Disclaimer

These slides are intended as presentation aids for the lecture. They contain information that would otherwise be to difficult or time-consuming to reproduce on the board. But they are incomplete, not self-explanatory, and are not always used in the order they appear in this presentation. As a result, these slides should not be used as a script for this course. I recommend you take notes during class, maybe on the slides themselves. It has been shown that taking notes improves learning success.

Reading for this set of slides

- Planning Algorithms (Steve LaValle)
  - 4 The Configuration Space (4.1 – 4.3)
  - 5 Sampling-based Motion Planning (5.1, 5.5, 5.6, also skim the remaining sections)

Please note that this set of slides is intended as support for the lecture, not as a stand-alone script. If you want to study for this course, please use these slides in conjunction with the indicated chapters in the text books. The textbooks are available online or in the TUB library (many copies that can be checked out for the entire semester. There are also some aspects of the lectures that will not be covered in the text books but can still be part of the homework or exam. For those It is important that you attend class or ask somebody about what was covered in class.

Robotics

Rapidly-Exploring Random Trees (RRTs)

TU Berlin

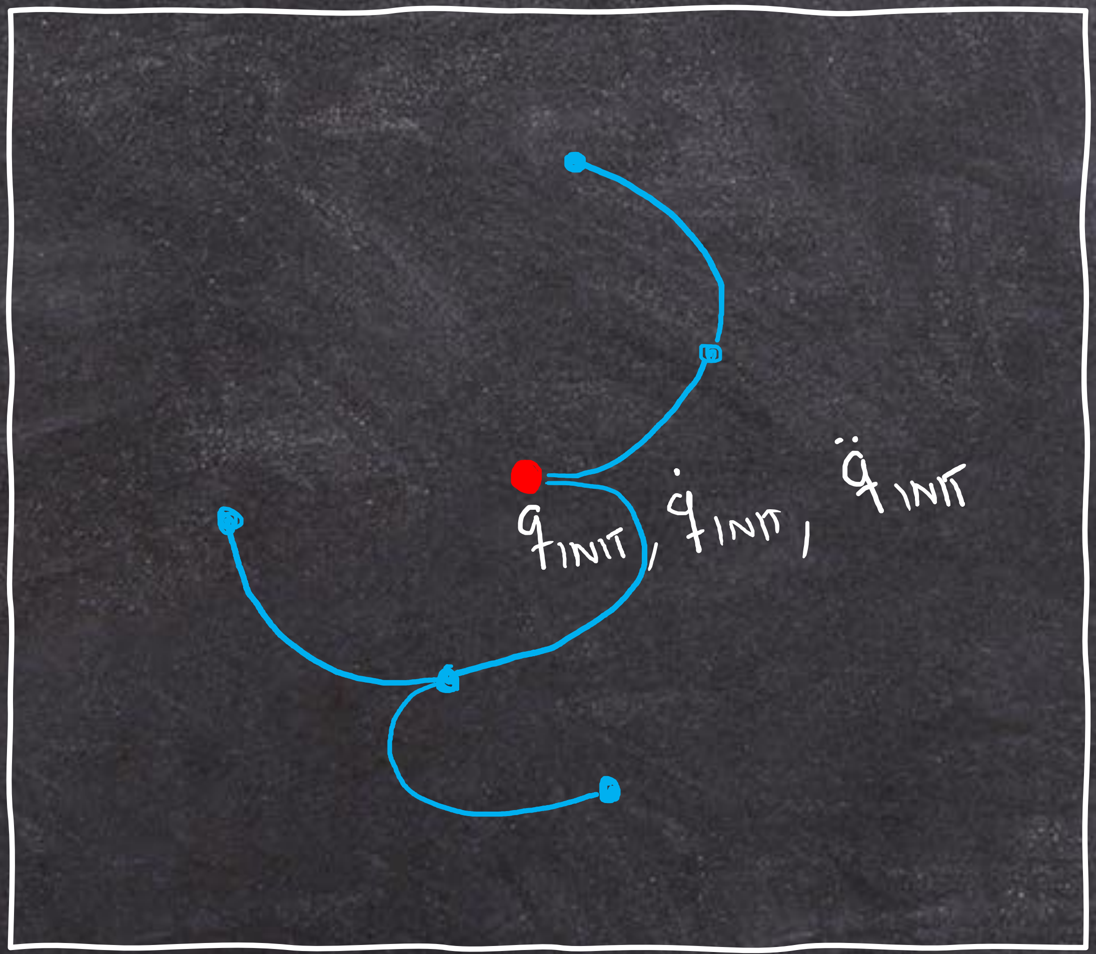
Oliver Brock

Rapidly-Exploring Random Trees

Steven M. LaValle

James Kuffner

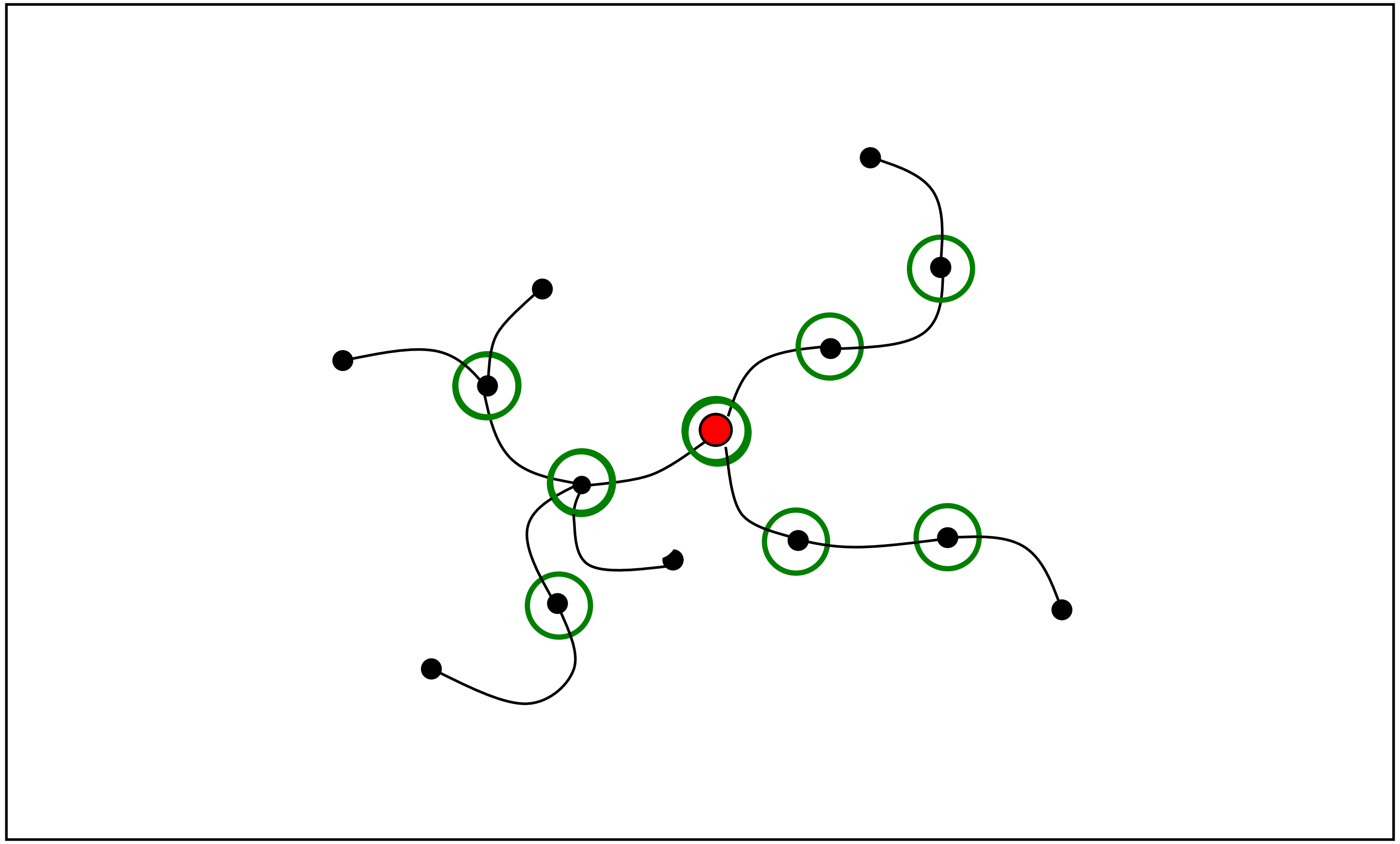
RAPIDLY-EXPLORING RANDOM TREE (RRT)



KINODYNAMIC PLANNING

1) RANDOM SAMPLE q_R
2) FIND CLOSEST V IN TREE
3) EXPAND TREE TOWARDS q_R
4) ADD NODE & EDGE TO TREE

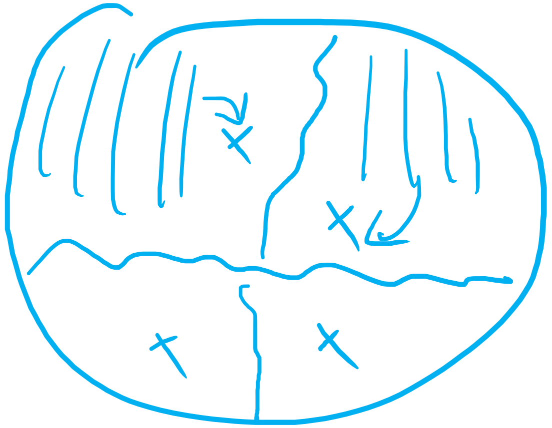
Rapidly-Exploring Random Trees (RRT)



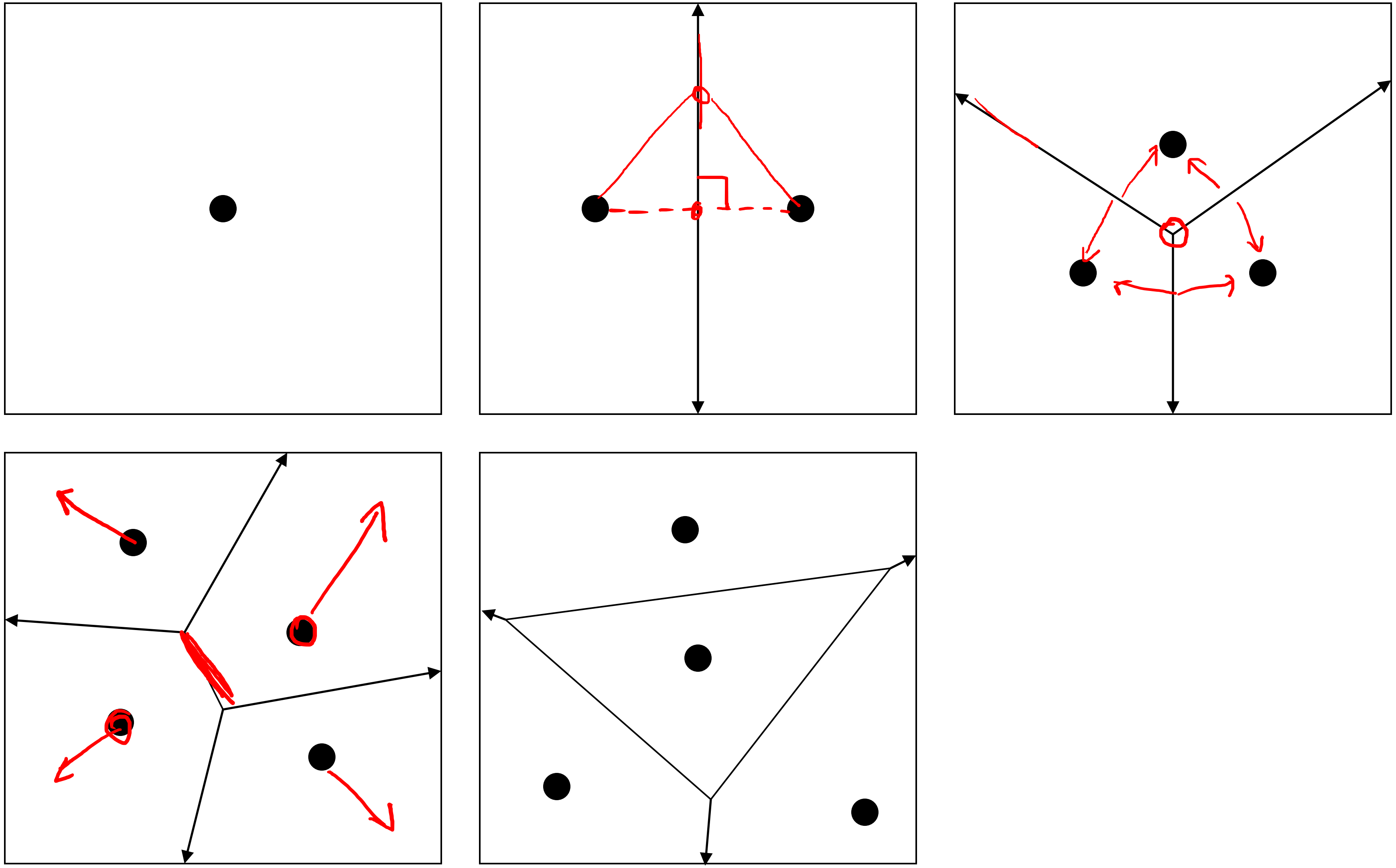
Sidebar: Voronoi Diagram

- The Post Office Problem: Which is the closest post office to every house? (Don Knuth)
- Given n **sites** in the plane
- Subdivision of plane based on proximity

Georgy Voronoi
1868-1908



Voronoi Diagram



Uses for Voronoi Diagram

- **Anthropology and Archeology** -- Identify the parts of a region under the influence of different neolithic clans, chiefdoms, ceremonial centers, or hill forts.
- **Astronomy** -- Identify clusters of stars and clusters of galaxies (Here we saw what may be the earliest picture of a Voronoi diagram, drawn by Descartes in 1644, where the regions described the regions of gravitational influence of the sun and other stars.)
- **Biology**, **Ecology**, **Forestry** -- Model and analyze plant competition ("Area potentially available to a tree", "Plant polygons")
- **Cartography** -- Piece together satellite photographs into large "mosaic" maps
- **Crystallography and Chemistry** -- Study chemical properties of metallic sodium ("Wigner-Seitz regions"); Modelling alloy structures as sphere packings ("Domain of an atom")
- **Finite Element Analysis** -- Generating finite element meshes which avoid small angles
- **Geography** -- Analyzing patterns of urban settlements
- **Geology** -- Estimation of ore reserves in a deposit using information obtained from bore holes; modelling crack patterns in basalt due to contraction on cooling
- **Geometric Modeling** -- Finding "good" triangulations of 3D surfaces
- **Marketing** -- Model market of US metropolitan areas; market area extending down to individual retail stores
- **Mathematics** -- Study of positive definite quadratic forms ("Dirichlet tesselation", "Voronoi diagram")
- **Metallurgy** -- Modelling "grain growth" in metal films
- **Meteorology** -- Estimate regional rainfall averages, given data at discrete rain gauges ("Thiessen polygons")
- **Pattern Recognition** -- Find simple descriptors for shapes that extract 1D characterizations from 2D shapes ("Medial axis" or "skeleton" of a contour)
- **Physiology** -- Analysis of capillary distribution in cross-sections of muscle tissue to compute oxygen transport ("Capillary domains")
- **Robotics** -- Path planning in the presence of obstacles
- **Statistics and Data Analysis** -- Analyze statistical clustering ("Natural neighbors" interpolation)
- **Zoology** -- Model and analyze the territories of animals

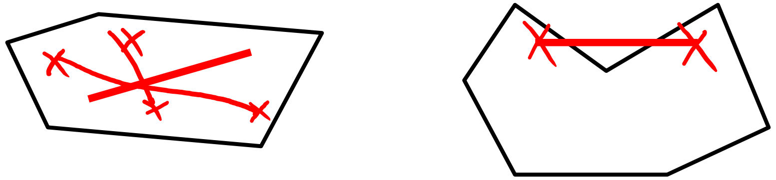
Facts about Voronoi

- A site has an unbounded region if and only if it lies on the convex hull of all sites.
- All Voronoi regions are convex.

- What is convex?

Sidebar: Convexity

- A polygon P is convex if a line connecting any two points $p_1, p_2 \in P$ is entirely contained in P.

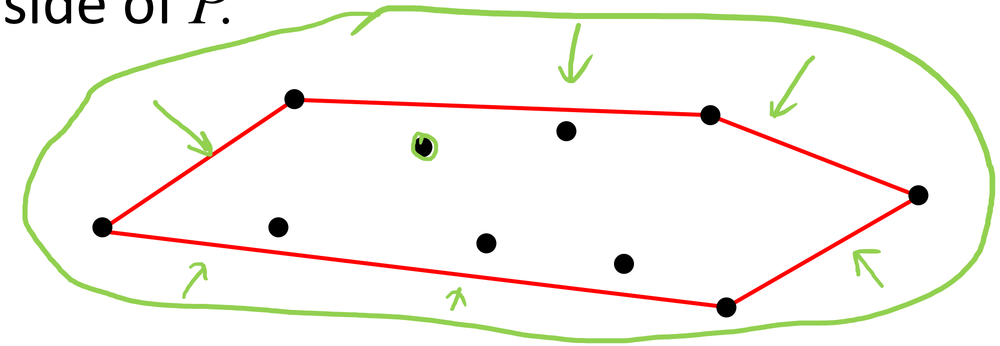


convex

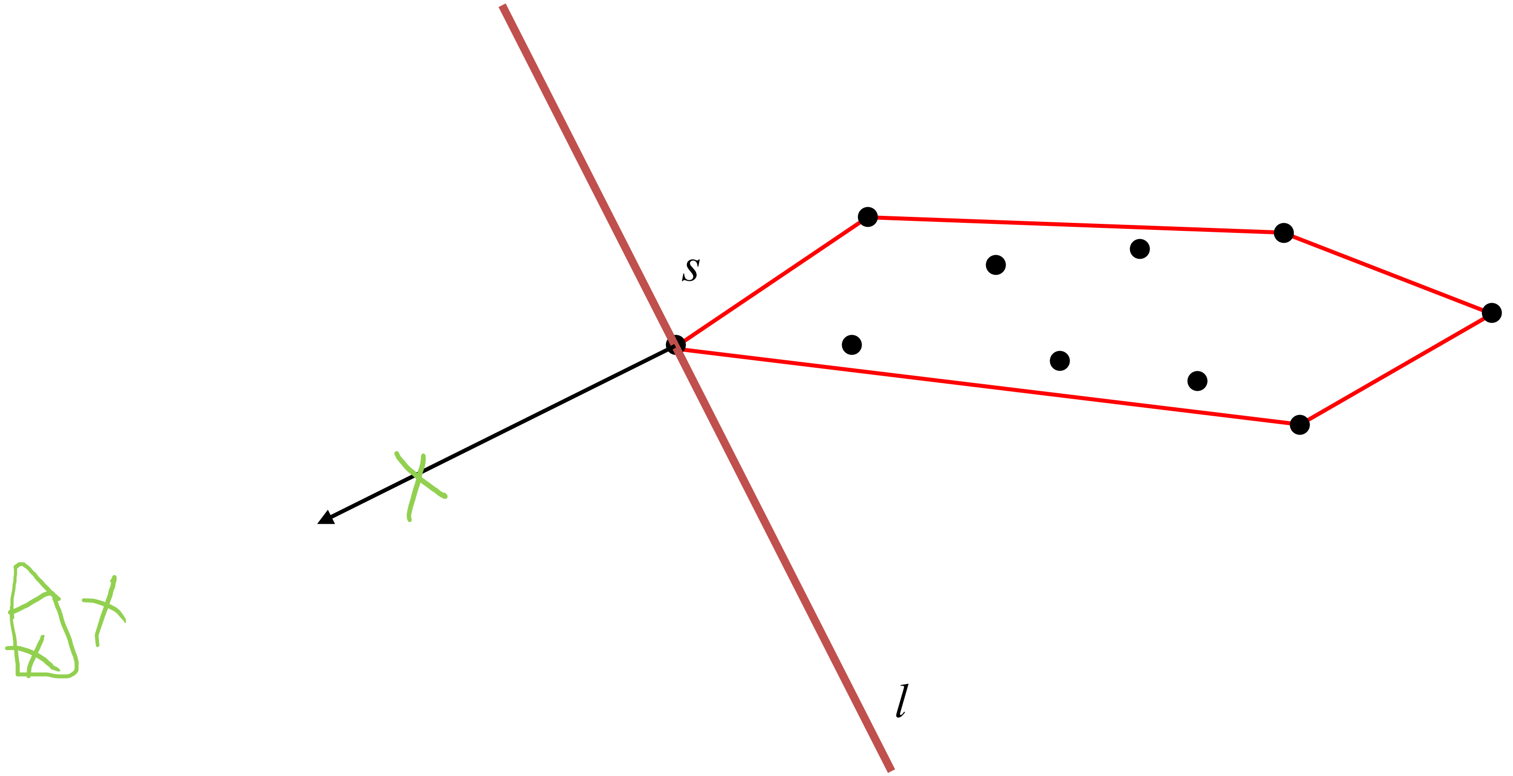
non-convex

CONCAVE

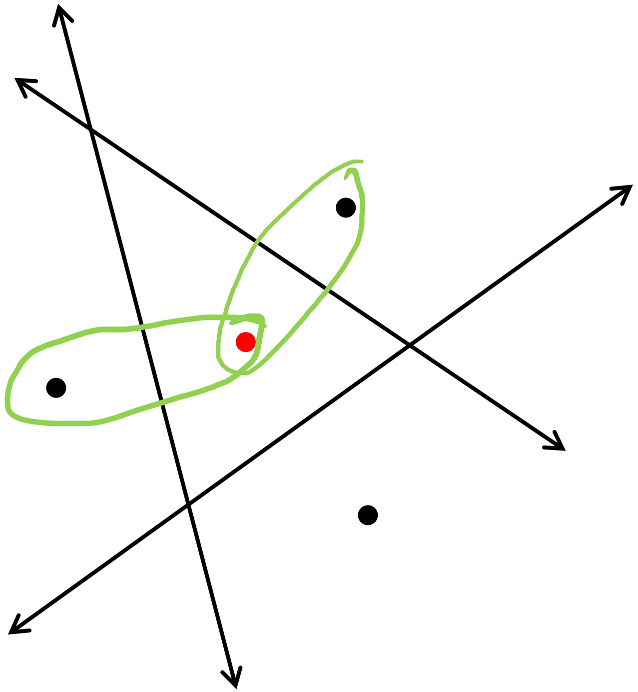
- The convex hull $\mathrm{hull}(S)$ of a set of points S is the smallest polygon P for which all points in S are either on the boundary or on the inside of P.



A site s has an unbounded region if and only if it lies on the convex hull of all sites.

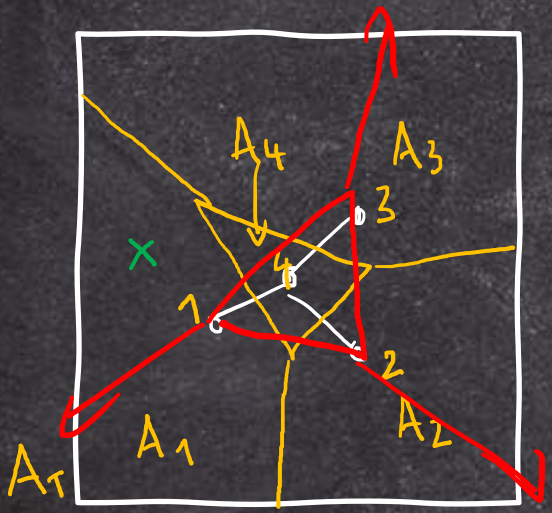


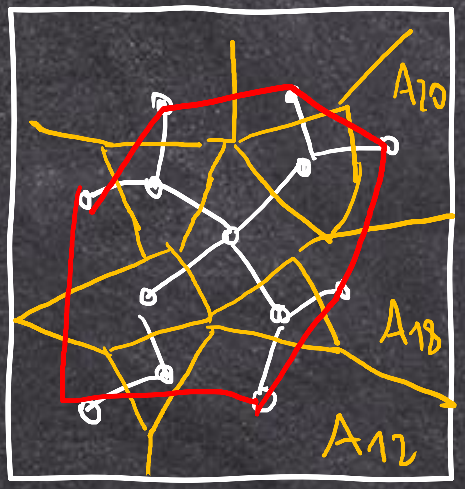
All Voronoi regions are convex.

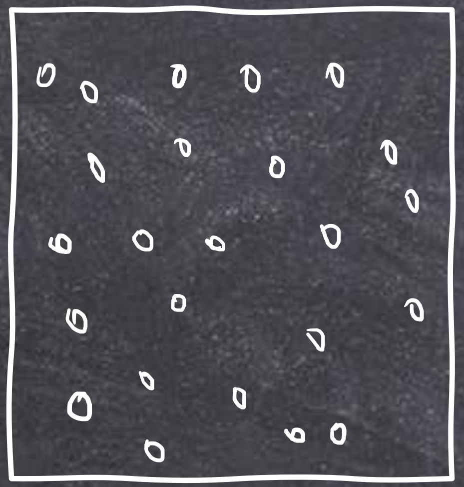


$$P(\text{EXPAND } V_1) = \frac{A_1}{A_T}$$

LOOKS UNIFORMLY SAMPLED



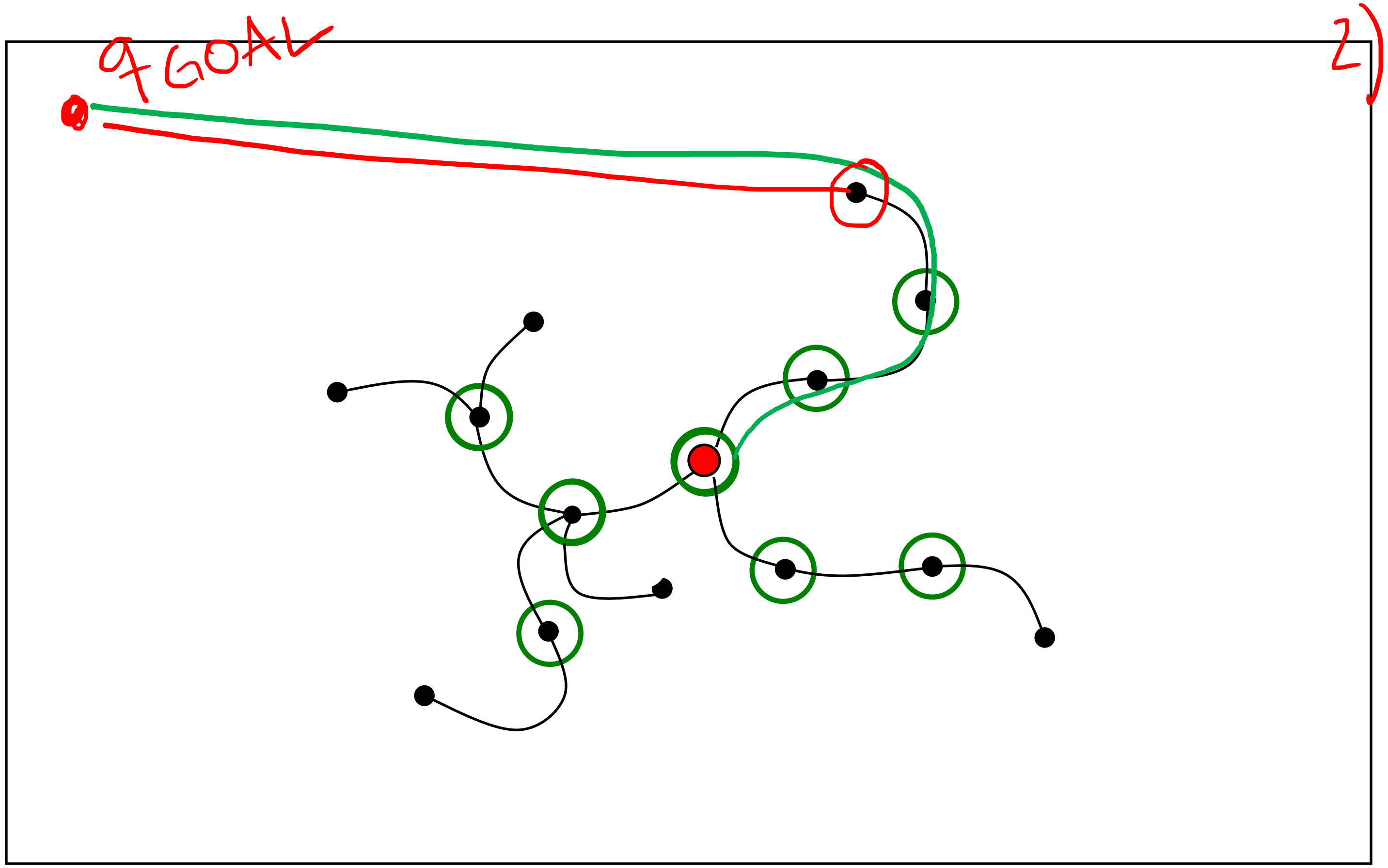




Rapidly-Exploring Random Trees

VORONOI BIAS

RRT Connect



1) EXPLORATION BASED ON VORONOI BIAS

2) EXPLOITATION (STRAIGHT SHOT)

Kinodynamic Planning with RRT

- Easy to integrate local planners for
  - Kinematic constraints
  - Dynamic constraints
- Expand C-space to state space for velocity representation (2 *d* dimensions)
- Requires known dynamic model (grasping!)
- Planning times for 7 dof ~ 10-20 seconds (holonomic, no dynamics)