Disclaimer

These slides are intended as presentation aids for the lecture. They contain information that would otherwise be to difficult or time-consuming to reproduce on the board. But they are incomplete, not self-explanatory, and are not always used in the order they appear in this presentation. As a result, these slides should not be used as a script for this course. I recommend you take notes during class, maybe on the slides themselves. It has been shown that taking notes improves learning success.

Reading for this set of slides

- Craig – Intro to Robotics (3rd Edition)
  - – 1 Introduction
  - – 2 Spatial descriptions and transformations (2.1 – 2.9)
  - – 3 Manipulator kinematics (3.1 – 3.6)
  - – 7 Trajectory generation

Please note that this set of slides is intended as support for the lecture, not as a stand-alone script. If you want to study for this course, please use these slides in conjunction with the indicated chapters in the text books. The textbooks are available online or in the TUB library (many copies that can be checked out for the entire semester. There are also some aspects of the lectures that will not be covered in the text books but can still be part of the homework or exam. For those It is important that you attend class or ask somebody about what was covered in class.

Robotics

Lyapunov Stability

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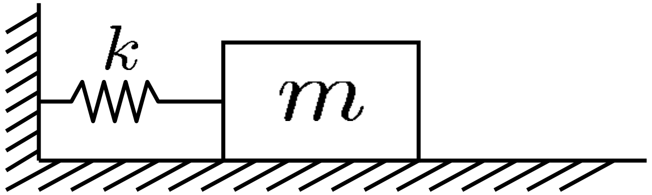
Stability Analysis

- In a linear system stability requires $k_v > 0$
- Assuming bounded disturbance we can make certain guarantees
- Analysis more complex in nonlinear systems
- Linearization is not always possible
  - inaccurate models
  - unknown models

Energy-Based Stability Analysis



$$m\ddot{x} + b\dot{x} + kx = 0$$

$$E = \frac{1}{2} m \, \dot{x}^2 + \frac{1}{2} k \, x^2$$

$$\begin{aligned}
\dot{E} &= m\ddot{x}\dot{x} + kx\dot{x} \\
&= (-b\dot{x} - kx)\dot{x} + kx\dot{x} \\
&= -b\dot{x}^2 \\
&< 0
\end{aligned}$$

Energy of system is reduced until it comes to rest at x = 0

Lyapunov Stability Theory

- Energy based example is an instance of Lyapunov method
- Applies to linear and nonlinear systems
- Stability analysis, but no performance analysis
- Aleksandr Mikhailovich Lyapunov, (1857-1918), friend of Markov and Chebychev

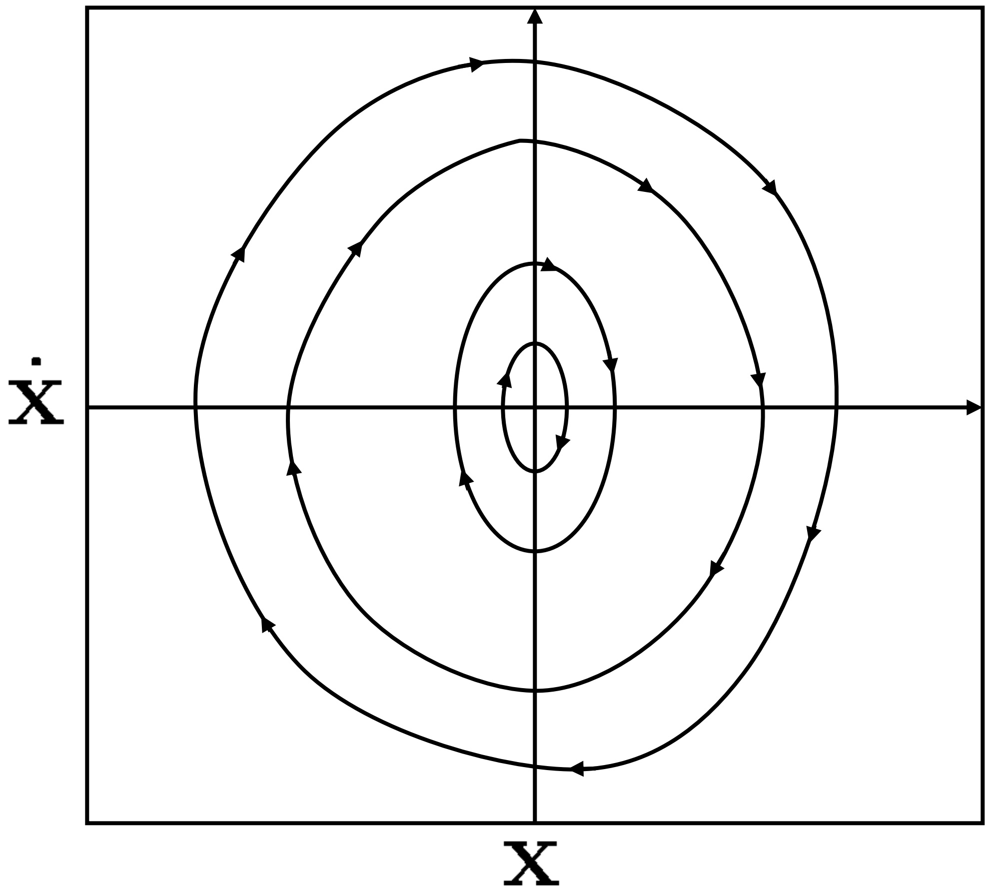
Lyapunov's Second Method

- Also called "direct" method
- Determines stability of differential equation

$$\dot{\mathbf{x}} = f(\mathbf{x})$$

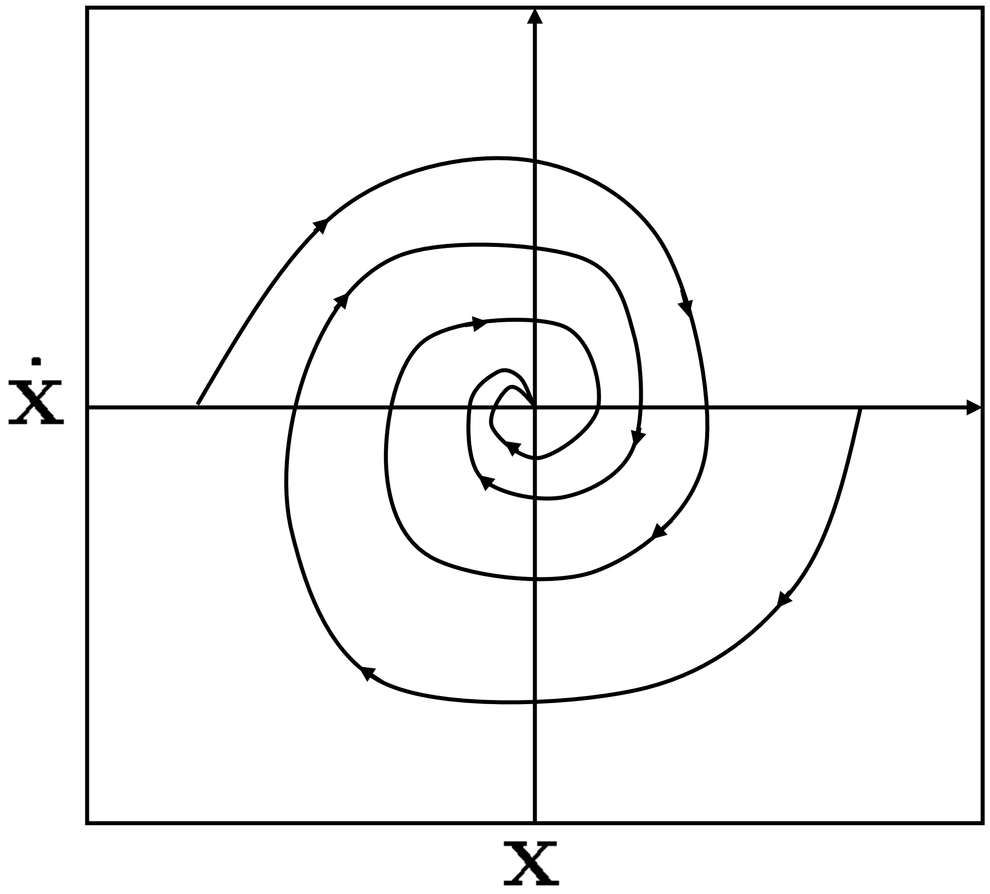
- Requires energy function $E(\mathbf{x})$
  - with continuous first partial derivatives
  - $\forall \mathbf{x} : E(\mathbf{x}) > 0$ except for $E(0) = 0$
  - and such that $\dot{E}(\mathbf{x}) \leq 0$
- Energy-like function that always decreases

Phase Plot: Lyapunov Stable

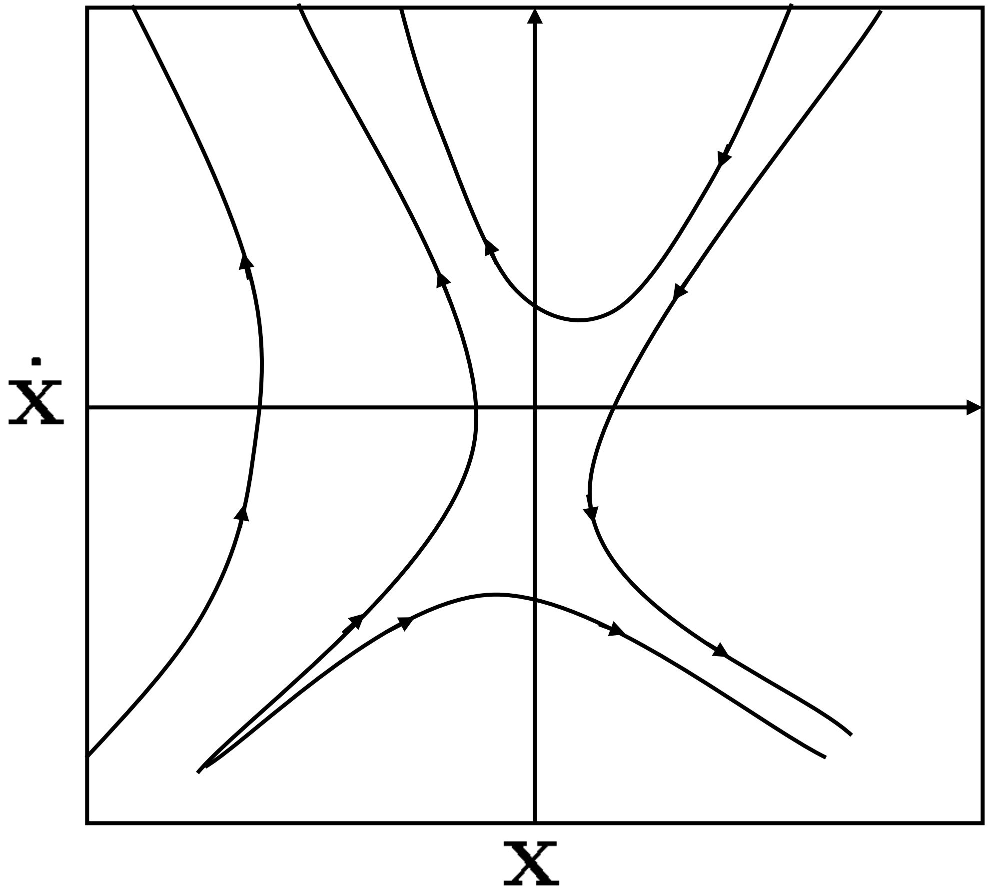


phase plot for $\dot{E}(\mathbf{x}) = 0$

Phase Plot: Asymptotically Stable



Phase Plot: Unstable



Stability of Computed Torque

$$M(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{v}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q}) = \tau$$

$$\tau = K_p \mathbf{e} - K_v \dot{\mathbf{q}} + \mathbf{G}(\mathbf{q})$$

$$M(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{v}(\mathbf{q}, \dot{\mathbf{q}}) + K_v \dot{\mathbf{q}} + K_p \mathbf{q} = K_p\, \mathbf{q}_d$$

Energy function:

$$E = \frac{1}{2}\dot{\mathbf{q}}^T M(\mathbf{q})\dot{\mathbf{q}} + \frac{1}{2}\mathbf{e}^T K_p \mathbf{e}$$

always positive because M, K_p positive definite

$$\begin{aligned} \dot{E} &= \frac{1}{2}\dot{\mathbf{q}}^T \dot{M}(\mathbf{q})\dot{\mathbf{q}} + \dot{\mathbf{q}}^T M(\mathbf{q})\ddot{\mathbf{q}} - \mathbf{e}^T K_p \dot{\mathbf{q}} \\ &= \frac{1}{2}\dot{\mathbf{q}}^T \dot{M}(\mathbf{q})\dot{\mathbf{q}} - \dot{\mathbf{q}}^T K_v \dot{\mathbf{q}} - \dot{\mathbf{q}}^T \mathbf{v}(\mathbf{q}, \dot{\mathbf{q}}) \\ &= -\dot{\mathbf{q}}^T K_v \dot{\mathbf{q}} \end{aligned}$$

$$\frac{1}{2}\dot{\mathbf{q}}^T \dot{M}(\mathbf{q})\dot{\mathbf{q}} = \dot{\mathbf{q}}^T \mathbf{v}(\mathbf{q}, \dot{\mathbf{q}})$$

always non-positive for K_v positive definite

Asymptotic Stability?

$$\dot{E} = -\dot{\mathbf{q}}^T K_v \dot{\mathbf{q}} = 0 \quad \Rightarrow \quad \ddot{\mathbf{q}} = \dot{\mathbf{q}} = 0$$

$$M(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{v}(\mathbf{q}, \dot{\mathbf{q}}) + K_v\dot{\mathbf{q}} + K_p\mathbf{q} = K_p\,\mathbf{q}_d$$

$$K_p\mathbf{e} = 0 \quad \Rightarrow \quad \mathbf{e} = 0$$

YES!

Lyapunov’s “First” Method

- Called indirect method of Lyapunov
- Uses linearization for nonlinear systems
- Stability of local linearization determines stability of original nonlinear equations
- We won’t look at it here…