Robotics: Final Examination
22. Februar 2012

Brock
Eppner / Höfer

Name: ....

First name: ....

Matr.-Nr.: ....

Time: 75 Minuten

- Please write clearly and do **not** use a red pen.
- Write your name and your student ID (Matrikelnummer) on *all* pages *now*.

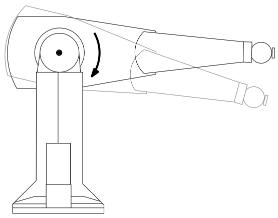
	Points	Score
1	6	
2	5	
3	9	
4	11	
5	8	
6	6	
7	10	
8	7	
Σ	62	

1 (6 points): Control

Solution

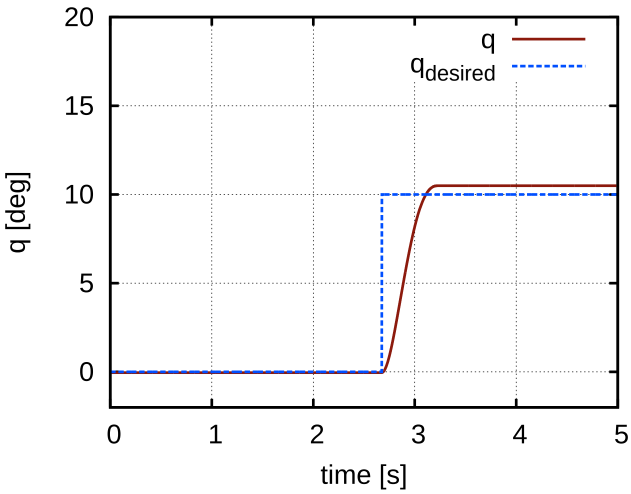
1.1. (6 points) Imagine the PUMA robot moving as shown in the figure below.

Only the second joint is actuated using a PD controller, all other joints are rigid. The following three plots show the manipulator's response to a 10 degree step. What are the most likely control parameters in each scenario, and how is the damping behavior called?
Each control parameter set (k_p, k_v) corresponds to exactly one of the plots.



☐ $k_p = 4000$, $k_v = 40$
⊗ $k_p = 400$, $k_v = 40$
☐ $k_p = 100$, $k_v = 40$

☐ Over-damped
☐ Under-damped
⊗ Critically damped



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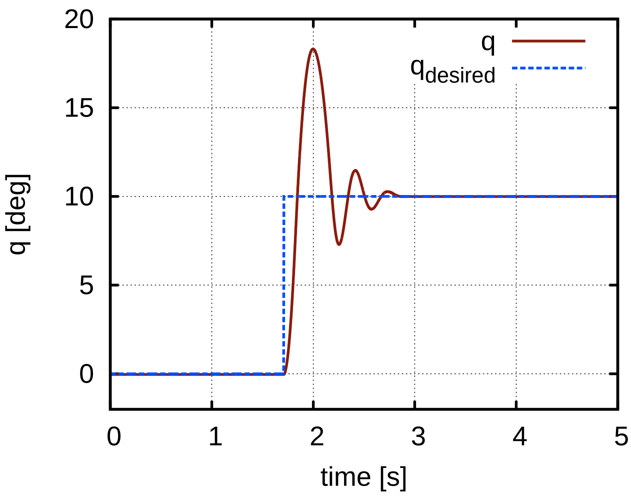

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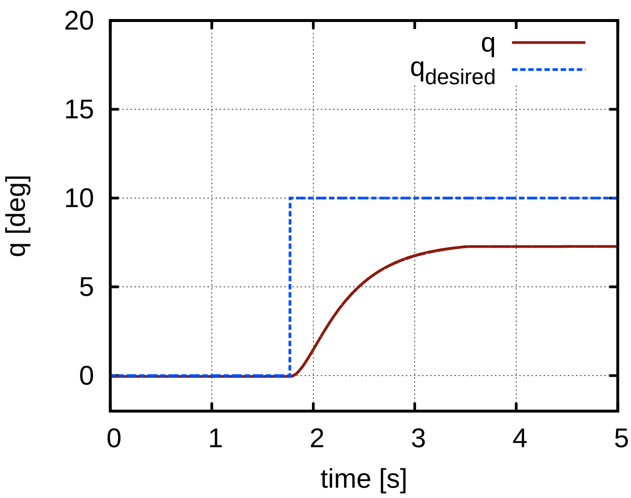


2 (5 points): Dynamics

2.1. (5 points) Explain the configuration space inertia matrix of a manipulator with 7 degrees of freedom. What are its dimensions?

Solution:
7×7

Give an intuition for the meaning of its diagonal entries M_{ii}.

Solution:
The diagonal elements M_{ii} describe the inertia ($\frac{kg}{m^2}$) seen by joint j. Usually the first diagonal elements, corresponding to the robot's waist and shoulder joints, are large since large motions of these joints involves rotation of the heavy upper- and lower-arm links.

Give an intuition for the meaning of its off-diagonal entries $M_{ij}, i \neq j$.

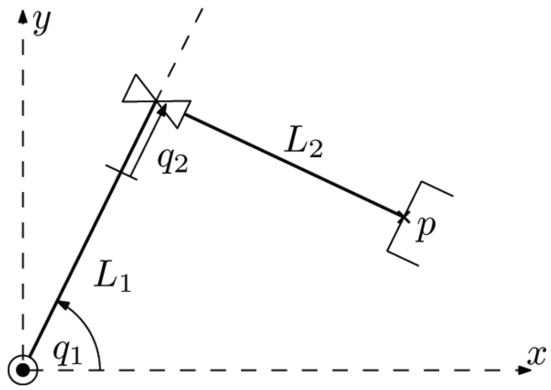
Solution:
The off-diagonal terms M_{ij} represent coupling of acceleration from joint j to the generalized force on joint i.

Is it symmetric? Describe intuitively, why or why not.

Solution:
Yes, it is symmetric. Intuition: actio == reactio. The torque/force exerted on joint i if joint j is accelerated is the same as vice versa.

3 (9 points): Forward Kinematics

3.1. (6 points) Consider the depicted 2-DoF kinematic chain consisting of one revolute joint q_1 and one prismatic joint q_2. The link lengths are given by L_1 and L_2.



Complete the following forward kinematics (given as a homogeneous transform) ${}^0T_E(\mathbf{q})$ of the end-effector as a function of the joint variables q_1 and q_2.

$$\begin{pmatrix} \sin(q_1) & \_\_\_\_\_\_\_\_\_\_ & \_\_\_\_\_\_\_\_\_\_ \\ -\cos(q_1) & \sin(q_1) & \_\_\_\_\_\_\_\_\_\_ \\ 0 & 0 & 1 \end{pmatrix}$$

Solution:

$$\begin{matrix} \cos(q_1 - \frac{\pi}{2}) & -\sin(q_1 - \frac{\pi}{2}) & (L_1 + q_2)\cos(q_1) + L_2\cos(q_1 - \frac{\pi}{2}) \\ \sin(q_1 - \frac{\pi}{2}) & \cos(q_1 - \frac{\pi}{2}) & (L_1 + q_2)\sin(q_1) + L_2\sin(q_1 - \frac{\pi}{2}) \\ 0 & 0 & 1 \end{matrix} = \begin{matrix} \sin(q_1) & \cos(q_1) & (L_1 + q_2)\cos(q_1) + L_2\sin(q_1) \\ -\cos(q_1) & \sin(q_1) & (L_1 + q_2)\sin(q_1) - L_2\cos(q_1) \\ 0 & 0 & 1 \end{matrix}$$

3.2. (3 points) Rotation matrices and Euler angles are alternative representations for the end-effector orientation. Name two main differences between those representations in three-dimensional space. For naming the differences you might consider the number of parameters to be stored, interpolation behavior, singularities, etc.

Solution:

Pose as a vector and Euler-Angles for orientation

Parameters to be stored: Euler angles are more efficient / concise (3 vs 9 numbers)

Interpolation behaviour:

- *You cannot interpolate directly between matrices*
- *You can interpolate between angles but the interpolation behavior is much wore than with e.g. quaternions*

Singularities:

- *Rotation matrix is orthonormal, so it is never singular*
- *Euler angles: If e.g. we use XYZ convention, it can happen that after the first (second) rotation around X (Y), the Y (Z) axis aligns with the previous X (Y) axis, resulting in a rotation in a degenerated 2-D space.*

Other properties possible

4 (11 points): Jacobian

A 3-DoF kinematic chain is depicted below. It consists of three revolute joints q_1, q_2 and q_3 and three links L_1, L_2 and L_3. We assume that all links are of equal length, i.e. $L = L_1 = L_2 = L_3$.

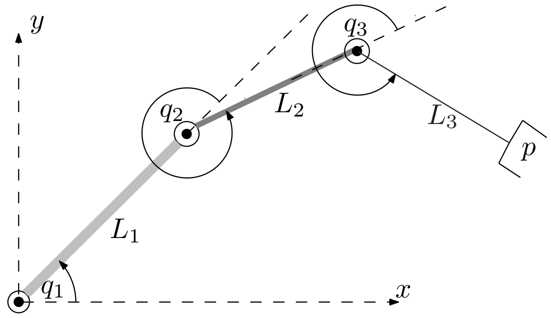


Figure 1: 3-DoF kinematic chain

The forward kinematics of the end-effector are given as follows:

$$\mathbf{p} = \begin{pmatrix} x \\ y \\ \theta \end{pmatrix} = \begin{pmatrix} L(c_1 + c_{12} + c_{123}) \\ L(s_1 + s_{12} + s_{123}) \\ q_1 + q_2 + q_3 \end{pmatrix}$$

where $s_{ij} = \sin(q_i + q_j)$ and $c_{ij} = \cos(q_i + q_j)$.

4.1. (2 points) Let $\mathbf{J}$ be the 3×3 Jacobian of the given kinematic chain. Give an intuition for the meaning of $\mathbf{J}_{1,1}$, i.e. the entry in the first column of the first row of the Jacobian!

Solution:

$\mathbf{J}_{1,1}$ states to which degree a change in q_1 (configuration space) affects a change in x (operational space).

4.2. (3 points) Calculate the 3×3 Jacobian $\mathbf{J}$ for the given kinematic chain.

Hints: $\sin'(x) = \cos(x)$ $\quad \cos'(x) = -\sin(x)$ $\quad (f \circ g)' = (f' \circ g) \cdot g'$

Solution:

$$\begin{pmatrix} -L(s_1 + s_{12} + s_{123}) & -L(s_{12} + s_{123}) & -Ls_{123} \\ L(c_1 + c_{12} + c_{123}) & L(c_{12} + c_{123}) & Lc_{123} \\ 1 & 1 & 1 \end{pmatrix}$$

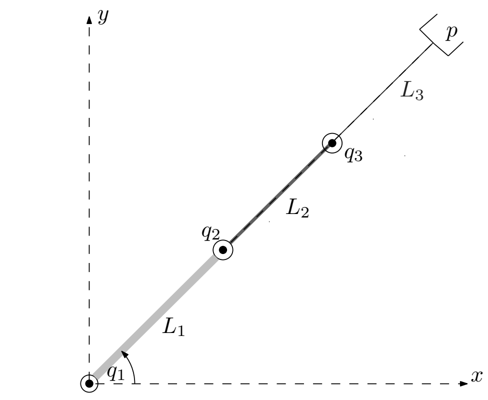
4.3. (6 points) Below, we depicted three configurations of the kinematic chain from Figure 1.

To answer the questions below it suffices to use your intuition. But you can of course also make use of the Jacobian you calculated in 4.2.

Hint: The value of angle q_1 does not affect the rank of the Jacobian of the depicted configurations.

Configuration 1

Is the Jacobian in this configuration singular?
⊗ yes
☐ no



What is the rank of the Jacobian for this configuration?

Solution:

1

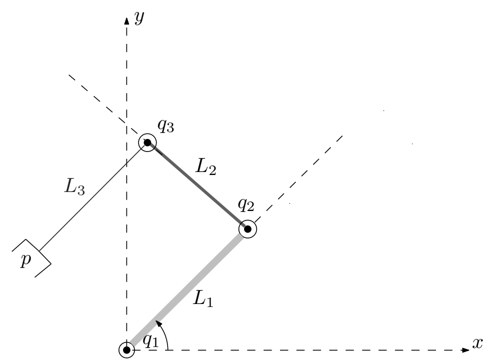
What movements are impossible for the robot to execute instantaneously in this configuration?

Solution:

The robot can neither change its position towards the center point nor the orientation of the EE

Configuration 2

Is the Jacobian in this configuration singular?
☐ yes
⊗ no



What is the rank of the Jacobian for this configuration?

Solution:

3

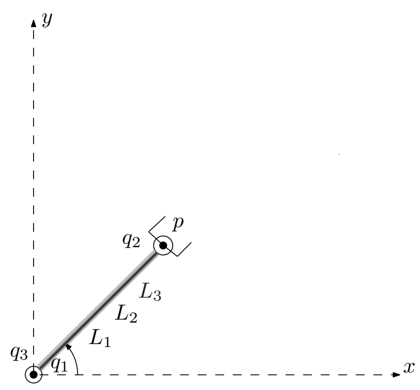
What movements are impossible for the robot to execute instantaneously in this configuration?

Solution:

Jacobian is regular! The robot can instantaneously change all of its DoF

Configuration 3

Is the Jacobian in this configuration singular?
⊗ yes
☐ no



What is the rank of the Jacobian for this configuration?

Solution:

2

What movements are impossible for the robot to execute instantaneously in this configuration?

Solution:

The first and the second columns become linearly dependent → Jacobian becomes singular, i.e. q_1 and q_2 have the same effect on the configuration change. Therefore, the robot cannot (instantaneously) change its position towards the (-,-) nor to the (+,+) quadrant (i.e. one spatial direction)

5 (8 points): Trajectory Generation

5.1. **(2 points)** Consider a robot with a single degree of freedom that you want to control from its current position q_{curr} to an arbitrary position q_{des} in joint space. You are only allowed to use a cubic polynomial to interpolate between q_{curr} and q_{des}. A cubic polynomial has four coefficients. What parameters of motion are appropriate for specifying these coefficients?

Solution:

4 parameters: initial and terminal position, initial and terminal velocity

If in addition you want to specify the initial and final acceleration of the motion, you would have to use a polynomial of which degree?

Solution:

We need 6 parameters, so a quintic polynomial (or higher).

5.2. **(6 points)** Imagine a single-joint robot that should be controlled from $x(t = 0) = 0$ to $x(t = 5) = 1$, with $\dot{x}(0) = \dot{x}(5) = 0$. Draw the position, velocity and acceleration function over time of a trajectory that employs a linear segment with parabolic blends. You have to draw the graphs only qualitatively, not quantitatively (e.g. exact blend times do not matter). Mark where the blend segments start and finish in each plot.

Solution:

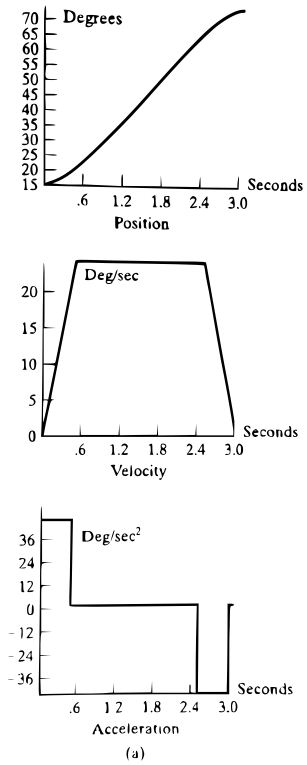


Figure 2: Example solution from Craig textbook

6 (6 points): Visual Servoing

6.1. **(3 points)** Imagine a system designed to perform visual servoing on an L-shaped block (see figure below). The goal of the system is to track the L-shaped block with the end-effector in order to grasp it later on.

The system is designed as follows: A visual sensor consisting of two cameras is mounted on a tripod. A visual processing unit calculates the 6D pose (position and orientation) ${}^{B}\mathbf{X}$ of the L-shaped block in the base frame.

Given the pose of the L-shaped block, we calculate the difference between the desired pose and the current pose $e(\mathbf{X}) = {}^{B}\mathbf{X}^{*} - {}^{B}\mathbf{X}$. We choose the desired pose ${}^{B}\mathbf{X}^{*}$ such that the end-effector is closely behind the block.

Finally, we control the end-effector using an operational space controller which minimizes the pose error $e(\mathbf{X})$.

Left) The robotic hand adapts to the manual perturbation of the L-shaped block using visual servoing.
Right) Schema of the presented visual servoing system.

What characterizations apply to the presented visual servoing application?

There are several correct answers. You get 1 point for a correct answer, for a wrong answer 1 point is subtracted. Negative scores are not possible.

☐ Monocular eye-to-hand
⊗ Binocular eye-to-hand
☐ Monocular eye-in-hand
☐ Binocular eye-in-hand
⊗ End-point open-loop
☐ End-point closed-loop
☐ Image-based
⊗ Position-based

6.2. **(3 points)** As you know the image Jacobian relates changes of the camera pose to changes in the camera image. Consider an image Jacobian which maps two point features (each one given by two coordinates, u_i and v_i, $i \in \{1, 2\}$) to the 6-DoF position of the camera (in operational space). Hence, the image Jacobian has dimensionality 4×6 and its null space has dimensionality 2.

Which property holds for camera motions in the null space of the image Jacobian?

Hint: The null space of a matrix $\mathbf{A}$ is defined as the set of all vectors $\mathbf{x}$ such that $\mathbf{A}\mathbf{x} = \mathbf{0}$.

Solution:

All motions of the camera that do not result in a change of position of the two point features in the image lie in the null space of the image Jacobian.

Describe a specific such motion which is part of the null space of this image Jacobian.

(You might consider illustrating your explanation with a drawing.)

Solution:

(Special) case 1: If the two points are collinear with the viewing axis of the camera the matrix looses rank and translations as well as rotations along the viewing axis project to the null space.

Case 2: Consider the case when the line l connecting the two points is orthogonal to the camera's viewing axis, l is parallel to the camera's z-coordinate (up) and the viewing axis points directly to the middle-point C of l (i.e. the points are equidistant to the camera with distance). Then the camera can move (tangentially) on a circle whose radius is the camera's distance to C and which is oriented such that it lies on the x-y-plane of the camera.

Case 3: The previous case also works if the the points are not aligned with any of the camera's axes; however, then the circle also has a non-aligned orientation.

Case 4: Consider case 2. If the camera performs a rotation downwards or upwards (distance between points increases) and at the same time compensates this motion by a translation towards the front (distance between points decreases)

General case: Finally, all the cases also work if the points do not lie on a line which is orthogonal to the viewing axis.

7 (10 points): Monte-Carlo Localization

7.1. **(4 points)** Assess the following statements about recursive estimation methods.

Non-parametric methods assume that the probability distribution of the current state is estimated from the data.

⊗ True
☐ False

The Kalman filter is a non-parametric recursive estimation method.

☐ True
⊗ False

Every Bayes filter includes the following steps: *prediction step*, *measurement update* and *resampling*.

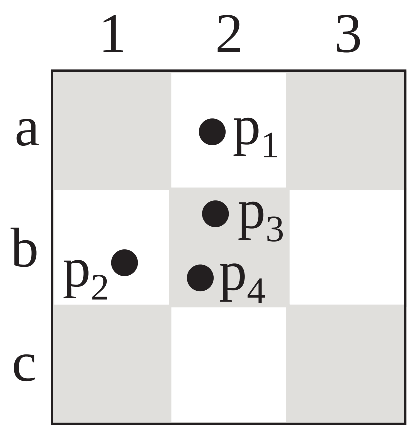
☐ True
⊗ False

In the *prediction step* of a Bayes filter, we estimate the posterior probability of the current state given the last measurement and the last action.

☐ True
⊗ False

7.2. **(6 points)** A particle filter is used to determine the position of a robot $[x, y]$ on a 3×3 checkerboard. The robot's orientation is not taken into account.

The figure below shows the current state of the particle filter. Each black dot represents a particle.



Either, the robot is standing on a gray cell (G) or it is standing on a white cell (W).

The robot is equipped with a sensor which measures the intensity of the ground below the robot. The sensor returns one of two values: *dark* or *light*.

If the robot is standing on a gray cell the sensor measures *dark* with a probability $\frac{9}{10}$:
$p(dark|G) = \frac{9}{10}$

If the robot is standing on a white cell the sensor measures *light* with a probability $\frac{8}{10}$:
$p(light|W) = \frac{8}{10}$

Now assume the sensor measures *dark*.

Calculate the accumulated, normalized importance factor of each square $w(a1)$, $w(a2)$, $w(b1)$ and $w(b2)$ according to the sensor reading and the particle distribution given in the figure above.

Hint: *Normalized* means that all importance factors sum up to one.

Solution:
The prior is not needed since it is implicitly included in the particle distribution, but we need the counter probabilities:
$p(light|G) = 0.1, p(dark|W) = 0.2$
For every square we now calculate the probability of the sensor reading and multiply it with the number of particles in that square.

$$w(a1) =$$

Solution:

0

$$w(a2) =$$

Solution:

$p(dark|W) = 1 - p(gray|W) = 0.2 \Rightarrow w(a_1) = 0.2 \Rightarrow w_{norm}(a1) = \frac{1}{11}$

$$w(b1) =$$

Solution:

$p(dark|W) = 1 - p(gray|W) = 0.2 \Rightarrow w(b_1) = 0.2 \Rightarrow w_{norm}(b1) = \frac{1}{11}$

$$w(b2) =$$

Solution:

$p(dark|G) = 0.9 \Rightarrow w(b_2) = 2p(dark|G) = 1.8 \Rightarrow w_{norm}(b2) = \frac{9}{11}$

Solution:

Normalization: $1.8 + 0.2 + 0.2 = 2.2$

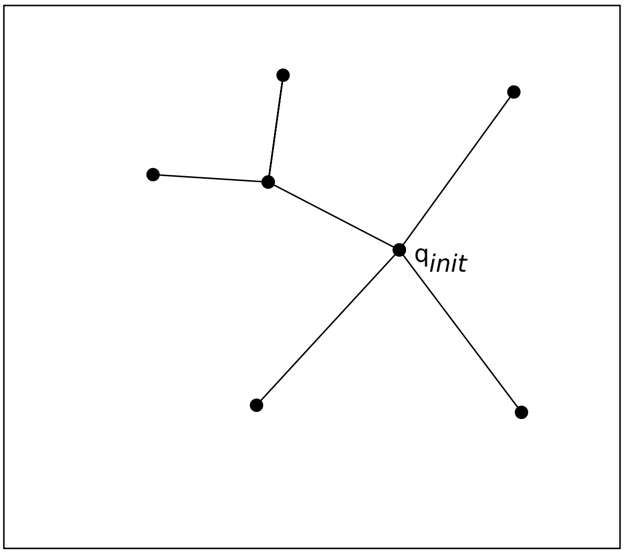
8 (7 points): Motion Planning

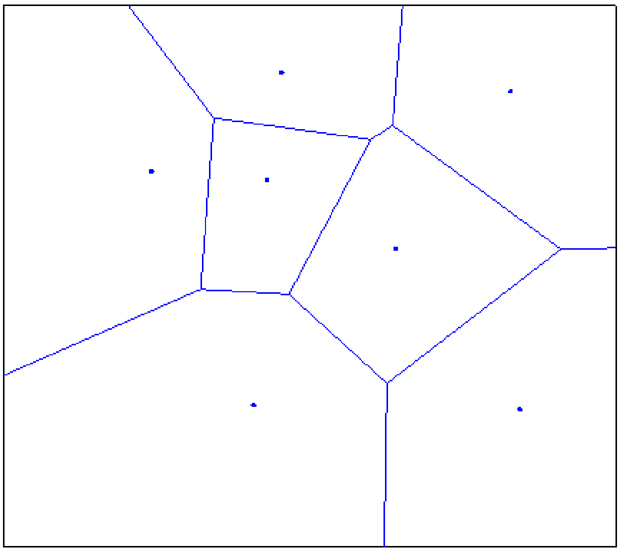
8.1. (2 points) Explain the difference between multi-query and single-query planning methods and give one example for each category.

Solution:

8.2. (2 points) After running the RRT algorithm for 6 iterations we end up with the tree shown below. Draw the Voronoi regions for all nodes and mark the node which is most likely expanded during the next iteration.

Solution:





8.3. (3 points) Describe briefly how the following motion generation methods balance exploration and exploitation.

Potential Field Method:

Solution:

Pure exploitation. (only exploration if global potential field is calculated?)

RRTExt (without connect step):

Solution:

Pure exploration.

PRM with Bridge Sampling:

Solution:

(Guided) exploration.