Disclaimer

These slides are intended as presentation aids for the lecture. They contain information that would otherwise be to difficult or time-consuming to reproduce on the board. But they are incomplete, not self-explanatory, and are not always used in the order they appear in this presentation. As a result, these slides should not be used as a script for this course. I recommend you take notes during class, maybe on the slides themselves. It has been shown that taking notes improves learning success.

Reading for this set of slides

- Please refer to your textbooks from high school or the Internet
- For a textbook on probability see *Introduction to Probability* by Dimitri P. Bertsekas and John N. Tsitsiklis

Please note that this set of slides is intended as support for the lecture, not as a stand-alone script. If you want to study for this course, please use these slides in conjunction with the indicated chapters in the text books. The textbooks are available online or in the TUB library (many copies that can be checked out for the entire semester. There are also some aspects of the lectures that will not be covered in the text books but can still be part of the homework or exam. For those It is important that you attend class or ask somebody about what was covered in class.

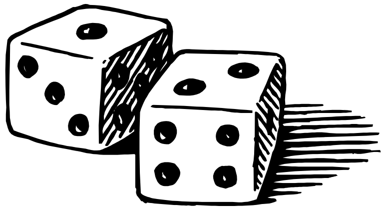
Robotics

Probability Review

TU Berlin

Oliver Brock

Probabilistic Model



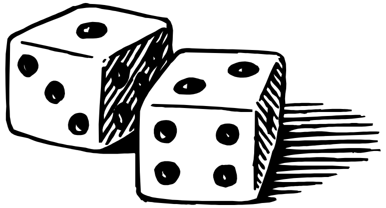
Sample Space $\Omega = \{1, 2, 3, 4, 5, 6\}$

Probability Law $L \ : \ A \subset \Omega \rightarrow 0 \leq \mathbf{P}(A) \leq 1$

$$
\begin{aligned}
\mathbf{P}(\{1\}) &= \frac{1}{6} \\
\mathbf{P}(\{2\}) &= \frac{1}{6} \\
\mathbf{P}(\{3\}) &= \frac{1}{6} \\
\mathbf{P}(\{1,2,3\}) &= \frac{1}{2} \\
\mathbf{P}(\{1,2,3,4,5,6\}) &= 1
\end{aligned}
$$

For a textbook on probability see *Introduction to Probability* by Dimitri P. Bertsekas and John N. Tsitsiklis

Probability Axioms



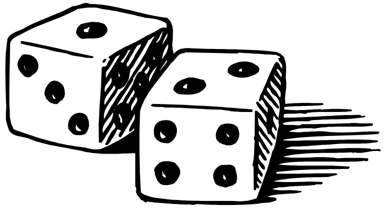
Nonnegativity $\forall A \subseteq \Omega \;:\; \mathbf{P}(A) \geq 0$

Additivity $\mathbf{P}(A \cup B) = \mathbf{P}(A) + \mathbf{P}(B)$ if $A \cap B = \emptyset$

Normalization $\mathbf{P}(\Omega) = 1$

Probability Law

(Properties)

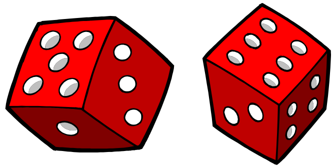


1. $A \subset B \;\Rightarrow\; \mathbf{P}(A) \leq \mathbf{P}(B)$

2. $\mathbf{P}(A \cup B) = \mathbf{P}(A) + \mathbf{P}(B) - \mathbf{P}(A \cap B)$

3. $\mathbf{P}(A \cup B) \leq \mathbf{P}(A) + \mathbf{P}(B)$

4. $\mathbf{P}(A \cup B \cup C) = \mathbf{P}(A) + \mathbf{P}(\bar{A} \cap B) + \mathbf{P}(\bar{A} \cap \bar{B} \cap C)$

Conditional Probability 

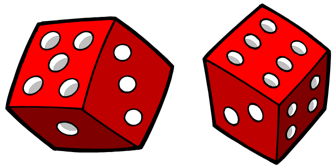
Conditional Probability allows us to reason about the outcome of an experiment based on partial information.

$$\text{Given:}\quad \overset{r_1}{\boxed{?}} + \overset{r_2}{\boxed{?}} = 9 \qquad \mathrm{P}(r_1 = 6) = ?$$

$$A_{\Sigma=9} = \{(3,6),(4,5),(5,4),(6,3)\}$$

$$\mathrm{P}(1^{\text{st}}\text{of two rolls is 6} \mid \text{sum of 2 rolls is 9}) = \tfrac{1}{4}$$

More Formally



$$\mathbf{P}(1^{\text{st}}\text{of two rolls is 6} \mid \text{sum of 2 rolls is 9}) = \frac{1}{4}$$

$$\mathbf{P}(A_{r_1=6} \mid A_{\Sigma=9}) = \frac{1}{4}$$

$$A_{r_1=6} = \{(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$$

$$A_{\Sigma=9} = \{(3,6),(4,5),(5,4),(6,3)\}$$

$$A_{r_1=6} \cap A_{\Sigma=9} = \{(6,3)\}$$

$$\boxed{\mathbf{P}(A|B) = \frac{\mathbf{P}(A \cap B)}{\mathbf{P}(B)}} = \frac{\mathbf{P}(A_{r_1=6} \cap A_{\Sigma=9})}{\mathbf{P}(A_{\Sigma=9})} = \frac{\frac{1}{36}}{\frac{4}{36}} = \frac{1}{4}$$

Another Example

- Fair coin is thrown 3 times
- A = {more heads than tails come up}
- B = {1st toss is a head}
- $P(A|B)$?
- $P(B) = 4/8$
- $P(A \cap B) = 3/8$
- $P(A|B) = 3/4$

Derivation of Bayes' Rule

Definition of Conditional Probability

$$\mathbf{P}(A|B) = \frac{\mathbf{P}(A \cap B)}{\mathbf{P}(B)} \qquad \mathbf{P}(B|A) = \frac{\mathbf{P}(B \cap A)}{\mathbf{P}(A)}$$

Multiplying both sides with denominator

$$\mathbf{P}(A \cap B) = \mathbf{P}(A|B)\,\mathbf{P}(B) \qquad \mathbf{P}(B \cap A) = \mathbf{P}(B|A)\,\mathbf{P}(A)$$

Set intersection is commutative

$$\mathbf{P}(A \cap B) = \mathbf{P}(B \cap A)$$

We equate the equations…

$$\mathbf{P}(B)\,\mathbf{P}(A|B) = \mathbf{P}(A)\,\mathbf{P}(B|A)$$

And devide by P(B) to arrive at Bayes' formula

$$\mathbf{P}(A|B) = \frac{\mathbf{P}(A)\,\mathbf{P}(B|A)}{\mathbf{P}(B)}$$

Interpretation of Bayes' Rule

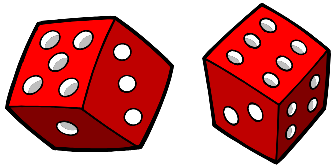
$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

$$P(\text{belief}|\text{sensory input}) = \frac{P(\text{sensory input}|\text{belief})P(\text{belief})}{P(\text{sensory input})}$$

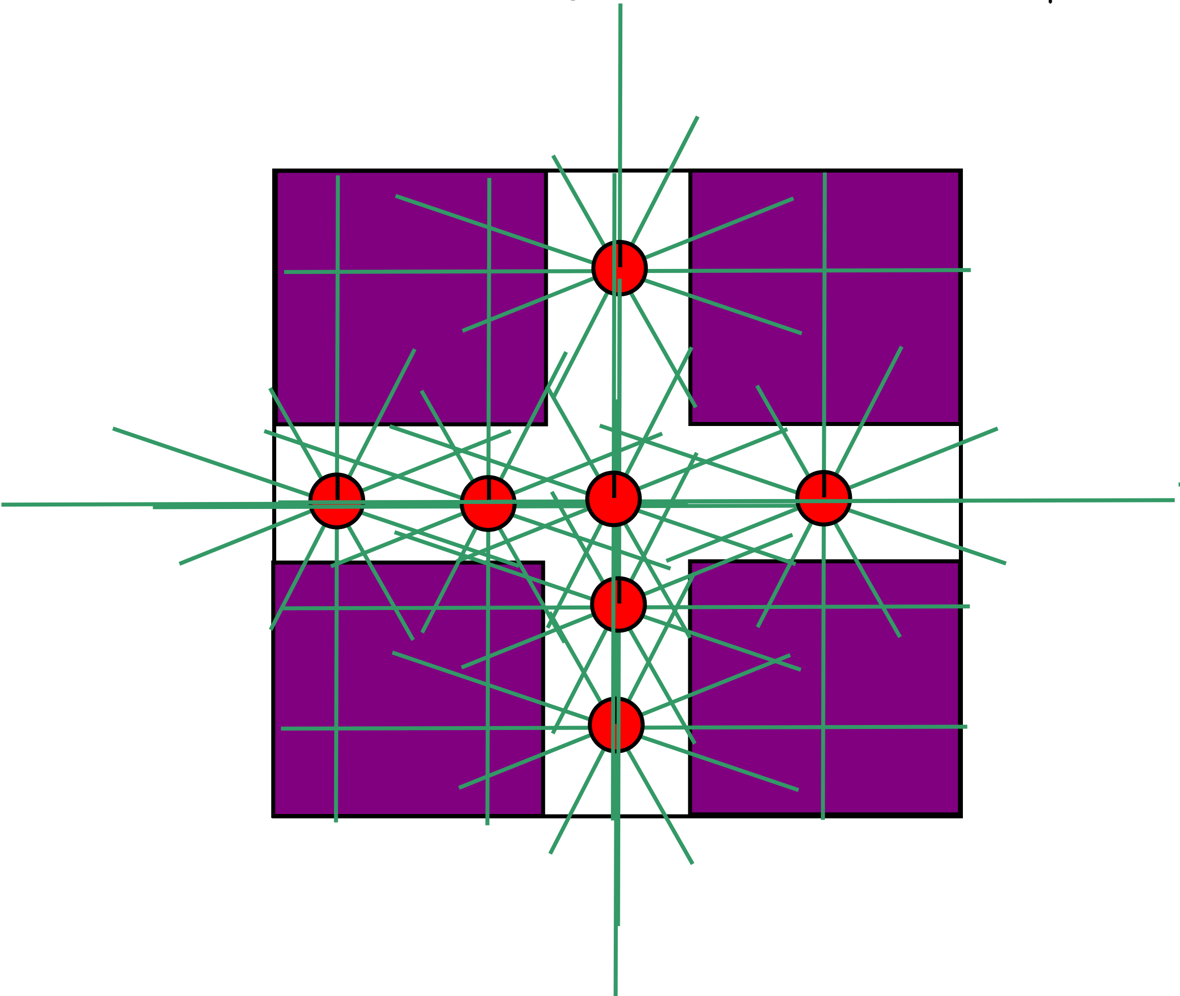
$$P(\text{model}|\text{data}) = \frac{P(\text{data}|\text{model})P(\text{model})}{P(\text{data})}$$

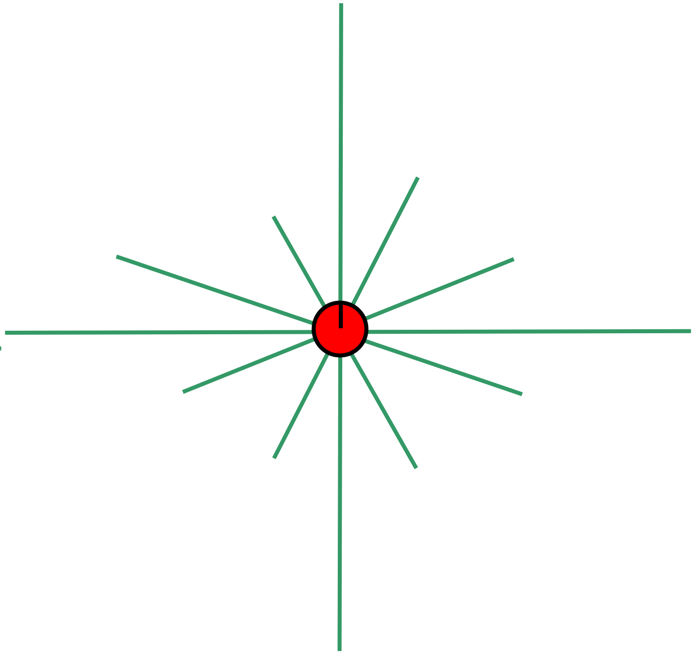
$$P(\text{hyptothesis}|\text{evindence}) = \frac{P(\text{evidence}|\text{hypothesis})P(\text{hypothesis})}{P(\text{evidence})}$$

Applying it...

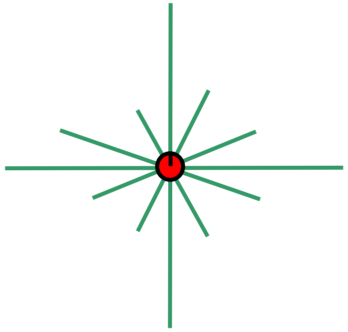


$$\mathrm{P}(\text{configuration} \mid \text{sensory information})$$





Problem!



$\mathrm{P}(\text{configuration} \mid \text{sensory information})$

Too Many Configurations!

Reversing the Condition

$$\mathbf{P}(A|B) = \frac{\mathbf{P}(A)\,\mathbf{P}(B|A)}{\mathbf{P}(B)}$$

Not all configurations!

(causal)

$$\mathbf{P}(\text{config}|\text{sensor}) = \frac{\mathrm{P}(\text{config})\,\mathrm{P}(\text{sensor}|\text{config})}{\mathrm{P}(\text{sensor})}$$

(diagnostic)

Summary

- Sample Space
- Probability Law
- Conditional Probability

$$\mathbf{P}(A|B) = \frac{\mathbf{P}(A \cap B)}{\mathbf{P}(B)}$$

- Bayes' Rule

$$\mathbf{P}(A|B) = \frac{\mathbf{P}(A)\,\mathbf{P}(B|A)}{\mathbf{P}(B)}$$