Disclaimer

These slides are intended as presentation aids for the lecture. They contain information that would otherwise be to difficult or time-consuming to reproduce on the board. But they are incomplete, not self-explanatory, and are not always used in the order they appear in this presentation. As a result, these slides should not be used as a script for this course. I recommend you take notes during class, maybe on the slides themselves. It has been shown that taking notes improves learning success.

Reading for this set of slides

- Craig – Intro to Robotics (3rd Edition)
  - Chapter 5.10

Please note that this set of slides is intended as support for the lecture, not as a stand-alone script. If you want to study for this course, please use these slides in conjunction with the indicated chapters in the text books. The textbooks are available online or in the TUB library (many copies that can be checked out for the entire semester. There are also some aspects of the lectures that will not be covered in the text books but can still be part of the homework or exam. For those It is important that you attend class or ask somebody about what was covered in class.

Robotics

The Jacobian and Forces

TU Berlin

Oliver Brock

What the Jacobian can do…

$$\delta \mathbf{x} = J(\mathbf{q})\ \delta \mathbf{q}$$

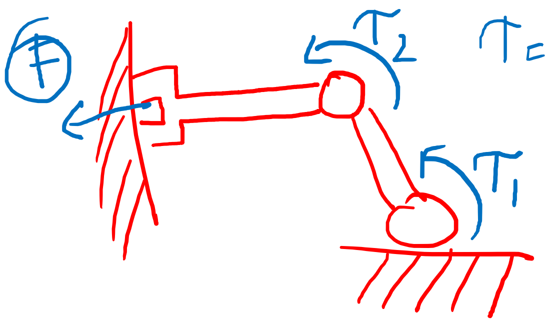
$$\dot{\mathbf{x}} = J(\mathbf{q})\ \dot{\mathbf{q}}$$

$$\delta \mathbf{q} = J(\mathbf{q})^{-1}\ \delta \mathbf{x}$$

Jacobian and Forces

work = force · distance

virtual work: virtual displacements with real force (or vice versa)



$\tau = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$

$$\underline{\mathbf{F}_{(6\times 1)}}\ \delta\mathbf{x} = \tau\,\delta\mathbf{q}$$

$\delta q = J(q)^{-1}\,\delta x$

$$\mathbf{F}^T\ \delta\mathbf{x} = \tau^T\delta\mathbf{q}$$ using matrix notation

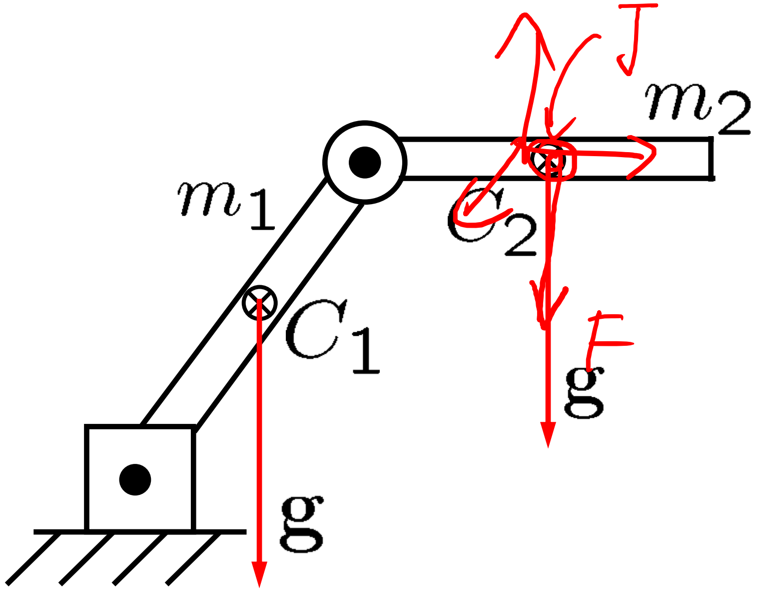
$$\mathbf{F}^T\ J\delta\mathbf{q} = \tau^T\delta\mathbf{q}$$ using $\delta\mathbf{x} = J(\mathbf{q})\ \delta\mathbf{q}$

$$\mathbf{F}^T\ J = \tau^T$$

$$\tau = J^T\mathbf{F}$$

C-SPACE ← WORKSPACE

Gravity Reloaded: With Jacobian



Linear part of Jacobian at Center of Mass 1

$$\tau_{m_1} = -J_{v_1}^T \overbrace{(\mathbf{g}\; m_1)}^{F}$$

$$\tau_{m_2} = -J_{v_2}^T (\mathbf{g}\; m_2)$$

LINKS

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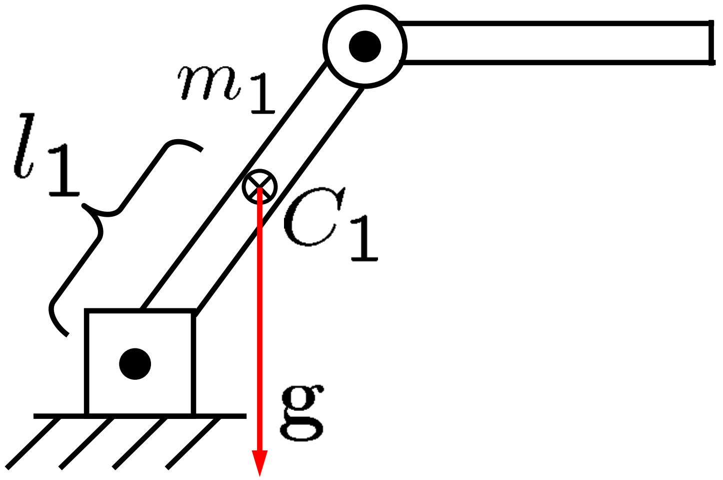
$$\mathbf{G} = -\sum_{i=0}^{n} J_{v_i}^T (\mathbf{g}\; m_i)$$

Gravity Example



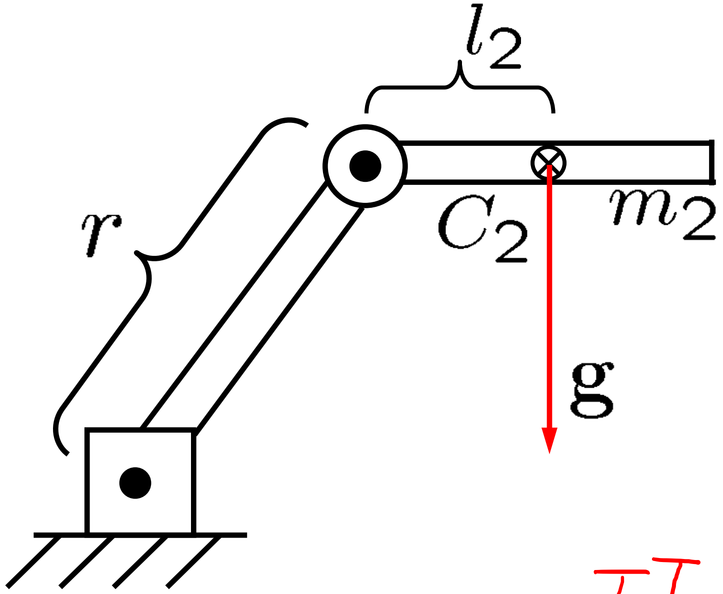
$$\tau_{m_1} = -J_{c_1}^T \; (\mathbf{g}\; m_1)$$

$$^0\mathbf{p}_{C_1} = \begin{pmatrix} l_1\, c_1 \\ l_1\, s_1 \end{pmatrix}$$

$$^0J_{v_1} = \begin{bmatrix} -l_1\, s_1 & 0 \\ l_1\, c_1 & 0 \end{bmatrix}$$

$$\tau_{m_1} = -\overbrace{\begin{bmatrix} -l_1\, s_1 & l_1\, c_1 \\ 0 & 0 \end{bmatrix}}^{J^T} \begin{pmatrix} 0 \\ -\mathbf{g}\; m_1 \end{pmatrix} = \begin{pmatrix} l_1\, c_1\, \mathbf{g}\; m_1 \\ 0 \end{pmatrix}$$

Gravity Example cont.



$$\tau_{m_2} = -J_{c_2}^T \, (\mathbf{g}\, m_2)$$

$${}^0\mathbf{p}_{C_2} = \begin{pmatrix} r\, c_1 + l_2\, c_{12} \\ r\, s_1 + l_2\, s_{12} \end{pmatrix}$$

$${}^0J_{v_2} = \begin{bmatrix} -r\, s_1 - l_2\, s_{12} & -l_2\, s_{12} \\ r\, c_1 + l_2\, c_{12} & l_2\, c_{12} \end{bmatrix}$$

J^T

$$\tau_{m_2} = - \begin{bmatrix} -r\, s_1 - l_2\, s_{12} & r\, c_1 + l_2\, c_{12} \\ -l_2\, s_{12} & l_2\, c_{12} \end{bmatrix} \begin{pmatrix} 0 \\ -\mathbf{g}\, m_2 \end{pmatrix} =$$

$$\begin{pmatrix} (r\, c_1 + l_2\, c_{12})\, \mathbf{g}\, m_2 \\ l_2\, c_{12}\, \mathbf{g}\, m_2 \end{pmatrix}$$

What the Jacobian can do…

$$\dot{\mathbf{x}} = J(\mathbf{q})\,\dot{\mathbf{q}}$$ ✓

$$\delta\mathbf{x} = J(\mathbf{q})\,\delta\mathbf{q}$$ ✓

$$\dot{\mathbf{q}} = J(\mathbf{q})^{-1}\,\dot{\mathbf{x}}$$ ✓✗

$$\delta\mathbf{q} = J(\mathbf{q})^{-1}\,\delta\mathbf{x}$$ ✓✗

$$\tau = J^T\mathbf{F}$$ ✓

My favorite formula!