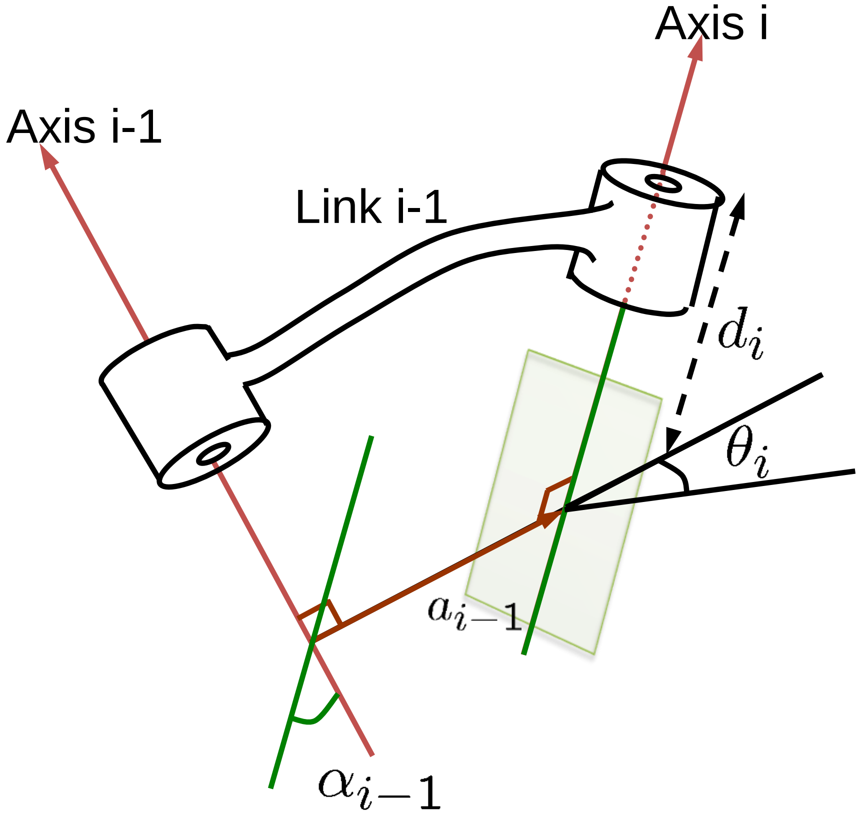
Denavit-Hartenberg Parameters

- Minimum number of parameters
- Standardized nomenclature
- Efficient calculation

Link Description and Connections



a_{i-1}

link length

perpendicular to both axes

not unique for parallel axes

α_{i-1}

link twist

angle measured about vector a_{i-1}

θ_i

joint angle

measured about axis ***i*** joint variable for revolute, fixed for prismatic

d_i

link offset

measured along axis ***i*** joint variable for prismatic, fixed for revolute

Frame Attachment

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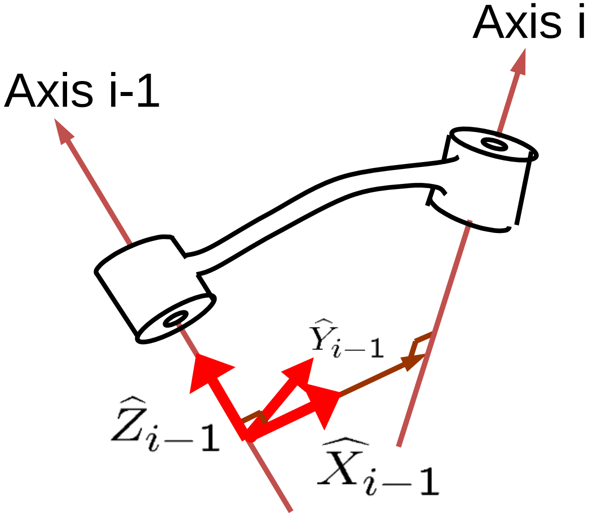
1. Identify joint axes; consider *i-1* and *i*
2. Identify common perpendicular
3. Label frame origin at perpendicular (or intersection)
4. Assign Z axis *i* along joint axis
5. Assign X axis *i* along perpendicular;
   if joint axes intersect, orthogonal to the axes plane
6. Complete frame by adding Y axis (right-hand-rule)
7. Assign {0} to match {1}
8. Choose end-effector frame {*n*}



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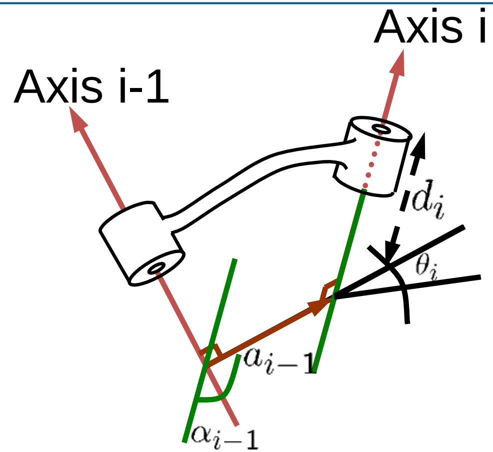
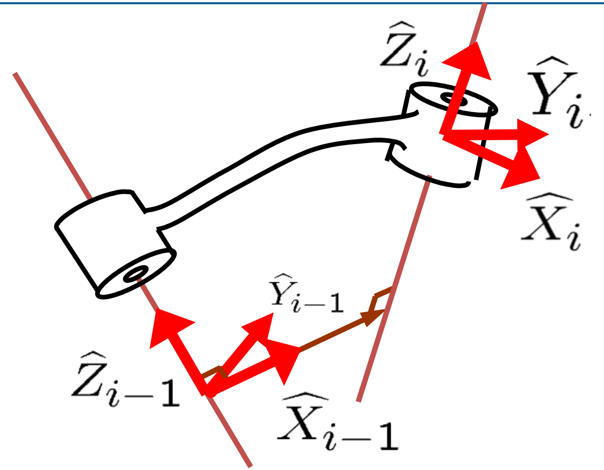
First and Last Link (0 and n)

- Frame 0 is reference (world) frame
  - Origin and Z axis coincide with frame 1
  - $\alpha_0 = a_0 = 0$
  - d_i or $\theta_i = 0$ depending on joint type
- Frame n is end-effector frame
  - X axis is aligned with X axis of frame n-1
  - d_i or $\theta_i = 0$ depending on joint type
- Maximize zeros in DH parameters

Denavit-Hartenberg (DH) parameters





α_i = the angle between $\hat{Z}_i$ and $\hat{Z}_{i+1}$ measured about $\hat{X}_i$

a_i = the distance from $\hat{Z}_i$ to $\hat{Z}_{i+1}$ measured along $\hat{X}_i$

d_i = the distance from $\hat{X}_{i-1}$ to $\hat{X}_i$ measured along $\hat{Z}_i$

θ_i = the angle between $\hat{X}_{i-1}$ and $\hat{X}_i$ measured about $\hat{Z}_i$

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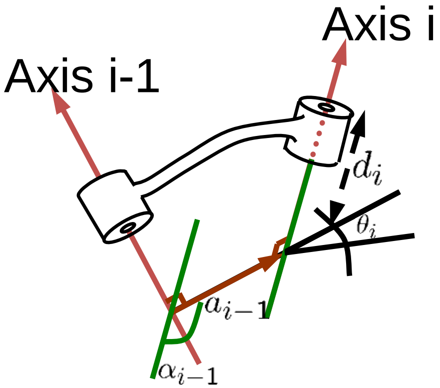
Frame to Frame Transformation



$$
{}^{i-1}_{\;\;i}T = R_X(\alpha_{i-1})\, D_X(a_{i-1})\, R_Z(\theta_i)\, D_Z(d_i)
$$

$$
= \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

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DH: Non-uniqueness

- Convention does not result in a unique frame assignment:

  - When aligning Z_i with axis i, there are two choices with respect to the direction (+ / -)
  - If joint axes intersect, there are two choices with respect to the direction of X_i
  - When {i} and {i+1} are parallel, the origin location for {i} is arbitrary (though usually chosen such that $d_i = 0$)
  - When prismatic joints are present, there is some freedom in frame assignment