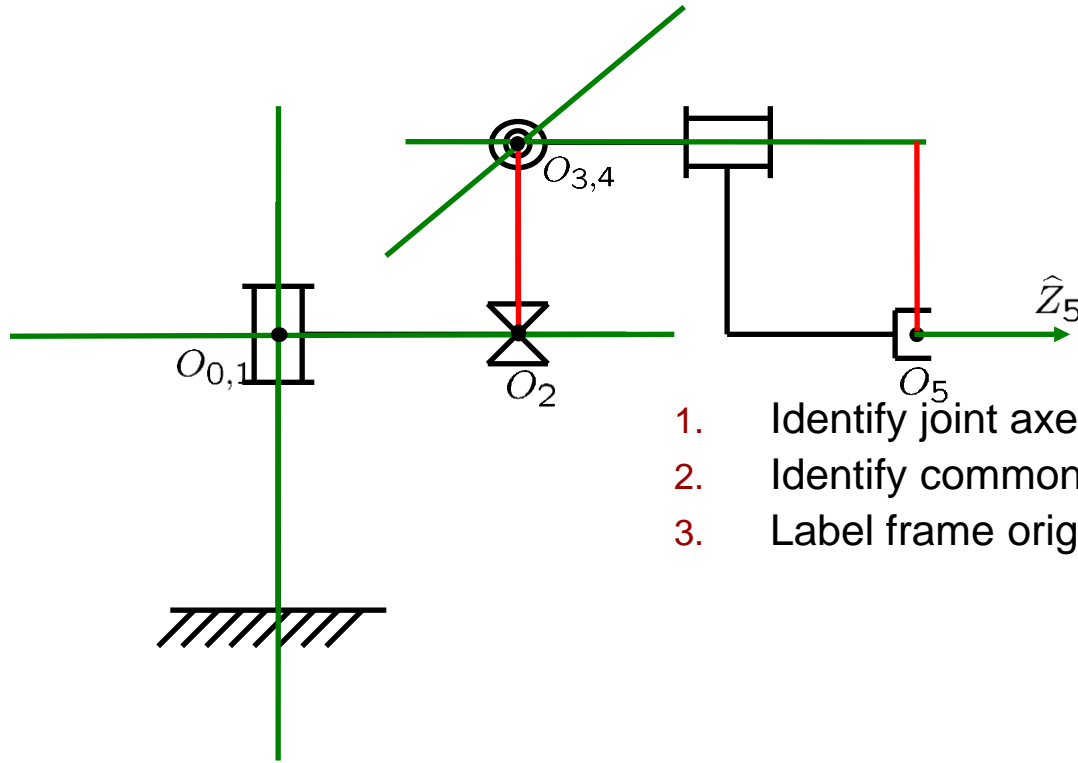
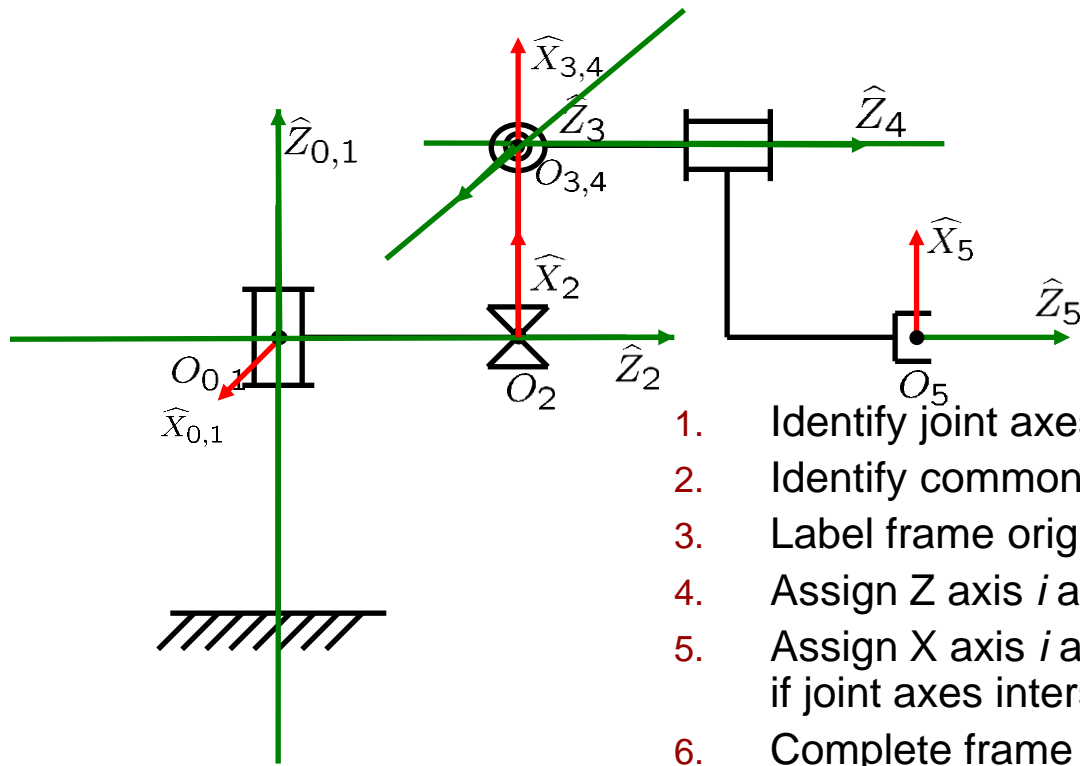


Example with $n=5$



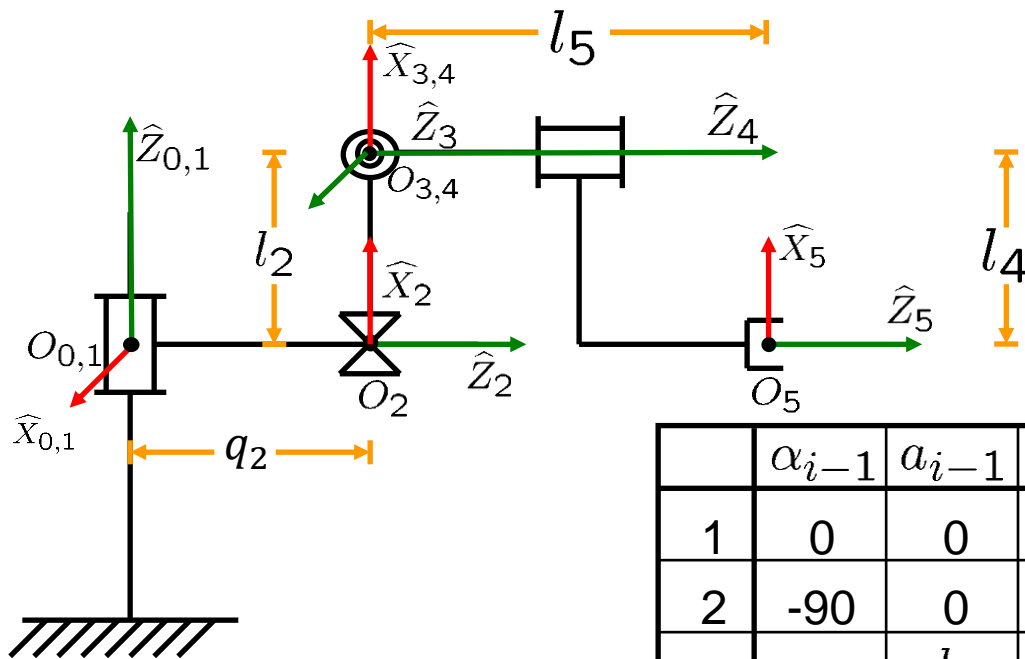
1. Identify joint axes; consider $i-1$ and i
2. Identify common perpendicular
3. Label frame origin at perpendicular (or intersection)

Example with n=5



1. Identify joint axes; consider $i-1$ and i
2. Identify common perpendicular
3. Label frame origin at perpendicular (or intersection)
4. Assign Z axis i along joint axis
5. Assign X axis i along perpendicular; if joint axes intersect, orthogonal to the axes plane
6. Complete frame by adding Y axis (right-hand-rule)
7. Assign $\{0\}$ to match $\{1\}$
8. Choose end-effector frame $\{n\}$

Example with n=5



α_i = the angle between \hat{Z}_i and \hat{Z}_{i+1} measured about \hat{X}_i

a_i = the distance from \hat{Z}_i to \hat{Z}_{i+1} measured along \hat{X}_i

d_i = the distance from \hat{X}_{i-1} to \hat{X}_i measured along \hat{Z}_i

θ_i = the angle between \hat{X}_{i-1} and \hat{X}_i measured about \hat{Z}_i

	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	q_1
2	-90	0	q_2	-90
3	-90	l_2	0	q_3
4	90	0	0	q_4
5	0	$-l_4$	l_5	0

	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	q_1
2	-90	0	q_2	-90
3	-90	l_2	0	q_3
4	90	0	0	q_4
5	0	$-l_4$	l_5	0

$${}^{i-1}_iT = R_X(\alpha_{i-1}) D_X(a_{i-1}) R_Z(\theta_i) D_Z(d_i)$$

$$= \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1}d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1}d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0_1T = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^1_2T = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q_2 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^2_3T = \begin{bmatrix} c_3 & -s_3 & 0 & l_2 \\ 0 & 0 & 1 & 0 \\ -s_3 & -c_3 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^3_4T = \begin{bmatrix} c_4 & -s_4 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s_4 & c_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^4_5T = \begin{bmatrix} 1 & 0 & 0 & -l_4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Summary



- ▶ Homogeneous transformation:
 - Describing positions and orientations in space and how to map between them
- ▶ Forward kinematics:
 - Where is the robot's i^{th} link or end-effector given the robot's configuration
- ▶ Denavit-Hartenberg parameters
 - Uniform procedure for defining the robot's kinematics in order to derive forward kinematics



Quiz time – DH params
