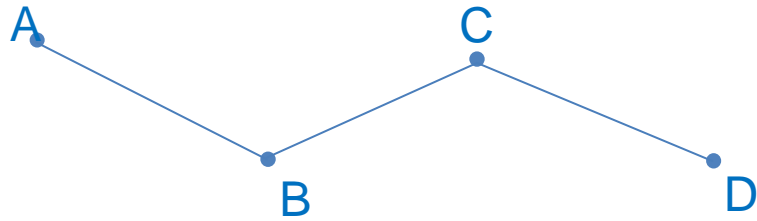


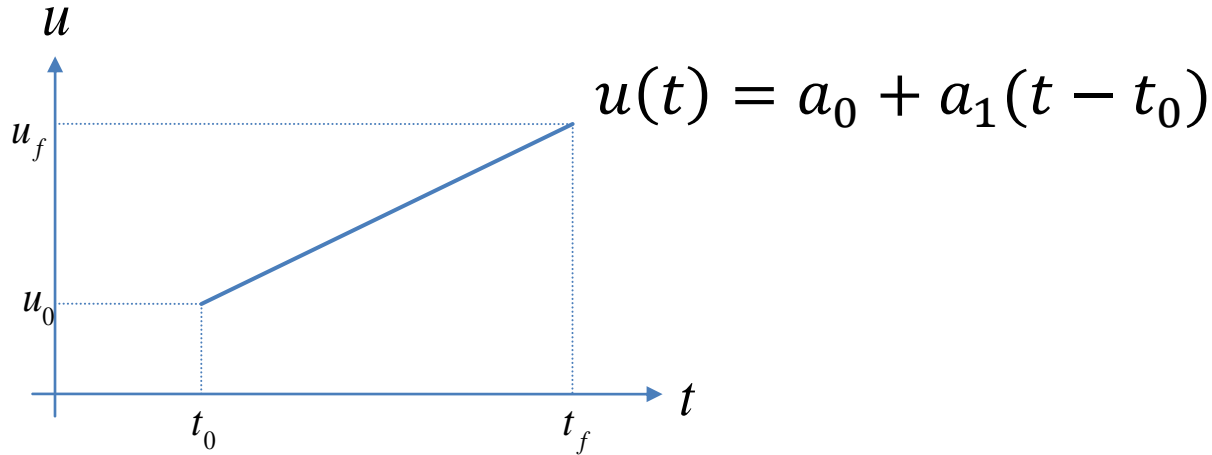
Calculating Trajectories

Candidate Curves

Linear



Linear Interpolation

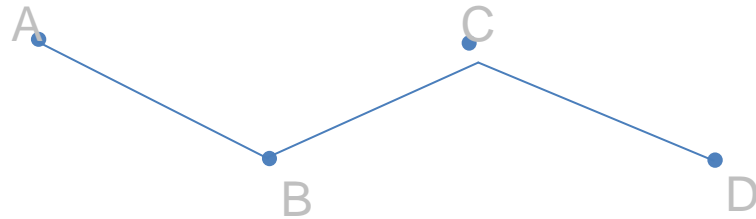


Two conditions: $u(t_0) = u_0$, $u(t_f) = u_f$

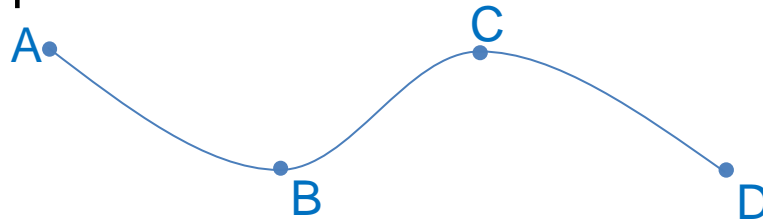
→ No control over velocities: discontinuities at beginning and end of motion require infinite acceleration!

Candidate Curves

Linear

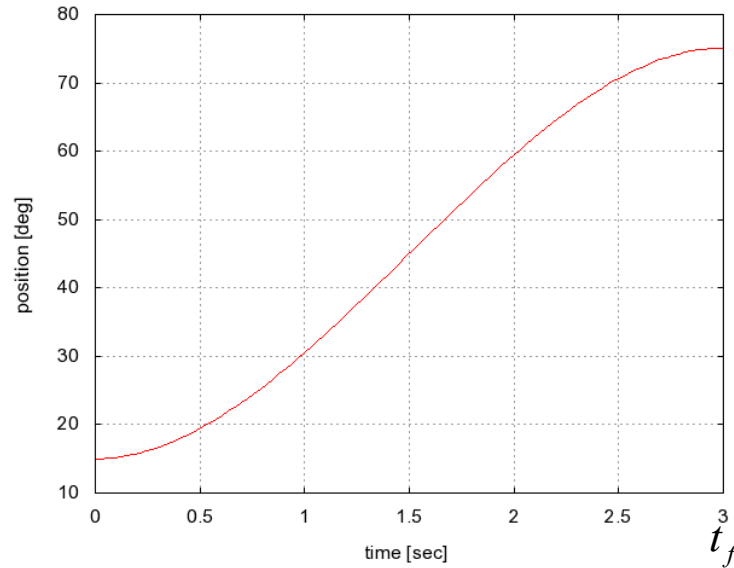


Polynomials/Splines



Cubic Splines

$$u(t) = a_0 + a_1t + a_2t^2 + a_3t^3$$

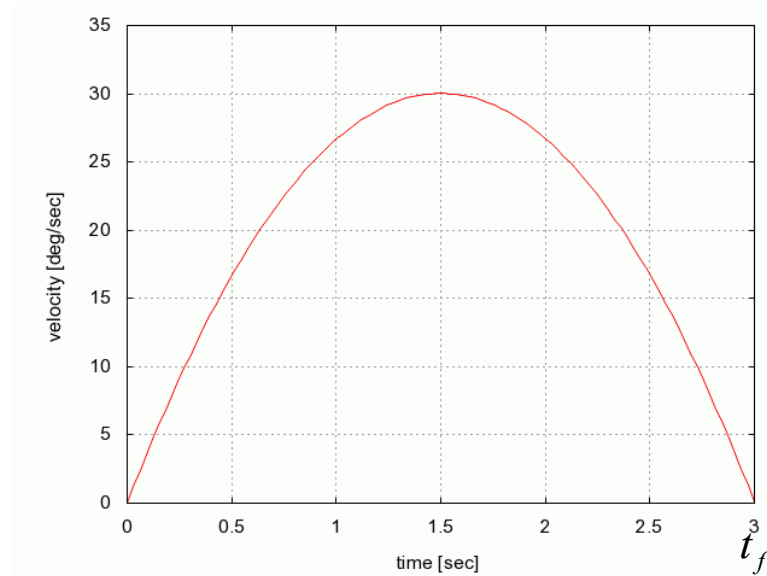


Initial Conditions:

$$u(0) = u_0 \quad u(t_f) = u_f$$

Cubic Splines

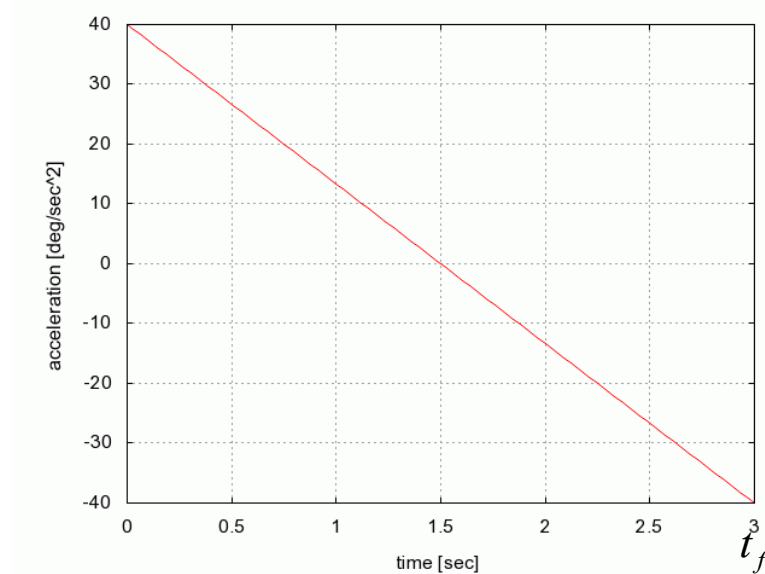
$$\dot{u}(t) = a_1 + 2a_2t + 3a_3t^2$$



Initial Conditions: $\dot{u}(0) = 0$ $\dot{u}(t_f) = 0$

Cubic Splines

$$\ddot{u}(t) = 2a_2 + 6a_3t$$



→ No control over acceleration
(use higher order polynomials (Quintics, Septics, ...))

Cubic Splines

$$u(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

Four conditions: $u(0) = u_0$ $u(t_f) = u_f$ $\dot{u}(0) = 0$ $\dot{u}(t_f) = 0$

Four unknowns: a_0, a_1, a_2, a_3

$$\Rightarrow u(t) = \underbrace{u_0}_{a_0} + \underbrace{\left(\frac{3}{t_f^2}\right)(u_f - u_0)t^2}_{a_2} - \underbrace{\left(\frac{2}{t_f^3}\right)(u_f - u_0)t^3}_{a_3}$$

Including Via Points (1)

Concatenate cubic splines, e.g. for including one via point:

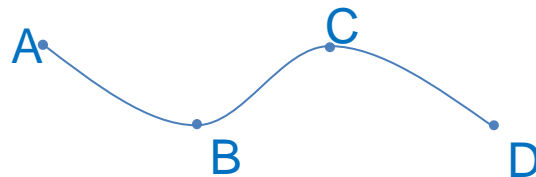
$$u_1(t) = a_0 + a_1t + a_2t^2 + a_3t^3$$

$$u_1(0) = u_0$$

$$u_1(t_{via}) = u_{via}$$

$$\dot{u}_1(0) = \dot{u}_0$$

$$\dot{u}_1(t_{via}) = \dot{u}_{via}$$



$$u_2(t) = b_0 + b_1(t - t_{via}) + b_2(t - t_{via})^2 + b_3(t - t_{via})^3$$

$$u_2(t_{via}) = u_{via}$$

$$u_2(t_f) = u_f$$

$$\dot{u}_2(t_{via}) = \dot{u}_{via}$$

$$\dot{u}_2(t_f) = \dot{u}_f$$

Including Via Points (2)

$$u_1(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

$$u_1(0) = u_0 \quad u_1(t_{via}) = u_{via}$$

$$\dot{u}_1(0) = \dot{u}_0 \quad \dot{u}_1(t_{via}) = \dot{u}_{via}$$

$$\Rightarrow a_0 = u_0 \quad a_2 = \frac{3}{t_{via}^2} (u_{via} - u_0) - \frac{2}{t_{via}} \dot{u}_0 - \frac{1}{t_{via}} \dot{u}_{via}$$

$$a_1 = \dot{u}_0 \quad a_3 = -\frac{2}{t_{via}^3} (u_{via} - u_0) + \frac{1}{t_{via}^2} (\dot{u}_0 + \dot{u}_{via})$$

Including Via Points (3)

$$u_2(t) = b_0 + b_1(t - t_{\text{via}}) + b_2(t - t_{\text{via}})^2 + b_3(t - t_{\text{via}})^3$$

$$u_2(t_{\text{via}}) = u_{\text{via}} \quad u_2(t_f) = u_f$$

$$\dot{u}_2(t_{\text{via}}) = \dot{u}_{\text{via}} \quad \dot{u}_2(t_f) = \dot{u}_f$$

$$\Rightarrow b_0 = u_{\text{via}} \quad b_2 = \frac{3}{(t_f - t_{\text{via}})^2} (u_f - u_{\text{via}}) - \frac{2}{t_f - t_{\text{via}}} \dot{u}_{\text{via}} - \frac{1}{t_f - t_{\text{via}}} \dot{u}_f$$

$$b_1 = \dot{u}_{\text{via}} \quad b_3 = -\frac{2}{(t_f - t_{\text{via}})^3} (u_f - u_{\text{via}}) + \frac{1}{(t_f - t_{\text{via}})^2} (\dot{u}_{\text{via}} + \dot{u}_f)$$

How to choose the velocity at
the via point?

How To Choose Velocities At Via Points

$$u_2(t) = b_0 + b_1(t - t_{\text{via}}) + b_2(t - t_{\text{via}})^2 + b_3(t - t_{\text{via}})^3$$

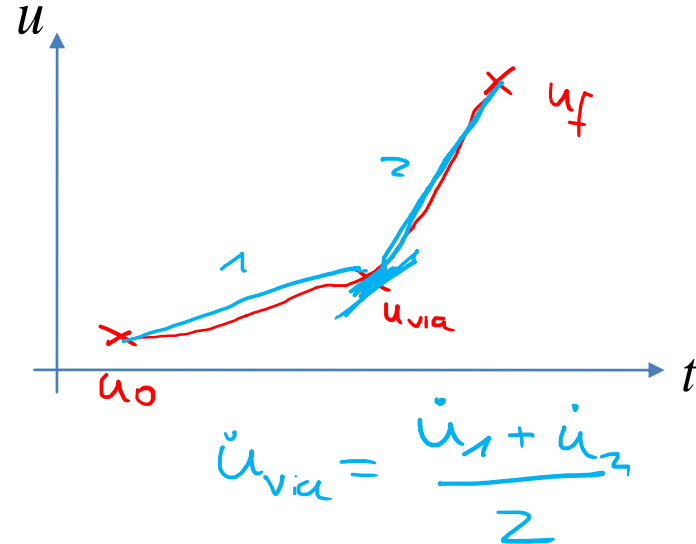
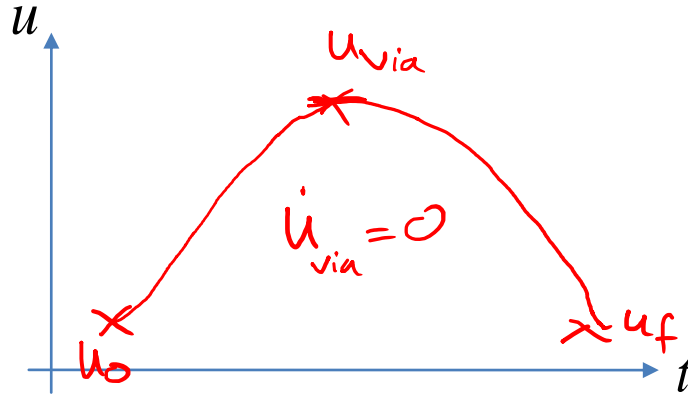
$$u_2(t_{\text{via}}) = u_{\text{via}} \quad u_2(t_f) = u_f$$

$$\dot{u}_2(t_{\text{via}}) = \dot{u}_{\text{via}} \quad \dot{u}_2(t_f) = 0$$

1. Let user specify
2. Use a heuristic (see assignment)
3. Alter the boundary conditions:
Remove velocity constraints and
force acceleration and velocity to be continuous

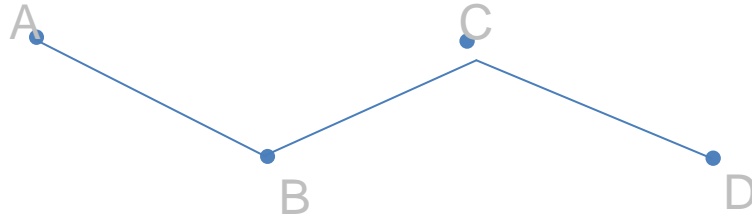
Velocity Heuristic

- ▶ Sign changes
- ▶ Sign stays the same

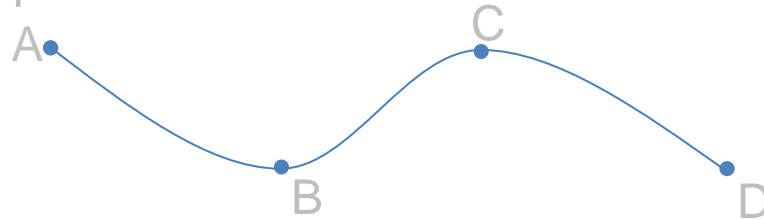


Candidate Curves

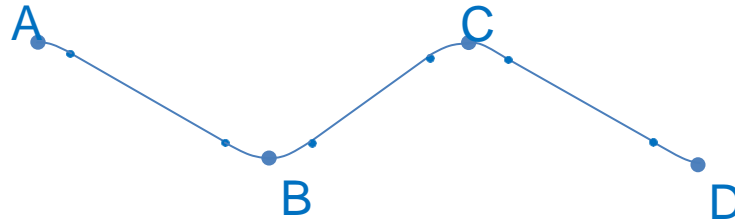
Linear



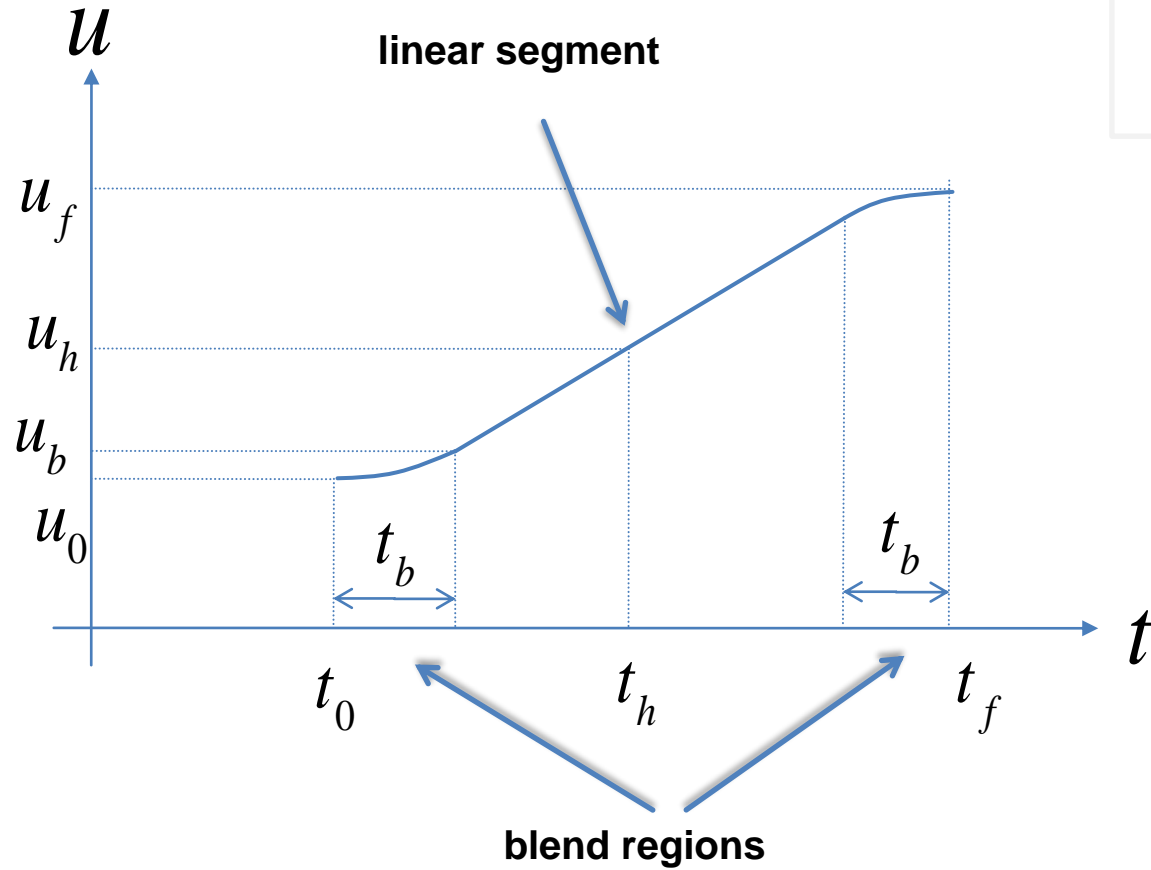
Polynomials/Splines



Linear with polynomial blends



Linear with Parabolic Blends (1)



Linear with Parabolic Blends (2)

- Idea: Blends with constant acceleration

$$u(t) = \frac{1}{2} \ddot{u} t^2 + u_0$$

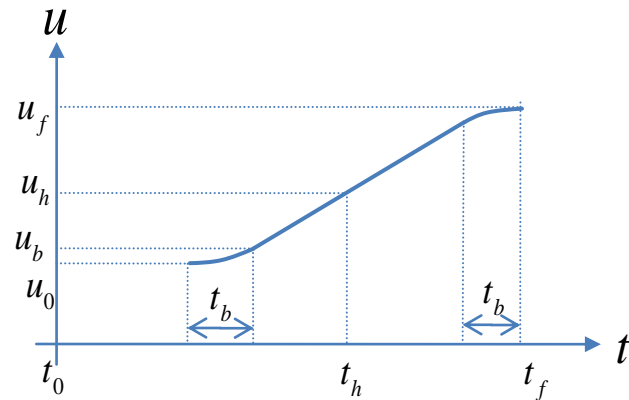
- Velocity at transition t_b :

$$\ddot{u} t_b = \frac{u_h - u_b}{t_h - t_b}$$

- Distance u_b :

$$u_b = u_0 + \frac{1}{2} \ddot{u} t_b^2$$

- Solve for unknowns




```
239 void njtrackControl(GlobalVariables& gv)
240 {
241     floatControl(gv); // Remove this line when you implement this controller
242 }

299 void proj1Control(GlobalVariables& gv)
300 {
301     floatControl(gv); // Remove this line when you implement proj1Control
302 }
303
304 void proj2Control(GlobalVariables& gv)
305 {
306     floatControl(gv); // Remove this line when you implement proj2Control
307 }
308
309 void proj3Control(GlobalVariables& gv)
310 {
311     floatControl(gv); // Remove this line when you implement proj3Control
312 }
```