Disclaimer

These slides are intended as presentation aids for the lecture. They contain information that would otherwise be to difficult or time-consuming to reproduce on the board. But they are incomplete, not self-explanatory, and are not always used in the order they appear in this presentation. As a result, these slides should not be used as a script for this course. I recommend you take notes during class, maybe on the slides themselves. It has been shown that taking notes improves learning success.

Reading for this set of slides

- Planning Algorithms (Steve LaValle)
  - 6 Combinatorial Motion Planning (6.1 – 6.3)
  - 8 Feedback Motion Planning (8.1, 8.2)
- Please refer to the slides for potential fields and vehicle kinematics

Please note that this set of slides is intended as support for the lecture, not as a stand-alone script. If you want to study for this course, please use these slides in conjunction with the indicated chapters in the text books. The textbooks are available online or in the TUB library (many copies that can be checked out for the entire semester. There are also some aspects of the lectures that will not be covered in the text books but can still be part of the homework or exam. For those It is important that you attend class or ask somebody about what was covered in class.

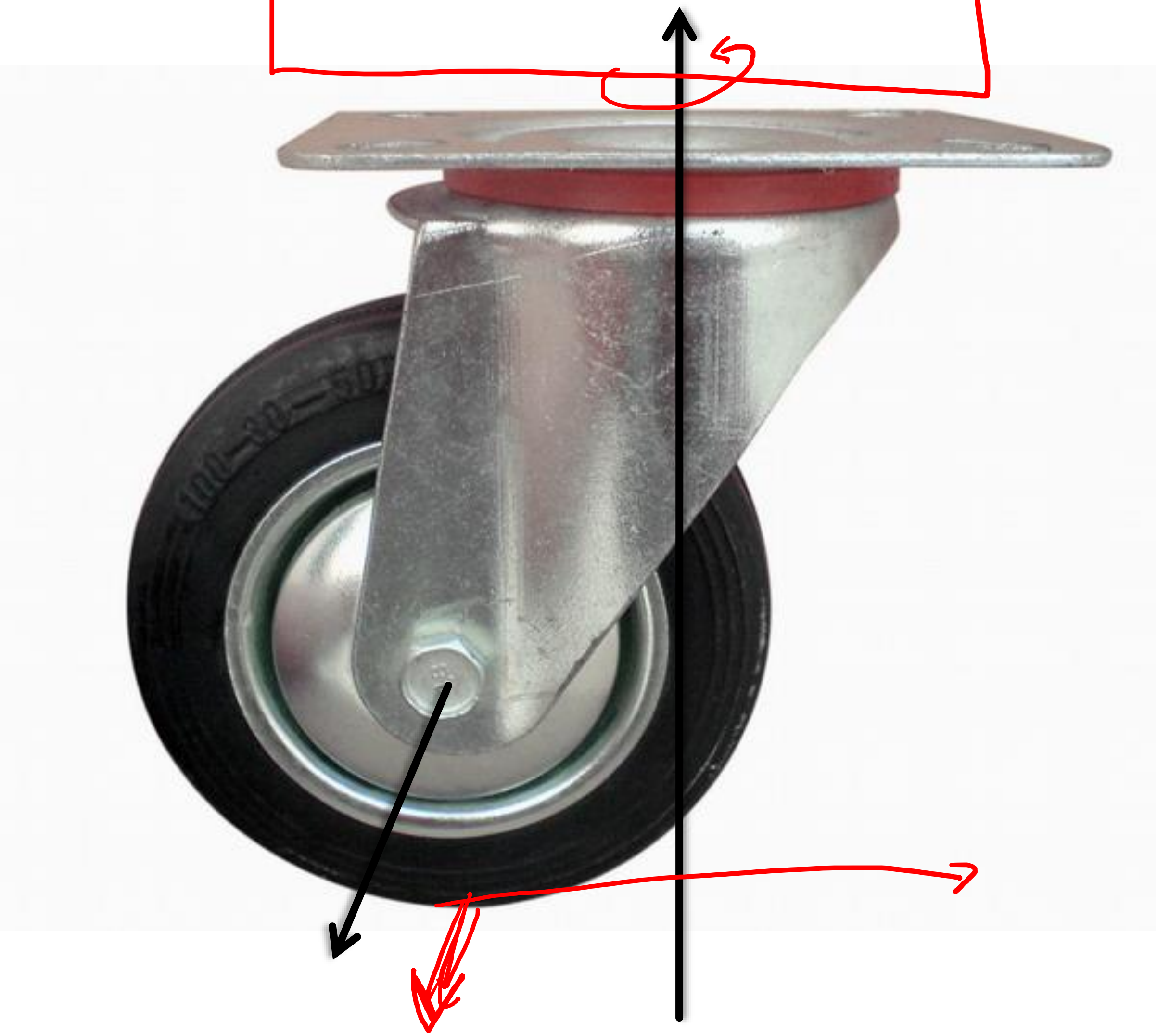
Robotics

Mobile Robots – Kinematics

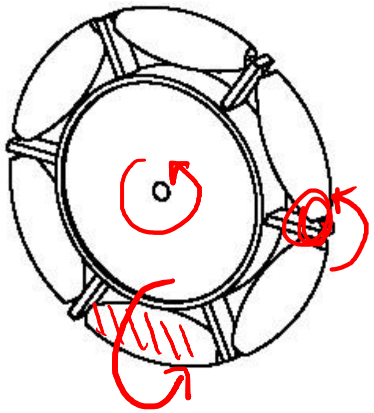
TU Berlin

Oliver Brock

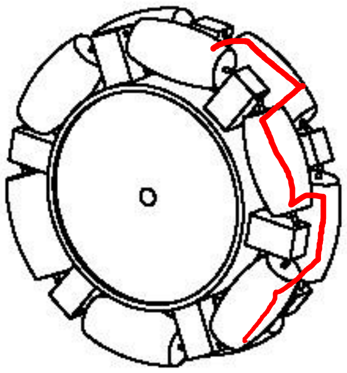
Caster Wheel



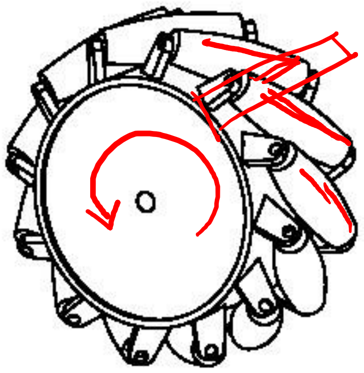
Wheels



Universal

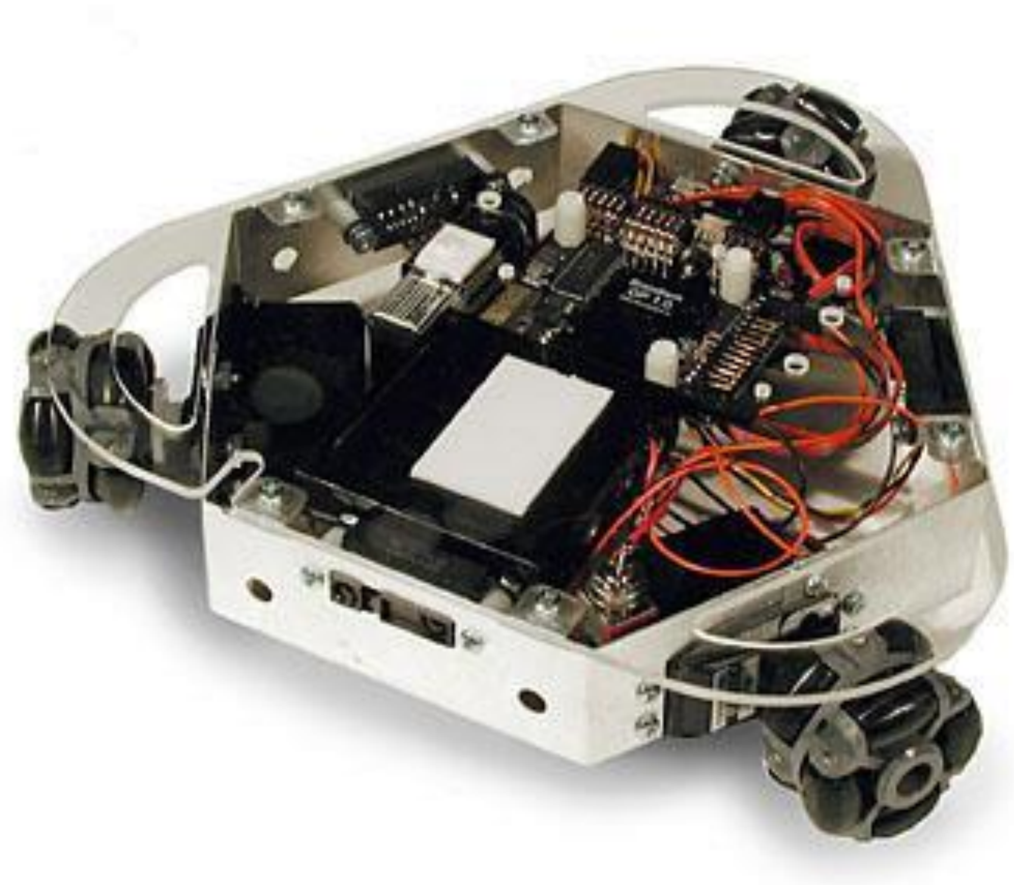


Double Universal



Swedish

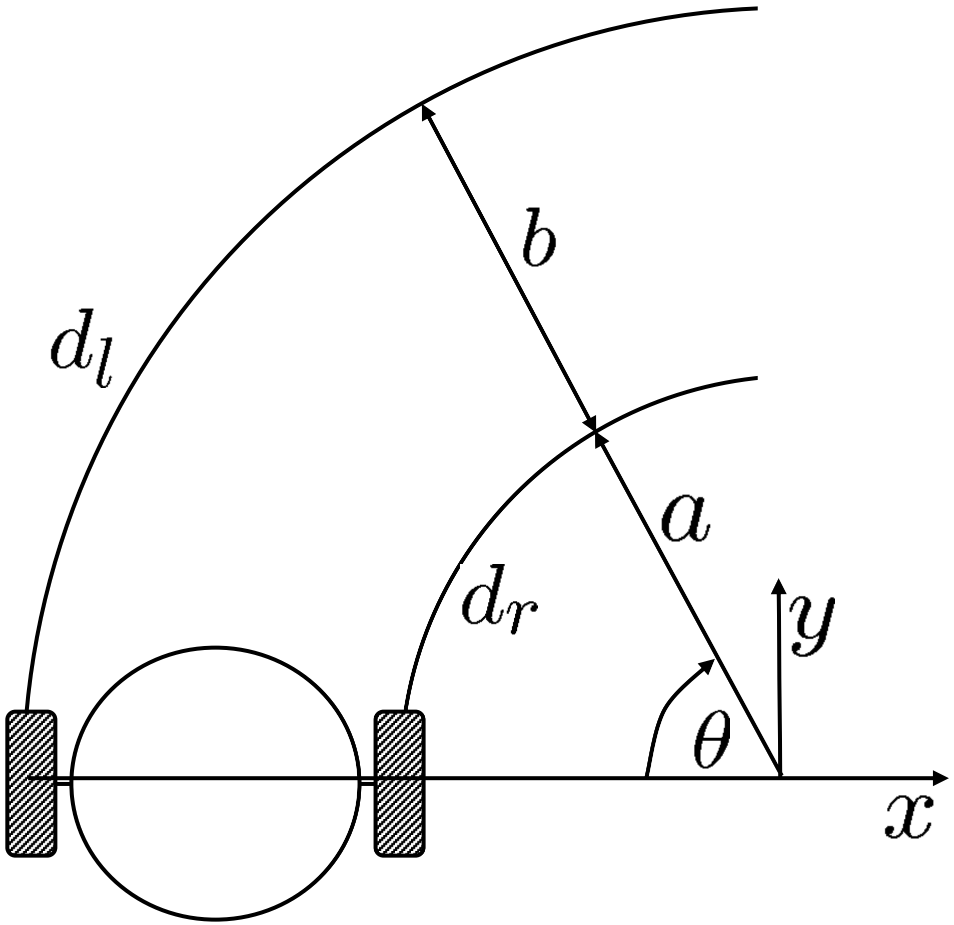
Getting Rid of Nonholonomicity



Drive Systems

- **Differential Drive**
- **Synchro-Drive**
- Holonomic Delta Robot
- Tricycle
- Ackerman Steering

Differential Drive



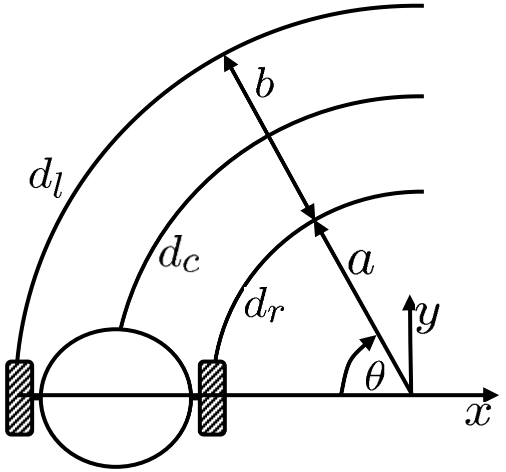
Circumference of a circle:

$$c = 2\pi r$$

$$2\pi = \frac{c}{r}$$

$$\theta = \frac{d_r}{a} = \frac{d_l}{a+b}$$

Differential Drive cont.



$$\theta = \frac{d_r}{a} = \frac{d_l}{a+b}$$

$$(a+b)\, d_r = a\, d_l$$

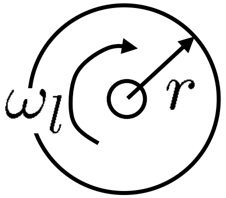
$$a = b\, \frac{d_r}{d_l - d_r}$$

$$\theta = \frac{d_l - d_r}{b}$$

$$d_c = \frac{d_l + d_r}{2}$$

$$\boxed{\begin{aligned} \omega &= \frac{v_l - v_r}{b} \\ v &= \frac{v_l + v_r}{2} \end{aligned}}$$

$$v_l = \omega_l\, r$$



Differential Drive cont. II

$$\omega = \frac{v_l - v_r}{b} \Rightarrow v_r = v_l - \omega b$$

$$v = \frac{v_l + v_r}{2} \Rightarrow v_r = 2v - v_l$$

$$v_l - \omega b = 2v - v_l$$

$$\boxed{\begin{aligned} v_l &= v + \frac{\omega b}{2} \\ v_r &= v - \frac{\omega b}{2} \end{aligned}}$$

Differential Drive Summary

$$v = \frac{v_l + v_r}{2}$$
$$\omega = \frac{v_l - v_r}{b}$$

$$\omega_l = \frac{v_l}{r} \qquad v_l = v + \frac{\omega b}{2}$$
$$\omega_r = \frac{v_r}{r} \qquad v_r = v - \frac{\omega b}{2}$$

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{pmatrix} v \\ \omega \end{pmatrix}$$

Kinematic Equations of Motion

$$\begin{pmatrix} v \\ \omega \end{pmatrix} = r \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{b} & -\frac{1}{b} \end{bmatrix} \begin{pmatrix} \omega_l \\ \omega_r \end{pmatrix}$$

$$\begin{pmatrix} \omega_l \\ \omega_r \end{pmatrix} = \frac{1}{r} \begin{bmatrix} 1 & \frac{b}{2} \\ 1 & -\frac{b}{2} \end{bmatrix} \begin{pmatrix} v \\ \omega \end{pmatrix}$$

Differential Drive Example

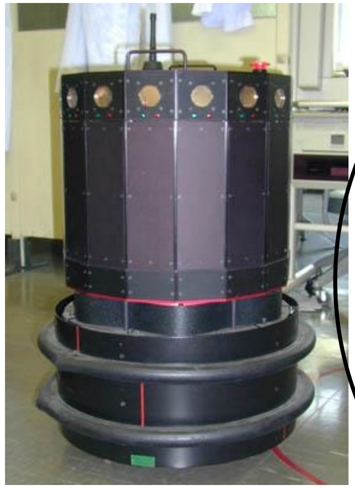
$$\begin{pmatrix} \omega_l \\ \omega_r \end{pmatrix} = \frac{1}{r} \begin{bmatrix} 1 & \frac{b}{2} \\ 1 & -\frac{b}{2} \end{bmatrix} \begin{pmatrix} v \\ \omega \end{pmatrix}$$

v=0, $\omega \neq 0$?   v$\neq$0, ω=0 ?

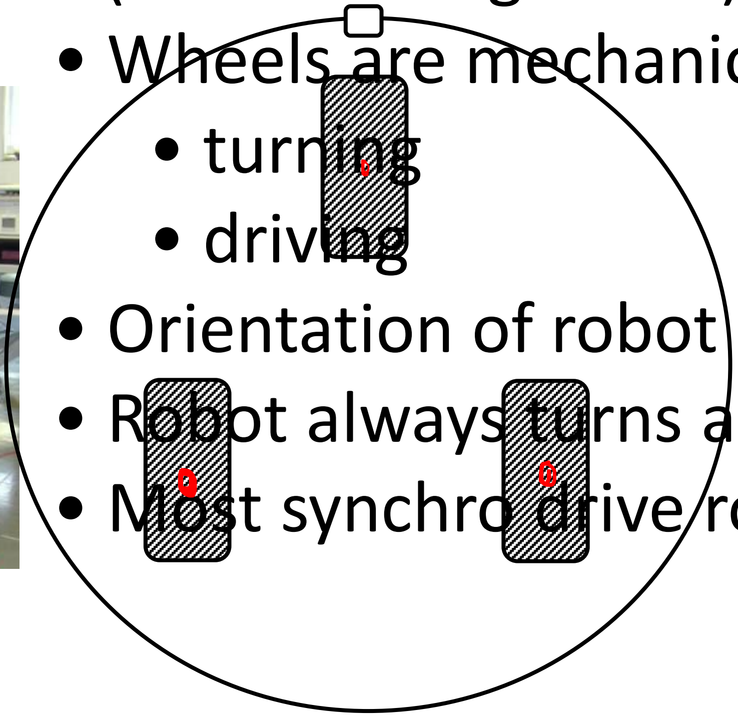
Wheel radius: 0.1m
Wheel base: 0.4m
Desired velocity: 0.5m/s
Desired turning velocity: 0.3rad/s

$$10 \begin{bmatrix} 1 & 0.2 \\ 1 & -0.2 \end{bmatrix} \begin{pmatrix} 0.5 \\ 0.3 \end{pmatrix} = \begin{pmatrix} 5.6 \\ 4.4 \end{pmatrix}$$

Synchro Drive



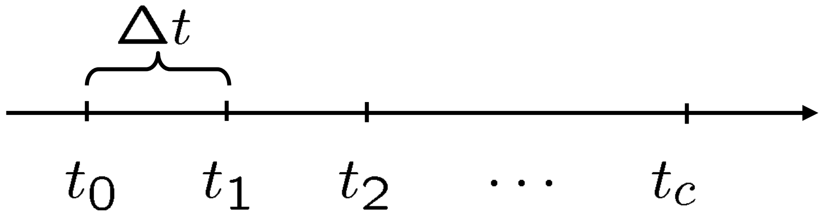
- Motivation: direct drive robots and tricycles are not very stable (wheel arrangement)
- Wheels are mechanically synchronized
  - turning
  - driving
- Orientation of robot is fixed
- Robot always turns about its center
- Most synchro drive robots have turret



Synchro Drive cont.

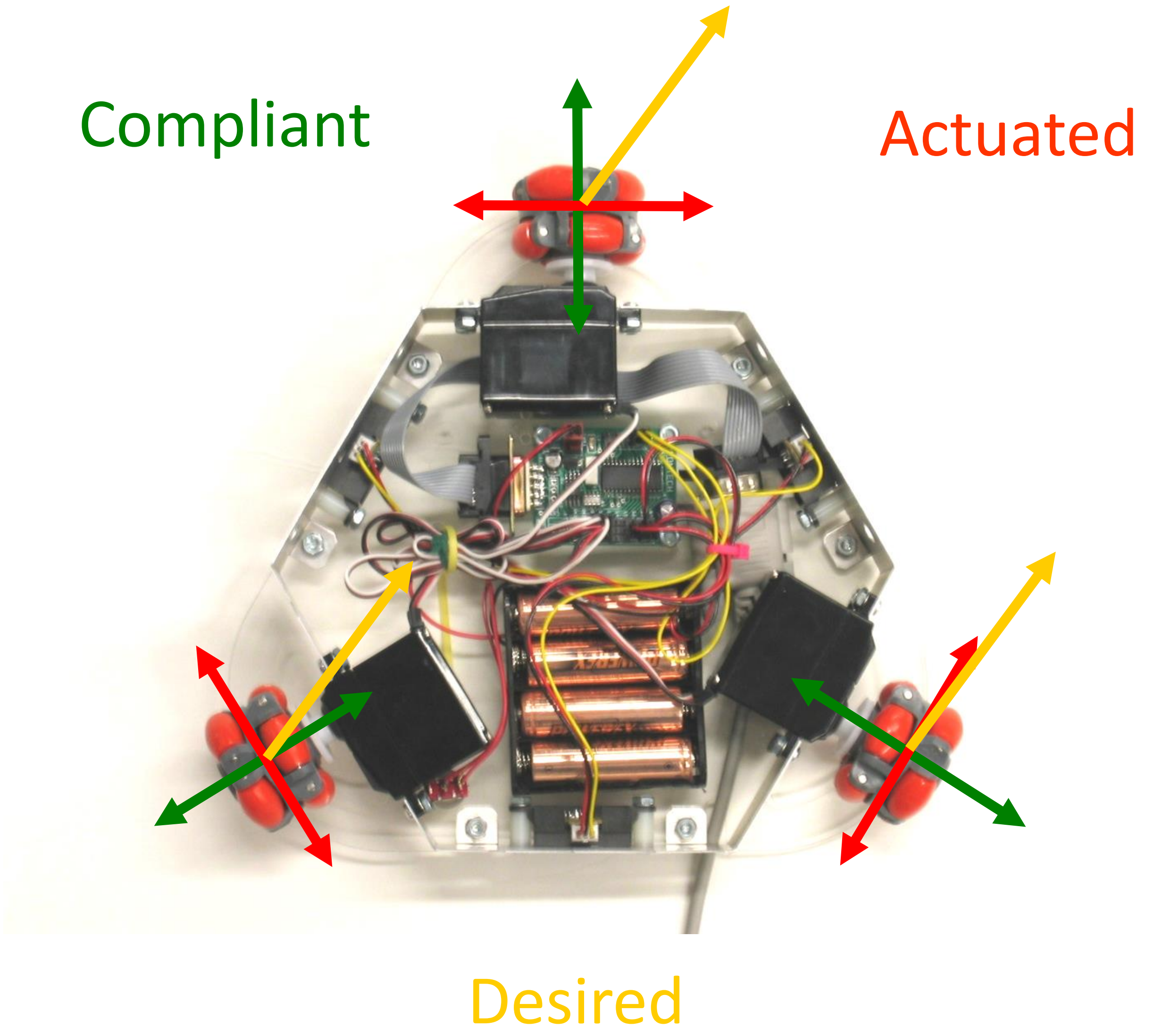
$$
\begin{aligned}
x(t_c) &= x(t_0) + \int_{t_0}^{t_c} v(t) \cdot \cos\theta(t)\, dt \\
y(t_c) &= y(t_0) + \int_{t_0}^{t_c} v(t) \cdot \sin\theta(t)\, dt \\
v(t_c) &= v(t_0) + \int_{t_0}^{t_c} \dot{v}(t) dt \\
\theta(t_c) &= \theta(t_0) + \int_{t_0}^{t_c} \dot{\theta}(t) dt
\end{aligned}
$$

Synchro Drive cont. II



$$
\begin{aligned}
x(t_c) &= x(t_0) + \sum_{t_0}^{t_c} v(t) \cdot \cos\theta(t)\, \Delta t \\
y(t_c) &= y(t_0) + \sum_{t_0}^{t_c} v(t) \cdot \sin\theta(t)\, \Delta t \\
v(t_c) &= v(t_0) + \sum_{t_0}^{t_c} \dot{v}(t) \Delta t \\
\theta(t_c) &= \theta(t_0) + \sum_{t_0}^{t_c} \dot{\theta}(t) \Delta t
\end{aligned}
$$

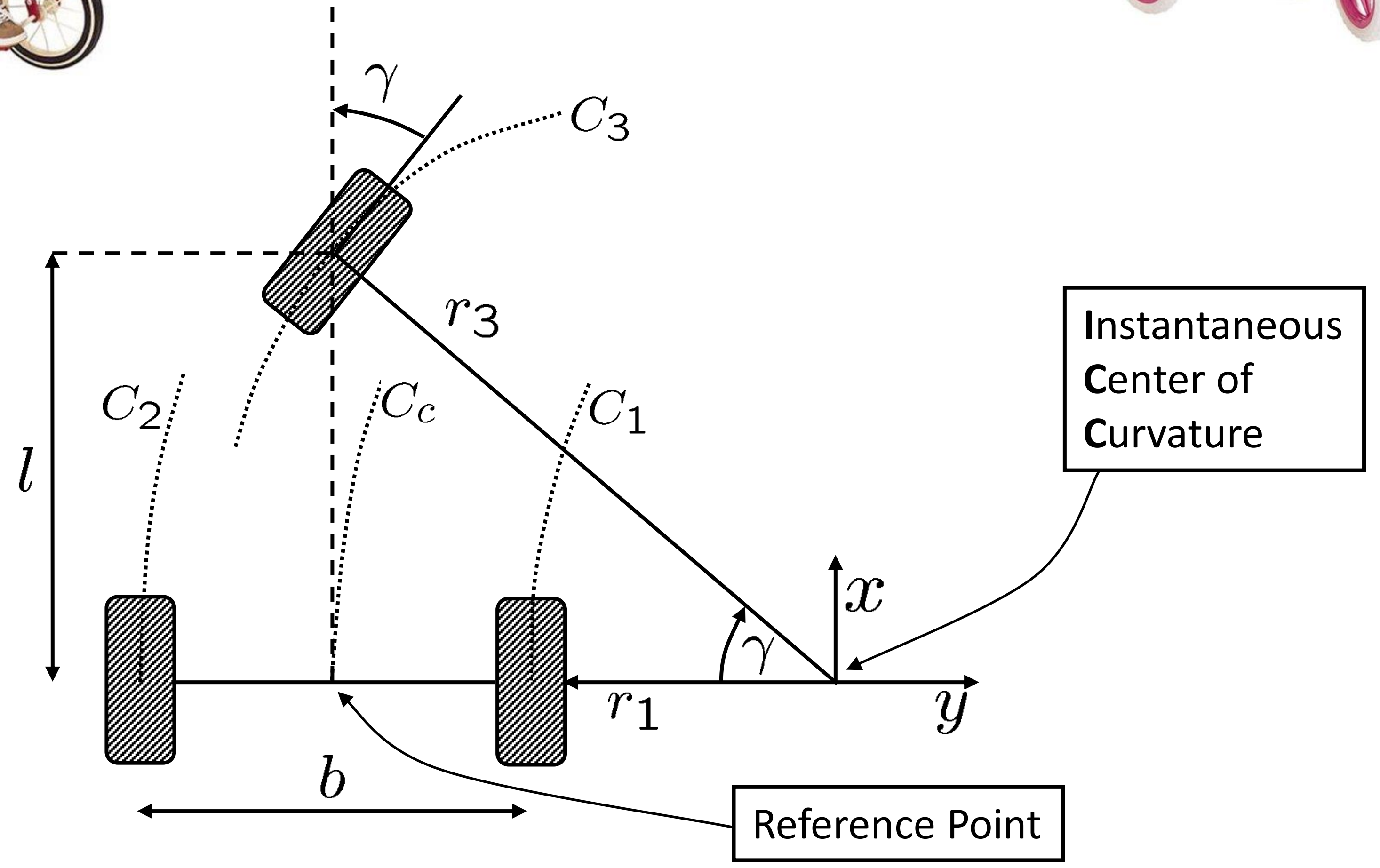
More Mobile Robot Kinematics in the Slides



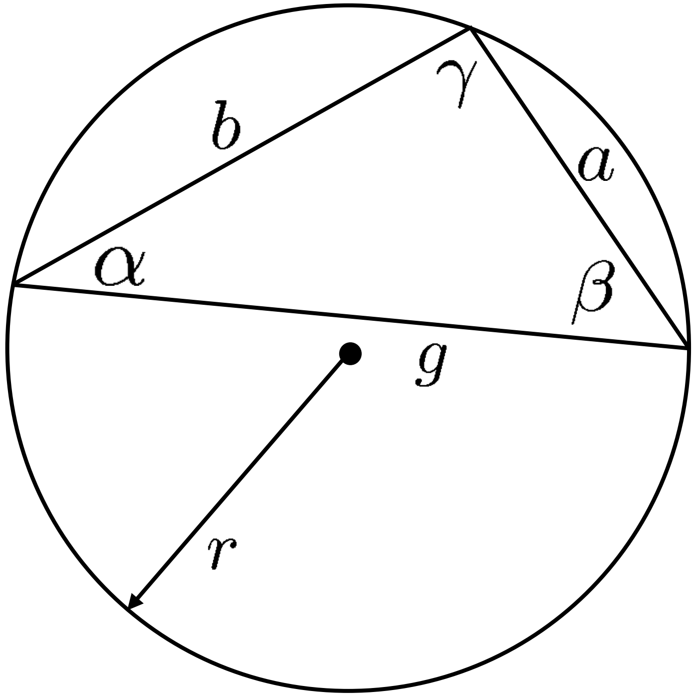
Tricycle

Ackermann Steering

Tricycle

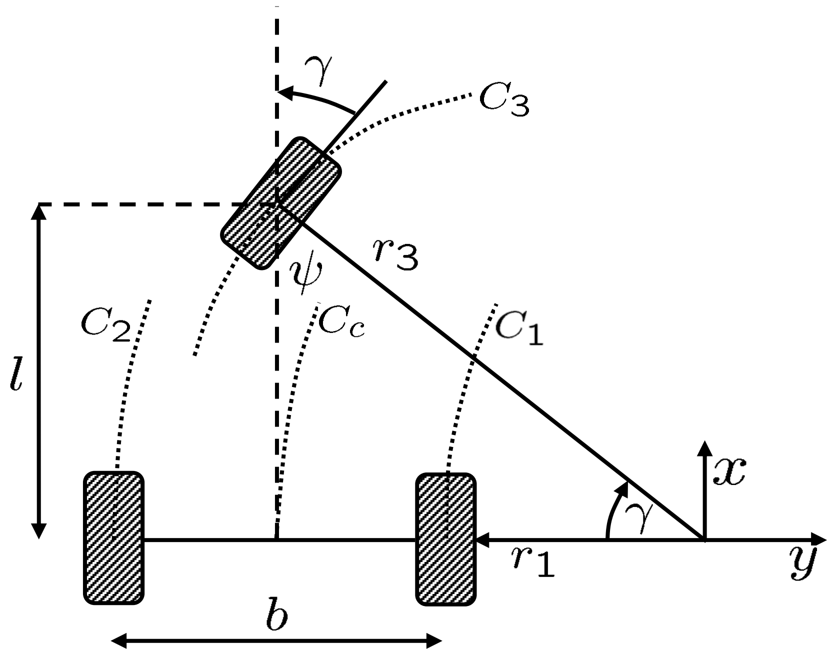

γ
C_3
r_3
Instantaneous Center of Curvature
C_2
C_c
C_1
l
x
γ
r_1
y
b
Reference Point

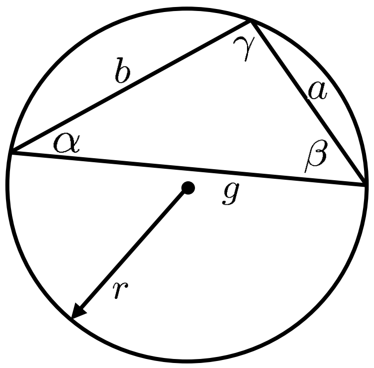
Sidebar: Law of Sines



$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{g}{\sin\gamma} = 2r$$

Tricycle cont.





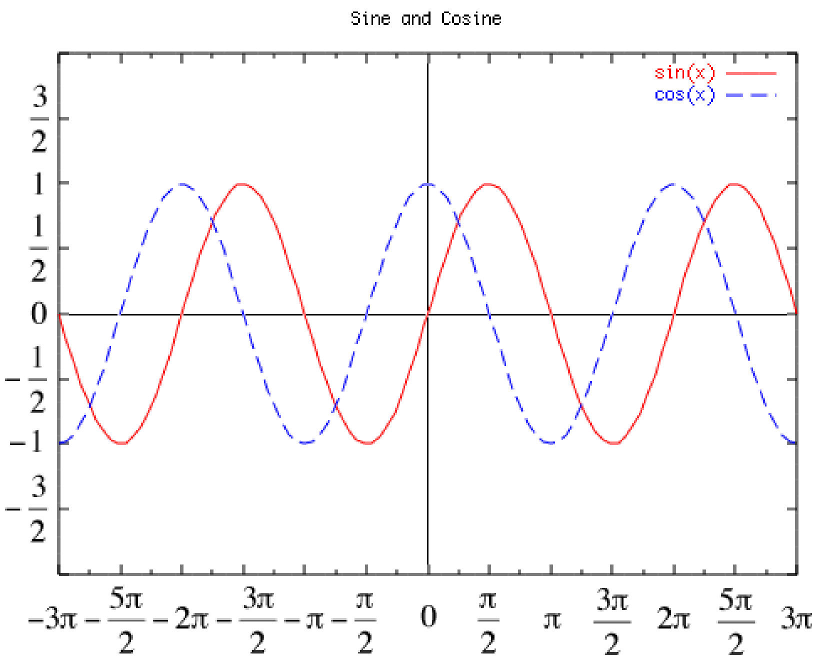
$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{g}{\sin\gamma} = 2r$$

$$\psi = \pi - \gamma - \frac{\pi}{2} = \frac{\pi}{2} - \gamma$$

$$\sin(\tfrac{\pi}{2} - \gamma) = -\sin(\gamma - \tfrac{\pi}{2}) = \cos\gamma$$

$$\frac{\sin\frac{\pi}{2}}{r_3} = \frac{\sin\gamma}{l} = \frac{\sin(\frac{\pi}{2}-\gamma)}{r_1+\frac{b}{2}} = \frac{\cos\gamma}{r_1+\frac{b}{2}}$$

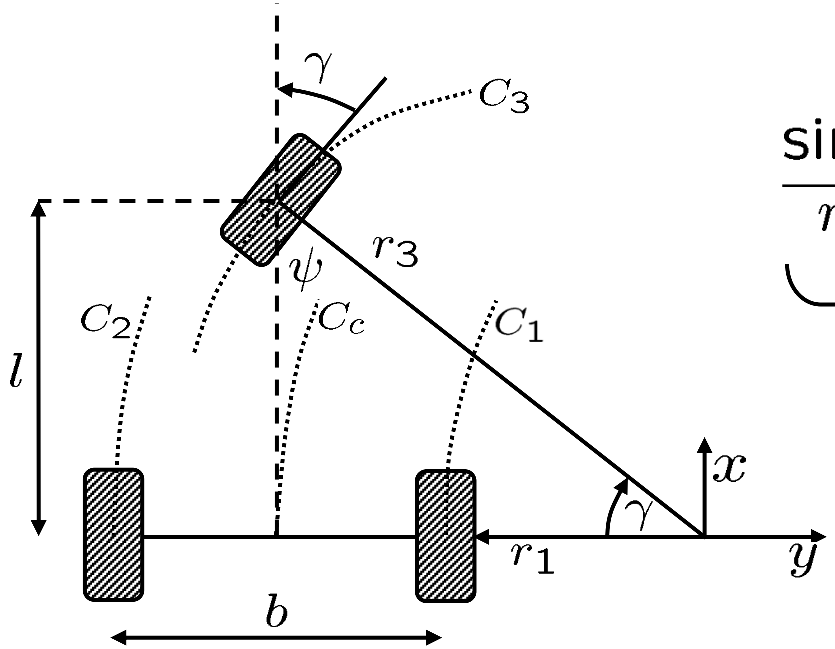
Sidebar: sin/cos identities



$$\sin\theta = -\sin(-\theta) = -cos(\theta + \frac{\pi}{2}) = \cos(\theta - \frac{\pi}{2})$$

$$\cos\theta = \cos(-\theta) = \sin(\theta + \frac{\pi}{2}) = -\sin(\theta - \frac{\pi}{2})$$

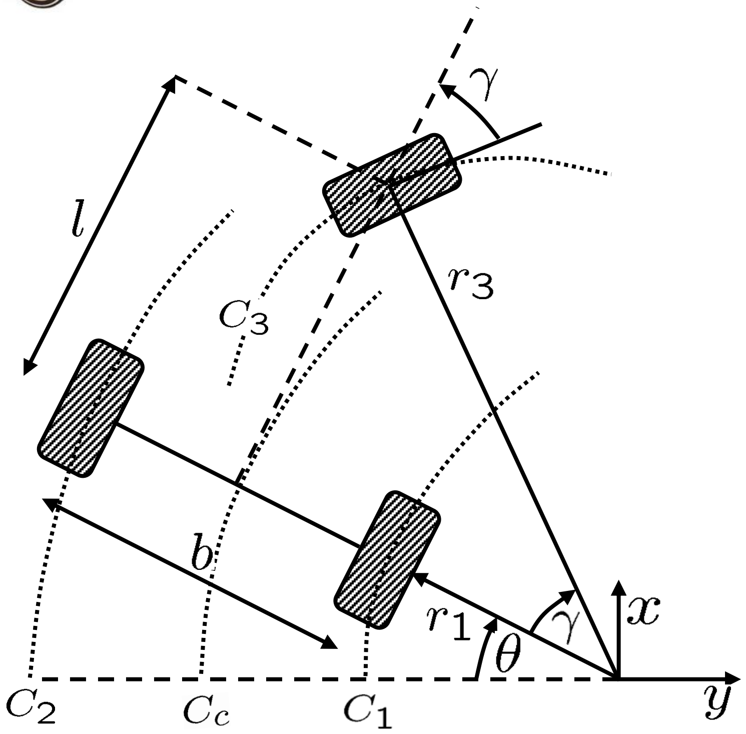
Tricycle cont. II



$$\underbrace{\frac{\sin\frac{\pi}{2}}{r_3} = \frac{\sin\gamma}{l}}_{r_3 = \frac{l}{\sin\gamma}} = \frac{\sin(\frac{\pi}{2}-\gamma)}{r_1+\frac{b}{2}} = \frac{\cos\gamma}{r_1+\frac{b}{2}}$$

$$\frac{\sin\gamma}{l} = \frac{\cos\gamma}{r_1+\frac{b}{2}} \Rightarrow r_1 = \frac{\cos\gamma}{\sin\gamma}l - \frac{b}{2}$$

Tricycle cont. III



$$r_1 = \frac{\cos\gamma}{\sin\gamma} l - \frac{b}{2}$$

$$r_3 = \frac{l}{\sin\gamma}$$

$$\theta = \frac{C_1}{r_1} \Rightarrow C_1 = \theta\, r_1\ , \quad C_2 = \theta(r_1 + b)$$

$$\theta = \frac{C_3}{r_3}$$

Tricycle cont. III

$$r_1 = \frac{\cos\gamma}{\sin\gamma} l - \frac{b}{2}$$

$$r_3 = \frac{l}{\sin\gamma}$$

$$\theta = \frac{C_3}{r_3}$$

$$C_1 = \theta r_1$$

$$C_2 = \theta (r_1 + b)$$

$$C_1 = \frac{C_3}{r_3}\left(\frac{\cos\gamma}{\sin\gamma} l - \frac{b}{2}\right)$$

$$C_1 = C_3\left(\cos\gamma - \frac{b}{2l}\sin\gamma\right)$$

$$C_2 = \frac{C_3}{r_3}\left(\frac{\cos\gamma}{\sin\gamma} l + \frac{b}{2}\right)$$

$$C_2 = C_3\left(\cos\gamma + \frac{b}{2l}\sin\gamma\right)$$

Tricycle cont. IV

$$C_1 = C_3 \left(\cos\gamma - \frac{b}{2l}\sin\gamma\right)$$

$$C_2 = C_3 \left(\cos\gamma + \frac{b}{2l}\sin\gamma\right)$$

From Differential Drive

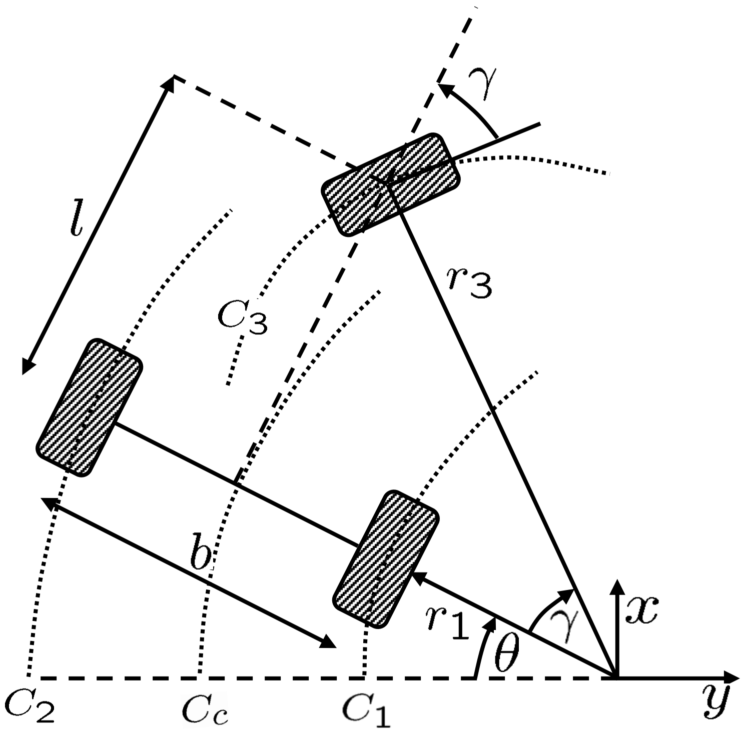
$$\left\{ \begin{aligned} \theta &= \frac{C_2 - C_1}{b} \\ r_1 &= b\frac{C_1}{C_2 - C_1} \end{aligned} \right.$$

$$\theta = \frac{C_3}{l}\sin\gamma$$

$$r_1 = l\,\frac{\cos\gamma}{\sin\gamma} - \frac{b}{2}$$

$$r_c = l\,\frac{\cos\gamma}{\sin\gamma}$$

Tricycle Summary



$$\theta = \frac{C_3}{l} \sin\gamma$$

$$r_1 = l\,\frac{\cos\gamma}{\sin\gamma} - \frac{b}{2}$$

$$r_2 = r_1 + b$$

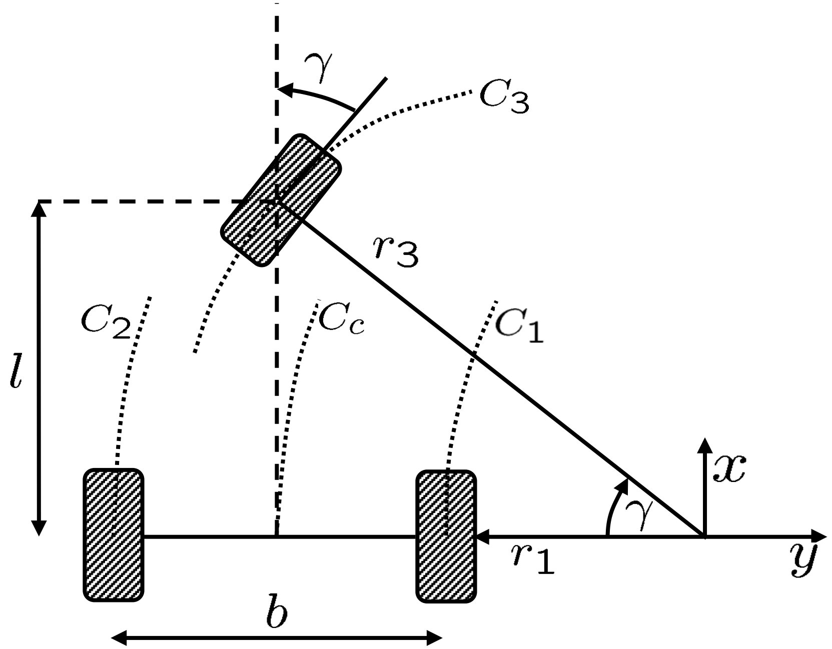
$$r_3 = \frac{l}{\sin\gamma}$$

$$r_c = l\,\frac{\cos\gamma}{\sin\gamma}$$

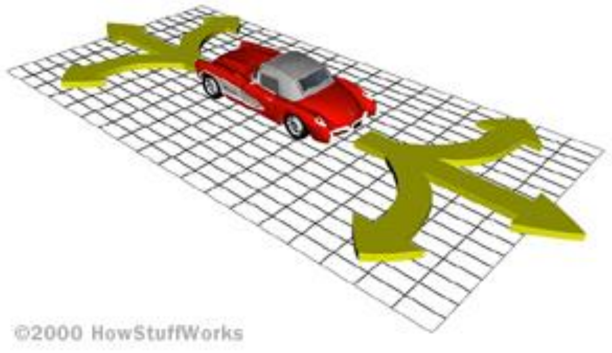
$$\omega = \frac{v_3}{l} \sin\gamma$$

$$v = \frac{v_1 + v_2}{2}$$

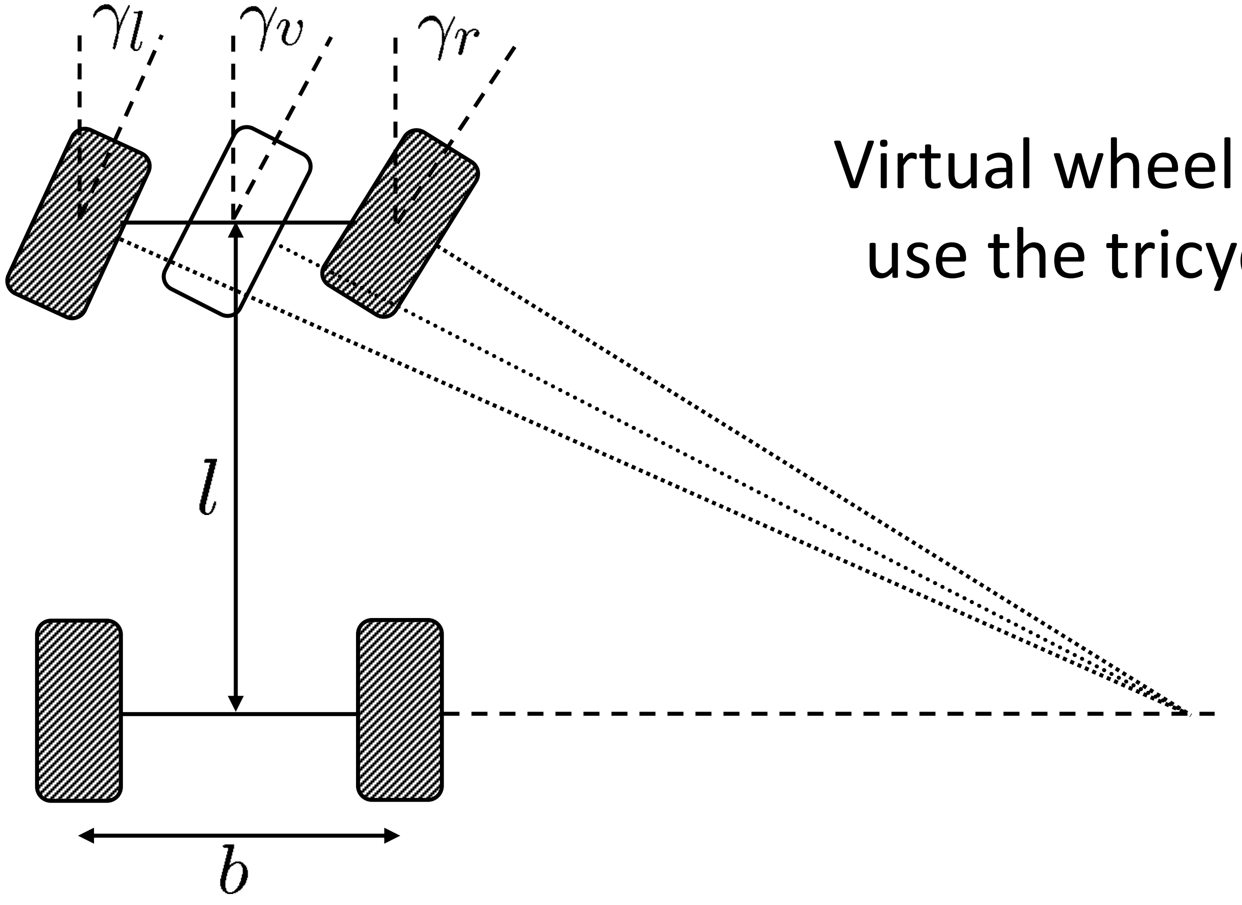
Tricycle in Local Frame



$$
\begin{aligned}
v_x(t) &= v_3(t)\cos\gamma(t) \\
v_y(t) &= 0 \\
\dot{\theta}(t) &= \frac{v_3(t)}{l}\sin\gamma(t)
\end{aligned}
$$

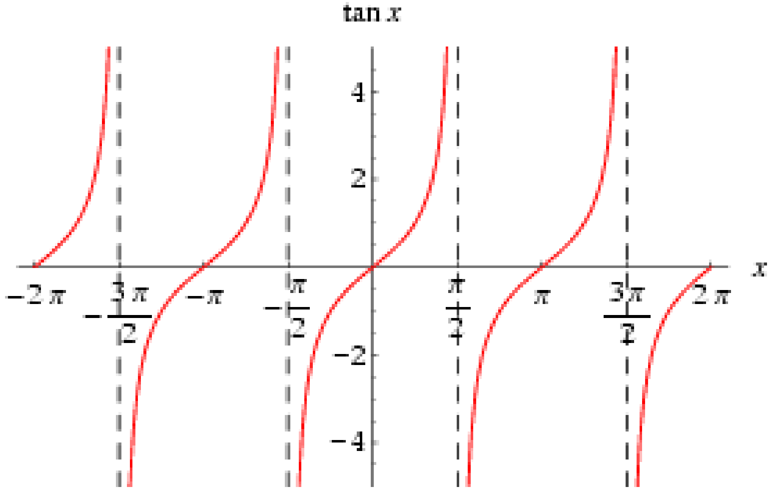


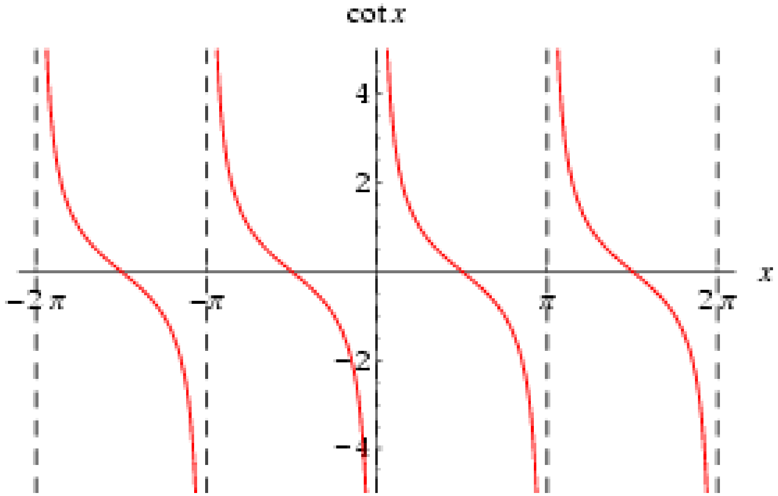
Ackerman Steering



Virtual wheel allows us use the tricycle case!

Sidebar: Cotangent



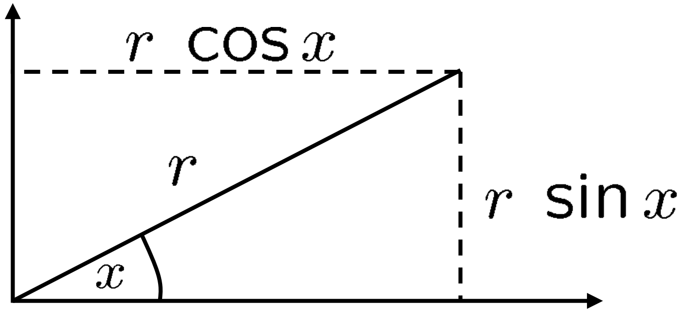


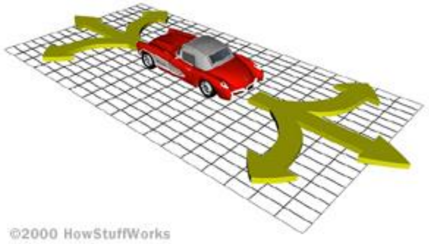
$$\tan x \equiv \frac{\sin x}{\cos x}$$

$$\cot x \equiv \frac{1}{\tan x}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{r\ \sin x}{r\ \cos x}$$

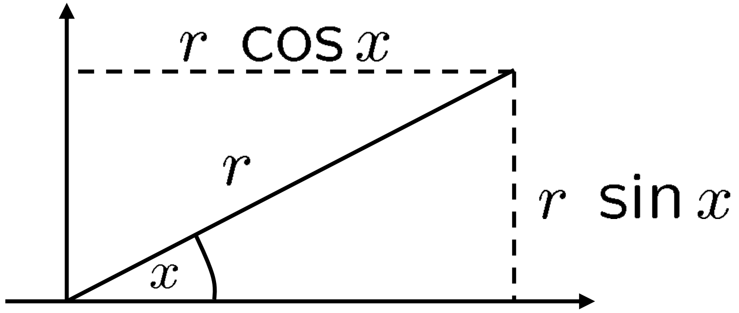
$$\cot x = \frac{\cos x}{\sin x} = \frac{r\ \cos x}{r\ \sin x}$$

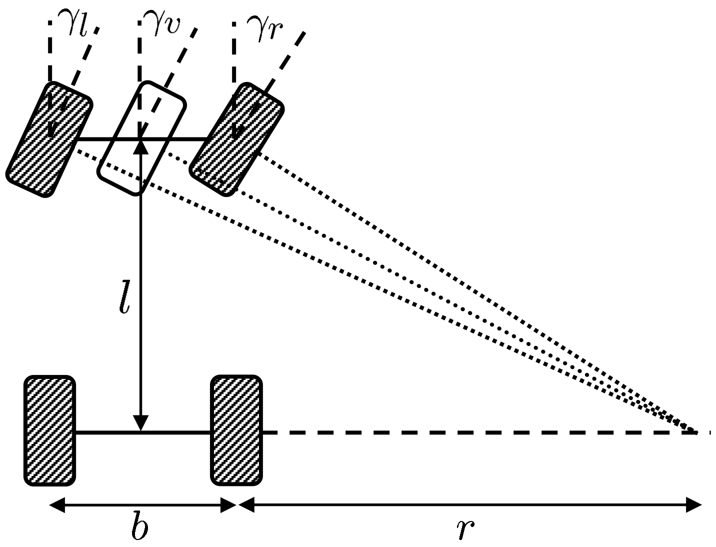




Ackerman Steering cont.

$$\cot x = \frac{\cos x}{\sin x} = \frac{r\ \cos x}{r\ \sin x}$$

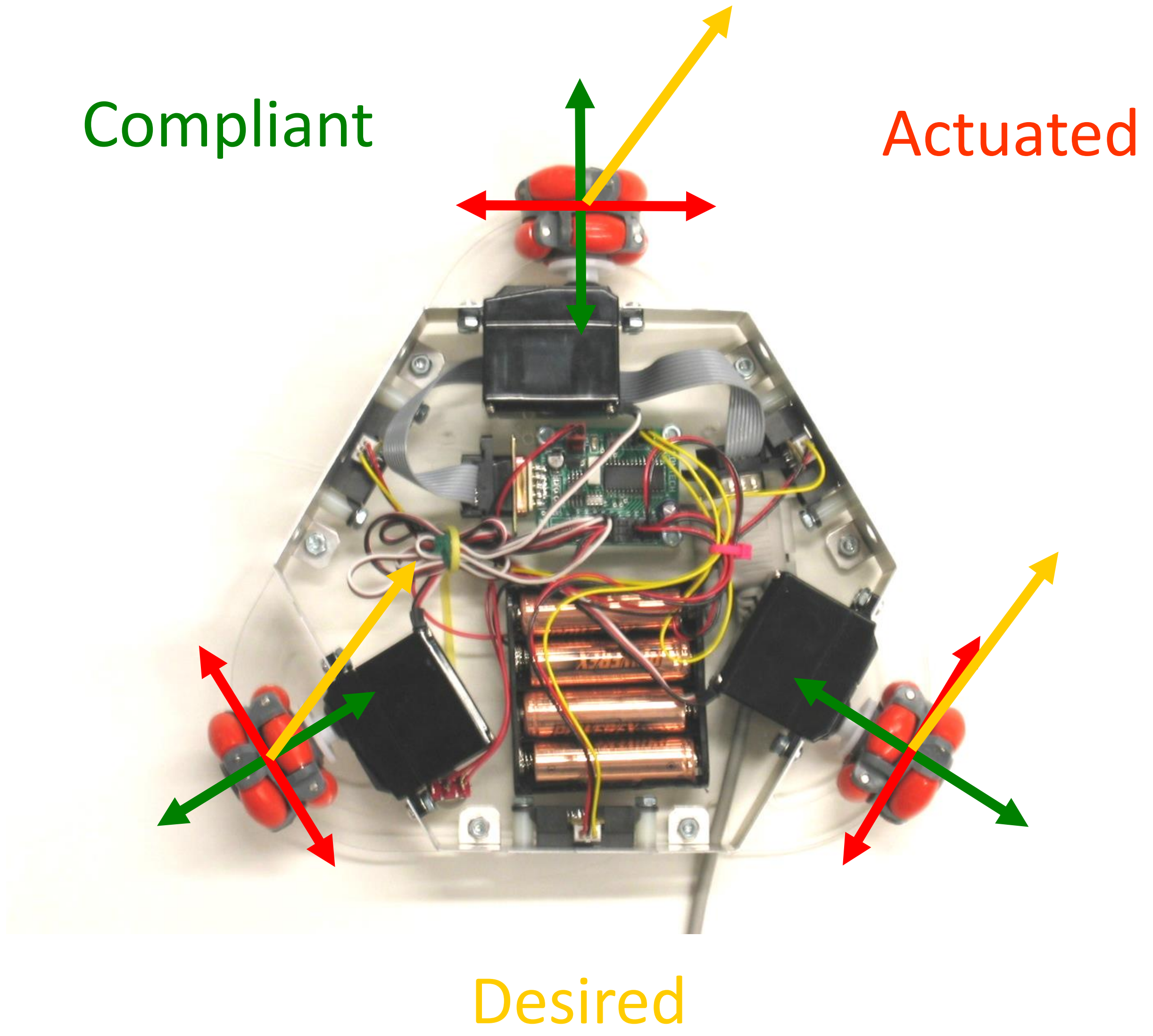




$$\begin{aligned}\cot \gamma_v = \frac{r + \frac{b}{2}}{l} &= \frac{r}{l} + \frac{b}{2l} \\ &= \cot \gamma_r + \frac{b}{2l} \\ &= \cot \gamma_l - \frac{b}{2l}\end{aligned}$$

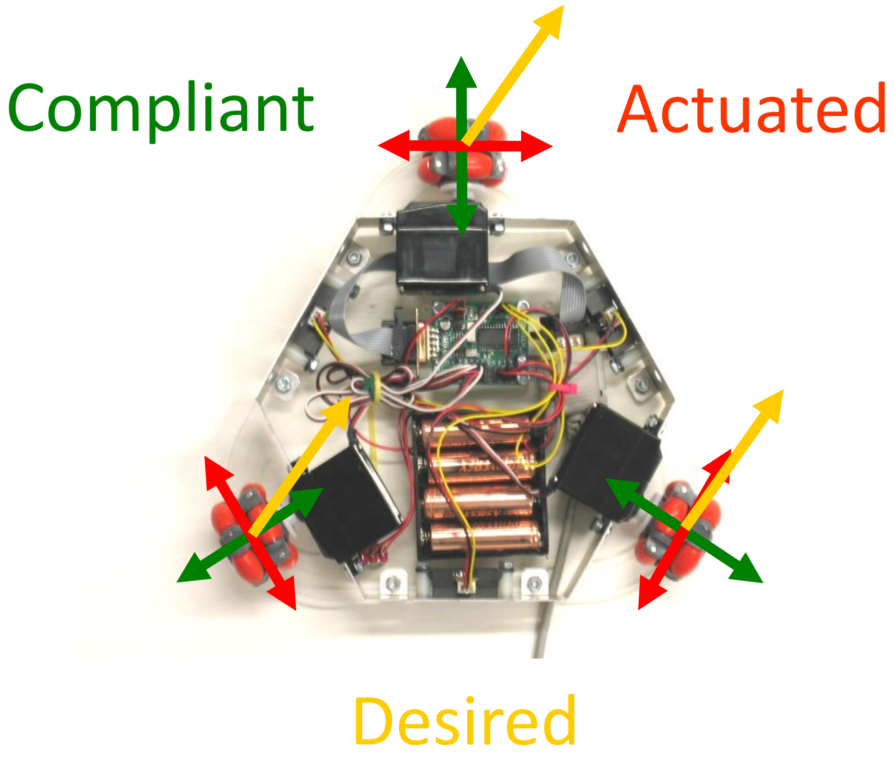
Now we can apply the tricycle case!

Achieving Holonomic Motion



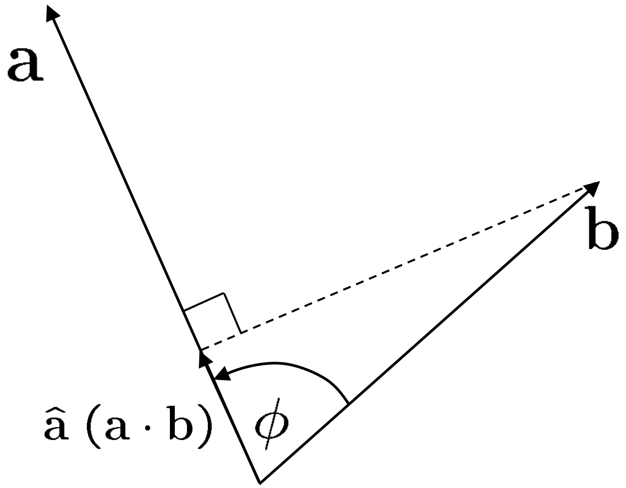
We need:

- Dot product
- Rolling Wheels



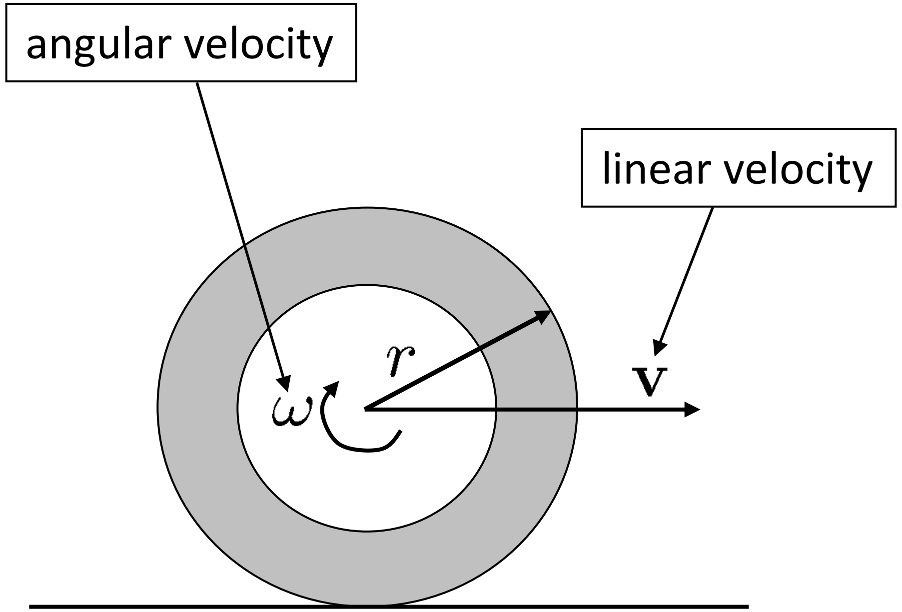
Sidebar: Dot Product

$$\mathbf{a} \cdot \mathbf{b} = \|\mathbf{a}\| \, \|\mathbf{b}\| \cos \phi$$
$$= \sum_{i=1}^{n} a_i b_i$$



$$\hat{\mathbf{a}} = \frac{\mathbf{a}}{\|\mathbf{a}\|}$$

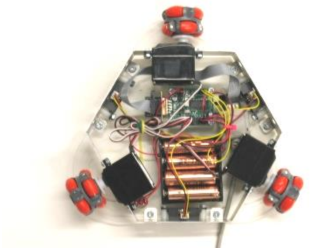
Sidebar: Rolling Wheels



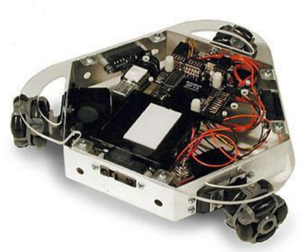
For a circle:

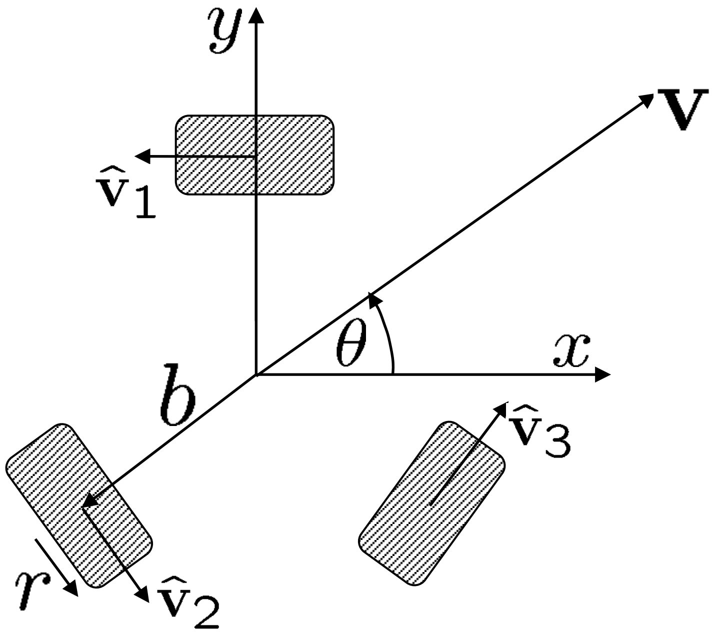
$$c = 2\pi r$$

$$\mathbf{v} = \omega \, r$$



Omniwheel





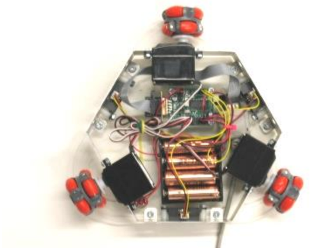
component of $\mathbf{v}$ along $\hat{\mathbf{v}}_1$: $\hat{\mathbf{v}}_1 \cdot \mathbf{v}$

expressed as angular velocity of the wheel: $\dfrac{\hat{\mathbf{v}} \cdot \mathbf{v}}{r}$

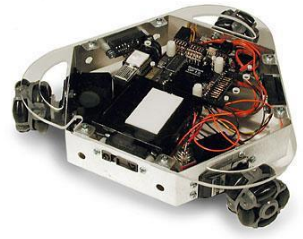
contribution of wheels to $\dot{\theta}$: $\dfrac{b\dot{\theta}}{r}$

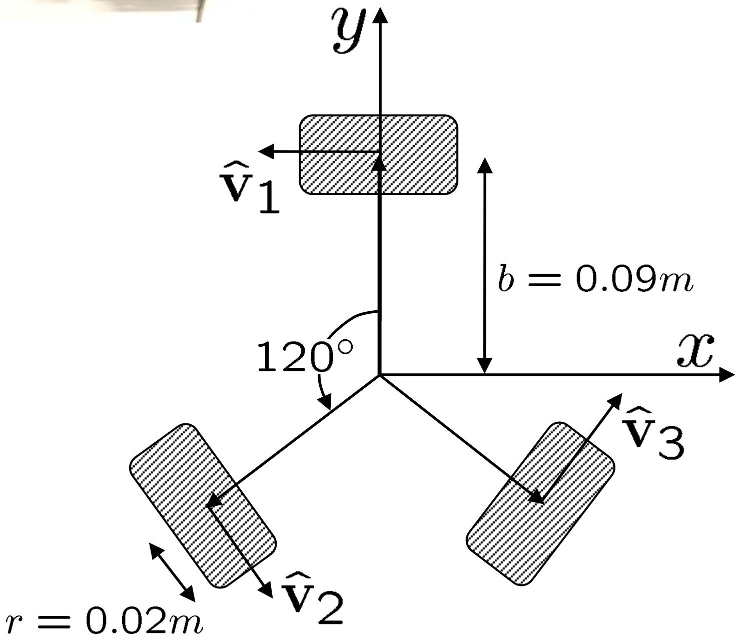
$$\begin{pmatrix} v_x \\ v_y \\ \dot{\theta} \end{pmatrix} \overset{?}{\Rightarrow} \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix}$$

$$\begin{aligned} \omega_1 &= \left(\hat{\mathbf{v}}_1 \cdot \mathbf{v} + b\,\dot{\theta}\right)/r \\ \omega_2 &= \left(\hat{\mathbf{v}}_2 \cdot \mathbf{v} + b\,\dot{\theta}\right)/r \\ \omega_3 &= \left(\hat{\mathbf{v}}_3 \cdot \mathbf{v} + b\,\dot{\theta}\right)/r \end{aligned}$$



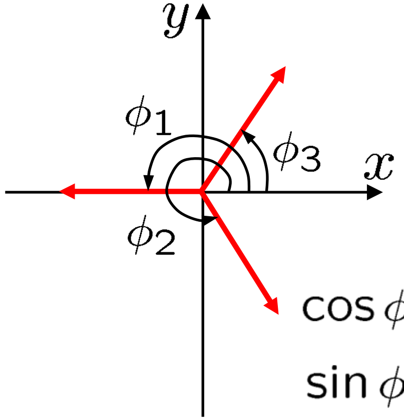
Delta Robot





$$\omega_1 = \left(\hat{\mathbf{v}}_1 \cdot \mathbf{v} + b\,\dot{\theta}\right)/r$$
$$\omega_2 = \left(\hat{\mathbf{v}}_2 \cdot \mathbf{v} + b\,\dot{\theta}\right)/r$$
$$\omega_3 = \left(\hat{\mathbf{v}}_3 \cdot \mathbf{v} + b\,\dot{\theta}\right)/r$$

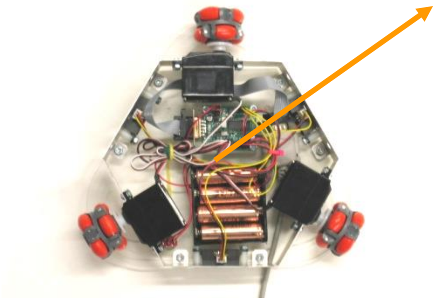
$$\begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} = \frac{1}{r} \begin{bmatrix} \cos\phi_1 & \sin\phi_1 & b \\ \cos\phi_2 & \sin\phi_2 & b \\ \cos\phi_3 & \sin\phi_3 & b \end{bmatrix} \begin{pmatrix} v_x \\ v_y \\ \dot{\theta} \end{pmatrix}$$



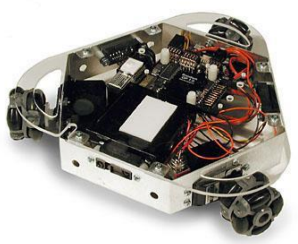
$$\begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} \approx 50 \begin{bmatrix} -1 & 0 & 0.09 \\ 0.5 & -0.866 & 0.09 \\ 0.5 & 0.866 & 0.09 \end{bmatrix} \begin{pmatrix} v_x \\ v_y \\ \dot{\theta} \end{pmatrix}$$

$\cos\phi_3 = 0.5$

$\sin\phi_3 = \frac{\sqrt{3}}{2} \approx 0.866$



Example



$$\begin{pmatrix} v_x \\ v_y \end{pmatrix} = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \|\mathbf{v}\|$$

$$= \begin{pmatrix} \cos 45^\circ \\ \sin 45^\circ \end{pmatrix} \sqrt{2} \cdot 0.05$$

$$= \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} \sqrt{2} \cdot 0.05$$

$$= \begin{pmatrix} 0.05 \\ 0.05 \end{pmatrix}$$

wheelbase = 0.09m
radius of wheels = 0.02m
desired velocity = $\sqrt{2}0.05$ m/s
desired heading = 45 degrees
desired angular velocity = 0.5 rad/s

$$\begin{pmatrix} -0.25 \\ 1.135 \\ 5.665 \end{pmatrix} = 50 \begin{bmatrix} -1 & 0 & 0.09 \\ 0.5 & -0.866 & 0.09 \\ 0.5 & 0.866 & 0.09 \end{bmatrix} \begin{pmatrix} 0.05 \\ 0.05 \\ 0.5 \end{pmatrix}$$

$$\begin{pmatrix} -4.75 \\ 3.165 \\ 1.165 \end{pmatrix} = 50 \begin{bmatrix} -1 & 0 & 0.09 \\ 0.5 & -0.866 & 0.09 \\ 0.5 & 0.866 & 0.09 \end{bmatrix} \begin{pmatrix} 0.05 \\ 0.05 \\ -0.5 \end{pmatrix}$$