Disclaimer

These slides are intended as presentation aids for the lecture. They contain information that would otherwise be to difficult or time-consuming to reproduce on the board. But they are incomplete, not self-explanatory, and are not always used in the order they appear in this presentation. As a result, these slides should not be used as a script for this course. I recommend you take notes during class, maybe on the slides themselves. It has been shown that taking notes improves learning success.

Reading for this set of slides

- Craig – Intro to Robotics (3rd Edition)
  - 1 Introduction
  - 2 Spatial descriptions and transformations (2.1 – 2.9)
  - 3 Manipulator kinematics (3.1 – 3.6)
  - 7 Trajectory generation

Please note that this set of slides is intended as support for the lecture, not as a stand-alone script. If you want to study for this course, please use these slides in conjunction with the indicated chapters in the text books. The textbooks are available online or in the TUB library (many copies that can be checked out for the entire semester. There are also some aspects of the lectures that will not be covered in the text books but can still be part of the homework or exam. For those It is important that you attend class or ask somebody about what was covered in class.

Robotics

Controlling a Real Robot

TU Berlin

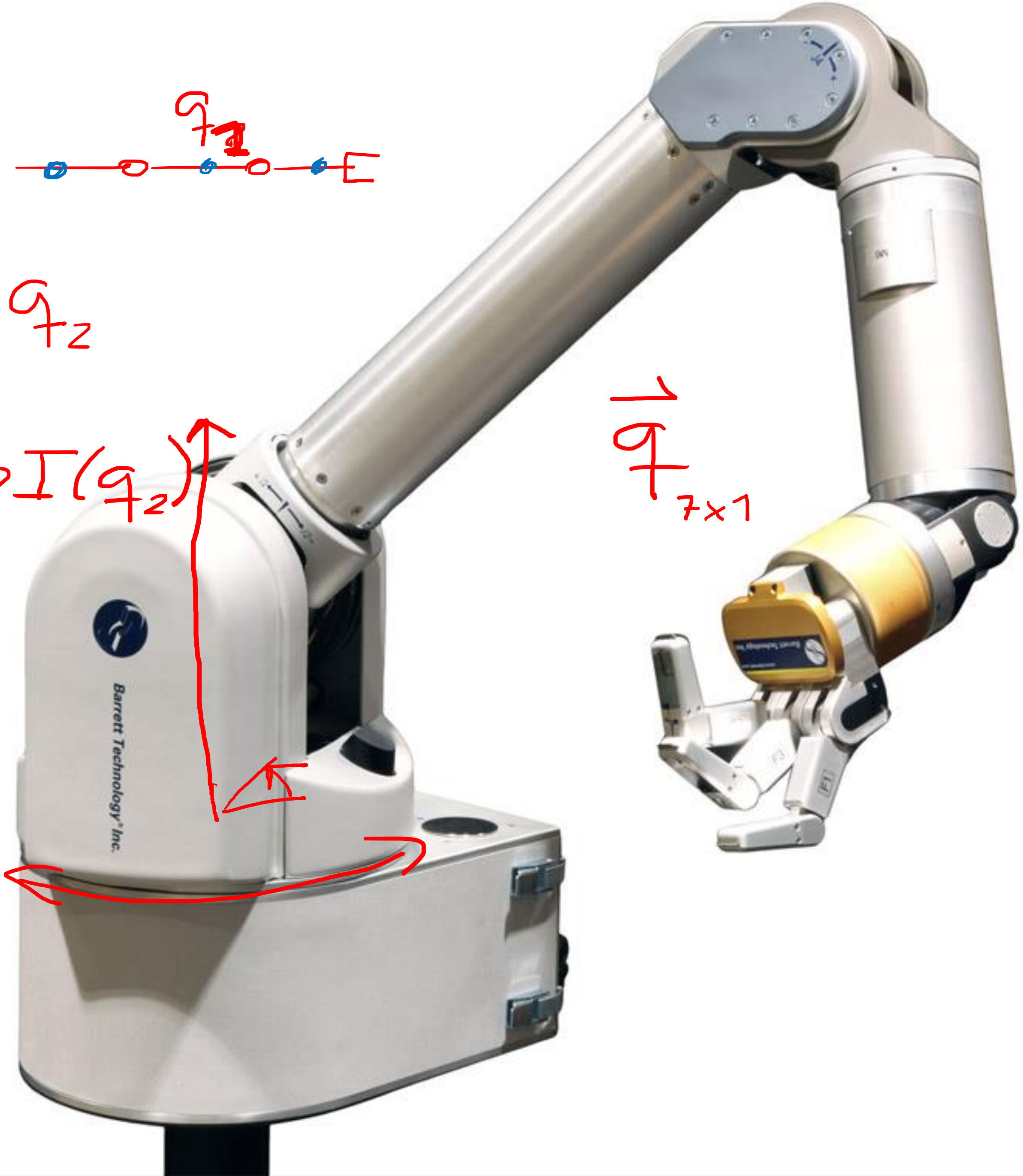
Oliver Brock

$\underline{T_1}$ q_{71} E

T_2 q_{72}

$I(q_1) \gg I(q_2)$

$T_1 \gg T_2$



$\vec{q}_{7\times1}$

Equations of Motion

scalar

$$f = m\ddot{x} + b\dot{x} + kx$$

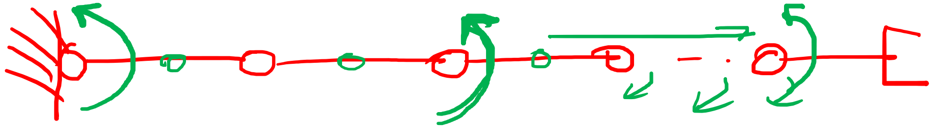
$$f = -k_p x - k_v\,\dot{x}$$

$$M(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{v}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q}) = \tau$$

MATRIX

CENTRIFUGAL

CORIOLIS



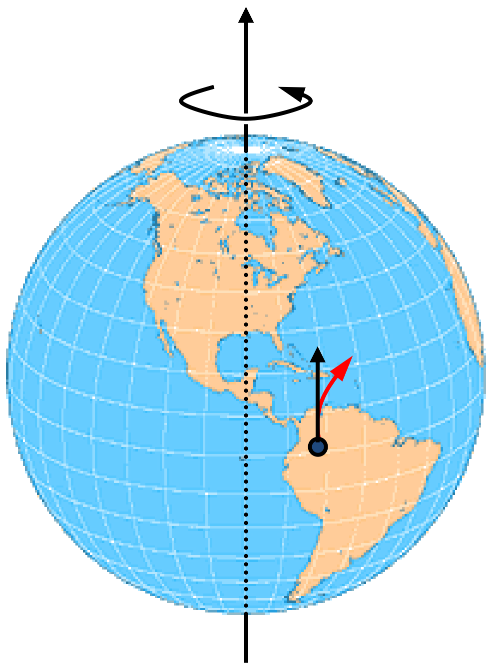
m_{11} : $m_{L1}, \dots, m_{L7}$

$\vec{q}$

m_{77} : m_{L7} , q_7

$$M_{7\times 7} = \begin{bmatrix} m_{11} & \cdots & m_{3,5} \\ & \ddots & \\ & & m_{77} \end{bmatrix}$$

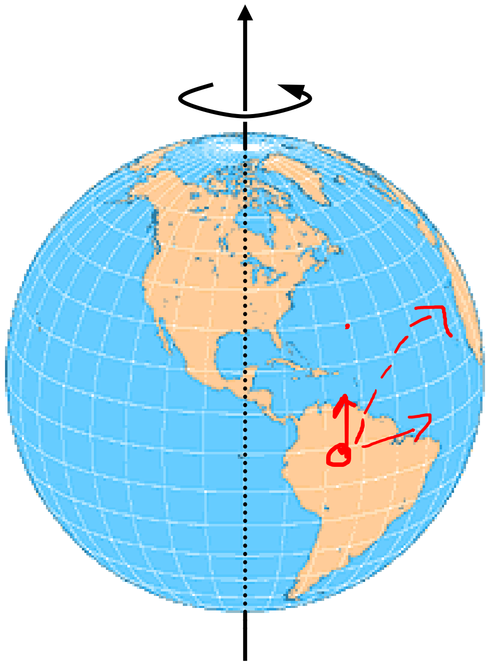
Our Planet – The Earth

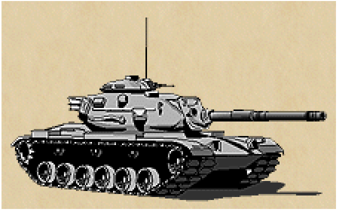


Velocity at equator:
1673 km/h
464 m/s
Radius of earth at equator:
6,378.14 km
Angular velocity of earth:
$|\Omega| = |v| / |r| = 464 / 6{,}378{,}140 =$
$7.27\ 10^{-5}$ rad/s =
0,00417 deg/s

Of course: 2π in 24 hours!

What are Coriolis Forces anyway?





$$\mathbf{F}_{\text{Coriolis}} = -2\, m\, \omega_{\{A\}} \times {}^{A}\mathbf{v}$$

Control Law Partitioning

model portion

$$f = \alpha f' + \beta$$

servo portion

$$m\ddot{x} + b\dot{x} + kx = \alpha f' + \beta$$

$$\alpha = m \qquad \beta = b\dot{x} + kx$$

$$\ddot{x} = f'$$

Unit Mass Controller

$$f' = -k_v \dot{x} - k_p x$$

with $\ddot{x} = f'$ yields

$$\ddot{x} + k_v \dot{x} + k_p x = 0$$

for critical damping: $k_v = 2\sqrt{k_p}$

independent of physical system!

Trajectory or Motion Control

A trajectory specifies as a function of time:

$$x_d(t), \dot{x}_d(t), \ddot{x}_d(t)$$

Error is defined as $e \overset{\text{def.}}{=} e(t) = x_d(t) - x(t)$

$$f' = \ddot{x} = \ddot{x}_d + k_v \dot{e} + k_p e$$

$$\ddot{e} + k_v \dot{e} + k_p e = 0$$

Disturbance Rejection

$$\cancel{\ddot{e}} + \cancel{k_v \dot{e}} + \underline{k_p}\, e = \underline{f_{\text{disturbance}}} \qquad G(q)$$

for bounded disturbances we can guarantee stability

Steady state error:

$$k_p\, e = f_{\text{disturbance}} \Rightarrow \underline{\underline{e}} = \frac{f_{\text{disturbance}}}{k_p}$$

Error will never be zero in the presence of a disturbance since k_p cannot be ∞

Integral Term gives PID Control

$$f' = \underbrace{\ddot{x}_d + k_v\,\dot{e} + k_p\,e} + k_i \int_{0\,/\,t-10}^{\infty\,/\,t} e\,dt$$

$$\ddot{e} + k_v\,\dot{e} + k_p\,e + k_i \int e\,dt = f_{\text{disturbance}}$$

P = proportional

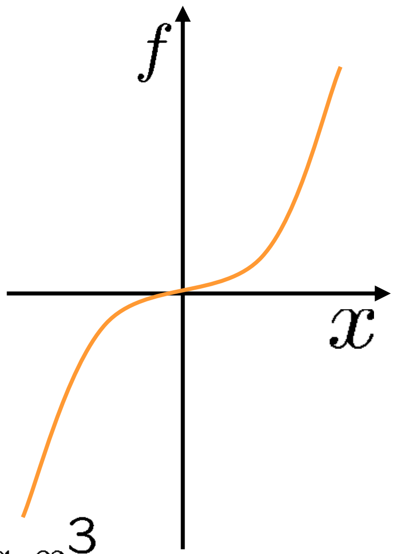
I = integral

D = derivative

control (feedback)

For simplicity, we will not consider the integral term.

Linearization using Partitioning



$f = q\, x^3$

$$m\, \ddot{x} + b\, \dot{x} + q\, x^3 = f$$

$$\alpha = m$$
$$\beta = b\, \dot{x} + q\, x^3$$

$$f' = \ddot{x}_d + k_v\, \dot{e} + k_p\, e$$

Linearization of Real Robot

α $\checkmark$ β

$$\underbrace{M(\mathbf{q})}_{\alpha}\underline{\underline{\ddot{\mathbf{q}}}}_{\tau'} + C(\mathbf{q})[\dot{\mathbf{q}}^2] + B(\mathbf{q})[\dot{\mathbf{q}}\dot{\mathbf{q}}] + \mathbf{G}(\mathbf{q}) = \tau$$

$$\tau = \alpha\tau' + \beta$$

$$\alpha = M(\mathbf{q})$$

$$\beta = C(\mathbf{q})[\dot{\mathbf{q}}^2] + B(\mathbf{q})[\dot{\mathbf{q}}\dot{\mathbf{q}}] + \mathbf{G}(\mathbf{q})$$

ROBOT DYNAMICS

$$\underline{\underline{\tau'}} = \ddot{\mathbf{q}}_d + \mathbf{k}_v\,\dot{\mathbf{e}} + \mathbf{k}_p\,\mathbf{e}$$

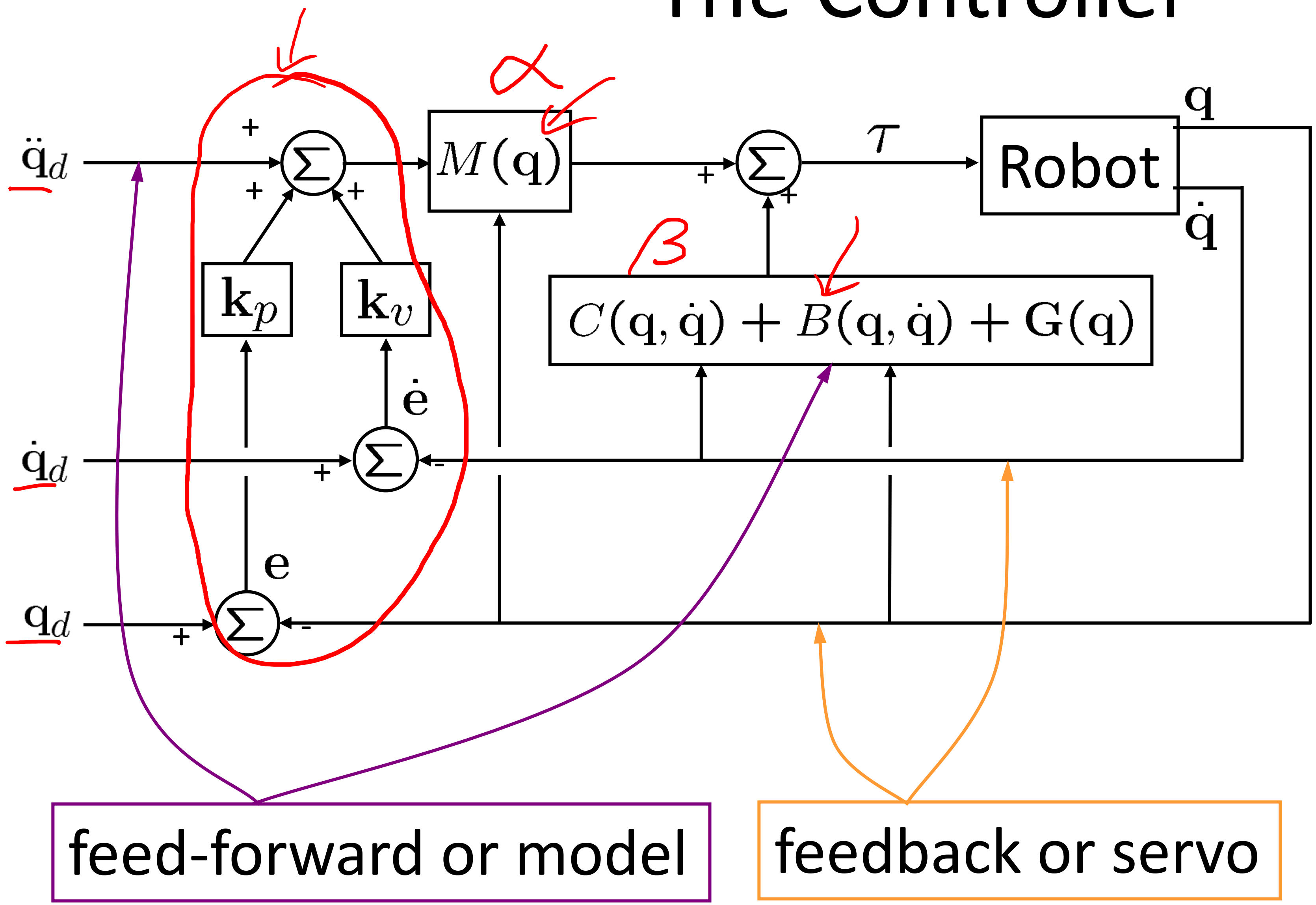
m=1

~~SPRING/MASS DAMPER~~

W/O ROBOT DYNAMICS

Note that we have gone from a single mass to a system of masses!

The Controller



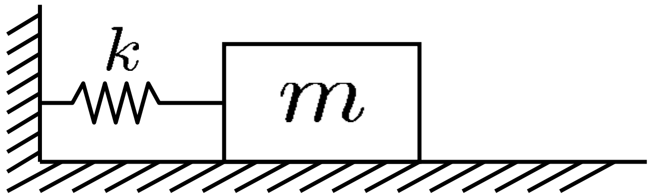
Recap

- Single mass with spring (and damper)
- Characteristics of motion
- Design controller to achieve desired behavior for linear system
- Partitioning for linearization of nonlinear system
- “Vectorization” for unified approach to *controlling a manipulator* with many d.o.f.

Stability Analysis

- In a linear system stability requires $k_v > 0$
- Assuming bounded disturbance we can make certain guarantees
- Analysis more complex in nonlinear systems
- Linearization is not always possible
  – inaccurate models
  – unknown models

Energy-Based Stability Analysis



$$m\ddot{x} + b\dot{x} + kx = 0$$

$$E = \frac{1}{2} m\,\dot{x}^2 + \frac{1}{2} k\,x^2$$

$$\begin{aligned} \dot{E} &= m\ddot{x}\dot{x} + kx\dot{x} \\ &= (-b\dot{x} - kx)\dot{x} + kx\dot{x} \\ &= -b\dot{x}^2 \\ &< 0 \end{aligned}$$

Energy of system is reduced until it comes to rest at x = 0

Equations of Motion

SMD -SYS

Joint Space

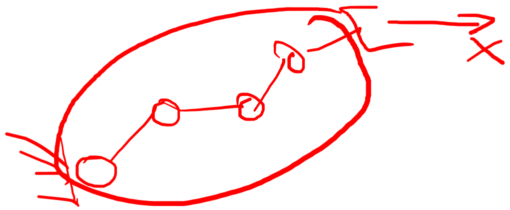
$$M(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{v}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q}) = \underline{\tau}$$

SMD -SYS

Operational Space

$$\Lambda(\mathbf{x})\ddot{\mathbf{x}} + \mu(\mathbf{x}, \dot{\mathbf{x}}) + \mathbf{p}(\mathbf{x}) = \mathbf{F}$$

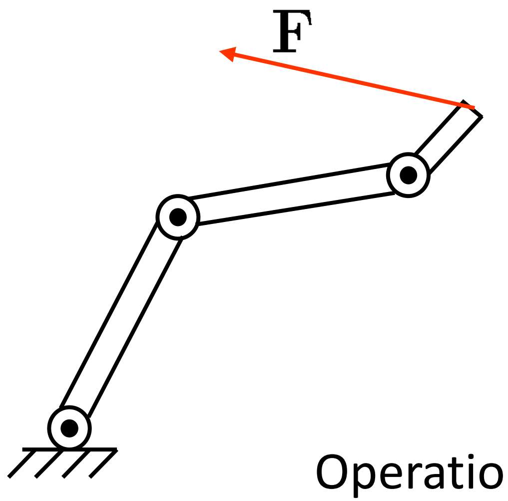
$$\Lambda = \begin{bmatrix} \Lambda_{xx} & & & \\ & \Lambda_{yy} & & \\ & & \Lambda_{zz} & \\ & & & \ddots \end{bmatrix}_{6\times 6}$$



Effector Inertia Matrix Λ

Joint space: $M(\mathbf{q})\ddot{\mathbf{q}} + C(\mathbf{q})[\dot{\mathbf{q}}^2] + B(\mathbf{q})[\dot{\mathbf{q}}\dot{\mathbf{q}}] + \mathbf{G}(\mathbf{q}) = \tau$

Inertia perceived at the joints



$m = ?$

Inertia perceived at effector?

Operational space inertia matrix $\Lambda(\mathbf{x})$