Robotics – Tutorial for Assignment #2

Denavit-Hartenberg parameters

by Előd Páll

Prerequisite Lecture Videos

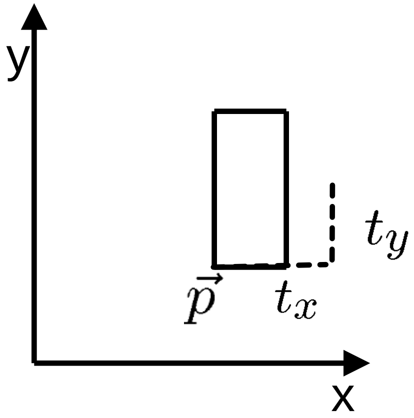
If you are not familiar with the following topic, you should watch the linked lecture videos.

Our Goal

Determine the end-effector pose of a robot arm using the Denavit-Hartenberg parameters

Translation

$$\vec{p} = \begin{pmatrix} p_x \\ p_y \end{pmatrix} \qquad \vec{t} = \begin{pmatrix} t_x \\ t_y \end{pmatrix}$$



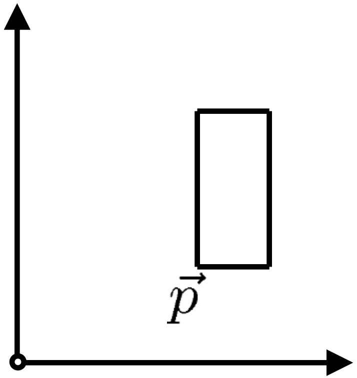
$$\vec{p}\,'$$

$$\vec{p}\,' = \vec{p} + \vec{t} = \begin{pmatrix} p_x + t_x \\ p_y + t_y \end{pmatrix}$$

Global Reference Coordinate System = World Frame

Rotation

$$\vec{p} = \begin{pmatrix} p_x \\ p_y \end{pmatrix}$$

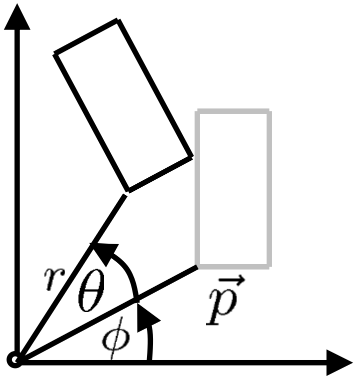


$$\vec{p}\,' = \ ?$$

Deriving the Rotation Matrix

$$
\begin{aligned}
p_x &= r \cdot \cos\phi \\
p_y &= r \cdot \sin\phi
\end{aligned}
$$

$$
\begin{aligned}
p'_x &= r \cdot \cos(\theta + \phi) \\
&= r \cdot \cos\phi \cdot \cos\theta - r \cdot \sin\phi \cdot \sin\theta \\
&= p_x \cdot \cos\theta - p_y \cdot \sin\theta \\
p'_y &= r \cdot \sin(\theta + \phi) \\
&= r \cdot \cos\phi \cdot \sin\theta + r \cdot \sin\phi \cdot \cos\theta \\
&= p_x \sin\theta + p_y \cos\theta
\end{aligned}
$$



$$\vec{p}\,' = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \cdot \vec{p}$$

$$\vec{p}\,' = R(\theta) \cdot \vec{p}$$

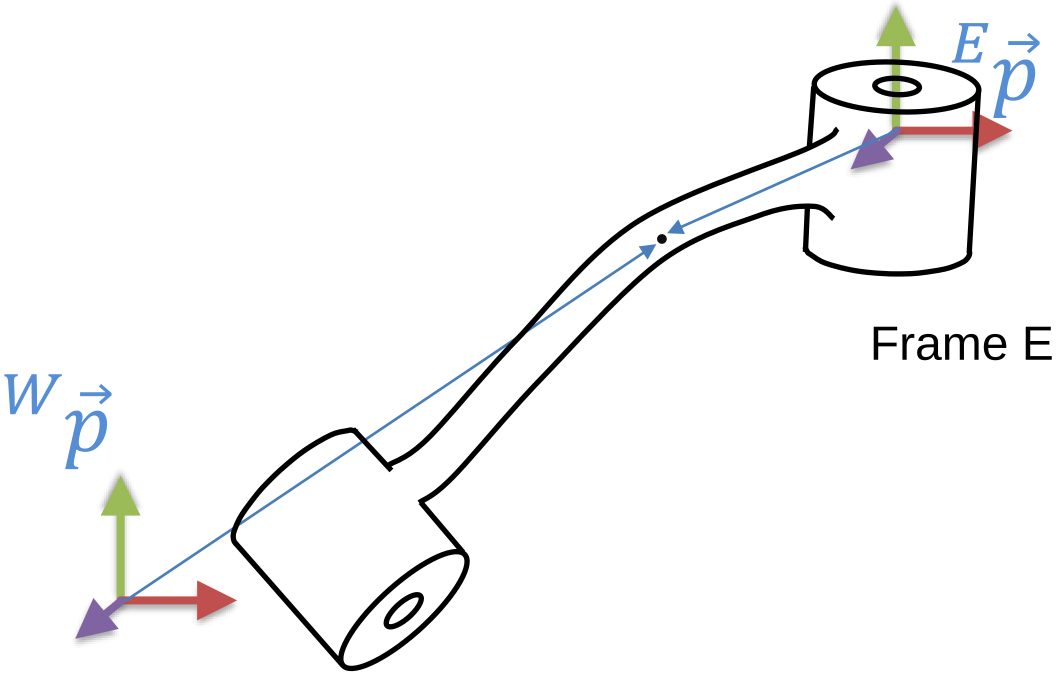
Homogeneous Transformations

$$\vec{p}\,' = \vec{p} + \vec{t} \qquad\qquad \vec{p}\,' = R(\theta) \cdot \vec{p}$$

$$\vec{p}\,' = \underbrace{R(\theta) \cdot \vec{p}}_{\text{Rotation first!}} + \vec{t}$$

$$\begin{pmatrix} p'_x \\ p'_y \\ 1 \end{pmatrix} = \begin{bmatrix} \multicolumn{2}{c}{R(\theta)} & t_x \\ & & t_y \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{pmatrix} p_x \\ p_y \\ 1 \end{pmatrix}$$

Frames

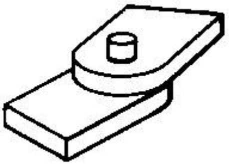


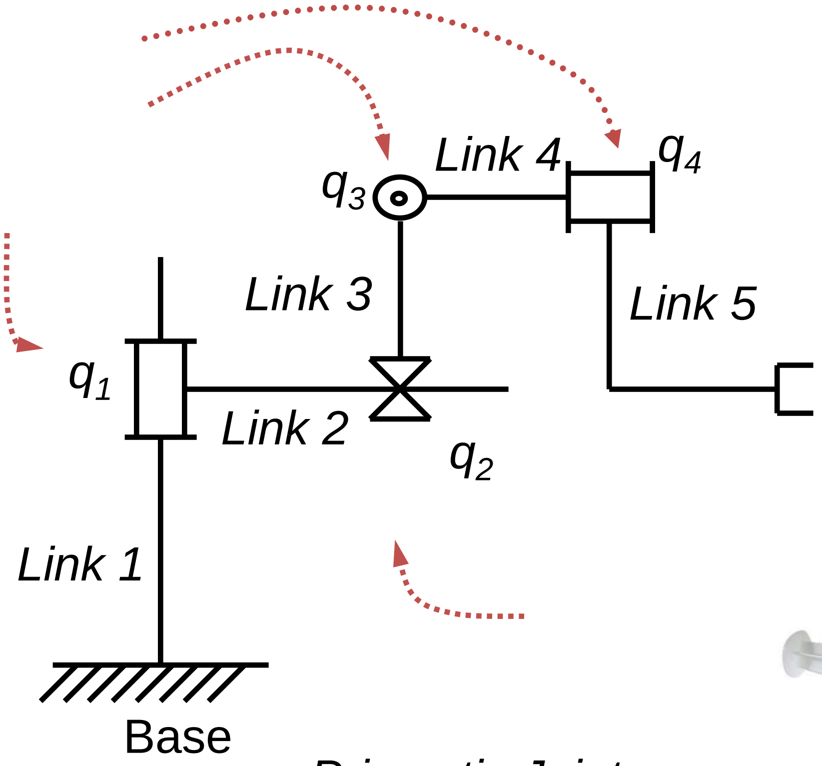
Frame E attached to a rigid body

World frame W

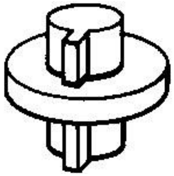
Kinematic Schematic of a Robot

Revolute Joint

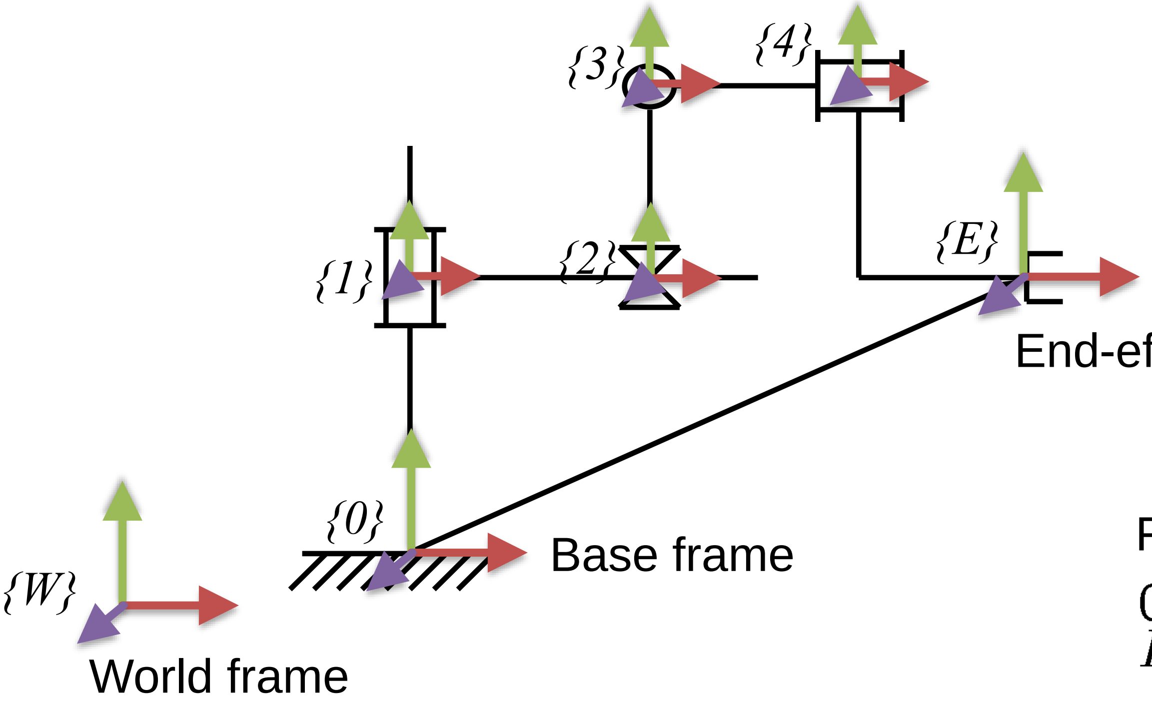




Prismatic Joint



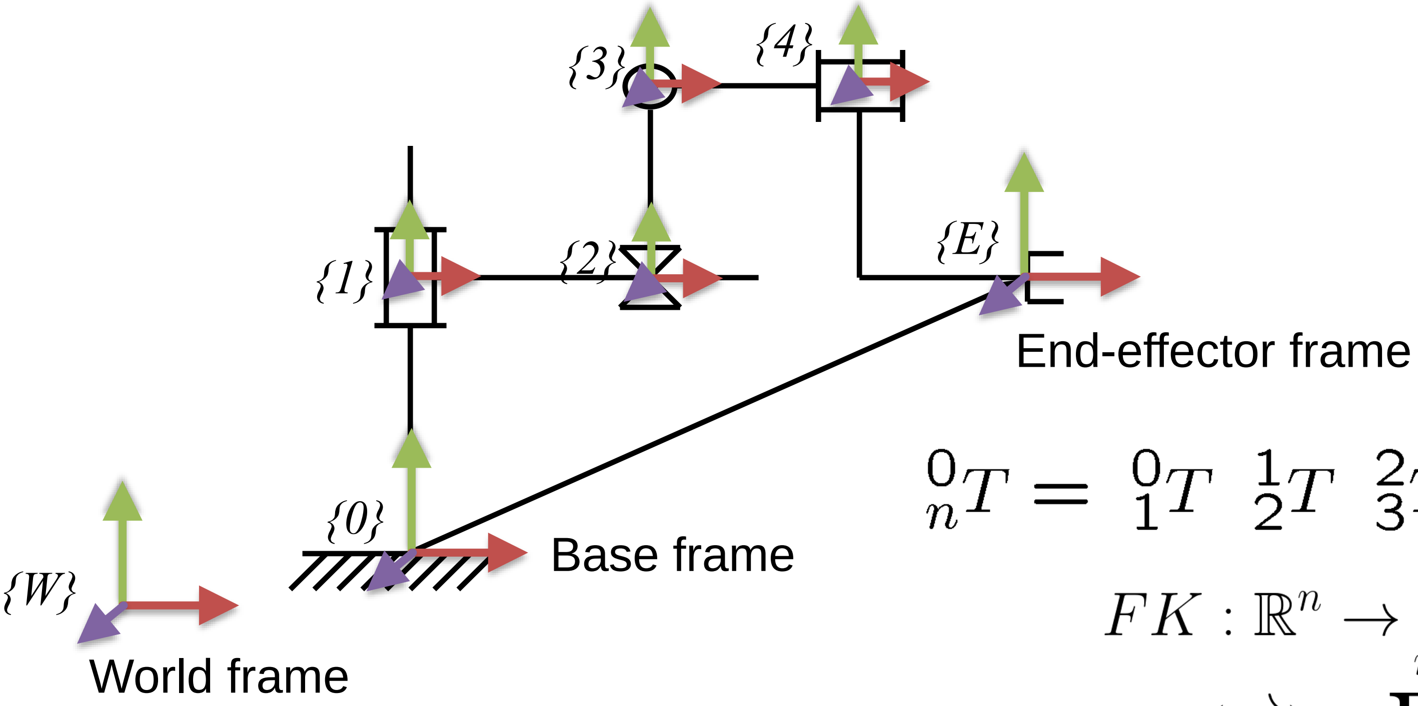
Kinematic Chains



Forward kinematics

$${}^{0}_{E}T = ?$$

Kinematic Chains



$$ {}^{0}_{n}T = {}^{0}_{1}T \; {}^{1}_{2}T \; {}^{2}_{3}T \quad \cdots \quad {}^{n-1}_{n}T $$

$$ FK : \mathbb{R}^n \rightarrow \mathbb{R}^6 $$

$$ FK(\vec{q}) = \prod_{i=1}^{n} {}^{i-1}_{i}T $$

Glossary

[Craig] Pp. 4

- Frame
  - Coordinate system attached rigidly to a body
  - Examples: World frame, base frame, end-effector frame
- Link
  - A rigid body that is part of the robot (any object)
- Joint
  - Movable connection between two links
  - Most common: Revolute, prismatic
- Degrees of Freedom
  - Amount of independent position variables the robot has (Usually amount of joints)
- End-effector
  - Free end at the end of the robot manipulator (e.g. a hand)
- Forward kinematics
  - Position and orientation of the end-effector relative to the base frame

Quiz time