Disclaimer

These slides are intended as presentation aids for the lecture. They contain information that would otherwise be to difficult or time-consuming to reproduce on the board. But they are incomplete, not self-explanatory, and are not always used in the order they appear in this presentation. As a result, these slides should not be used as a script for this course. I recommend you take notes during class, maybe on the slides themselves. It has been shown that taking notes improves learning success.

Reading for this set of slides

- Craig – Intro to Robotics (3rd Edition)
  – Chapters 2 and 3

Please note that this set of slides is intended as support for the lecture, not as a stand-alone script. If you want to study for this course, please use these slides in conjunction with the indicated chapters in the text books. The textbooks are available online or in the TUB library (many copies that can be checked out for the entire semester. There are also some aspects of the lectures that will not be covered in the text books but can still be part of the homework or exam. For those It is important that you attend class or ask somebody about what was covered in class.

Robotics

Track a Shape with the End-Effector

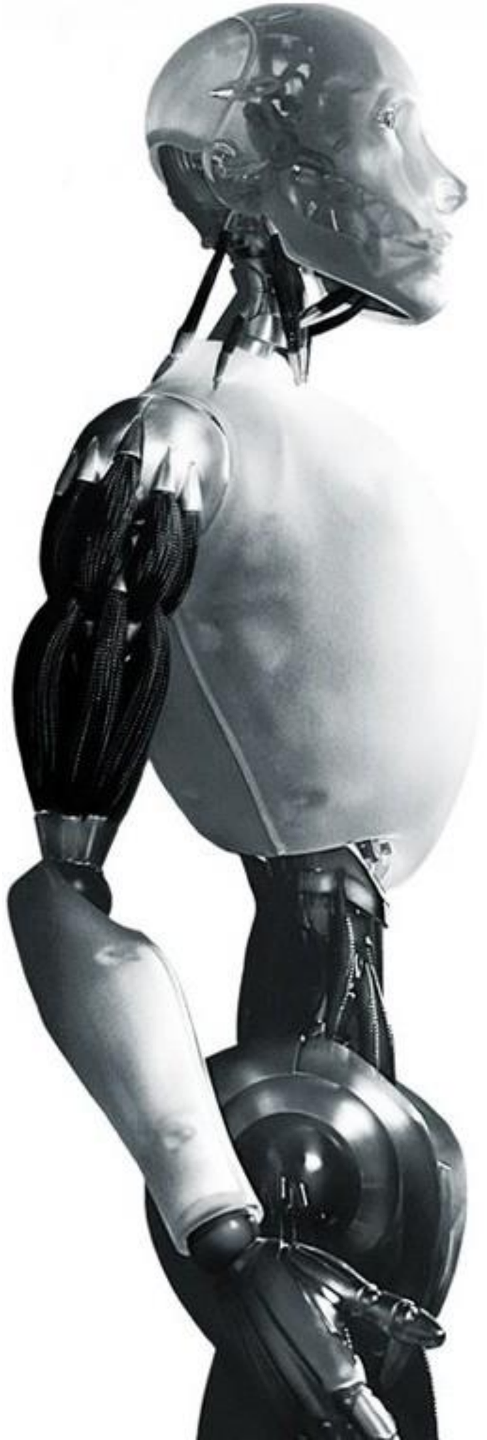
TU Berlin

Oliver Brock


PUMA 500
UNIMATE

What do we need now?

1. A way to specify a trajectory (task) for the end-effector
2. A control law to move the end-effector along the specified trajectory
3. But first: a way to determine the joint angles, given a desired pose of the end-effector – or: *a way to determine how to move the joints so that the error is reduced!*
4. To improve performance of the controller: dynamic compensation (feed-forward model of the robot's dynamics)

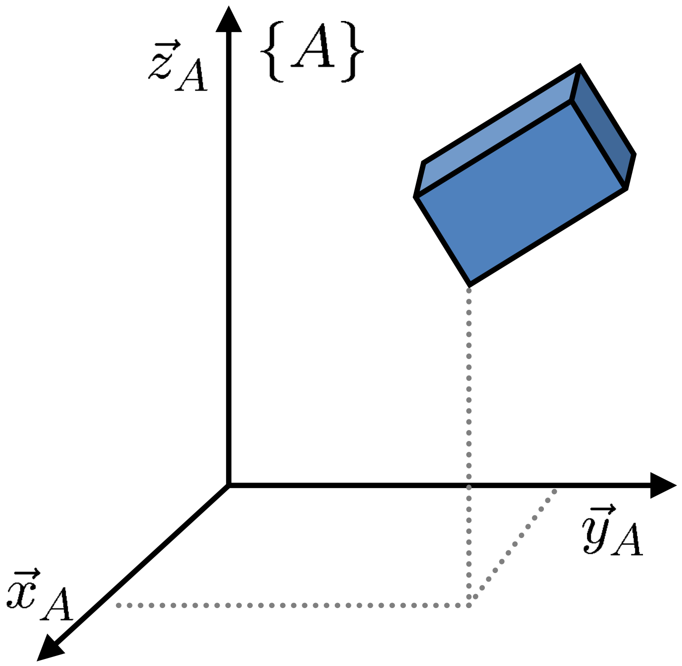


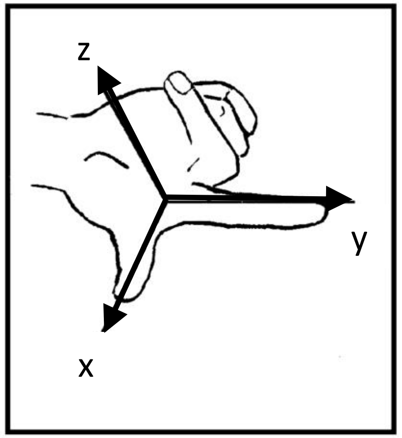
Robotics

Where is the End-Effector?
Forward Kinematics: Homogeneous Transforms, Spatial Representations

TU Berlin
Oliver Brock

Object Position and Orientation



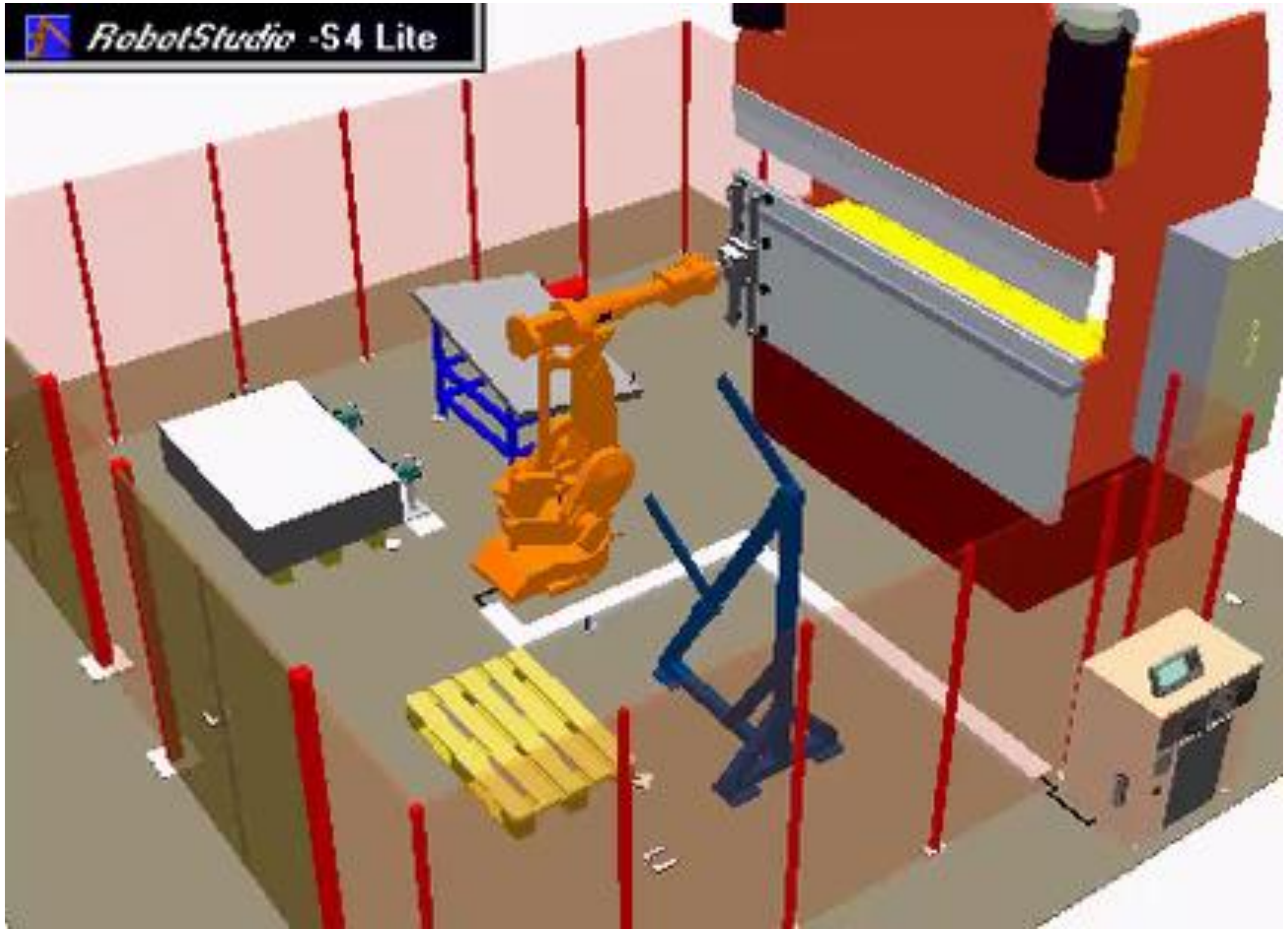


right-handed coordinate system

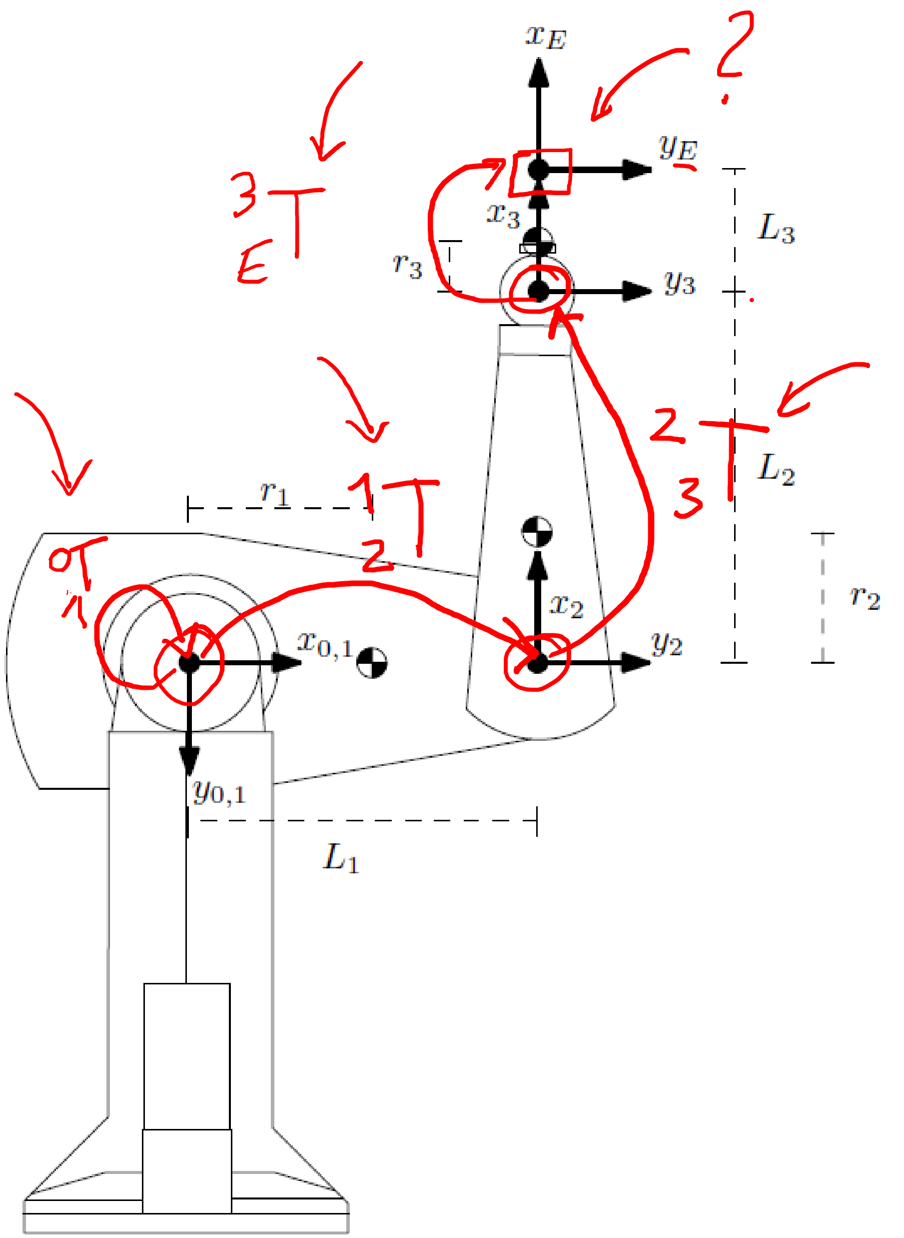
Kinematics

- Webster
  - Kinematics: branch of dynamics that deals with aspects of motion apart from considerations of mass and force
  - Dynamics: branch of mechanics that deals with forces and their relation primarily to the motion but sometimes also to the equilibrium of bodies
  - Mechanics: branch of physical science that deals with energy and forces and their effect on bodies
- Here: relationship between joint motion and motion of rigid bodies (links) without regard for the forces that cause it

Where is the end-effector?



Robert Bohlin



x_E
y_E
x_3
r_3
y_3
L_3
L_2
x_2
y_2
r_2
r_1
$x_{0,1}$
$y_{0,1}$
L_1
${}^{3}_{E}T$
${}^{2}_{3}T$
${}^{1}_{2}T$
${}^{0}_{1}T$
?

Configuration Space / Degrees of Freedom

- A configuration **q** is a minimal set of parameters required to uniquely specify the position and orientation (pose) of every point on the robot.

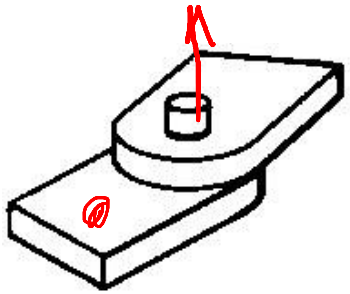
$$\mathbf{q} = \begin{pmatrix} q_1 \\ q_2 \\ \vdots \\ q_{n=6} \end{pmatrix} \qquad q_i = \begin{cases} \theta_i & \text{if dof } i \text{ is revolute} \\ d_i & \text{if dof } i \text{ is prismatic} \end{cases}$$

End-Effector Pose in Operational Space

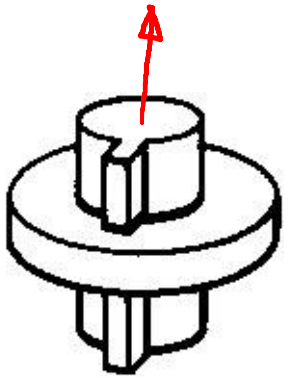
- A pose **x** is a (not necessarily minimal) set of parameters required to uniquely specify the position and orientation (pose) of the robot's end-effector. The choice of **x** depends on the task.

Example: $\mathbf{x} = \begin{pmatrix} x \\ y \\ z \\ \alpha \\ \beta \\ \gamma \end{pmatrix}$

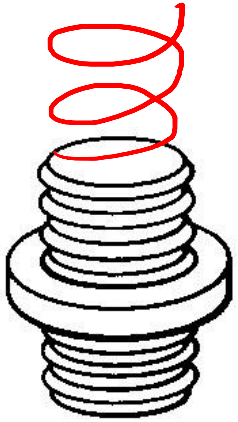
Types of Joints



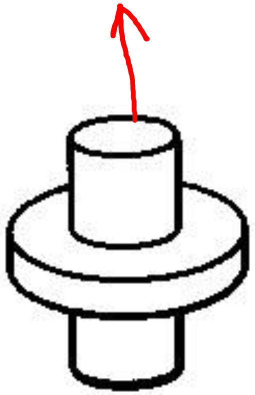
Revolute: 1 dof



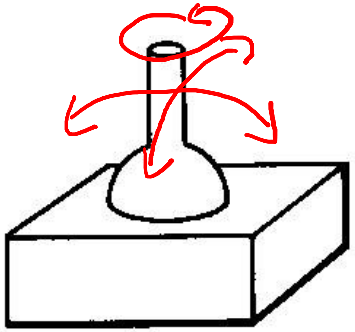
Prismatic: 1 dof



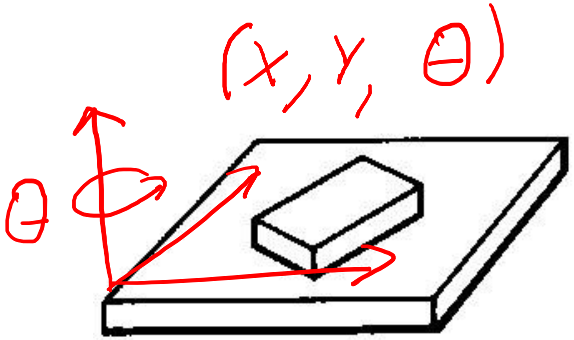
Screw: 1 dof



Cylindrical: 2 dof



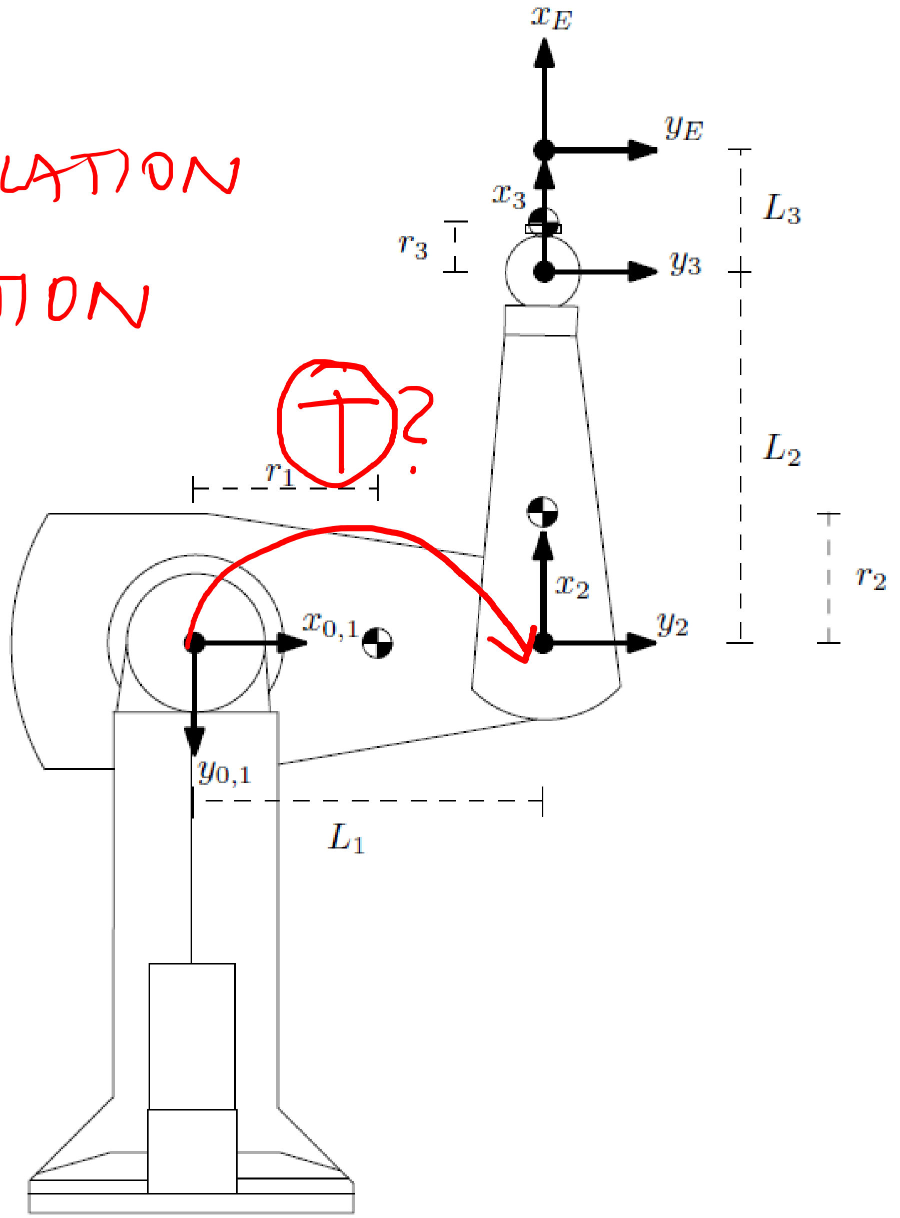
Spherical: 3 dof



Planar: 3 dof

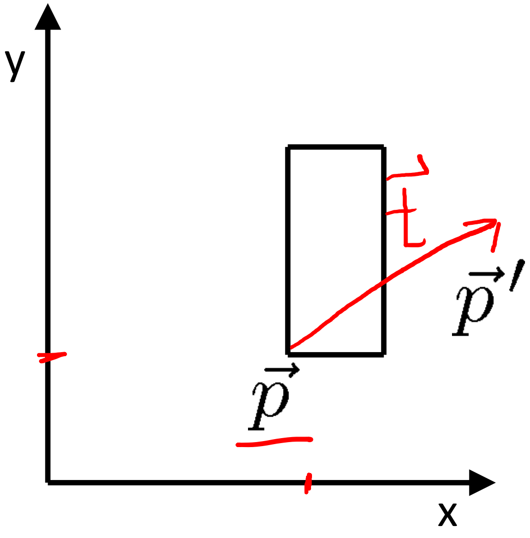
1) TRANSLATION

2) ROTATION



Translation

$$\vec{p} = \begin{pmatrix} p_x \\ p_y \end{pmatrix} \qquad \vec{t} = \begin{pmatrix} t_x \\ t_y \end{pmatrix}$$

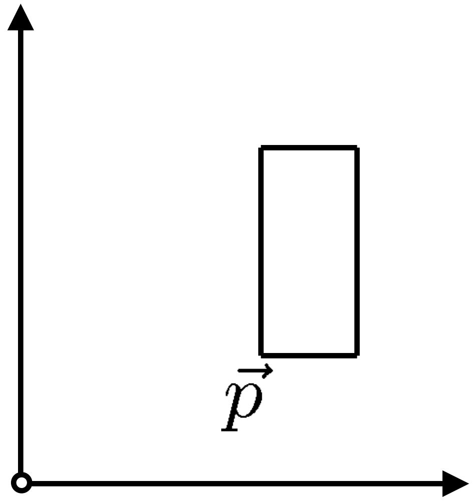


$$\vec{p}' = \vec{p} + \vec{t} = \begin{pmatrix} p_x + t_x \\ p_y + t_y \end{pmatrix}$$

Global Reference Coordinate System = World Frame

Rotation

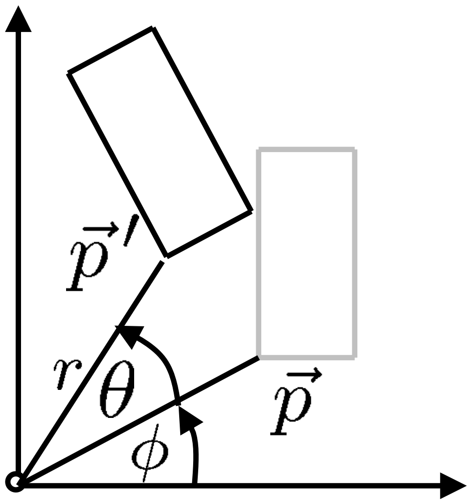
$$\vec{p} = \begin{pmatrix} p_x \\ p_y \end{pmatrix}$$



$$\vec{p}' = \text{?}$$

Deriving the Rotation Matrix

$$\begin{aligned} p_x &= r \cdot \cos\phi \\ p_y &= r \cdot \sin\phi \end{aligned}$$



$$\begin{aligned} p'_x &= r \cdot \cos(\theta + \phi) \\ &= r \cdot \cos\phi \cdot \cos\theta - r \cdot \sin\phi \cdot \sin\theta \\ &= p_x \cos\theta - p_y \sin\theta \\ p'_y &= r \cdot \sin(\theta + \phi) \\ &= r \cdot \cos\phi \cdot \sin\theta + r \cdot \sin\phi \cdot \cos\theta \\ &= p_x \sin\theta + p_y \cos\theta \end{aligned}$$

$$\vec{p}\,' = \left[\begin{array}{cc} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{array} \right] \cdot \vec{p}$$

$$\vec{p}\,' = R(\theta) \cdot \vec{p}$$

Useful equations for sines and cosines

$$\sin\theta = -\sin(-\theta) = -\cos\left(\theta + \frac{\pi}{2}\right) = \cos\left(\theta - \frac{\pi}{2}\right)$$

$$\cos\theta = \cos(-\theta) = \sin\left(\theta + \frac{\pi}{2}\right) = -\sin\left(\theta - \frac{\pi}{2}\right)$$

$$\begin{aligned}
\cos(\theta_1 + \theta_2) &= \cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2 \\
\sin(\theta_1 + \theta_2) &= \cos\theta_1 \sin\theta_2 + \sin\theta_1 \cos\theta_2 \\
\cos(\theta_1 - \theta_2) &= \cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2 \\
\sin(\theta_1 - \theta_2) &= \sin\theta_1 \cos\theta_2 - \cos\theta_1 \sin\theta_2
\end{aligned}$$

$$\cos^2\theta + \sin^2\theta = 1$$

Properties of Rotation Matrices

$$\vec{p}' = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \cdot \vec{p}$$

- Orthogonal
- Normal
- Determinant = 1
- $R^T = R^{-1}$

These properties also hold for rotation matrices in n dimensions!

Homogeneous Transformations

$$\vec{p}\,' = \vec{p} + \vec{t} \qquad\qquad \vec{p}\,' = R(\theta) \cdot \vec{p}$$

$$\vec{p}\,' = \underbrace{R(\theta) \cdot \vec{p}}_{\text{Rotation first!}} + \vec{t}$$

$$\begin{pmatrix} p'_x \\ p'_y \\ 1 \end{pmatrix} = \begin{bmatrix} \multicolumn{2}{c}{\multirow{2}{*}{$R(\theta)$}} & t_x \\ & & t_y \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{pmatrix} p_x \\ p_y \\ 1 \end{pmatrix}$$

3D Homogeneous Transforms

$$\begin{pmatrix} p'_x \\ p'_y \\ p'_z \\ 1 \end{pmatrix} = \begin{bmatrix} & & & t_x \\ & R(\theta) & & t_y \\ & & & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{pmatrix} p_x \\ p_y \\ p_z \\ 1 \end{pmatrix}$$

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \qquad R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Affine Transformations

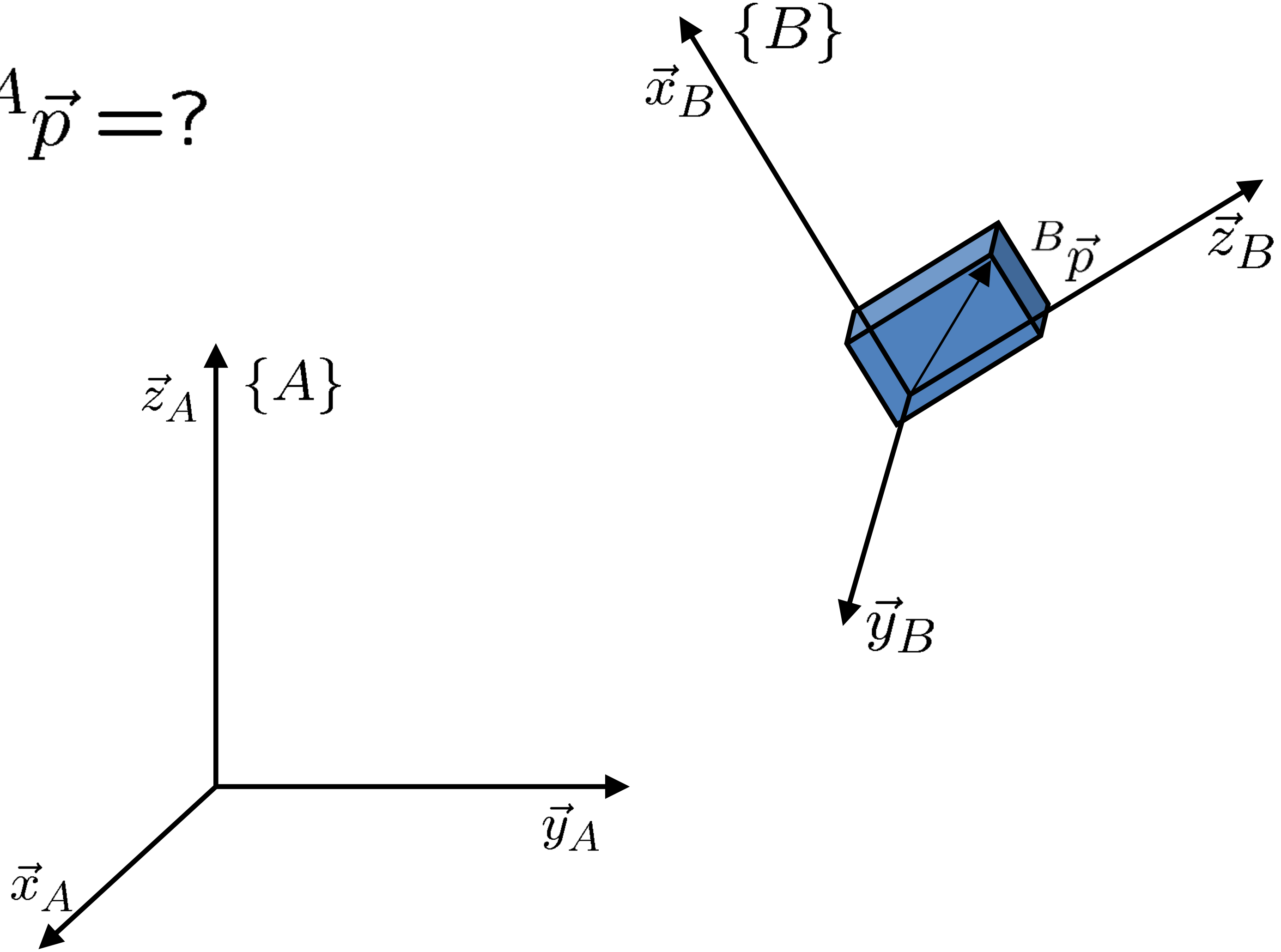
- Rotation
- Translation
- Scaling
- Sheering
- and any combination of them

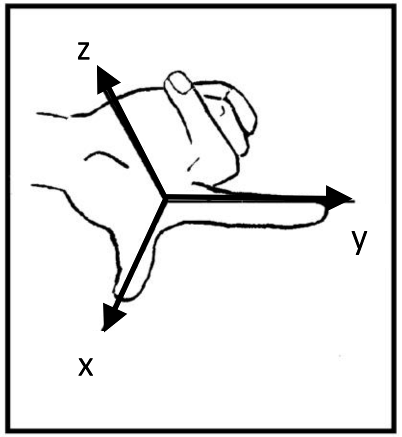
…are affine!

Affine transformations preserve collinearity and ratios of distances.

Frames

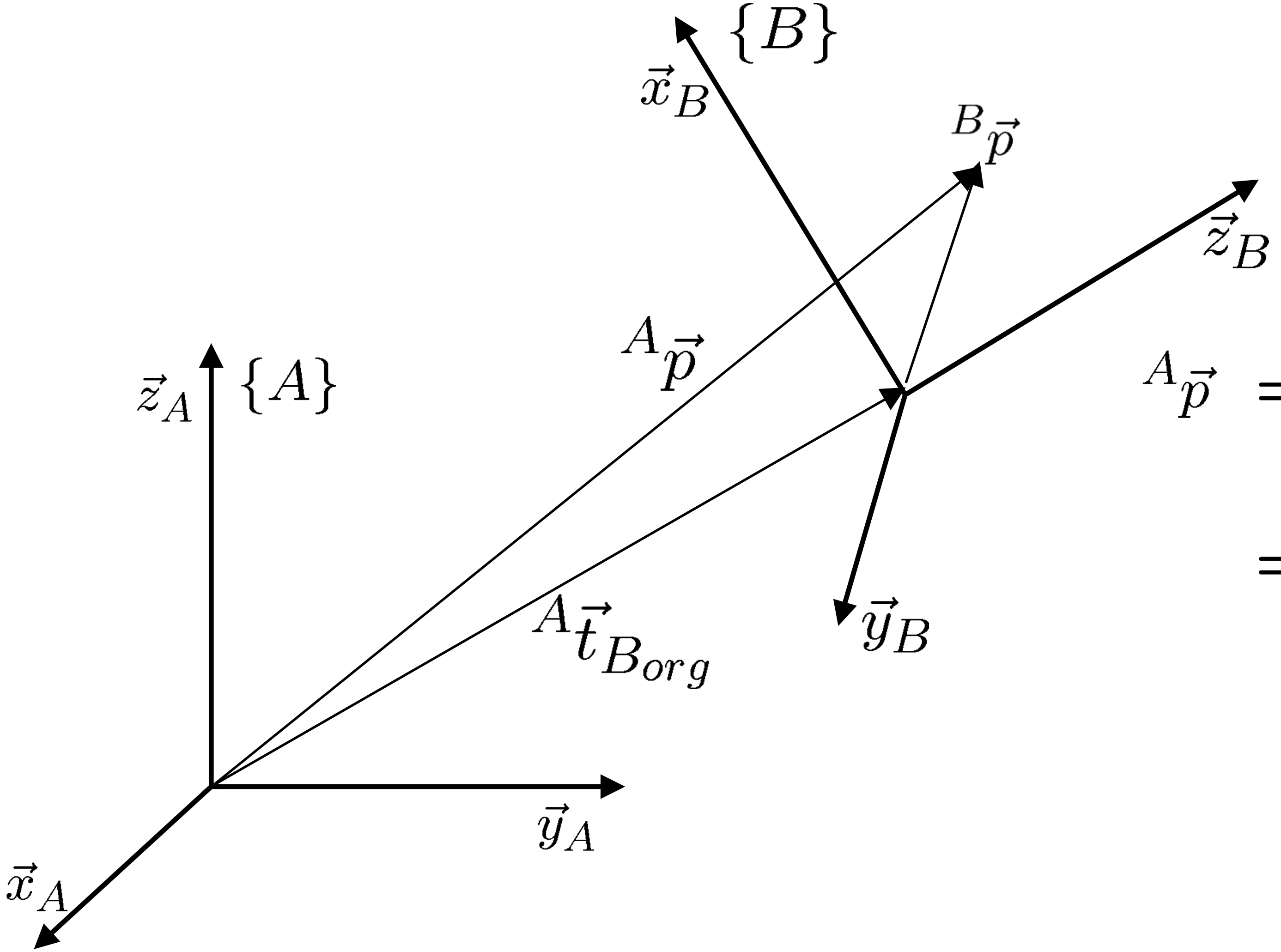
$$ {}^{A}\vec{p} = ? $$





right-handed coordinate system

HT to Map between Frames



$$
\begin{aligned}
{}^A\vec{p} &= {}^A_BT \; {}^B\vec{p} \\
&= \left[\begin{array}{cccc} & {}^A_BR & & {}^A\vec{t}_{B_{org}} \\ 0 & 0 & 0 & 1 \end{array} \right] {}^B\vec{p}
\end{aligned}
$$

How to compute R?

$$
{}^{A}_{B}R = \begin{bmatrix} {}^{A}\widehat{x}_B & {}^{A}\widehat{y}_B & {}^{A}\widehat{z}_B \end{bmatrix}
$$

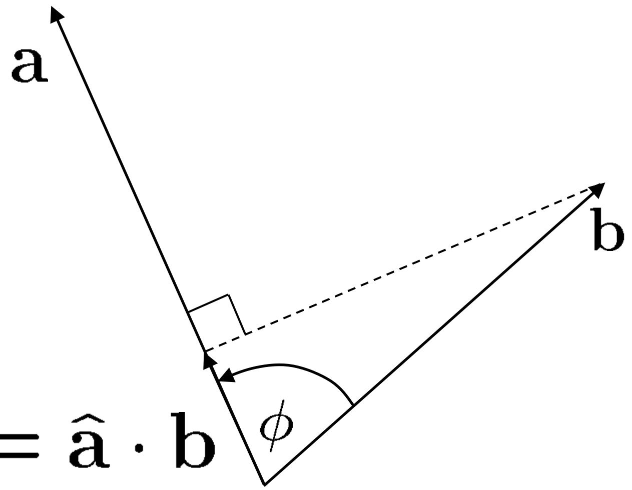
$$
= \begin{bmatrix} \widehat{x}_B \cdot \widehat{x}_A & \widehat{y}_B \cdot \widehat{x}_A & \widehat{z}_B \cdot \widehat{x}_A \\ \widehat{x}_B \cdot \widehat{y}_A & \widehat{y}_B \cdot \widehat{y}_A & \widehat{z}_B \cdot \widehat{y}_A \\ \widehat{x}_B \cdot \widehat{z}_A & \widehat{y}_B \cdot \widehat{z}_A & \widehat{z}_B \cdot \widehat{z}_A \end{bmatrix}
$$

Direction Cosines

Sidebar: Dot Product (Inner Product)

$$\mathbf{a} \cdot \mathbf{b} = \|\mathbf{a}\| \, \|\mathbf{b}\| \, \cos\phi$$

$$= \sum_{i=1}^{n} a_i b_i$$

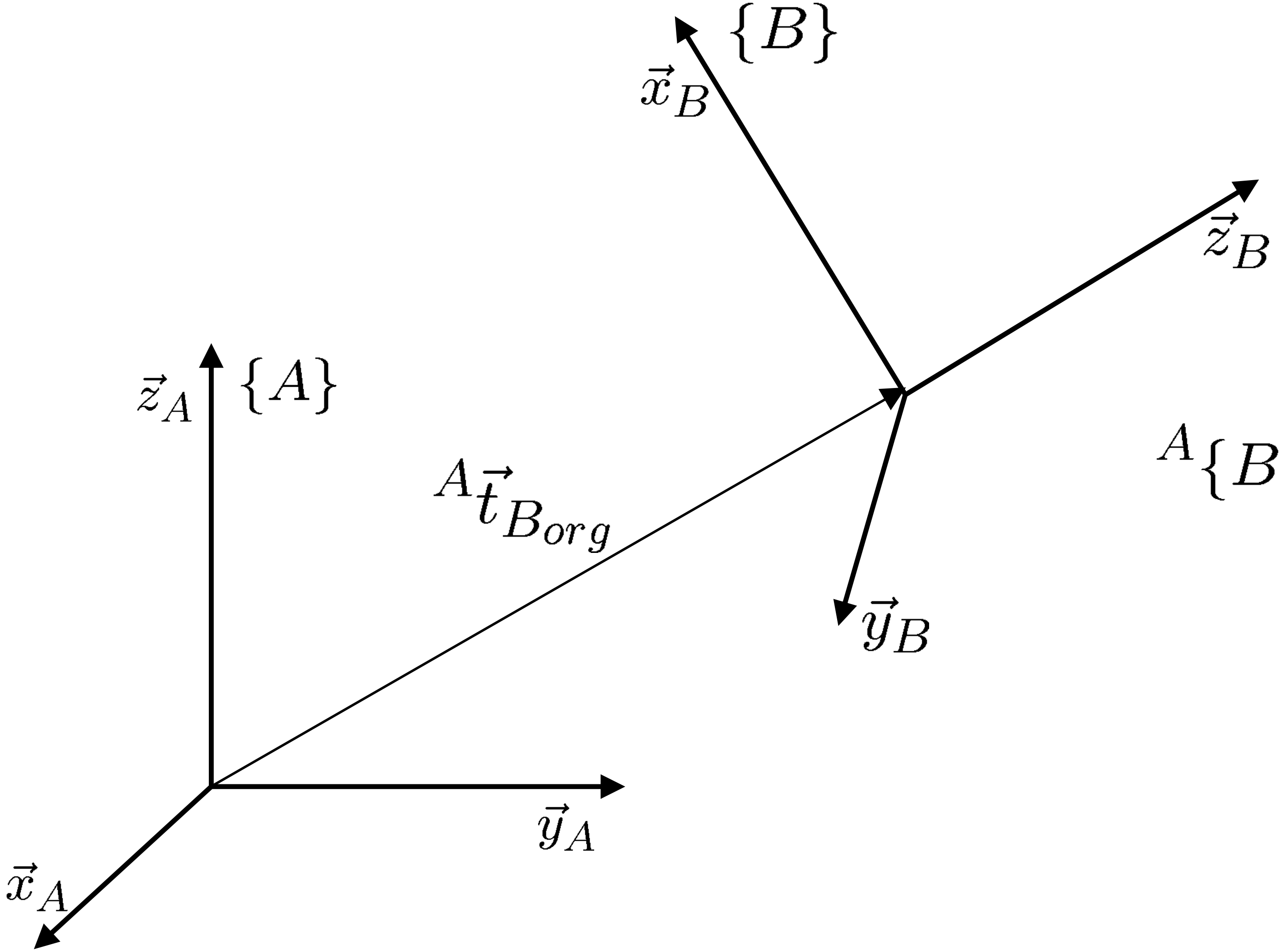


$$\mathbf{b_a} = \hat{\mathbf{a}} \cdot \mathbf{b}$$

$$\hat{\mathbf{a}} = \frac{\mathbf{a}}{\|\mathbf{a}\|}$$

$$\mathbf{a} \cdot \mathbf{b} = \mathbf{a_b} \, \|\mathbf{b}\| = \mathbf{b_a} \, \|\mathbf{a}\|$$

HT as Frame Descriptions



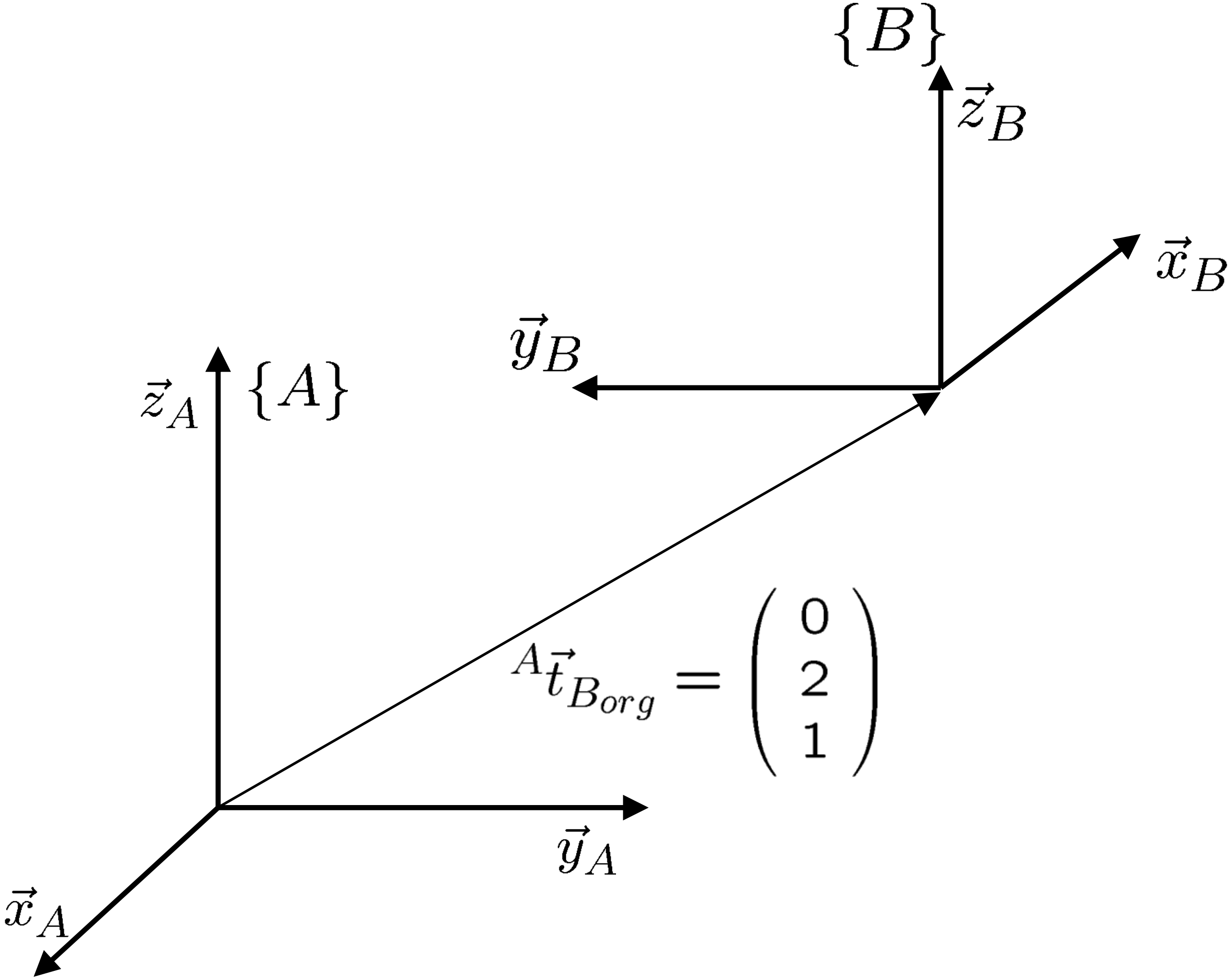
$$ {}^A\{B\} = \begin{bmatrix} & {}^A_BR & & {}^A\vec{t}_{B_{org}} \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Three Uses for HTs

1. Frame Description
2. Mapping between frames
3. Transform operator

We will be somewhat loose about vectors and their homogeneous counterparts.

HT: Describing a Frame

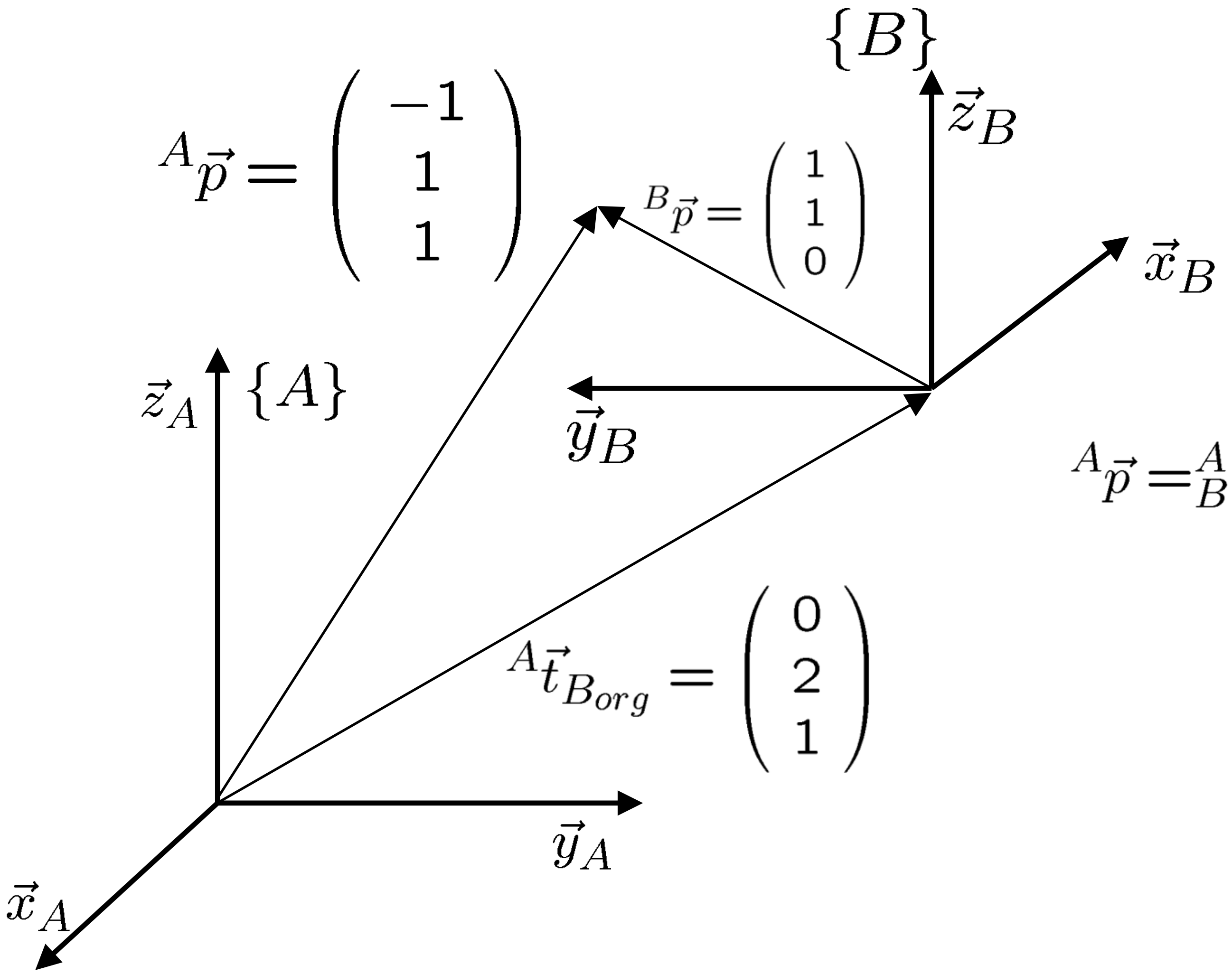


$${}^{A}\vec{t}_{B_{org}} = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$$

$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$${}^{A}\{B\} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

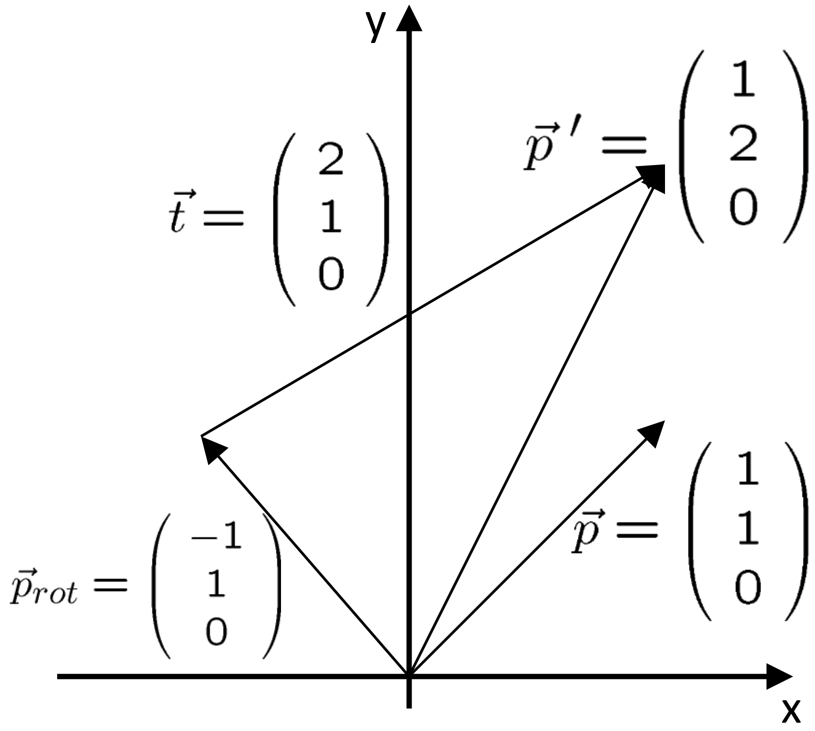
HT: Mapping between Frames



$$^{A}\vec{p} = {}^{A}_{B}T \; {}^{B}\vec{p} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

HT: Transforming a point

Rotate by $\pi/2$ about z and translate by (2,1,0)



$$\begin{aligned}
\vec{p}\,' &= \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 2 \\ \sin\theta & \cos\theta & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix} \\
&= \begin{bmatrix} 0 & -1 & 0 & 2 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix} \\
&= \begin{pmatrix} 1 \\ 2 \\ 0 \\ 1 \end{pmatrix}
\end{aligned}$$

Computing with HTs

WORLD FRAME

$$ {}^{A}_{F}T = \overbrace{{}^{A}_{\cancel{B}}T \; {}^{\cancel{B}}_{\cancel{C}}T \; {}^{\cancel{C}}_{\cancel{D}}T \; {}^{\cancel{D}}_{\cancel{E}}T \; {}^{\cancel{E}}_{F}T}^{*} $$

E-E

$$ {}^{A}\vec{p} = {}^{A}_{B}T \; {}^{B}_{C}T \; {}^{C}\vec{p} $$

${}^{A}_{C}T$

Matrix multiplication is NOT commutative!

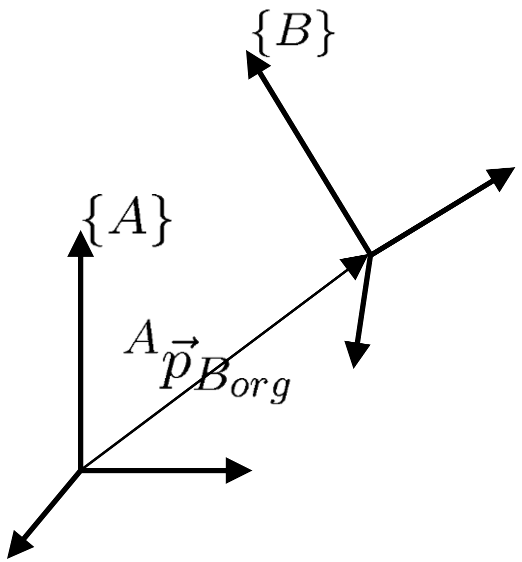
ORDER MATTERS !!!

Inverting ${}^{A}_{B}T$!

$$ {}^{A}\vec{p} = {}^{A}_{B}T \, {}^{B}\vec{p} = \begin{bmatrix} {}^{A}_{B}R & {}^{A}\vec{p}_{B_{org}} \\ 0 \quad 0 \quad 0 & 1 \end{bmatrix} {}^{B}\vec{p} $$

$$ {}^{B}_{A}R = {}^{A}_{B}R^{T} $$

$$ {}^{B}({}^{A}\vec{p}_{B_{org}}) = {}^{B}_{A}R \; {}^{A}\vec{p}_{B_{org}} + {}^{B}\vec{p}_{A_{org}} $$

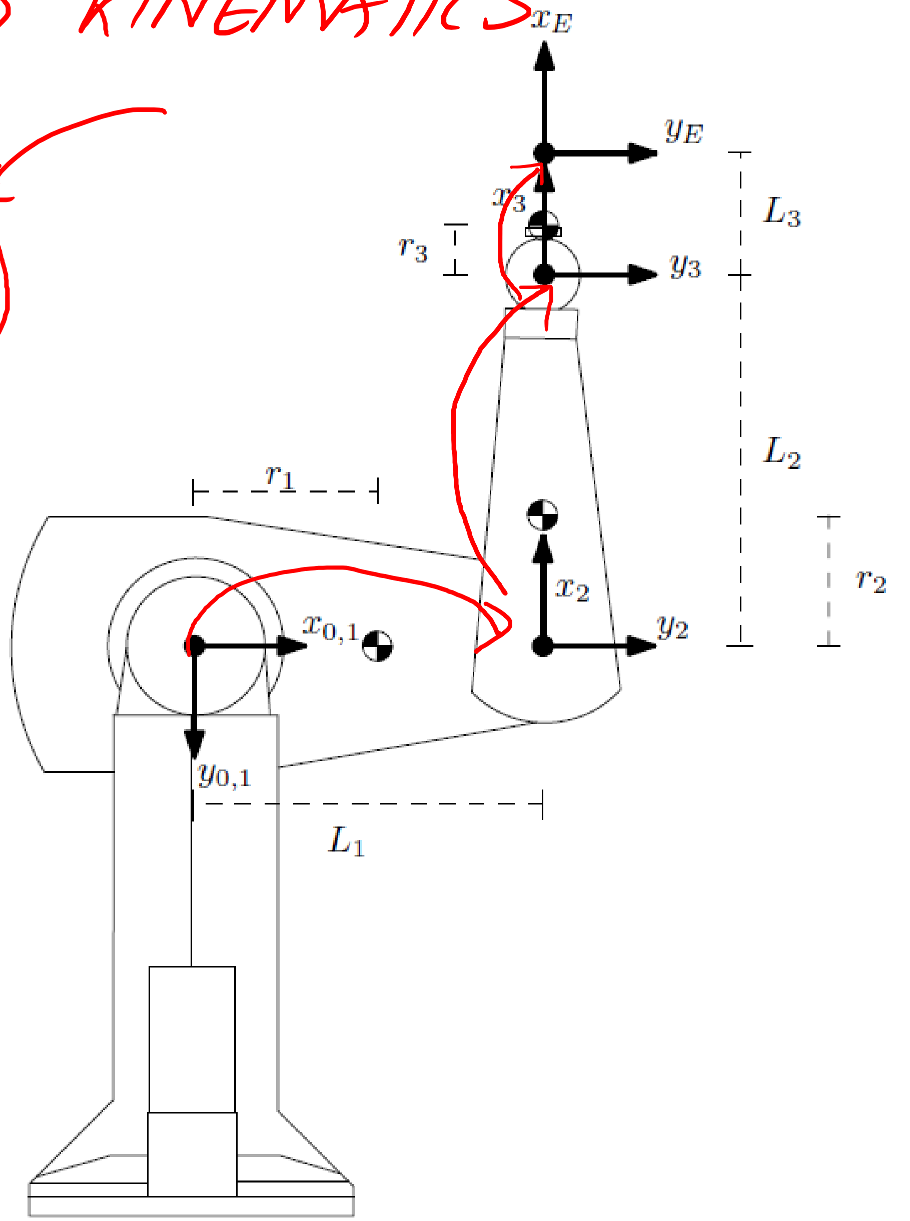


$$ {}^{B}\vec{p}_{A_{org}} = -{}^{B}_{A}R \; {}^{A}\vec{p}_{B_{org}} = -{}^{A}_{B}R^{T} \; {}^{A}\vec{p}_{B_{org}} $$

$$ {}^{B}_{A}T = \begin{bmatrix} {}^{A}_{B}R^{T} & -{}^{A}_{B}R^{T} \, {}^{A}\vec{p}_{B_{org}} \\ 0 \quad 0 \quad 0 & 1 \end{bmatrix} $$

FORWARD KINEMATICS

$$X \leftarrow {}^{0}_{EE}T(\vec{q})$$



9 numbers

$$\begin{bmatrix} R & \vec{t} \\ 0\ 0\ 0 & 1 \end{bmatrix} \leftarrow 3$$

$12 > 6$

Review

- Affine transformations
- Homogeneous transforms
  - describe frames
  - map between frames
  - transform point
- Operations with HTs
  - concatenation
  - inversion