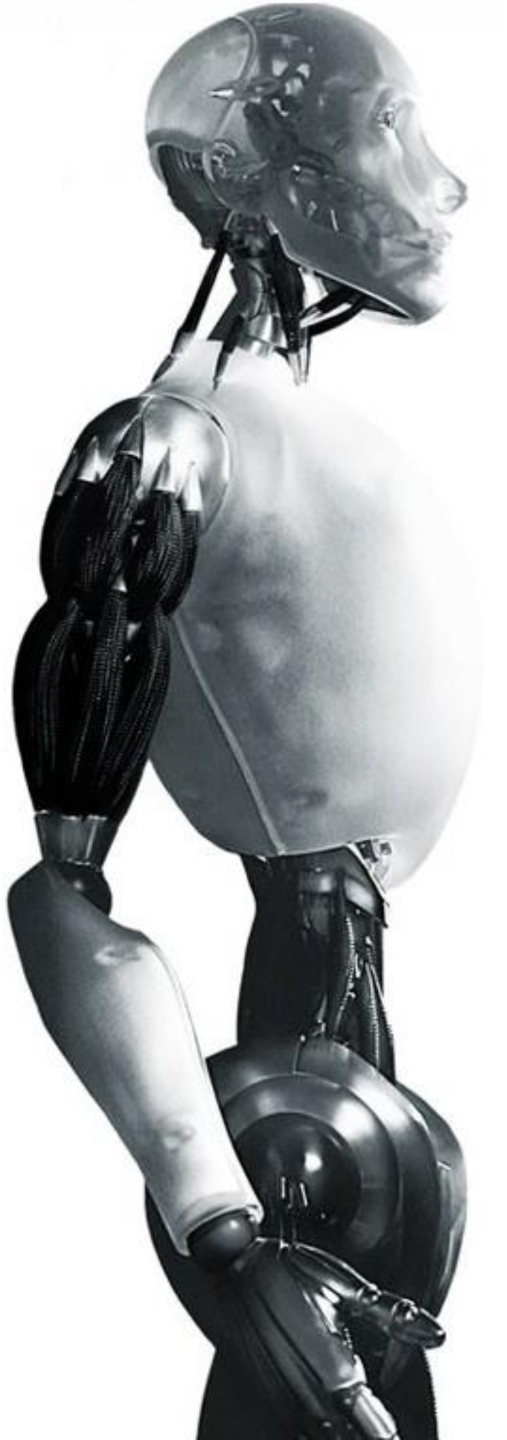
Disclaimer

These slides are intended as presentation aids for the lecture. They contain information that would otherwise be to difficult or time-consuming to reproduce on the board. But they are incomplete, not self-explanatory, and are not always used in the order they appear in this presentation. As a result, these slides should not be used as a script for this course. I recommend you take notes during class, maybe on the slides themselves. It has been shown that taking notes improves learning success.

Reading for this set of slides

- Probabilistic Robotics
  - Chapters 1-4, 7, 8-10 (please match the level of detail from the lectures, not all the material in these chapters is required)

Please note that this set of slides is intended as support for the lecture, not as a stand-alone script. If you want to study for this course, please use these slides in conjunction with the indicated chapters in the text books. The textbooks are available online or in the TUB library (many copies that can be checked out for the entire semester. There are also some aspects of the lectures that will not be covered in the text books but can still be part of the homework or exam. For those It is important that you attend class or ask somebody about what was covered in class.



Robotics

Monte Carlo localization (Derivation)

TU Berlin

Oliver Brock

Bayes' Rule

Definition of Conditional Probability

$$\mathbf{P}(A|B) = \frac{\mathbf{P}(A \cap B)}{\mathbf{P}(B)} \qquad \mathbf{P}(B|A) = \frac{\mathbf{P}(B \cap A)}{\mathbf{P}(A)}$$

Multiplying both sides with denominator

$$\mathbf{P}(A \cap B) = \mathbf{P}(A|B)\,\mathbf{P}(B) \qquad \mathbf{P}(B \cap A) = \mathbf{P}(B|A)\,\mathbf{P}(A)$$

Set intersection is commutative

$$\mathbf{P}(A \cap B) = \mathbf{P}(B \cap A)$$

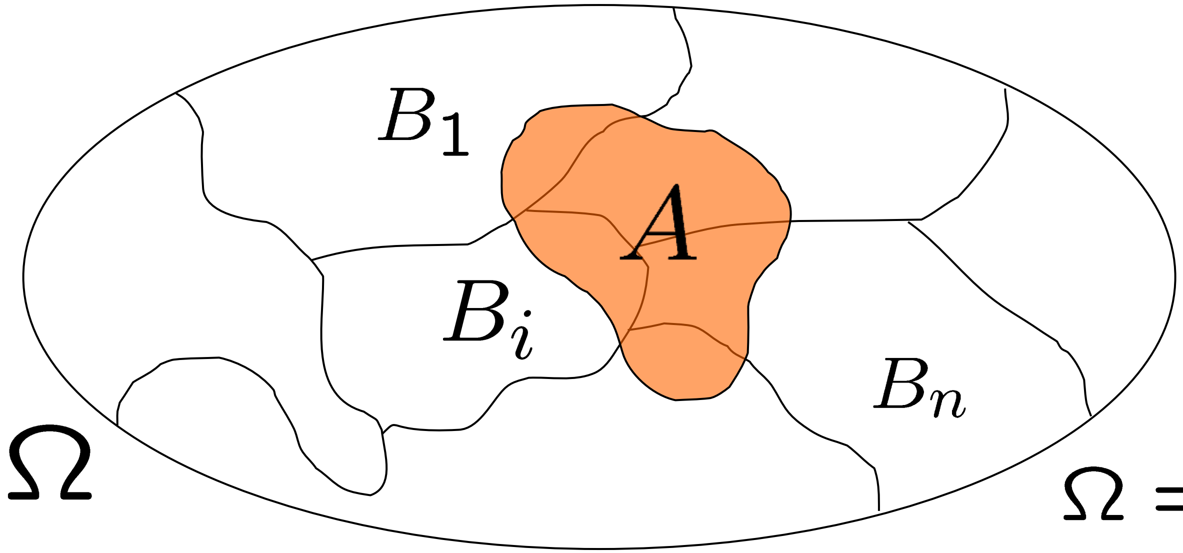
We equate the equations…

$$\mathbf{P}(B)\,\mathbf{P}(A|B) = \mathbf{P}(A)\,\mathbf{P}(B|A)$$

And devide by P(B) to arrive at Bayes' formula

$$\mathbf{P}(A|B) = \frac{\mathbf{P}(A)\,\mathbf{P}(B|A)}{\mathbf{P}(B)}$$

The Law of Total Probability



$$\Omega = \bigcup_i B_i$$

$$\mathbf{P}(A) = \sum_n \mathbf{P}(\underbrace{A \cap B_n})$$

Terminology

$$b_t(s_t) = p(s_t \mid o_0, \cdots, a_{t-1}, o_t, m)$$

$b_t(s_t)$ is the belief to be at time t in state s_t

o_t is the obersvation at time t

a_t is the action taken at time t

m is the map

Derivation: Step 1

$$\begin{aligned} b_t(s_t) &= p(s_t \,|\, o_0, \cdots, a_{t-1}, o_t, m) \\ &= \mu_t \; p(o_t \,|\, o_0, \cdots, a_{t-1}, s_t, m) \cdot p(s_t \,|\, o_0, \cdots, a_{t-1}, m) \end{aligned}$$

Using:

$$\mathbf{P}(A|B) = \frac{\mathbf{P}(A)\,\mathbf{P}(B|A)}{\mathbf{P}(B)}$$

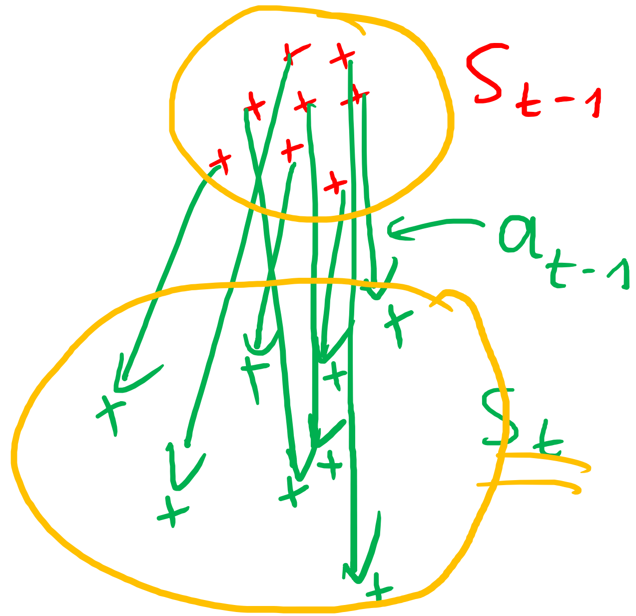
Derivation: Step 2

$$\begin{aligned} b_t(s_t) &= p(s_t \,|\, o_0, \cdots, a_{t-1}, o_t, m) \\ &= \mu_t \, p(o_t \,|\, o_0, \cdots, a_{t-1}, s_t, m) \cdot \\ &\quad p(s_t \,|\, o_0, \cdots, a_{t-1}, m) \\ &= \mu_t \, p(o_t \,|\, s_t, m) \cdot p(s_t \,|\, o_0, \cdots, a_{t-2}, o_{t-1}, m) \end{aligned}$$

Using the *Markov Property*: the observation o_t does only depend on the current state s_t, but not on any states before that.

Derivation: Step 3

$$
\begin{aligned}
b_t(s_t) &= p(s_t \mid o_0, \cdots, a_{t-1}, o_t, m) \\
&= \mu_t \, p(o_t \mid o_0, \cdots, a_{t-1}, s_t, m) \cdot p(s_t \mid o_0, \cdots, a_{t-1}, m) \\
&= \mu_t \, p(o_t \mid s_t, m) \cdot p(s_t \mid o_0, \cdots, a_{t-2}, o_{t-1}, m) \\
&= \mu_t \, p(o_t \mid s_t, m) \cdot \\
&\quad \int p(s_t \mid o_0, \cdots, a_{t-1}, s_{t-1}, m) \, p(s_{t-1} \mid o_0, \cdots, a_{t-1}, m) \, ds_{t-1}
\end{aligned}
$$



Using: $$\mathbf{P}(A) = \sum_n \mathbf{P}(A \cap B_n)$$

Derivation: Step 4

$$
\begin{aligned}
b_t(s_t) &= p(s_t \,|\, o_0, \cdots, a_{t-1}, o_t, m) \\
&= \mu_t \; p(o_t \,|\, o_0, \cdots, a_{t-1}, s_t, m) \cdot p(s_t \,|\, o_0, \cdots, a_{t-1}, m) \\
&= \mu_t \; p(o_t \,|\, s_t, m) \cdot p(s_t \,|\, o_0, \cdots, a_{t-1}, m) \\
&= \mu_t \; p(o_t \,|\, s_t, m) \cdot \\
&\quad \int p(s_t \,|\, \cancel{o_0, \cdots,} a_{t-1}, s_{t-1}, m) \; p(s_{t-1} \,|\, o_0, \cdots, a_{t-2}, o_{t-1} m) \; ds_{t-1} \\
&= \mu_t \; p(o_t \,|\, s_t, m) \cdot \\
&\quad \int p(s_t \,|\, a_{t-1}, s_{t-1}, m) p(s_{t-1} \,|\, o_0, \cdots, o_{t-1}, m) \; ds_{t-1}
\end{aligned}
$$

Using the Markov Property once more

Derivation: Step 5

$$
\begin{aligned}
b_t(s_t) &= p(s_t \mid o_0, \cdots, a_{t-1}, o_t, m) \\
&= \mu_t \, p(o_t \mid o_0, \cdots, a_{t-1}, s_t, m) \cdot p(s_t \mid o_0, \cdots, a_{t-1}, m) \\
&= \mu_t \, p(o_t \mid s_t, m) \cdot p(s_t \mid o_0, \cdots, a_{t-1}, m) \\
&= \mu_t \, p(o_t \mid s_t, m) \cdot \\
&\quad \int p(s_t \mid o_0, \cdots, a_{t-2}, o_{t-1}, s_{t-1}, m) \, p(s_{t-1} \mid o_0, \cdots, a_{t-1}, m) \, ds_{t-1} \\
&= \mu_t \, p(o_t \mid s_t, m) \cdot \\
&\quad \int p(s_t \mid a_{t-1}, s_{t-1}, m) \, \overbrace{p(s_{t-1} \mid o_0, \cdots, o_{t-1}, m)}^{b_{t-1}(s_{t-1})} \, ds_{t-1} \\
&= \mu_t \, \underbrace{\boxed{p(o_t \mid s_t, m)}}_{\text{SENSOR MODEL}} \cdot \\
&\quad \int \underbrace{p(s_t \mid a_{t-1}, s_{t-1}, m)}_{\text{MOTION MODEL}} \, b_{t-1}(s_{t-1}) \, ds_{t-1}
\end{aligned}
$$

RECURSIVE ESTIMATION

Towards Implementation

$$b_t(s_t) \;=\; \mu_t \, p(o_t \mid s_t, m) \cdot \int p(s_t \mid a_{t-1}, s_{t-1}, m) \, b_{t-1}(s_{t-1}) \, ds_{t-1}$$

Sensor Model 3

$$b_t(s_t) \;=\; \mu_t \boxed{p(o_t \mid s_t, m)} \cdot \sum_{\text{samples}} \boxed{p(s_t \mid a_{t-1}, s_{t-1}, m)} \, \boxed{b_{t-1}(s_{t-1})}$$

1 Motion model

Importance factor of sample s_{t-1} 2

Particle Filter Localization

- Represent continuous probability distribution by discrete set of samples *S* ✓ (particles)
- Samples have *importance factor*
- Initialize:
  - *m* samples with importance factor m^{-1} (uniform distribution) -> b_0
- Iterate:
  1. predict samples forward in time using the *motion model* $p(s_t \mid a_{t-1}, s_{t-1}, m)$
  2. assign importance factors based on *sensor model* $p(o_t \mid s_t, m)$
  3. re-sample to adapt distribution of samples to importance factors $b_{t-1}(s_{t-1})$

MM

SM

PREVIOUS BELIEF