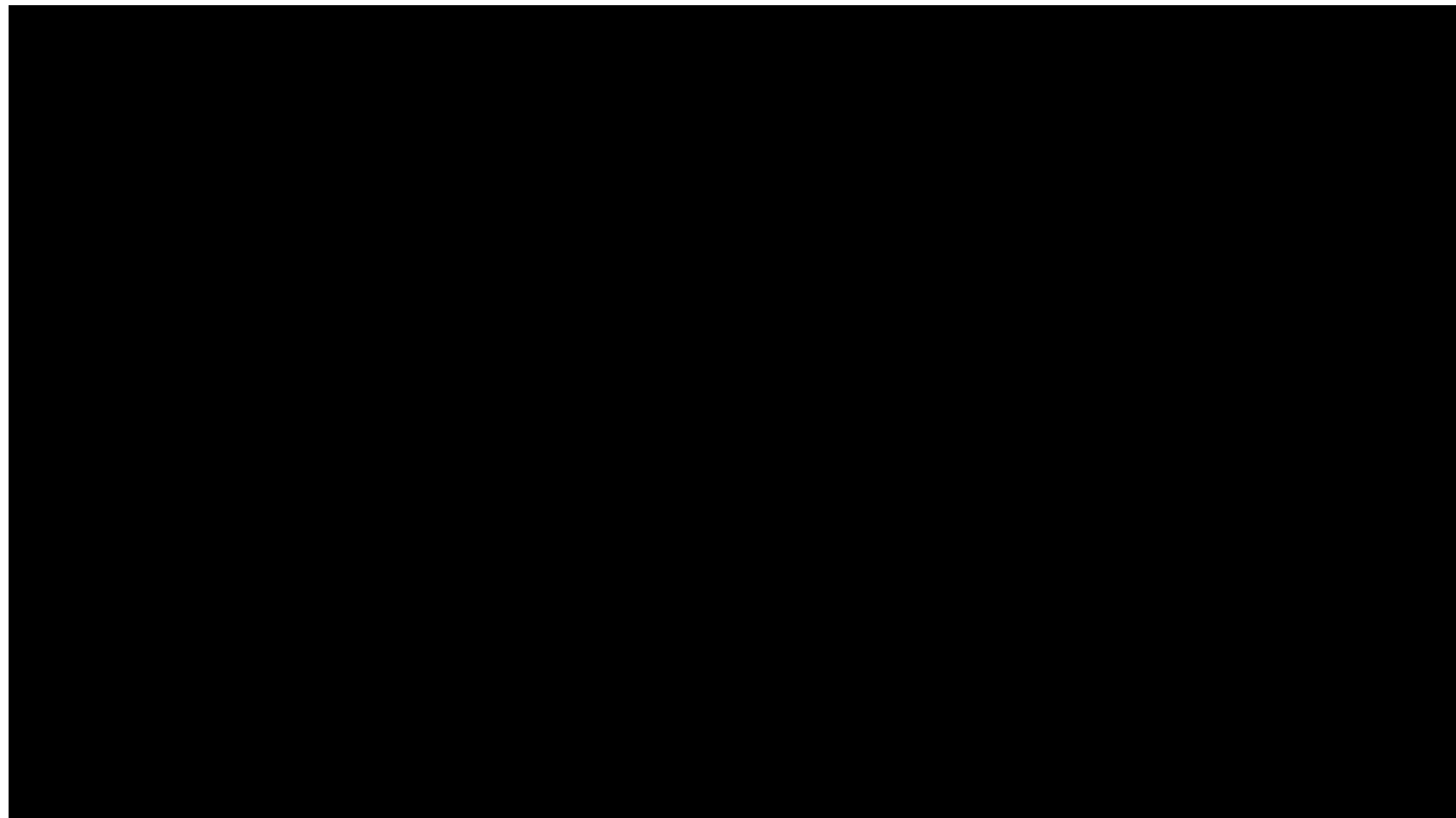

# Trajectory-Following Control

Servo-control law:

$$f' = \ddot{x}_d + k_v \dot{e} + k_p e$$

Trajectory:

$$[x_d(t), \dot{x}_d(t), \ddot{x}_d(t)]$$

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$$e = x_d - x$$

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# Trajectory-Following Control

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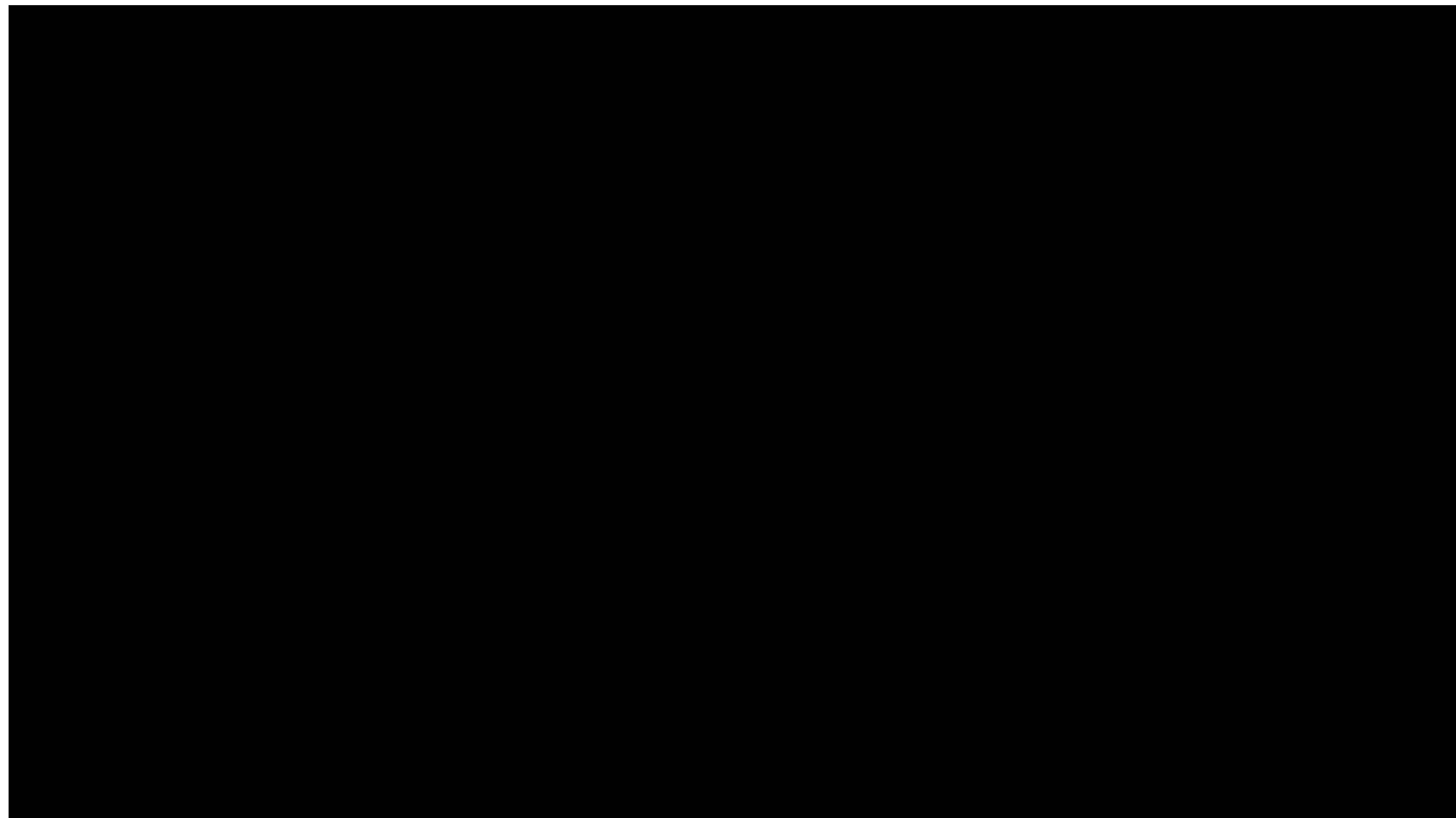
Remember:

$$f' := -k_v \dot{x} - k_p x$$



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Servo-control law:

$$f' = \ddot{x}_d + k_v \dot{e} + k_p e$$

Remember:

$$f' := -k_v \dot{x} - k_p x$$

reformulate

$$f = k_v (0 - \dot{x}) + k_p (0 - x)$$

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Trajectory:

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Remember:

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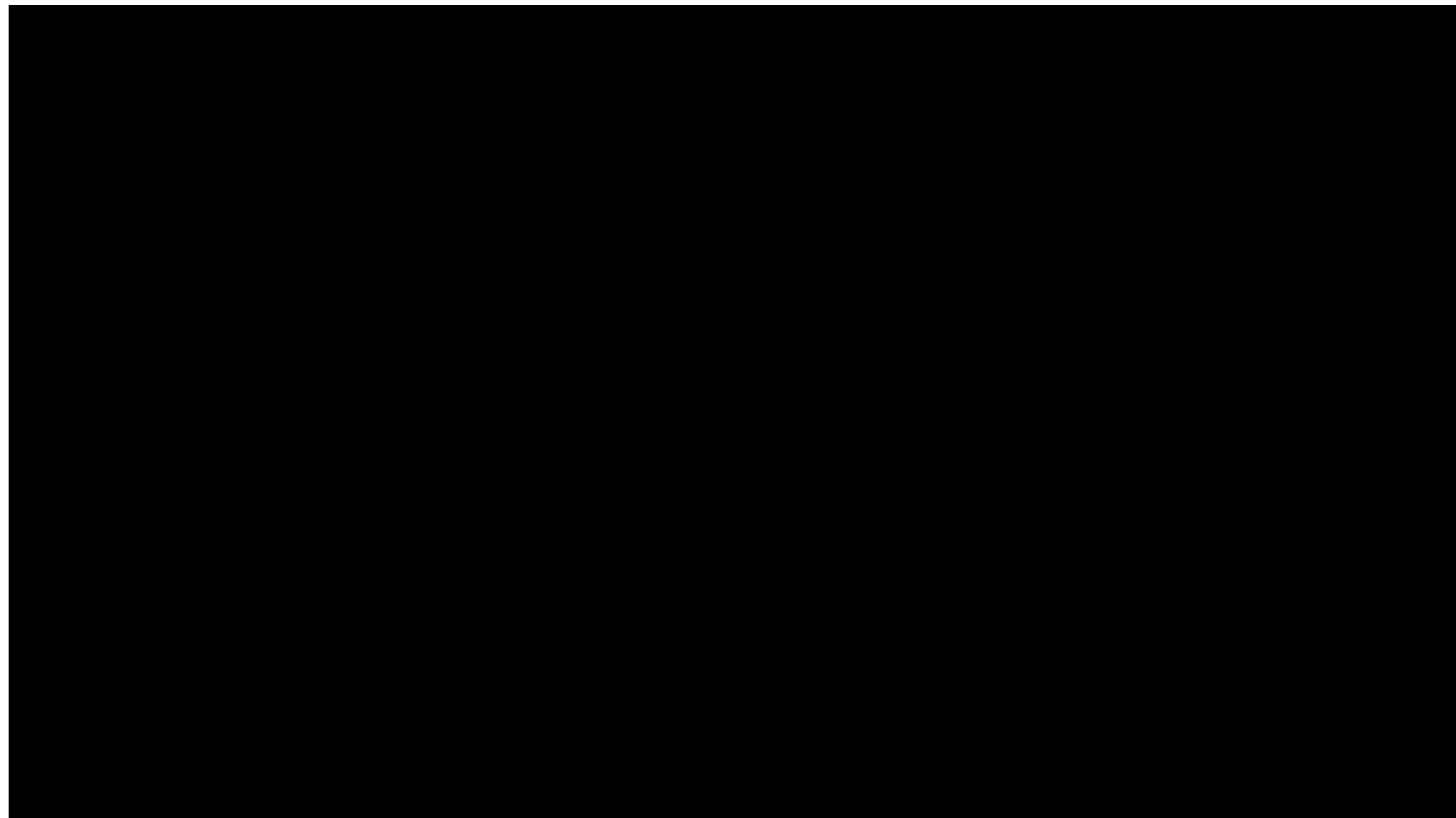
reformulate

$$f = k_v(\underset{\dot{x}_d}{0} - \dot{x}) + k_p(\underset{x_d}{0} - x)$$



# Trajectory-Following Control

Servo-control law:

$$f' = \ddot{x}_d + k_v \dot{e} + k_p e$$

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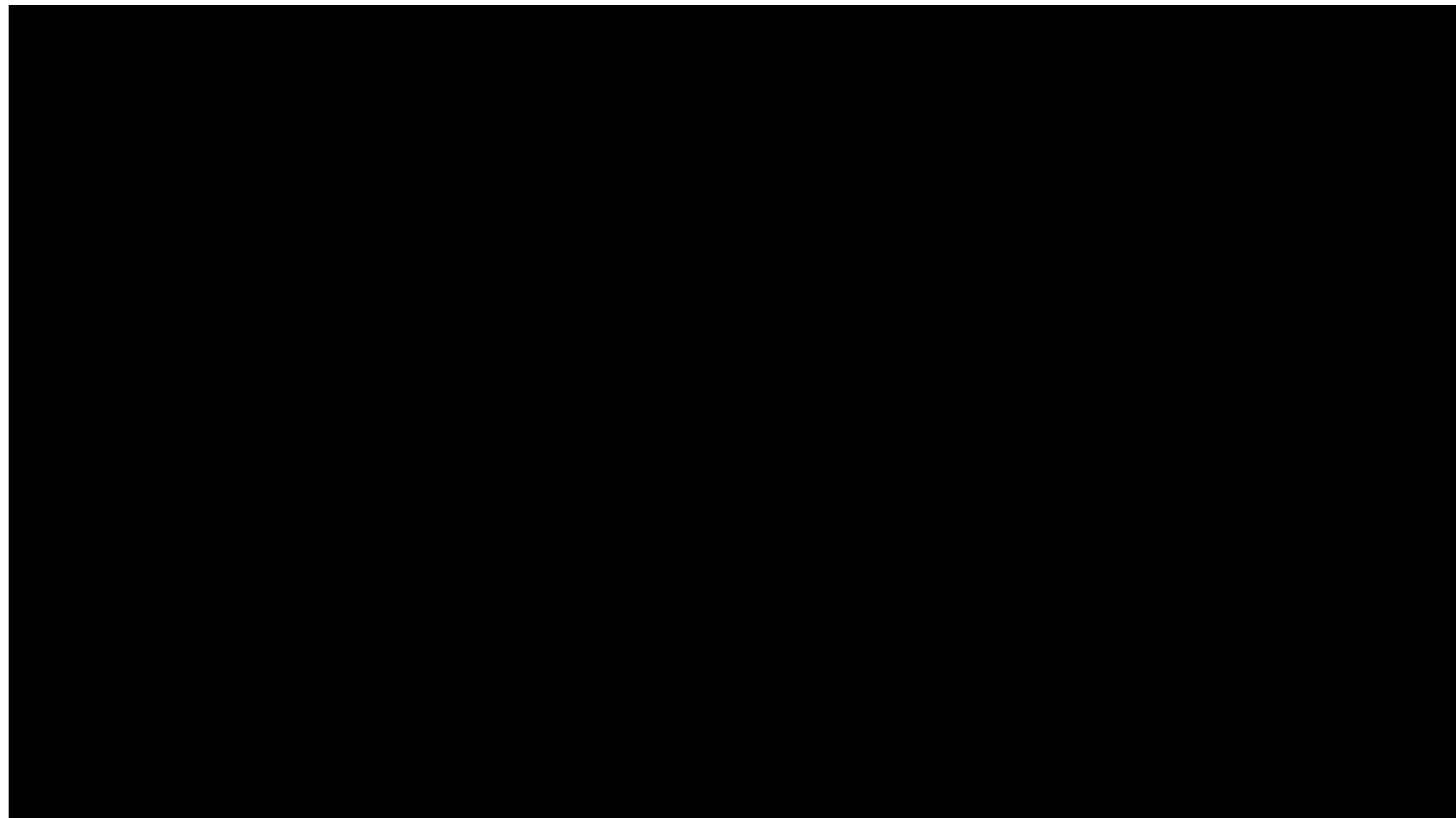

# Trajectory-Following Control

Servo-control law:

$$f' = \ddot{x}_d + k_v \dot{e} + k_p e$$

Control-law partitioning

$$m\ddot{x} = -kx - b\dot{x} + [f]$$

$$m\ddot{x} = -kx - b\dot{x} + [\alpha f' + \beta]$$

Trajectory:

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Insert definitions

$$m\ddot{x} = -kx - b\dot{x} + [m < \ddot{x}_d + k_v \dot{e} + k_p e > + b\dot{x} + kx]$$

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$$m\ddot{x} = m\ddot{x}_d + mk_v \dot{e} + mk_p e$$

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simplify

$$m\ddot{x} = m\ddot{x}_d + mk_v \dot{e} + mk_p e$$

$\ddot{e} = \ddot{x}_d - \ddot{x}$

$$\ddot{e} + k_v \dot{e} + k_p e = 0$$

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Remember:

For: $\ddot{x} + k_v \dot{x} + k_p x = 0$

Critical damping: $k_v = 2\sqrt{k_p}$

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$$m\ddot{x} = -kx - b\dot{x} + [m < \ddot{x}_d + k_v\dot{e} + k_p e > + b\dot{x} + kx]$$

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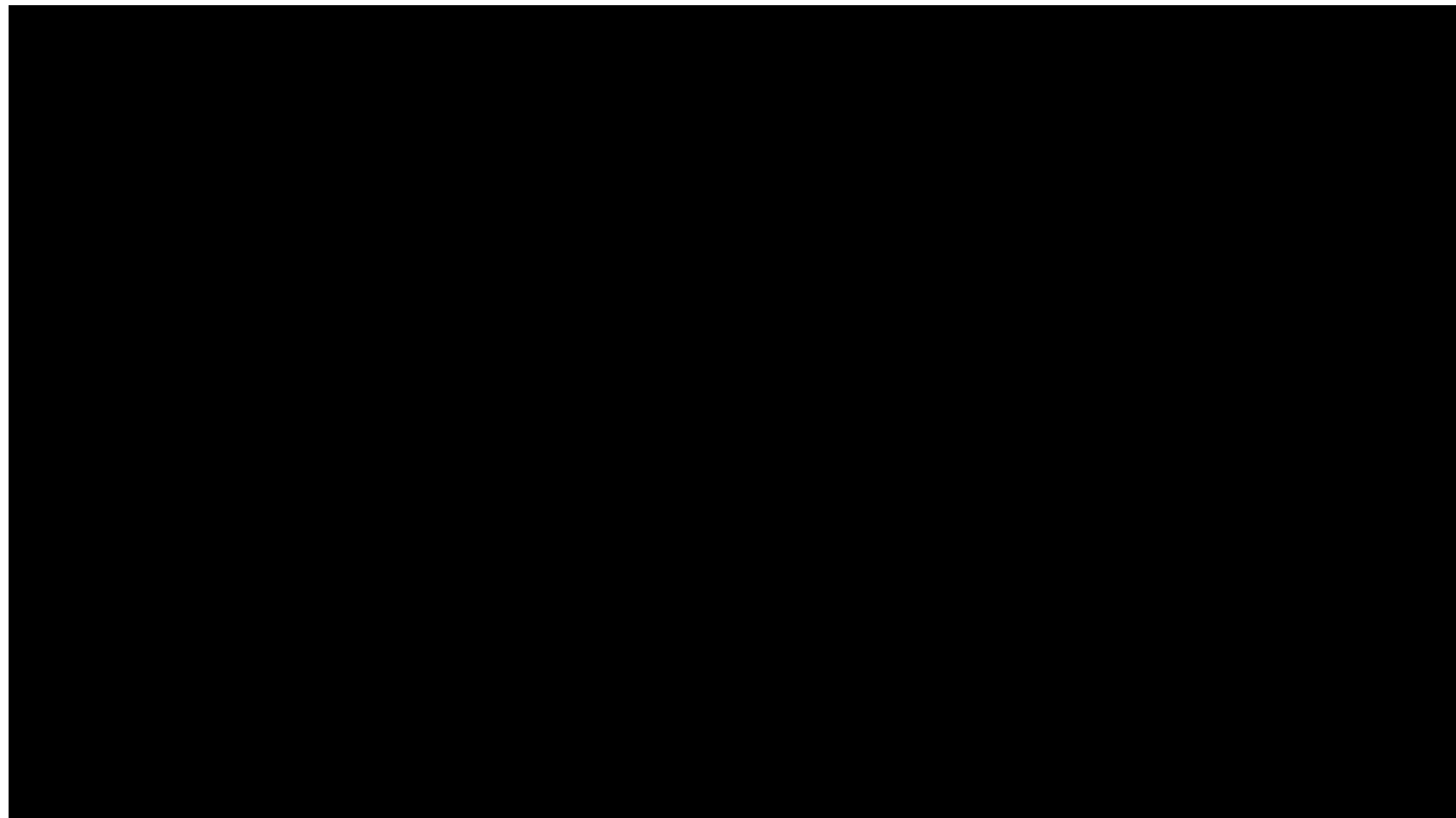

Remember:

For: $\ddot{x} + k_v\dot{x} + k_p x = 0$

Critical damping: $k_v = 2\sqrt{k_p}$

Outcome:

Critical damping:

$$k_v = 2\sqrt{k_p}$$

# Disturbance Rejection

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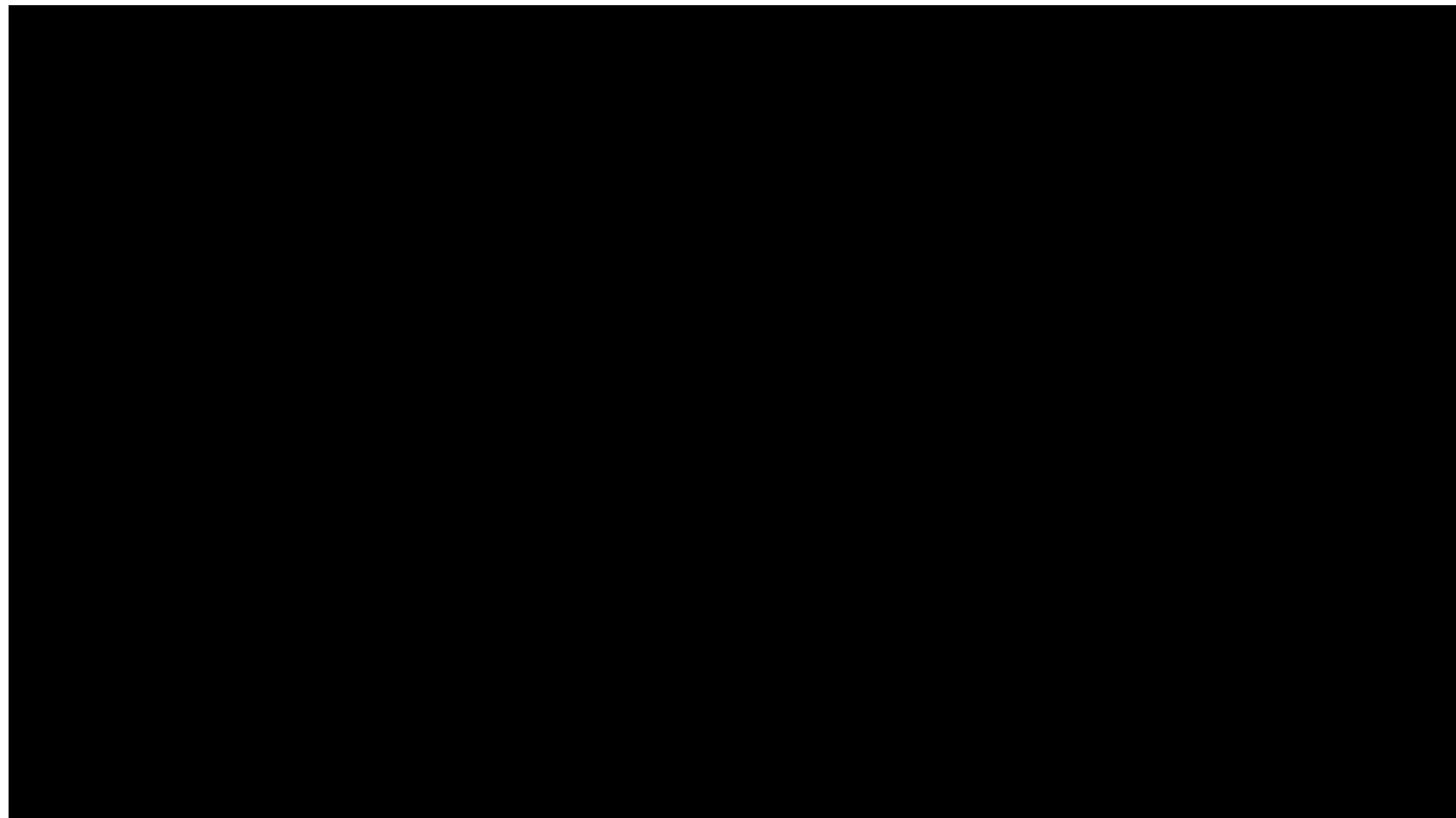
Error equation:

$$f_{\text{dist}} = \ddot{e} + k_v \dot{e} + k_p e$$



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Steady-state error:

$$\ddot{e} = \dot{e} = 0$$

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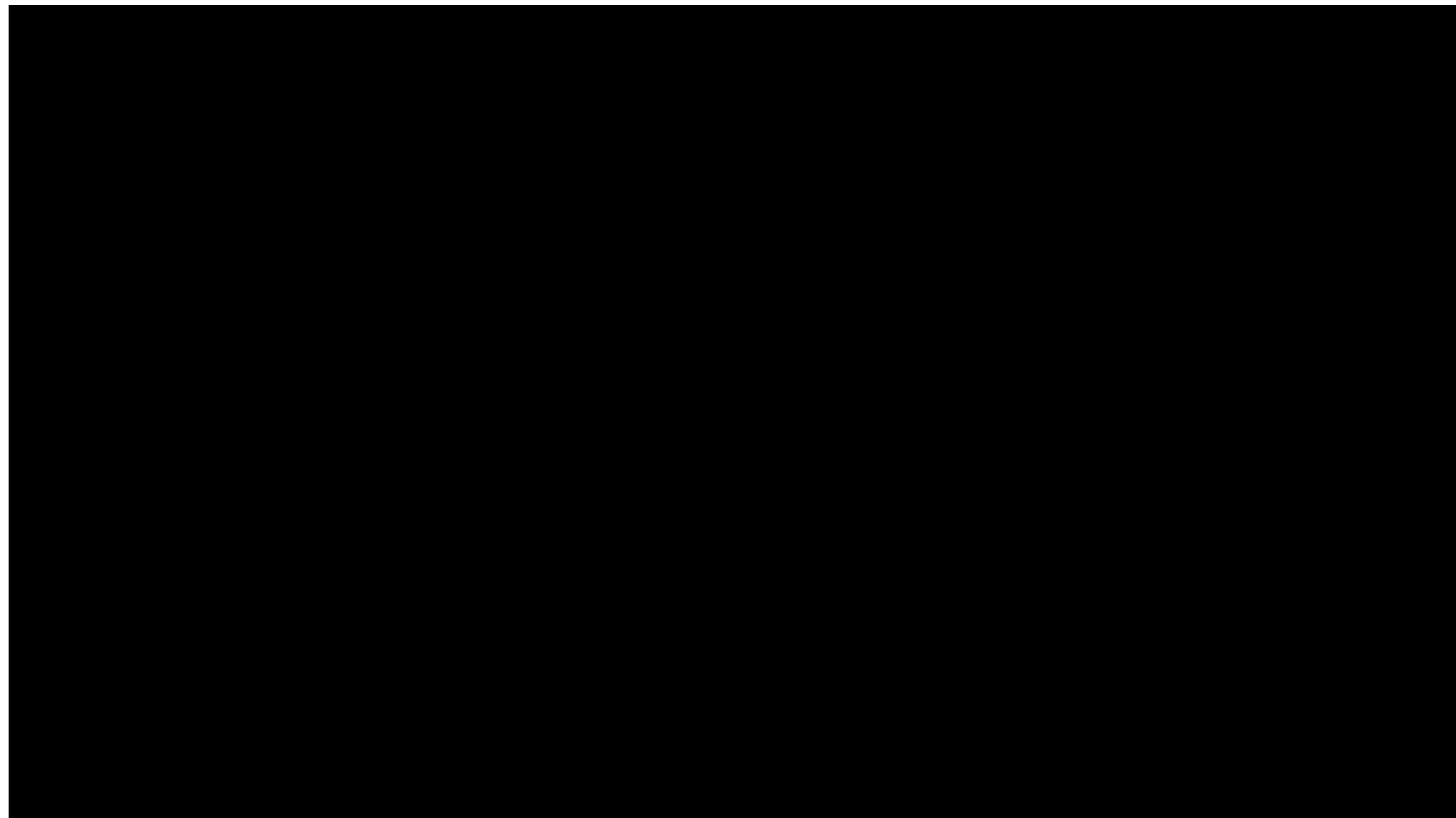

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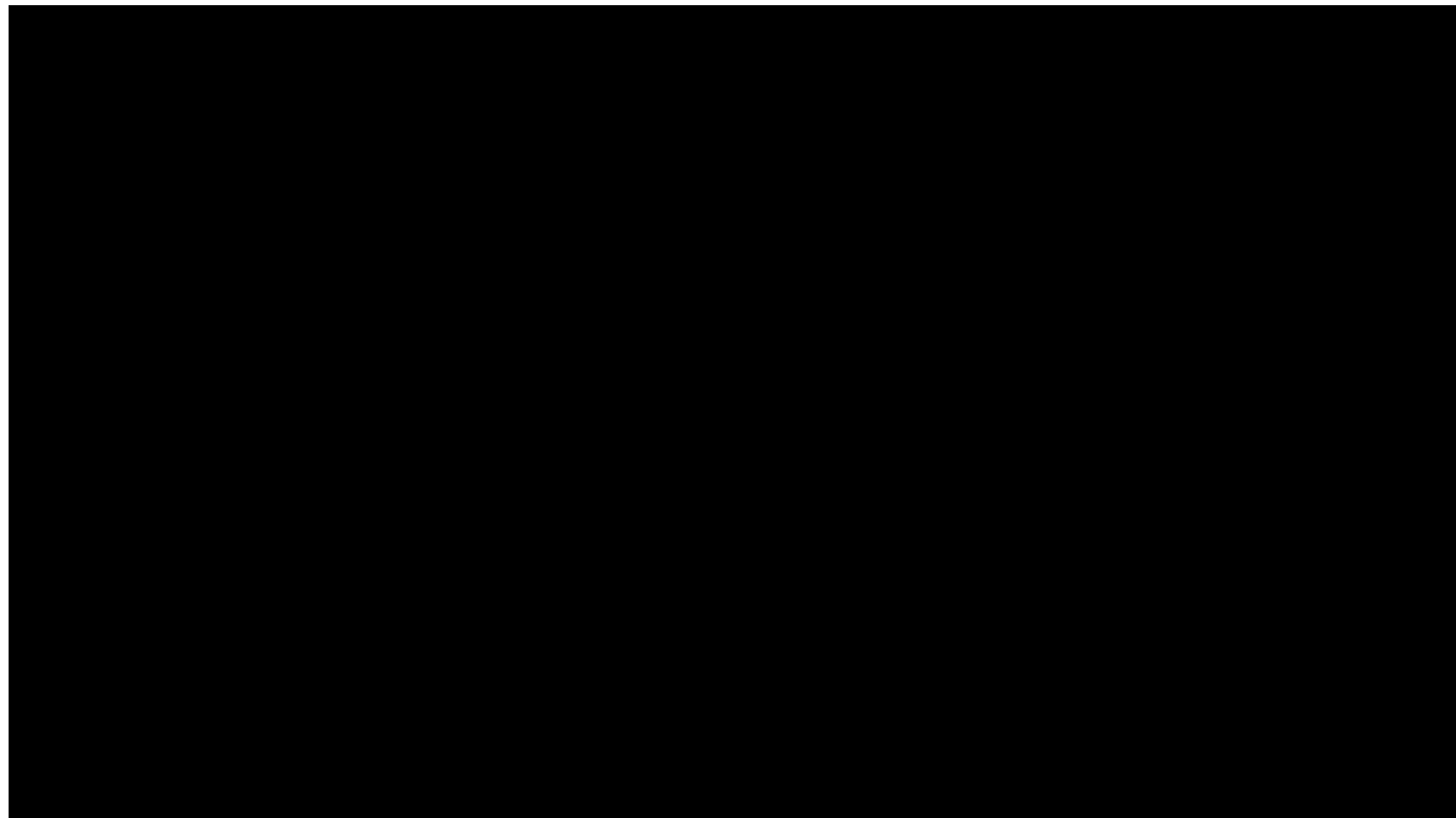
Steady-state error:

$$\ddot{e} = \dot{e} = 0$$

$$f_{\text{dist}} = k_p e$$

$$e = \frac{f_{\text{dist}}}{k_p}$$



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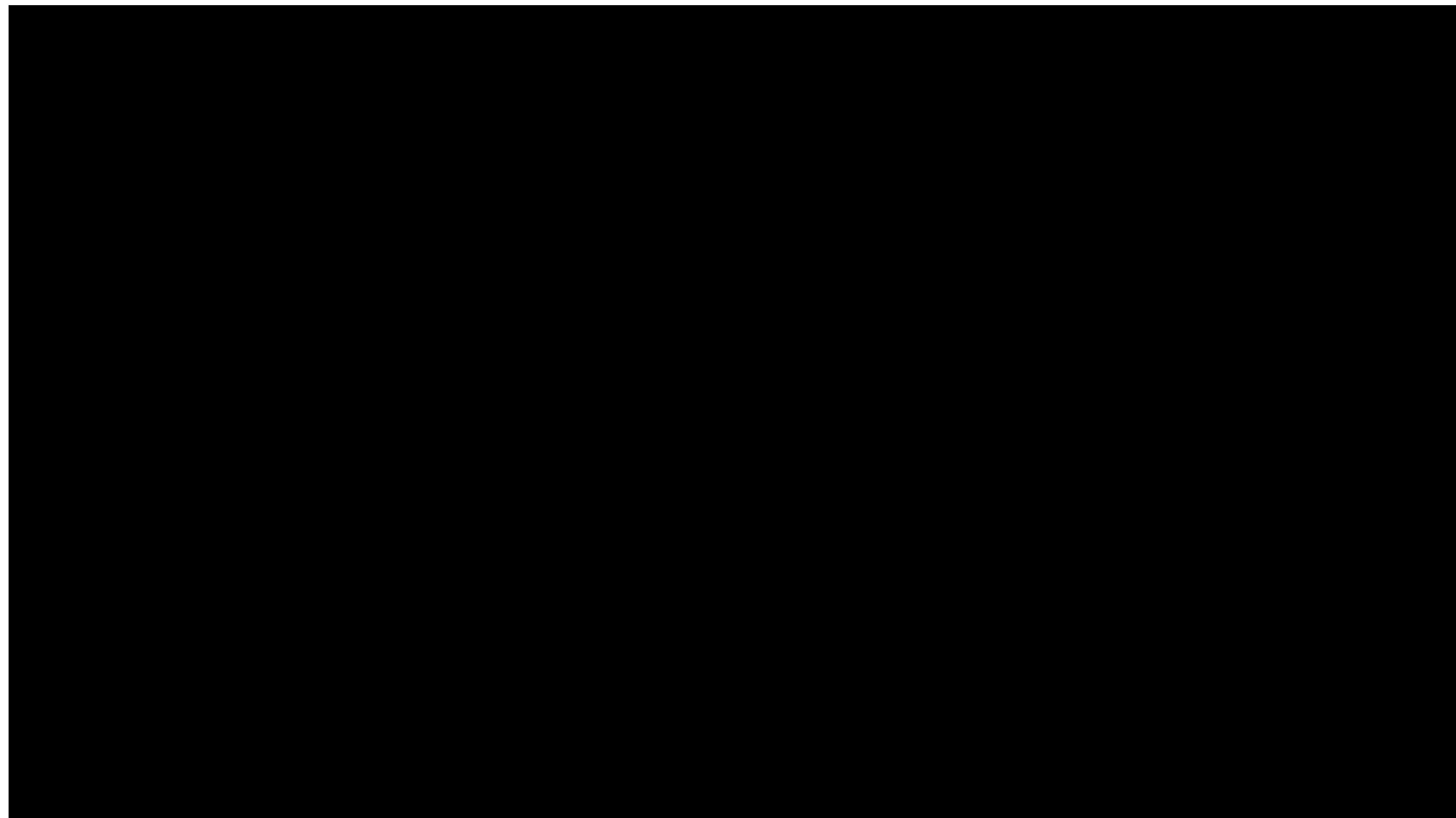

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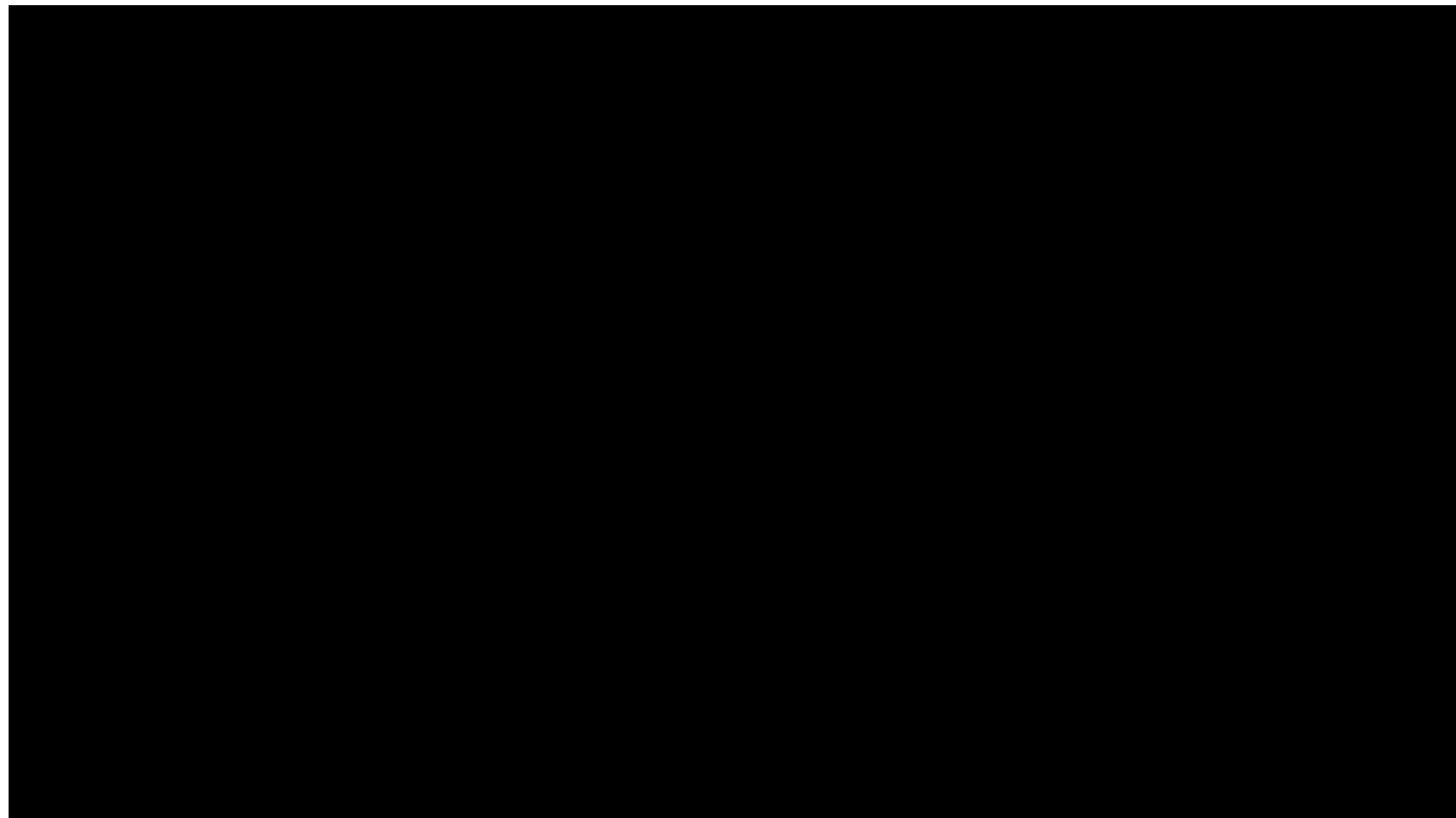

# Disturbance Rejection

Error equation: $f_{\text{dist}} = \ddot{e} + k_v \dot{e} + k_p e$

$$\ddot{e} = \ddot{x}_d - \ddot{x}$$

$$f_{\text{dist}} = \ddot{x}_d - \ddot{x} + k_v \dot{e} + k_p e$$

$$f_{\text{dist}} = \ddot{e} + k_v \dot{e} + k_p e$$

# Disturbance Rejection

Error equation: $f_{\text{dist}} = \ddot{e} + k_v \dot{e} + k_p e$

$$\ddot{x} = \ddot{x}_d + k_v \dot{e} + k_p e - f_{\text{dist}}$$

reformulate

$$f_{\text{dist}} = \ddot{x}_d - \ddot{x} + k_v \dot{e} + k_p e$$

$\ddot{e} = \ddot{x}_d - \ddot{x}$

$$f_{\text{dist}} = \ddot{e} + k_v \dot{e} + k_p e$$

# Disturbance Rejection

Error equation: $f_{\text{dist}} = \ddot{e} + k_v \dot{e} + k_p e$

$$\ddot{x} = f' - f_{\text{dist}}$$

$$f' = \ddot{x} + k_v \dot{e} + k_p x$$

$$\ddot{x} = \ddot{x}_d + k_v \dot{e} + k_p e - f_{\text{dist}}$$

reformulate

$$f_{\text{dist}} = \ddot{x}_d - \ddot{x} + k_v \dot{e} + k_p e$$

$$\ddot{e} = \ddot{x}_d - \ddot{x}$$

$$f_{\text{dist}} = \ddot{e} + k_v \dot{e} + k_p e$$

# Disturbance Rejection

Error equation: $f_{\text{dist}} = \ddot{e} + k_v \dot{e} + k_p e$

$$m\ddot{x} = mf' - mf_{\text{dist}}$$

mult by m

$$\ddot{x} = f' - f_{\text{dist}}$$

$f' = \ddot{x} + k_v \dot{e} + k_p x$

$$\ddot{x} = \ddot{x}_d + k_v \dot{e} + k_p e - f_{\text{dist}}$$

reformulate

$$f_{\text{dist}} = \ddot{x}_d - \ddot{x} + k_v \dot{e} + k_p e$$

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# Disturbance Rejection

Error equation: $f_{\text{dist}} = \ddot{e} + k_v \dot{e} + k_p e$

$$m\ddot{x} = -kx - b\dot{x} + [\alpha f' + \beta] - m f_{\text{dist}}$$

$\beta := b\dot{x} + kx$
$\alpha := m$

$$m\ddot{x} = m f' - m f_{\text{dist}}$$

mult by m

$$\ddot{x} = f' - f_{\text{dist}}$$

$f' = \ddot{x} + k_v \dot{e} + k_p x$

$$\ddot{x} = \ddot{x}_d + k_v \dot{e} + k_p e - f_{\text{dist}}$$

reformulate

$$f_{\text{dist}} = \ddot{x}_d - \ddot{x} + k_v \dot{e} + k_p e$$

$\ddot{e} = \ddot{x}_d - \ddot{x}$

$$f_{\text{dist}} = \ddot{e} + k_v \dot{e} + k_p e$$

# Wrap-Up

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**Recap on natural system**

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- Over-damped, under-damped, critically damped

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**Control-Law Partitioning**

- Recap on model-based, servo-based control portion

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- Recap on model-based, servo-based control portion
- Trajectory Following

# Wrap-Up

**Recap on natural system**

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- Reformulated equation of motion using forces

**Recap on Closed-Loop System**

**Control-Law Partitioning**

- Recap on model-based, servo-based control portion
- Trajectory Following
- Disturbance Rejection