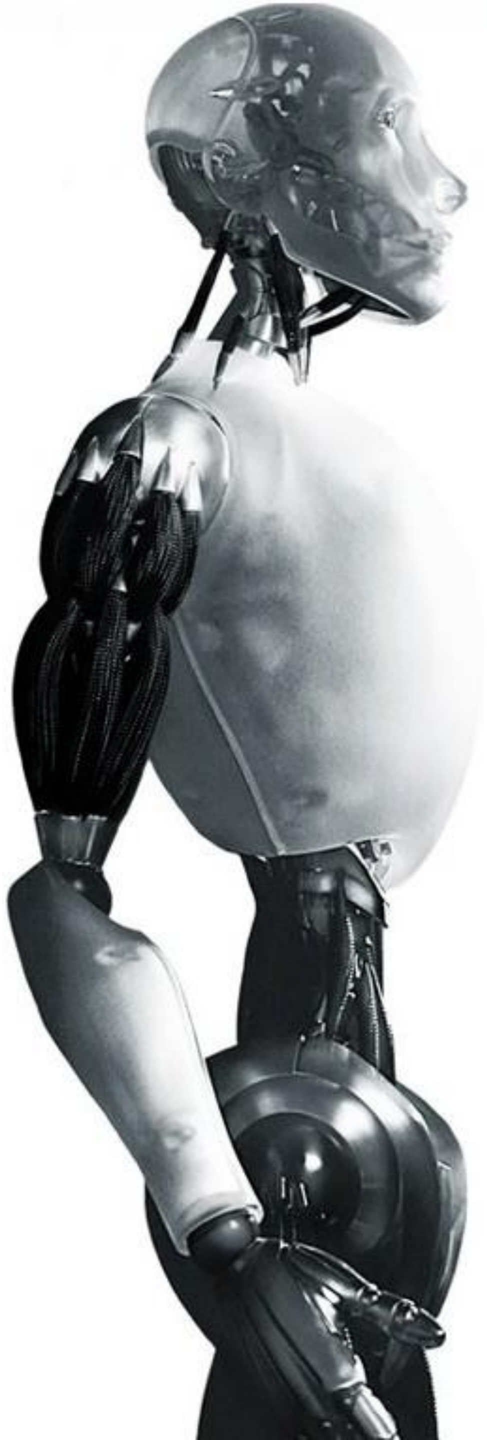
Disclaimer

These slides are intended as presentation aids for the lecture. They contain information that would otherwise be to difficult or time-consuming to reproduce on the board. But they are incomplete, not self-explanatory, and are not always used in the order they appear in this presentation. As a result, these slides should not be used as a script for this course. I recommend you take notes during class, maybe on the slides themselves. It has been shown that taking notes improves learning success.

Reading for this set of slides

- Planning Algorithms (Steve LaValle)
  - 6 Combinatorial Motion Planning (6.1 – 6.3)
  - 8 Feedback Motion Planning (8.1, 8.2)
- Please refer to the slides for potential fields and vehicle kinematics

Please note that this set of slides is intended as support for the lecture, not as a stand-alone script. If you want to study for this course, please use these slides in conjunction with the indicated chapters in the text books. The textbooks are available online or in the TUB library (many copies that can be checked out for the entire semester. There are also some aspects of the lectures that will not be covered in the text books but can still be part of the homework or exam. For those It is important that you attend class or ask somebody about what was covered in class.



Robotics

Generating Motion for Mobile Robots

TU Berlin

Oliver Brock

How do we get there?

Assumptions:

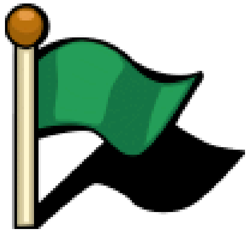
- Perfect knowledge of the world
- Perfect motion execution

How realistic is this?

NOT!

But let's just assume…

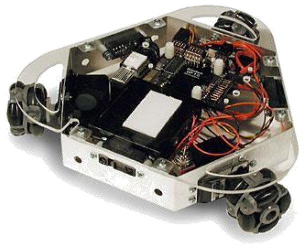
Heading



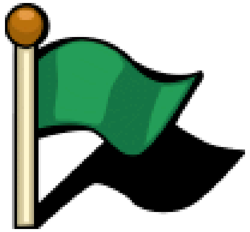
robot position

$$\mathbf{v} = {}^{0}\mathbf{g} - {}^{0}\mathbf{p}$$

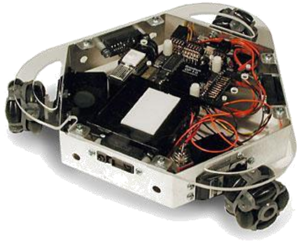
goal position



Problems with Heading

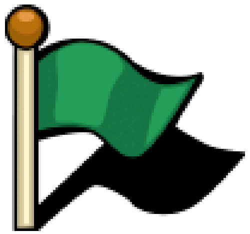


$$\mathbf{v} = {}^{0}\mathbf{g} - {}^{0}\mathbf{p}$$

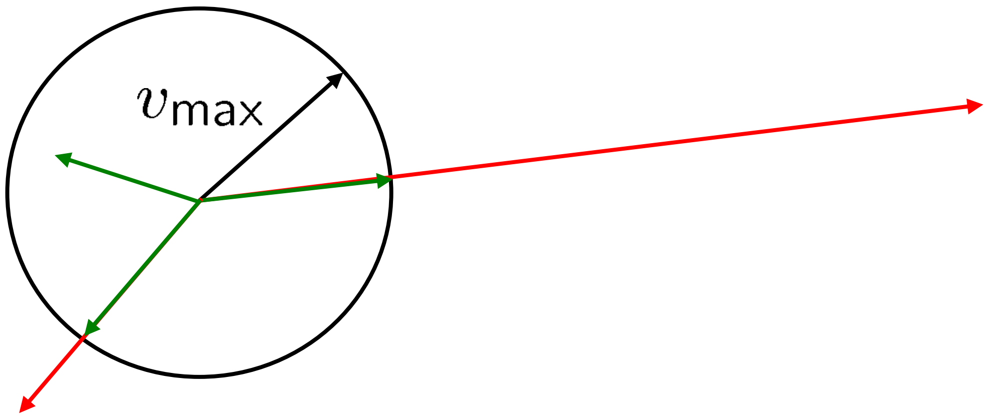


Problems:
- Obstacles
- Unlimited Velocity

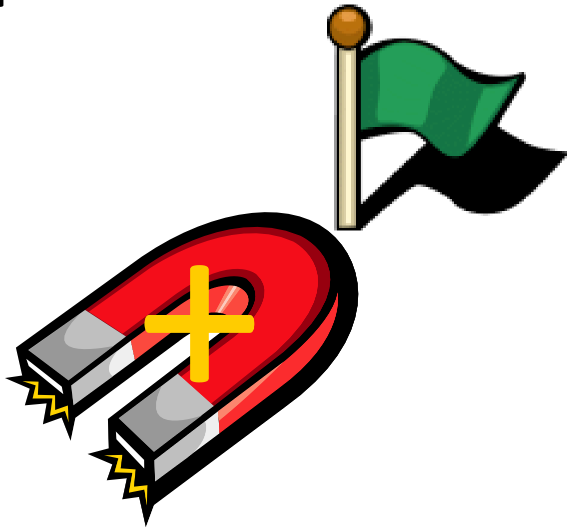
Saturating Velocity

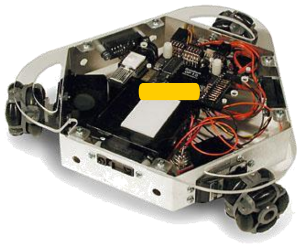


$$\mathbf{v} = \begin{cases} \mathbf{v} & \text{if } \|\mathbf{v}\| < v_{\mathsf{max}} \\ v_{\mathsf{max}} \dfrac{\mathbf{v}}{\|\mathbf{v}\|} & \text{otherwise} \end{cases}$$

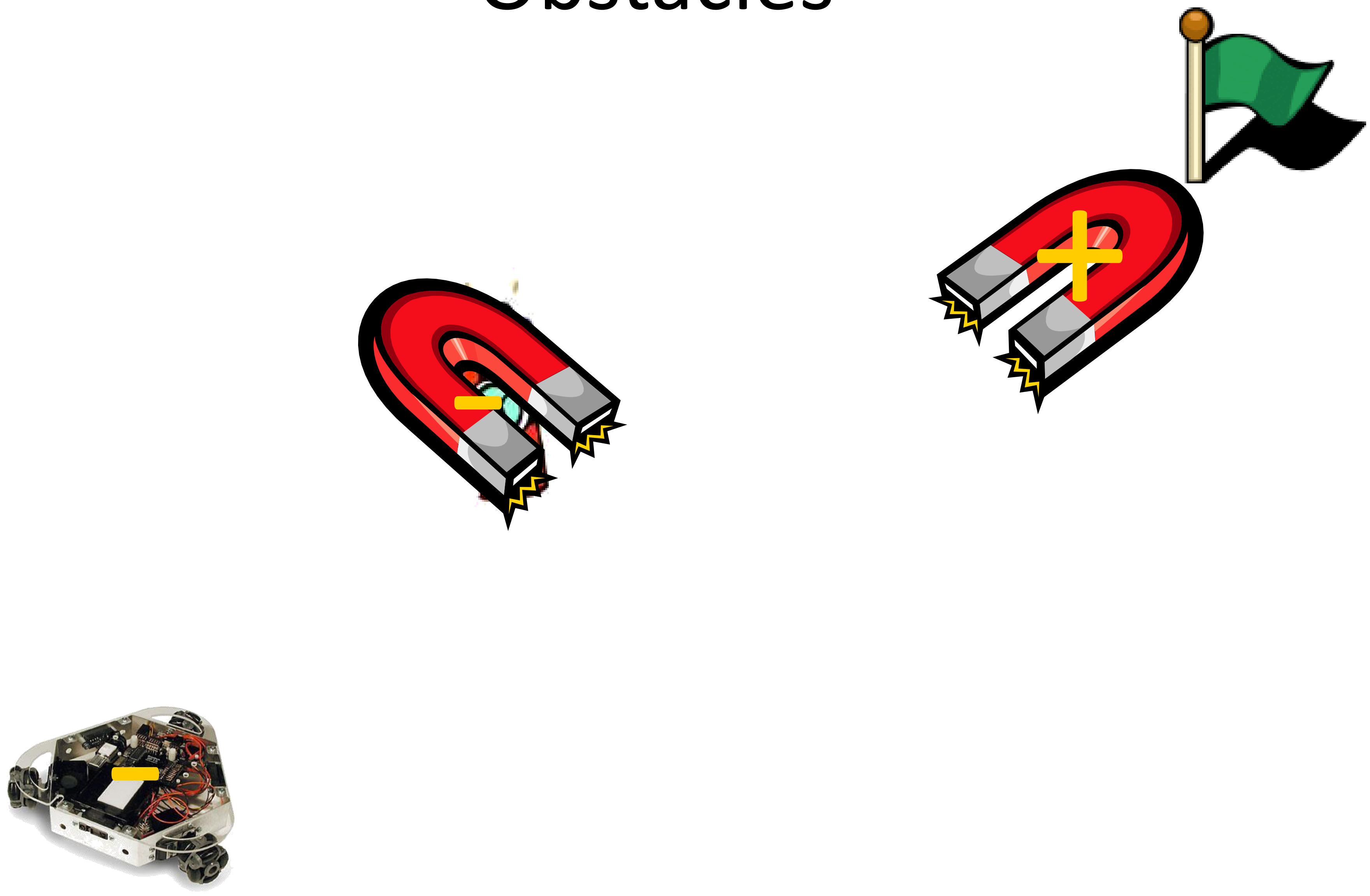


Physical Model

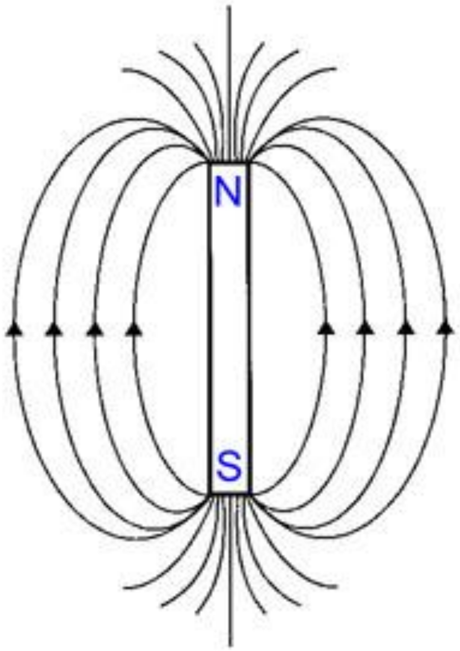




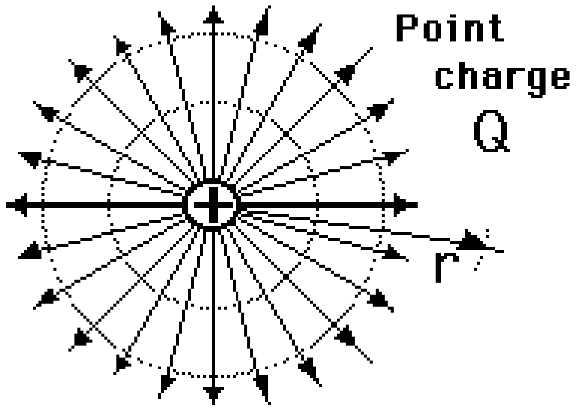
Obstacles



Electric Charges are better…

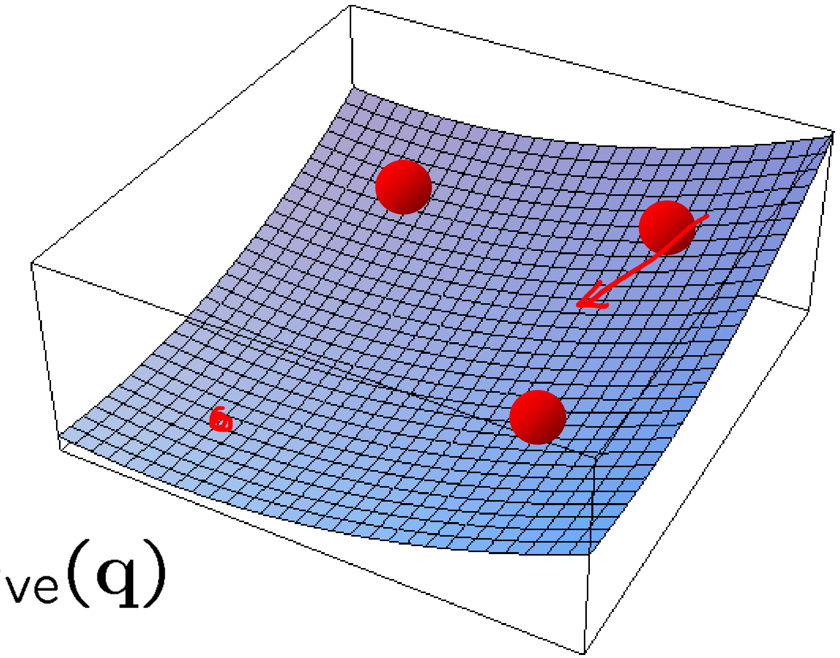


depends on
direction



$$\mathbf{F} = \frac{k \cdot q_1 \cdot q_2}{r^2}$$

Attractive Potential

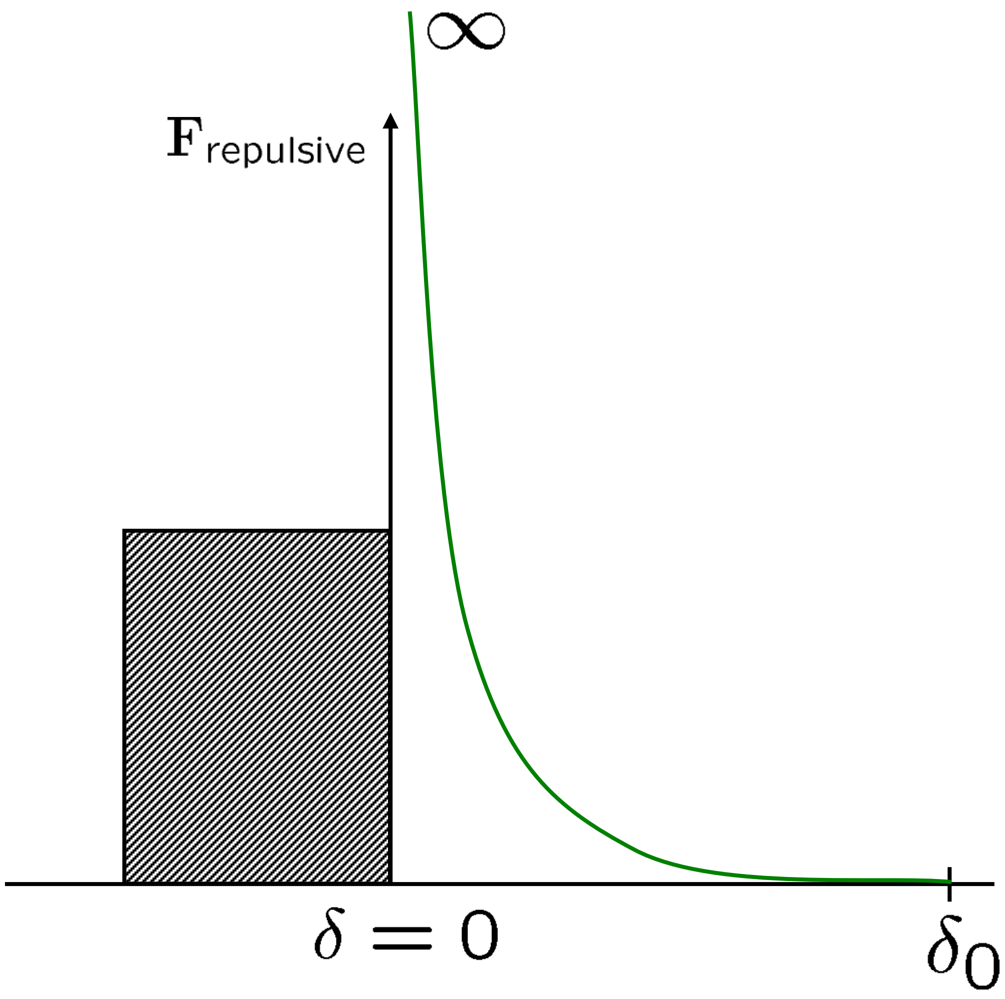


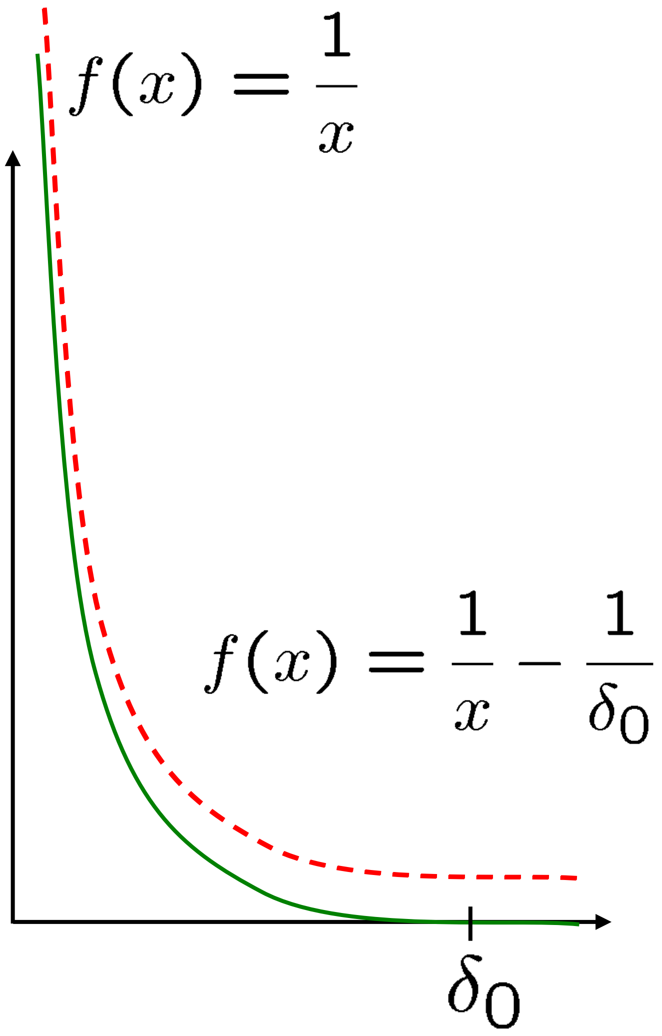
$$U_{\text{attractive}}(\mathbf{q}) = \frac{1}{2} k\, \delta^2_{\text{goal}}(\mathbf{q})$$

$$\begin{aligned}\mathbf{F}_{\text{attractive}}(\mathbf{q}) &= -\nabla U_{\text{attractive}}(\mathbf{q}) \\ &= -k\, \delta_{\text{goal}}(\mathbf{q})\end{aligned}$$

$$\mathbf{F}_{\text{charge}} = \frac{k \cdot q_1 \cdot q_2}{r^2}$$ NOT physically motivated!

Designing Repulsive Potential



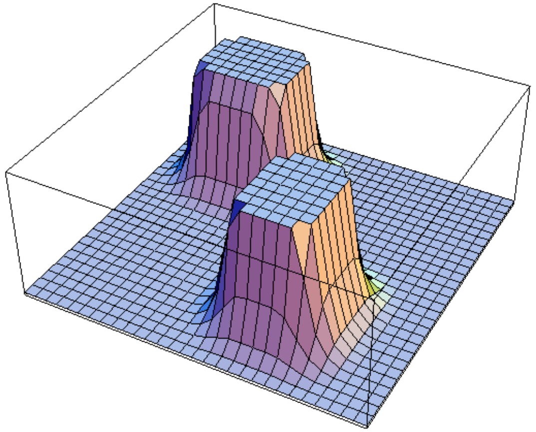


Repulsive Potential

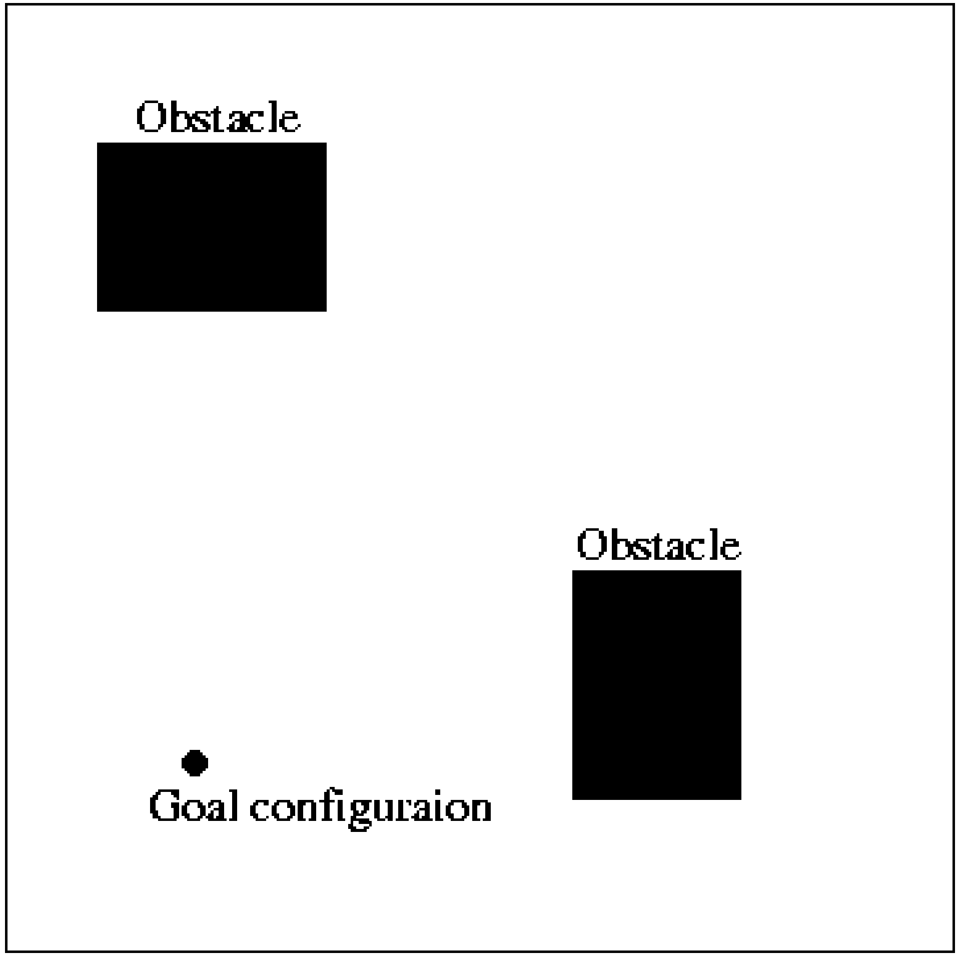
$$\mathbf{F}_{\text{repulsive}}(\mathbf{q}) = -\nabla U_{\text{repulsive}}(\mathbf{q})$$

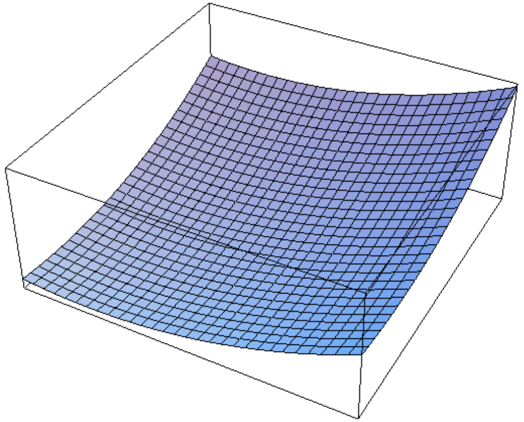
$$= \begin{cases} -k \left(\frac{1}{\delta_{\text{obstacle}}(\mathbf{q})} - \frac{1}{\delta_0} \right) & \text{if } \delta_{\text{obstacle}}(\mathbf{q}) < \delta_0 \\ 0 & \text{otherwise} \end{cases}$$

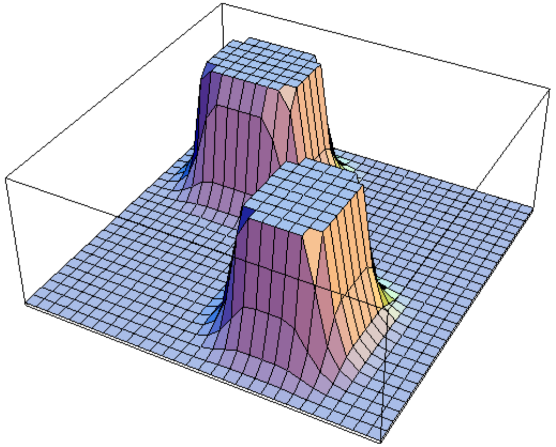
$$U_{\text{repulsive}}(\mathbf{q}) = \begin{cases} \frac{1}{2} k \left(\frac{1}{\delta_{\text{obstacle}}(\mathbf{q})} - \frac{1}{\delta_0} \right)^2 & \text{if } \delta_{\text{obstacle}}(\mathbf{q}) < \delta_0 \\ 0 & \text{otherwise} \end{cases}$$



Let's put it together



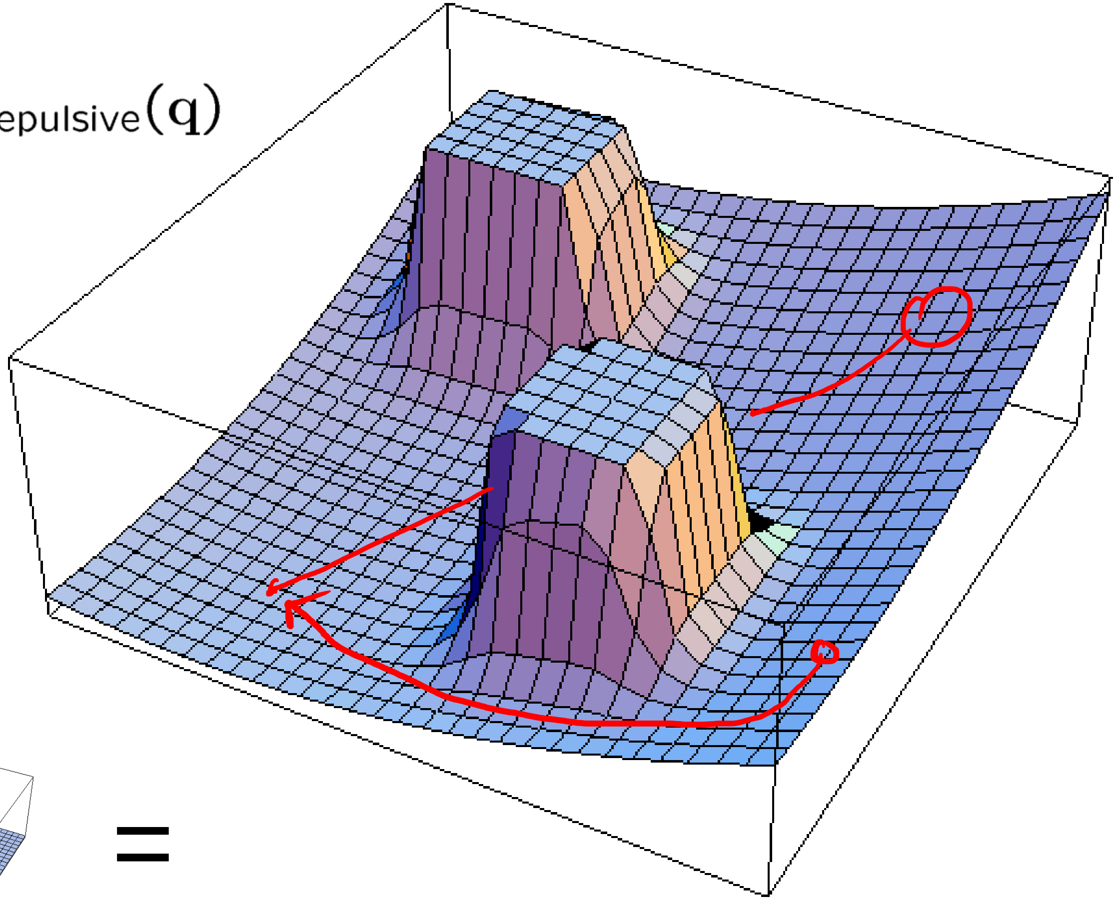


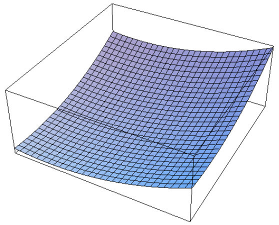


Artificial Potential Function

$$U(\mathbf{q}) = U_{\text{attractive}}(\mathbf{q}) + U_{\text{repulsive}}(\mathbf{q})$$

$$\mathbf{F}(\mathbf{q}) = -\nabla U(\mathbf{q})$$





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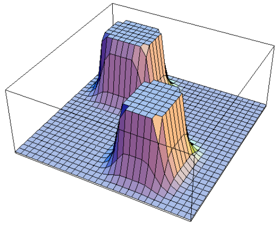

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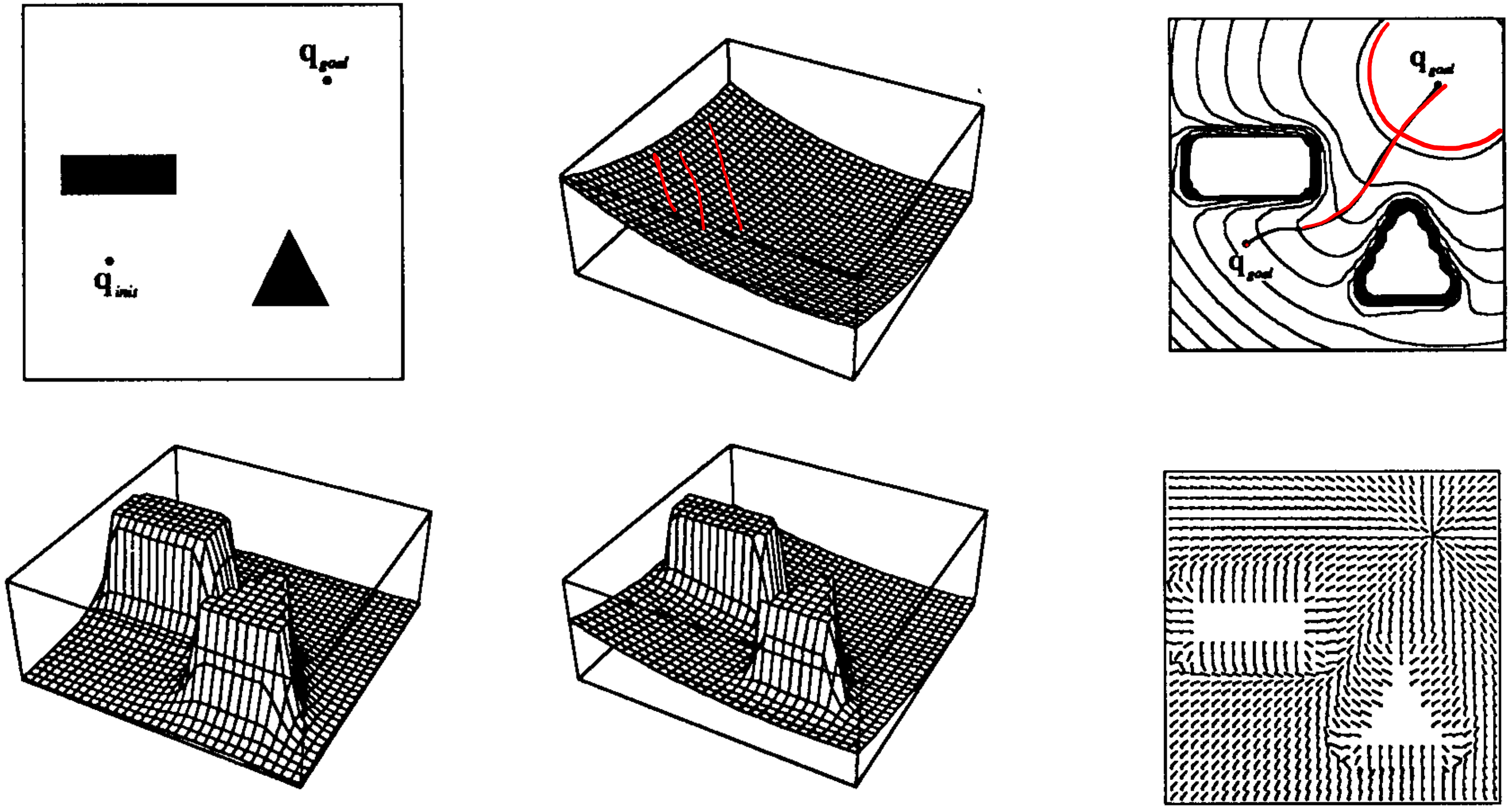
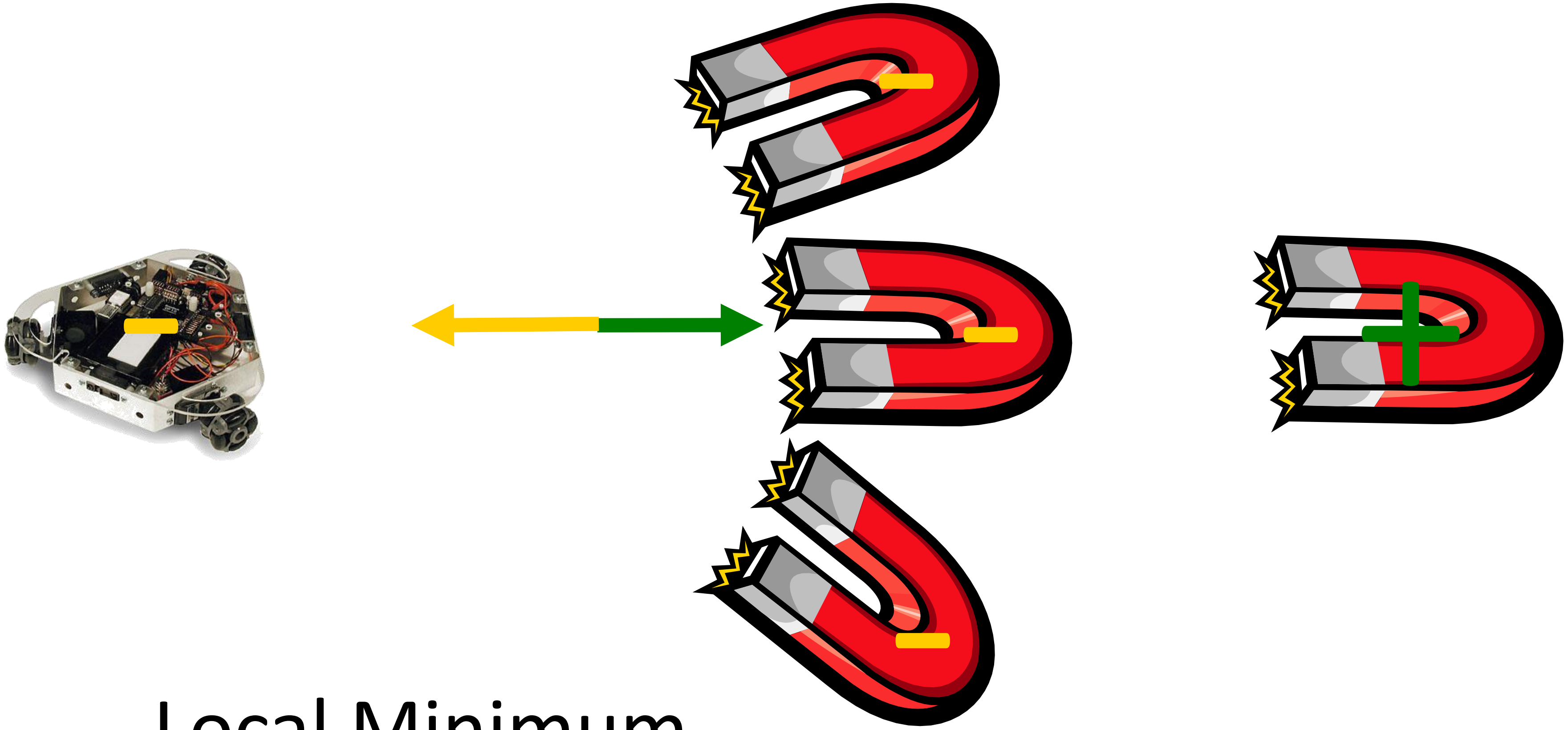
Potential Field Approach



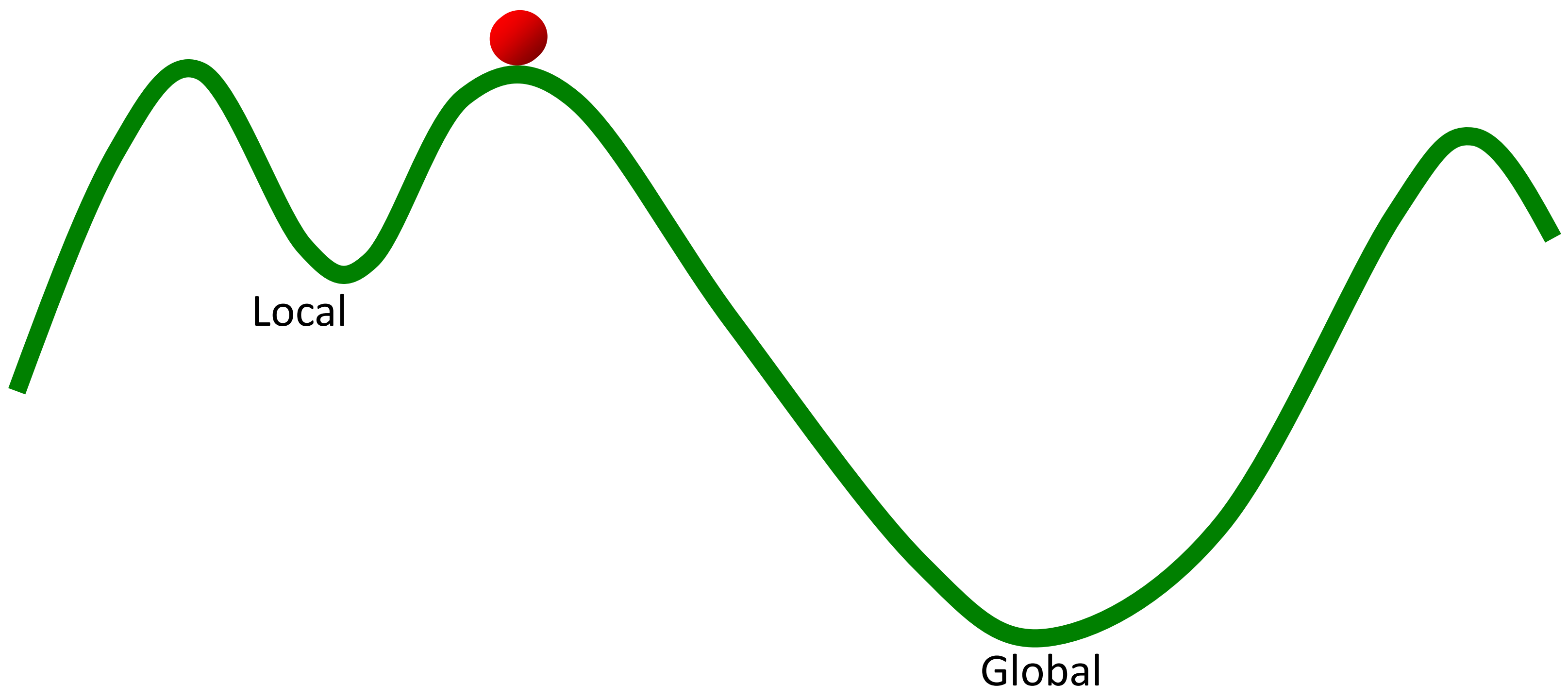
Jean-Claude Latombe

Getting Stuck

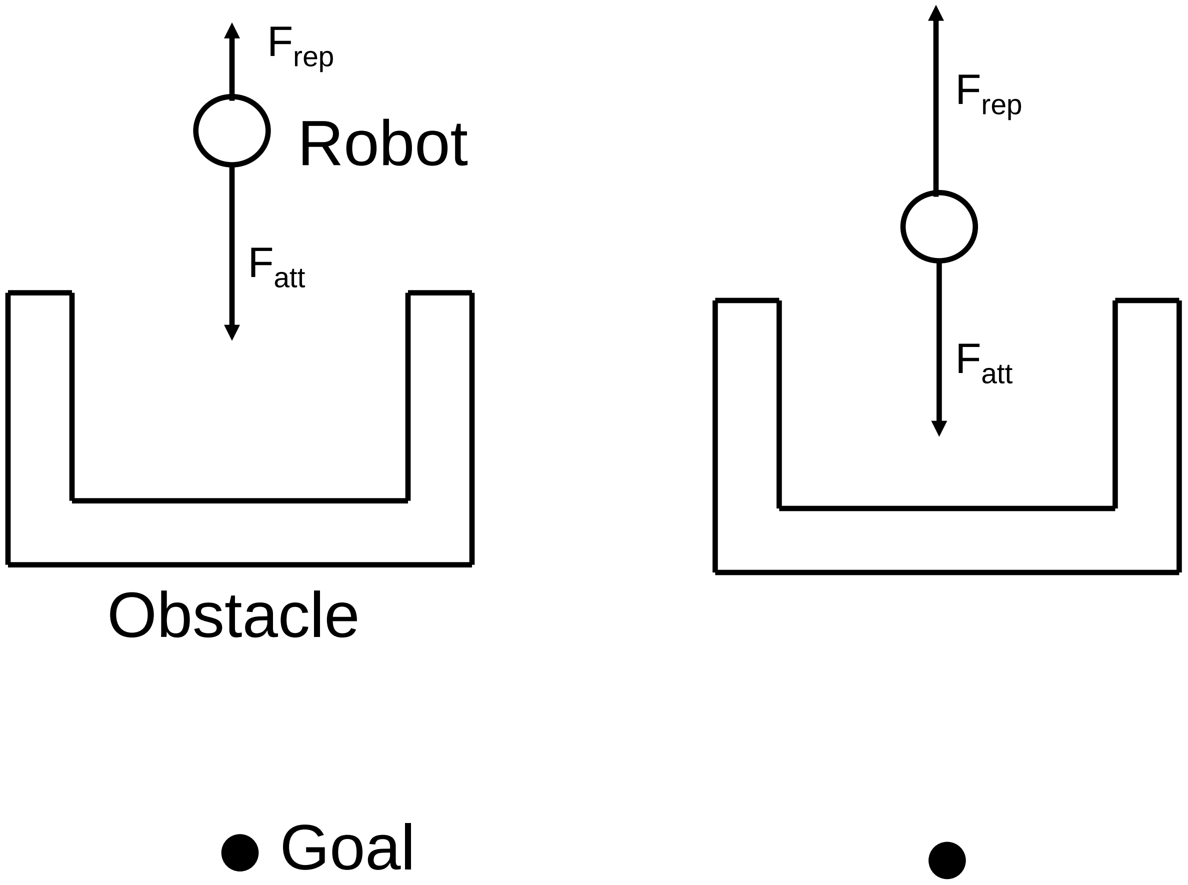


Local Minimum

Minima

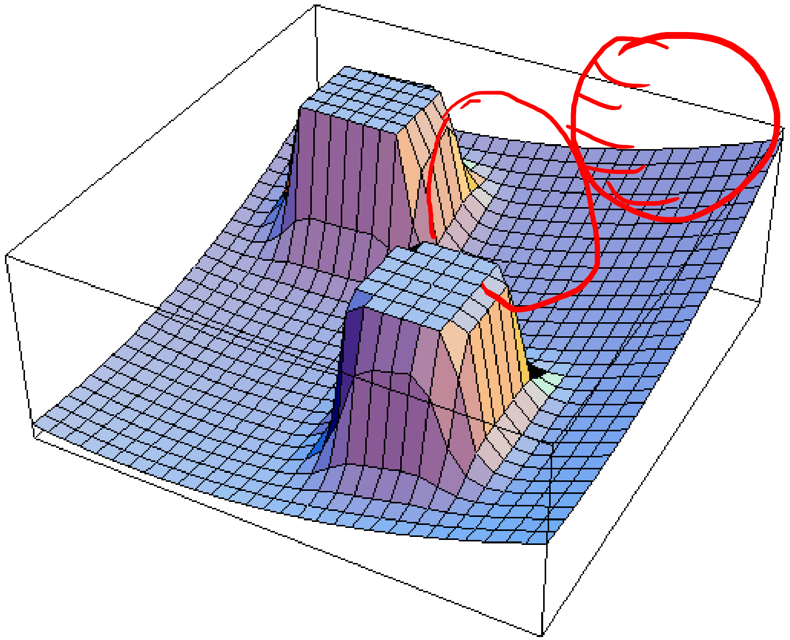


Local Minimum



Potential Field Approach

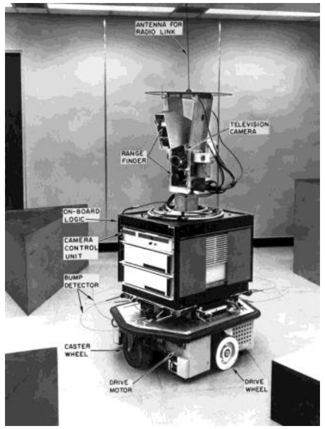
- Requirements:
  - Sensing
  - Odometry
- Pros:
  - Easy and efficient
- Cons
  - Local minima



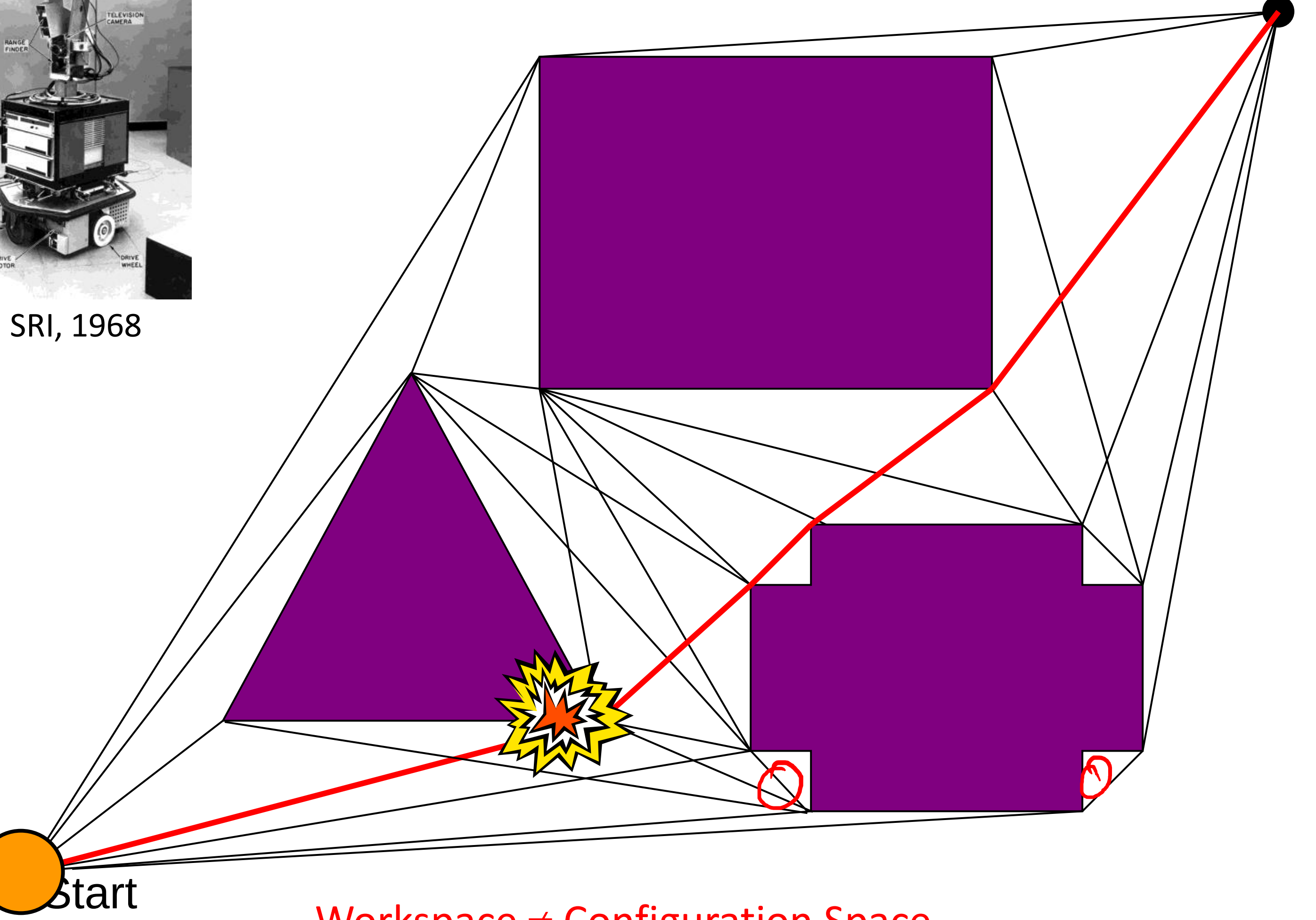
Overcoming Local Minima

- Why do they exist?
  – Definition of repulsive potential
  – Only uses local information
- How can we overcome them?
  – Use global information!

Visibility Map

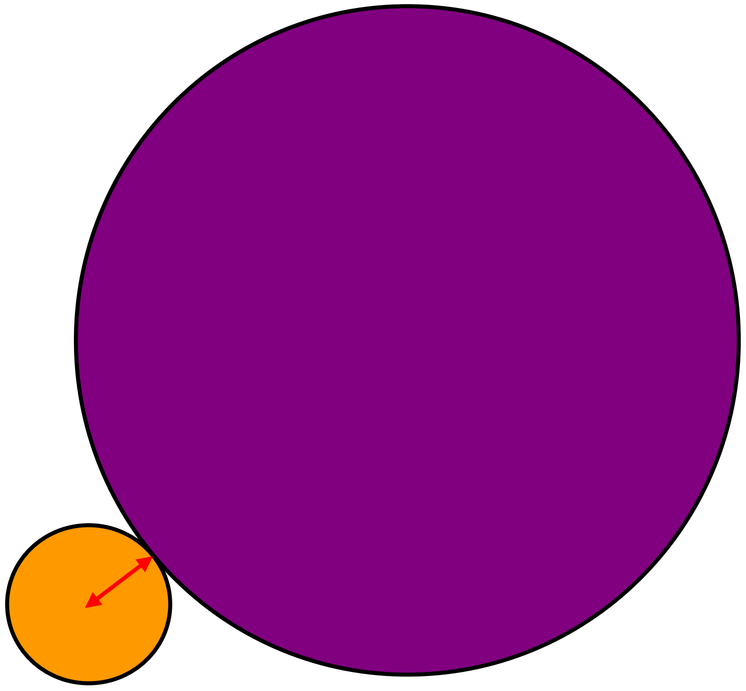


Shakey, SRI, 1968

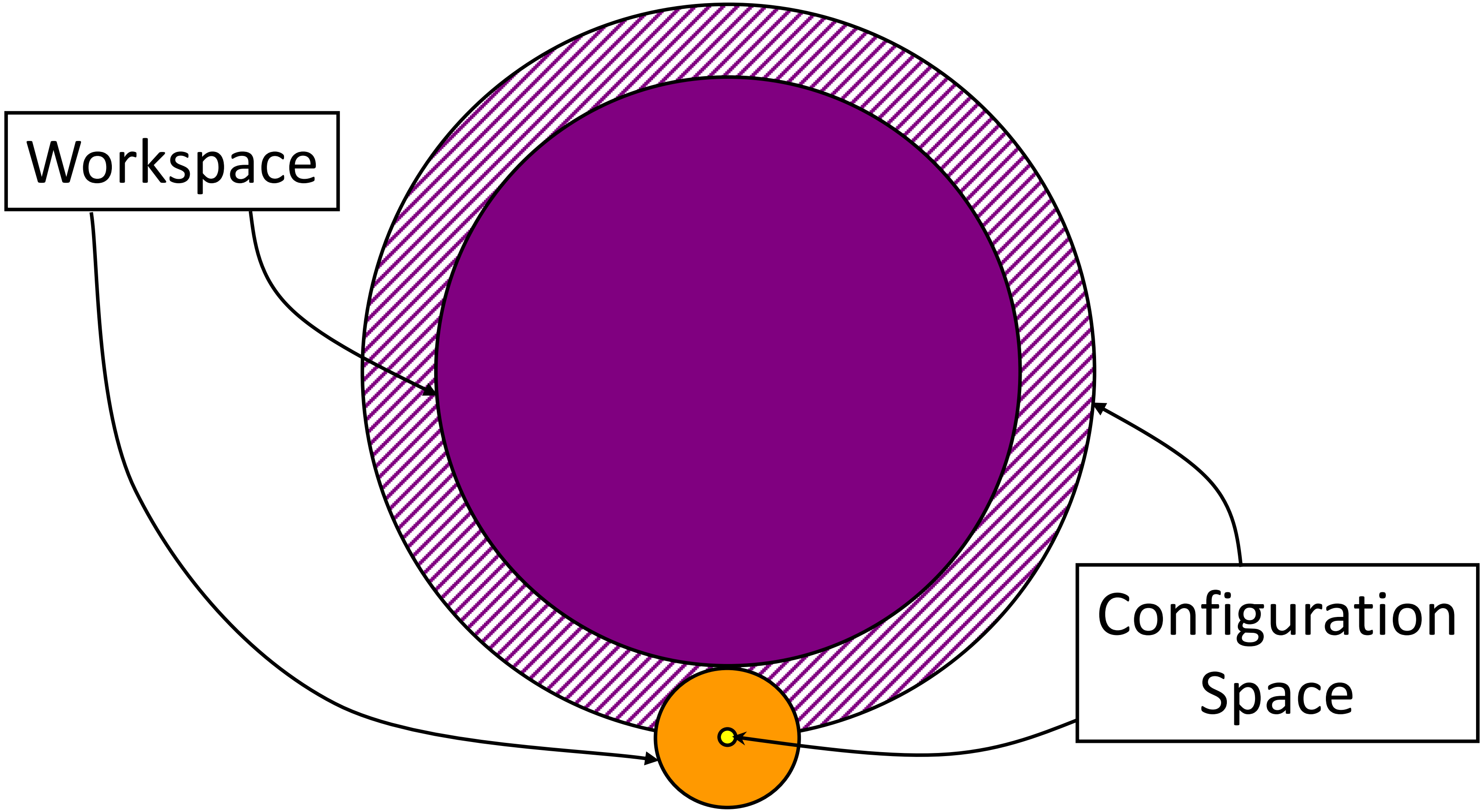


Workspace ≠ Configuration Space

Computing C-Space: Growing Obstacles

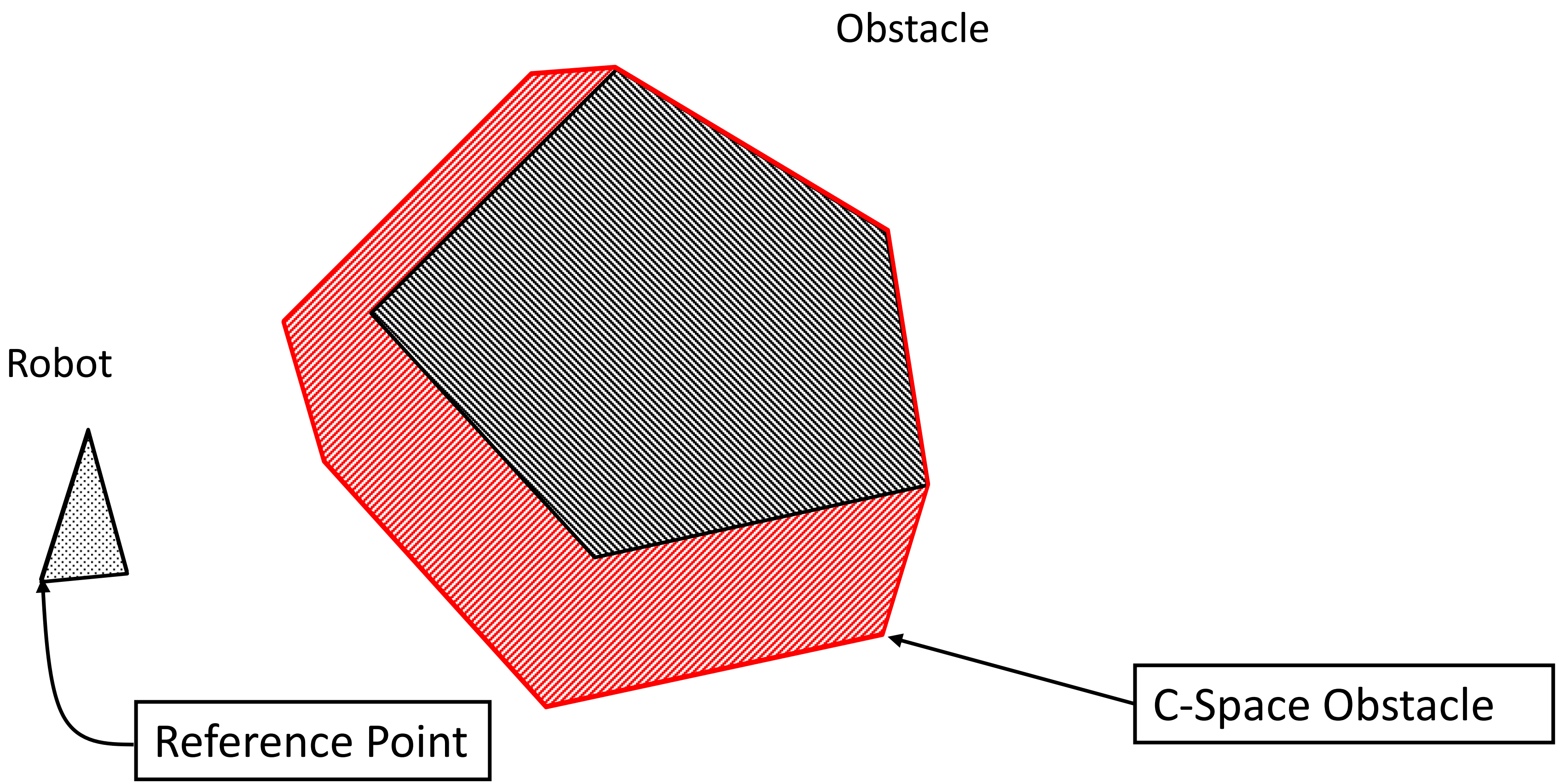


Sliding Along the Boundary



How about changing θ ?

Translational Case (Fixed θ)

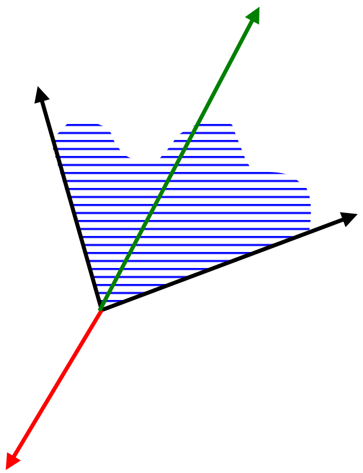


Sidebar: Linear Combination

$$a_1 \cdot \vec{x}_1 + a_2 \cdot \vec{x}_2 + \cdots + a_n \cdot \vec{x}_n$$

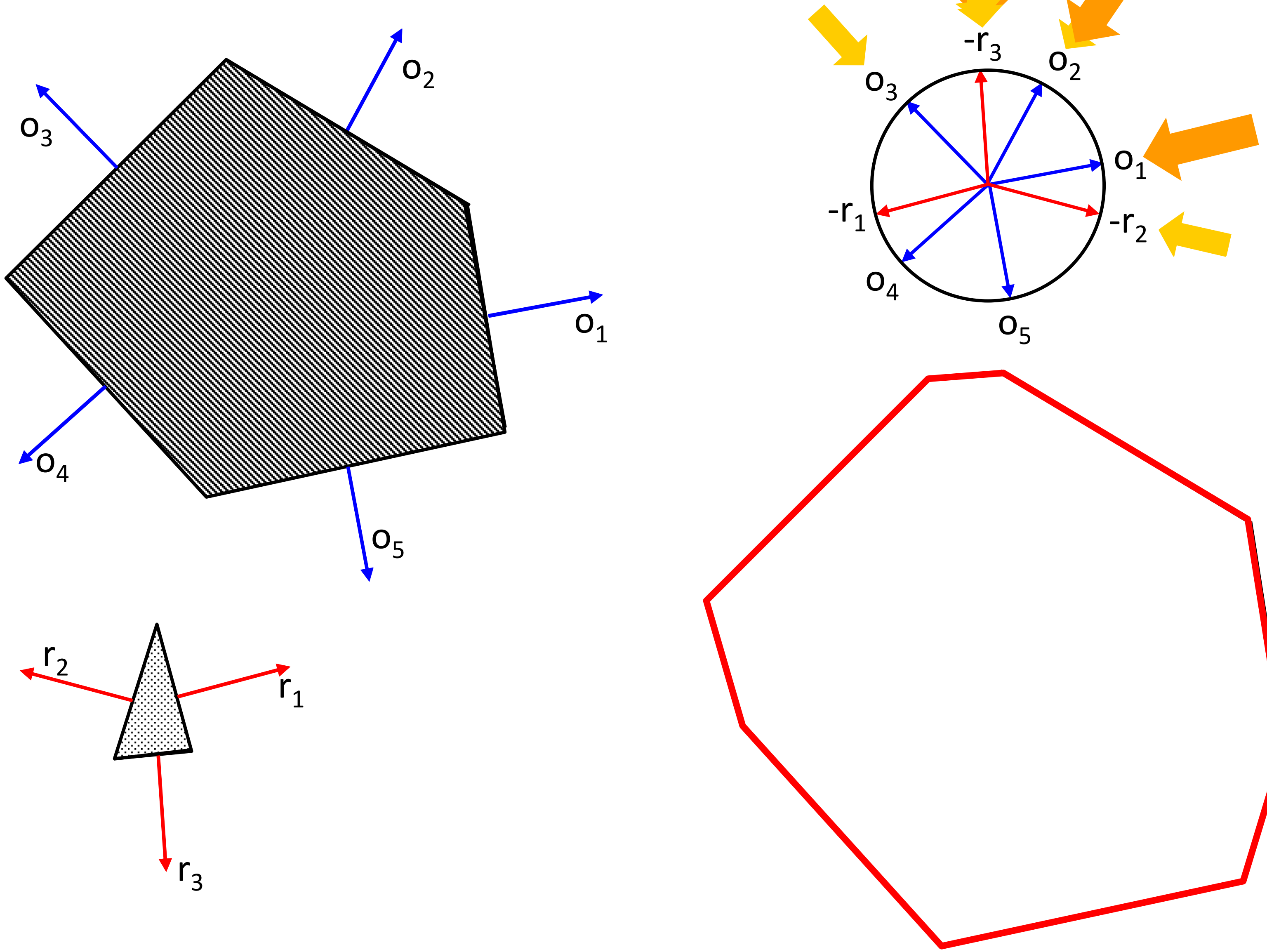
is called a *positive linear combination* if and only if

$$a_1, a_2, \cdots, a_n > 0$$

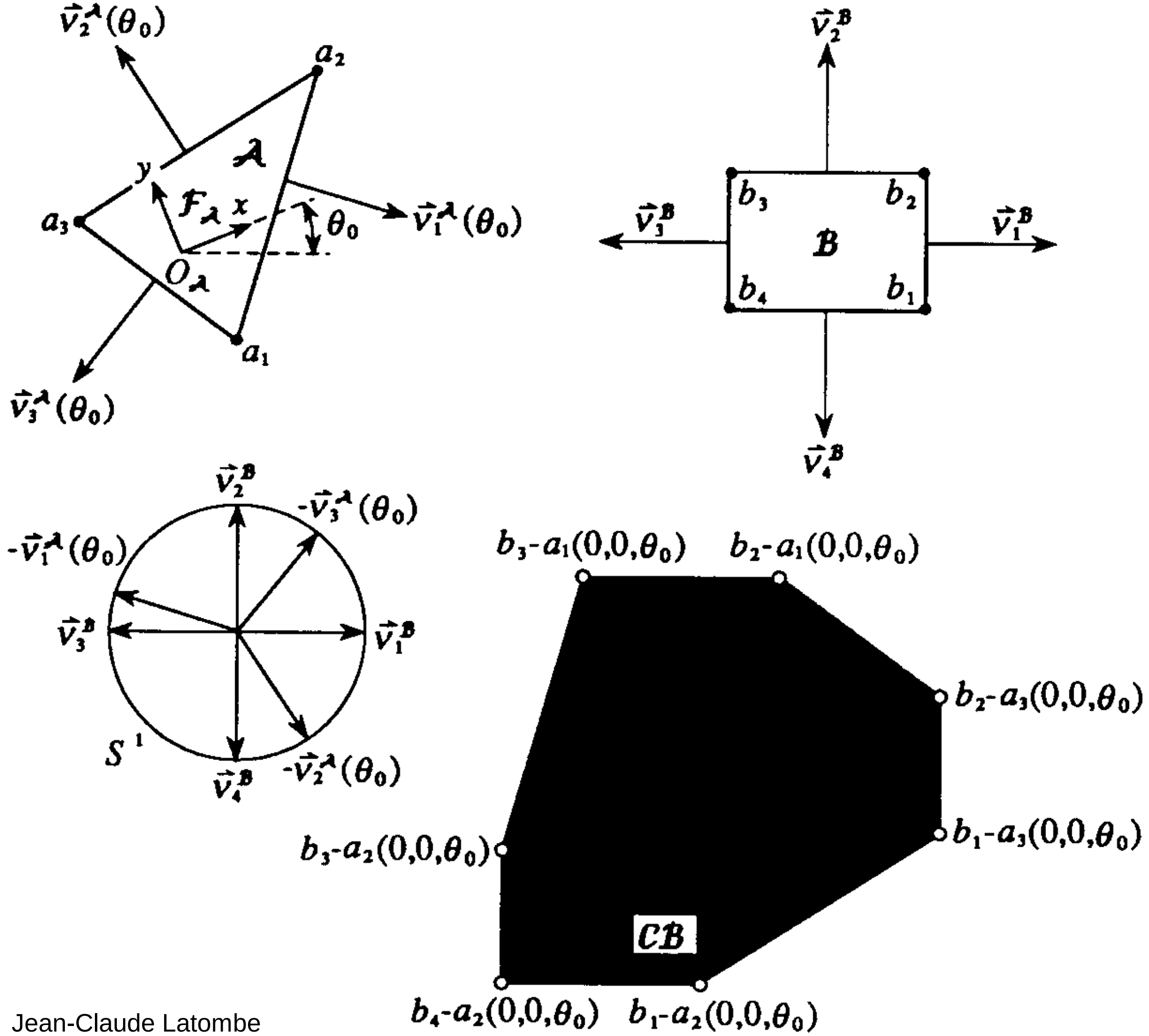


C-Obstacle

☑ positive linear combination



C-Obstacle Construction



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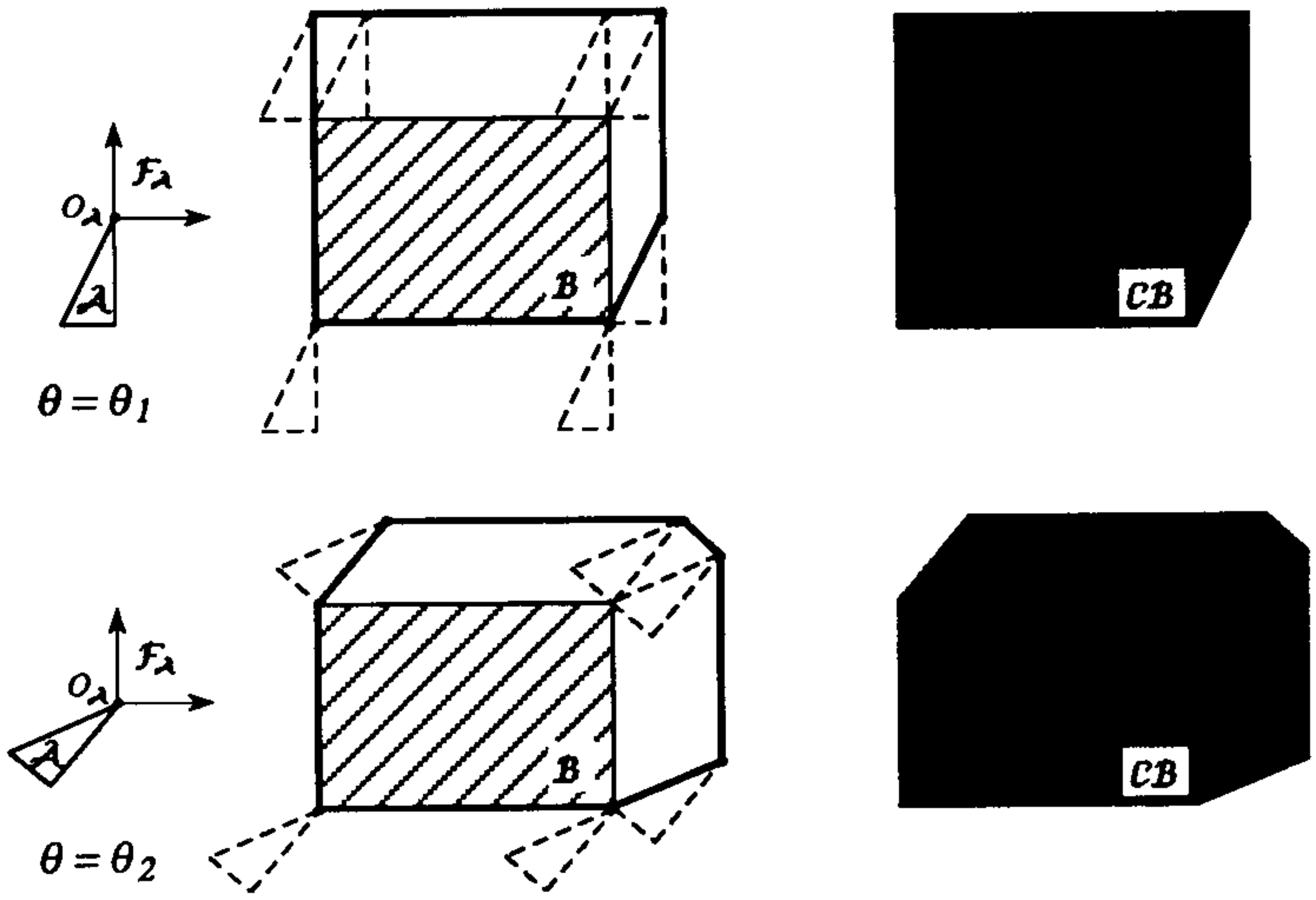
C-Obstacles for Varying θ



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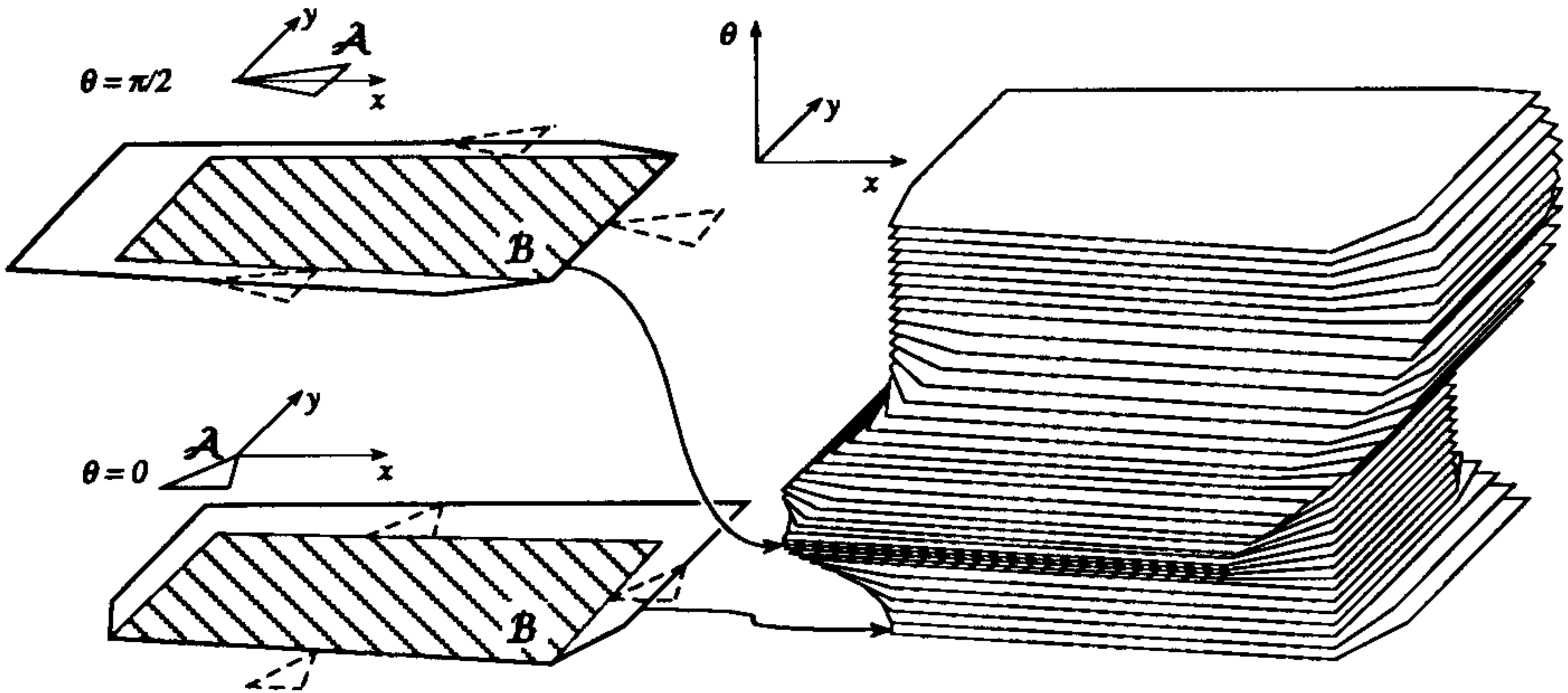
C-Obstacle in 3D



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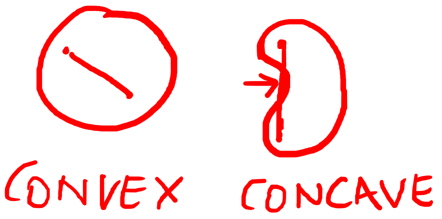
Okay, what next?

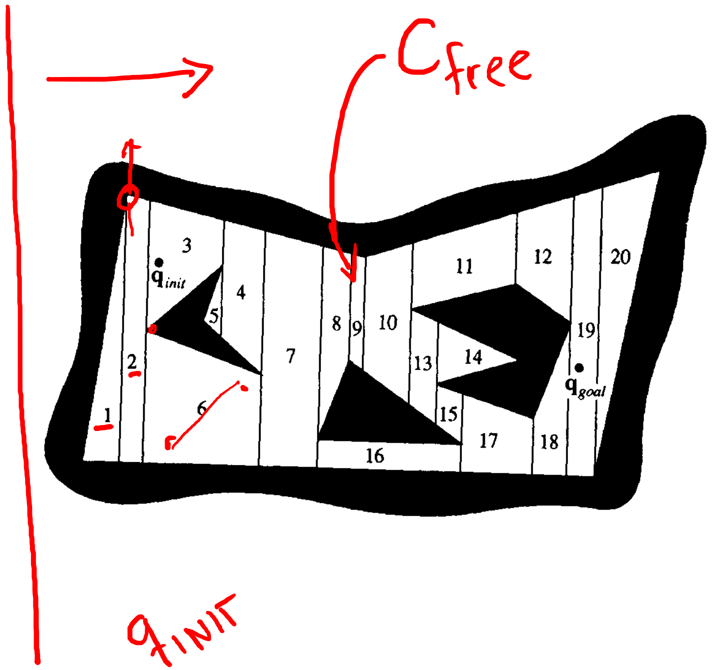
- We have computed C-space obstacles for polygonal robots in the plane.
- How can we actually compute a motion to the goal?
- Lesson from potential field approach:

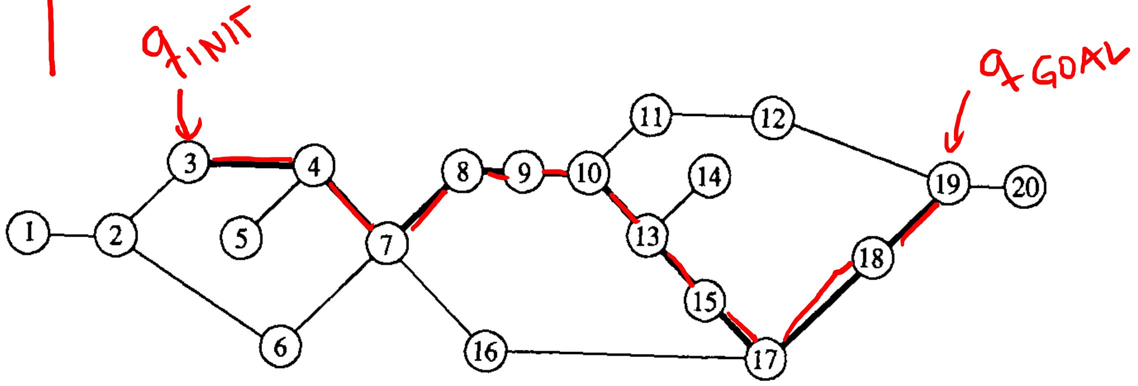
Global Information

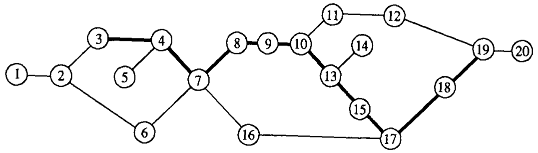
CONVEX

Exact Cell Decomposition

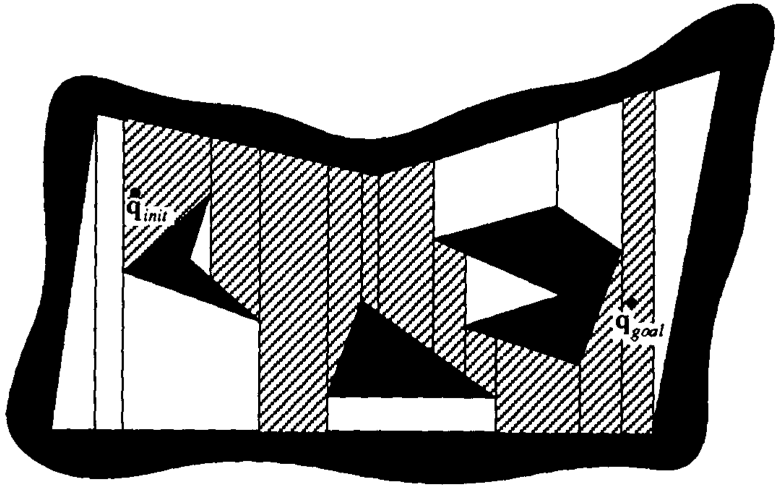


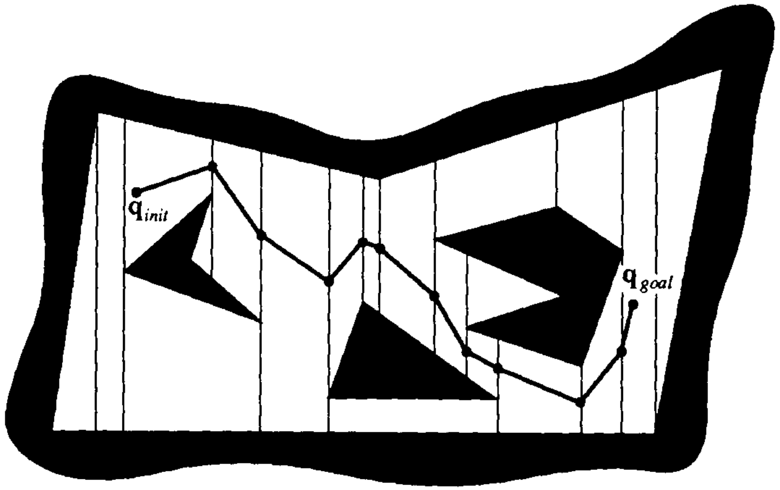






Exact Cell Decomposition





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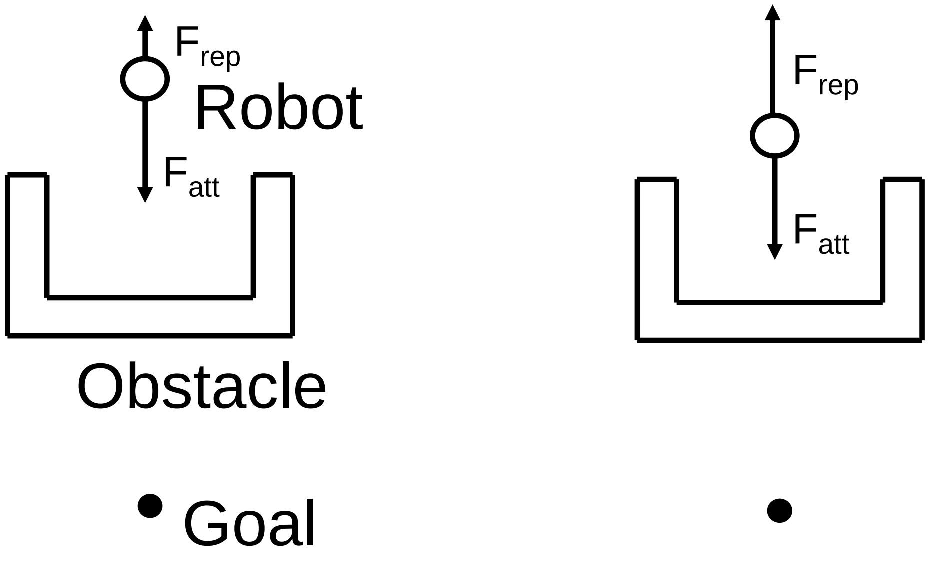
Adaptation of Dijkstra’s Algorithm

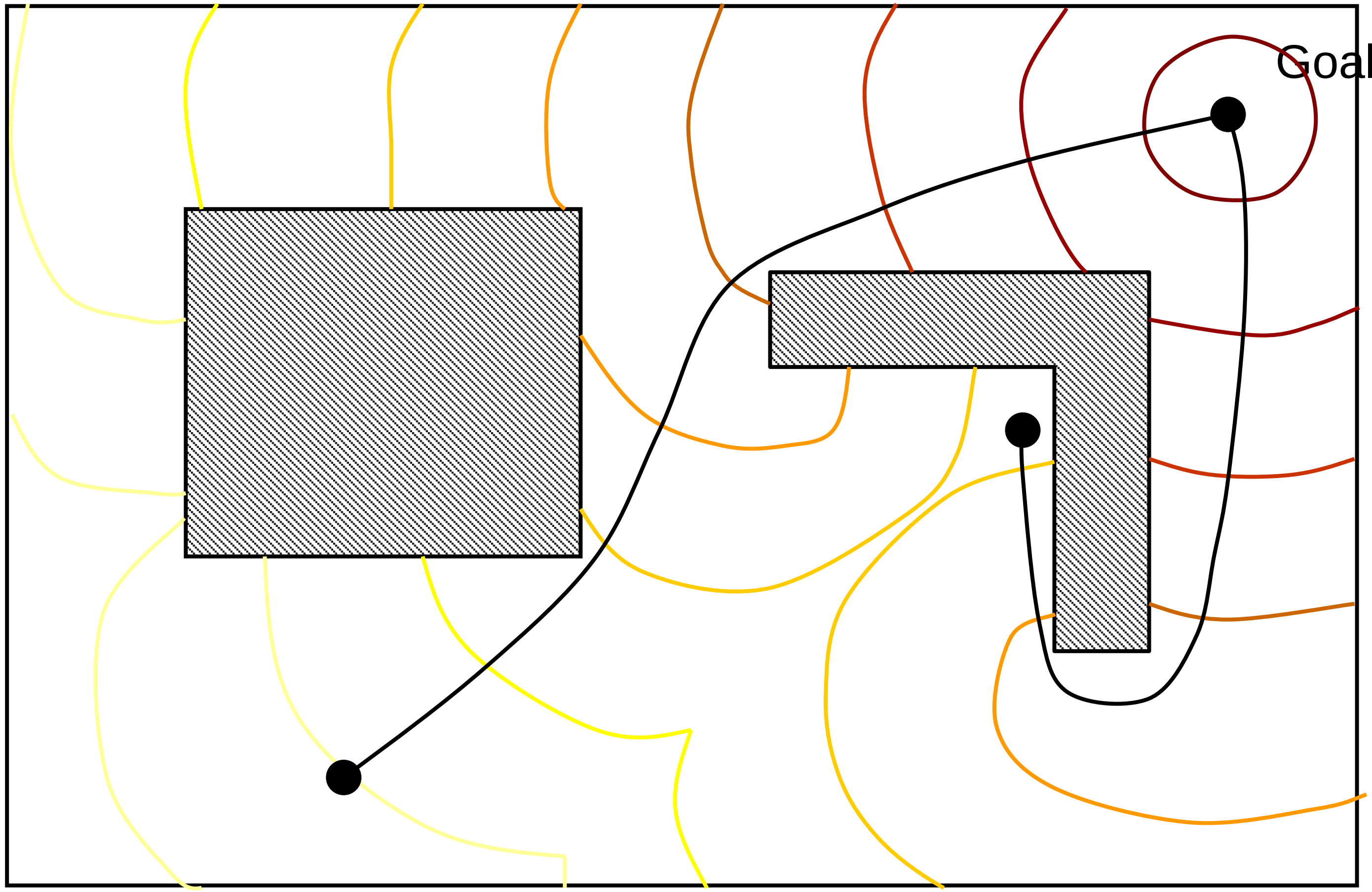
- For every undirected edge add two directed edges in opposite directions
- Determine weight of edge based on
  - distance
  - difficulty of passage
  - other properties of the space

Global Potential Functions

- Goal: avoid local minima
- Problem: requires global information
- Solution: **Navigation Function**



Navigation Function NF1



Wave Front Expansion

NF1 Real-World Scenario

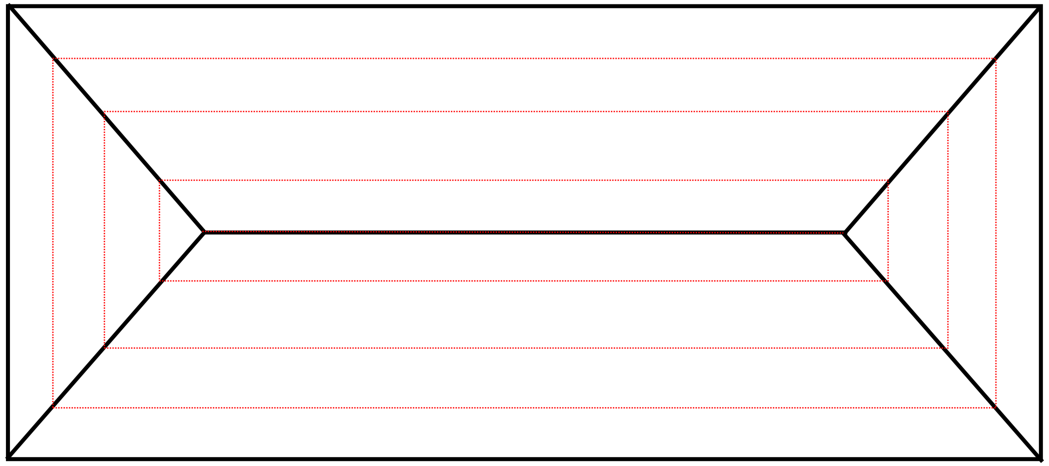
14	13	12	11	10	9	8	7	6	5	4	3	2	1	2 Goal
13	12	11	10	9	8	7	6	5	4	3	2	1	0	1
14	13						7	6	5	4	3	2	1	2
15	14						8						2	3
16	15						9	10	11	12			3	4
17	16						10	11	12	13			4	5
18	17						11	12	13	12			5	6
19	18	17	16	15	14	13	12	13	12	11			6	7
20	19	18	17	16	15	14	13	12	11	10	9	8	7	8
21	20	19	18	17	16	15	14	13	12	11	10	9	8	9

$O(n * m)$

$= O(n^2)$

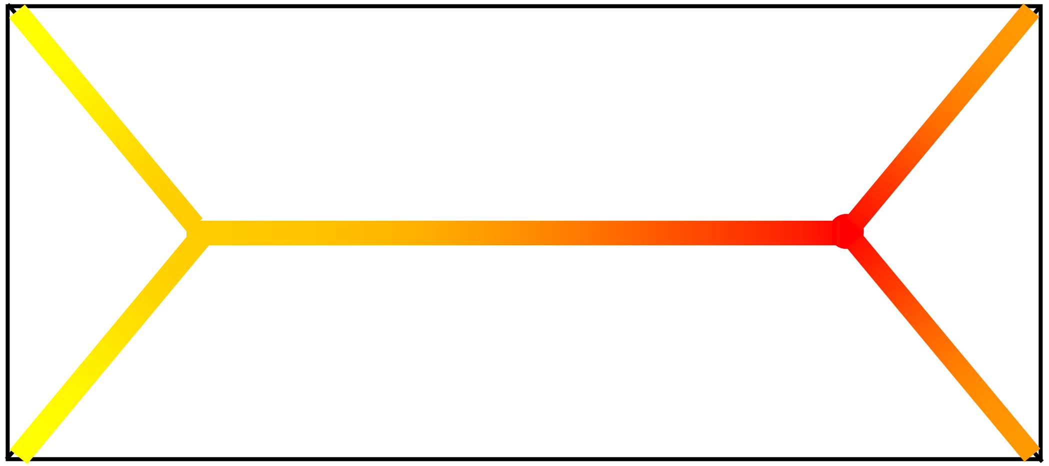
$O(n^d)$ "d" DOF

NF2 – Step 1



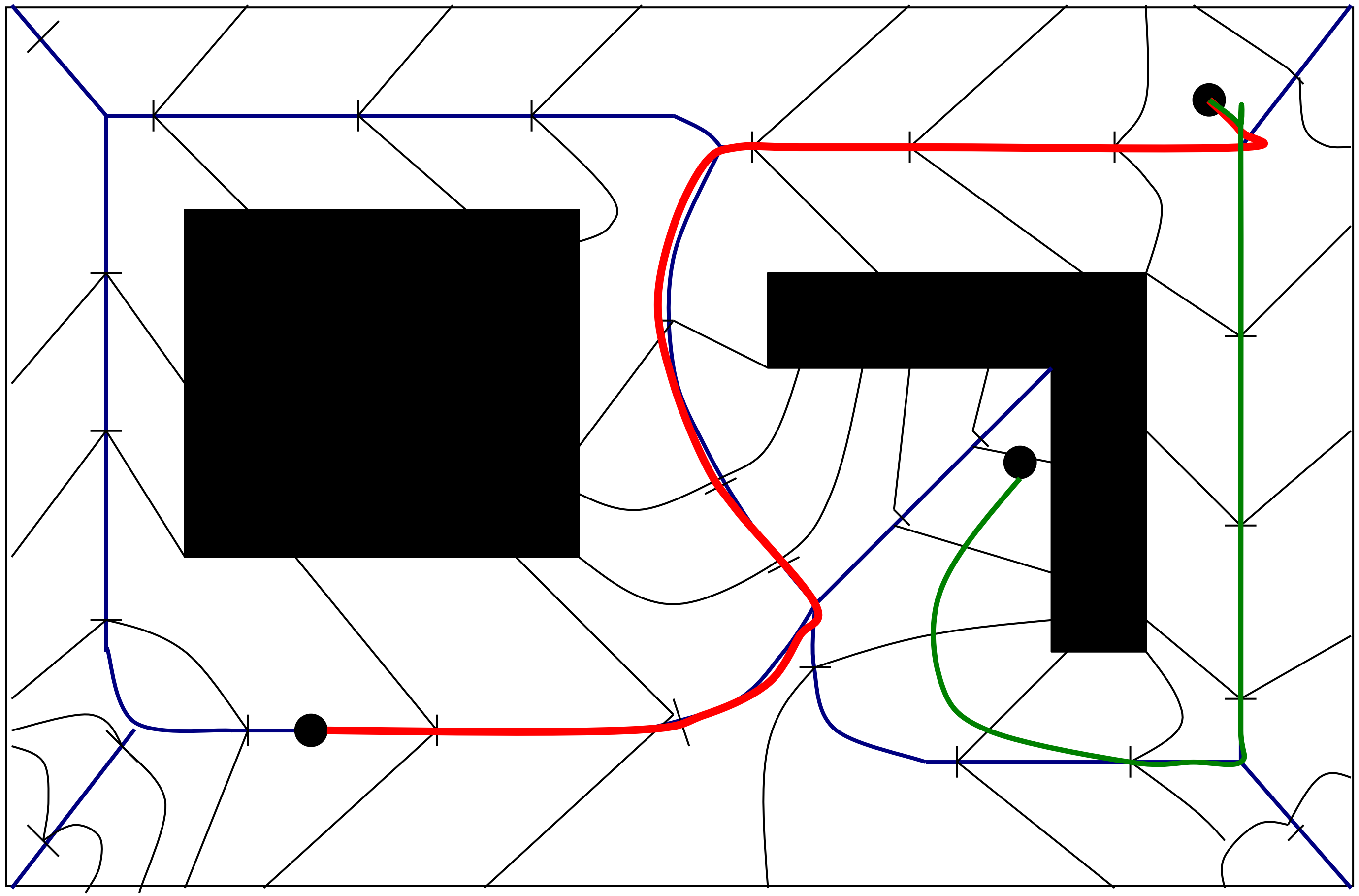
Compute medial axis with wave front expansion

NF2 – Step 2



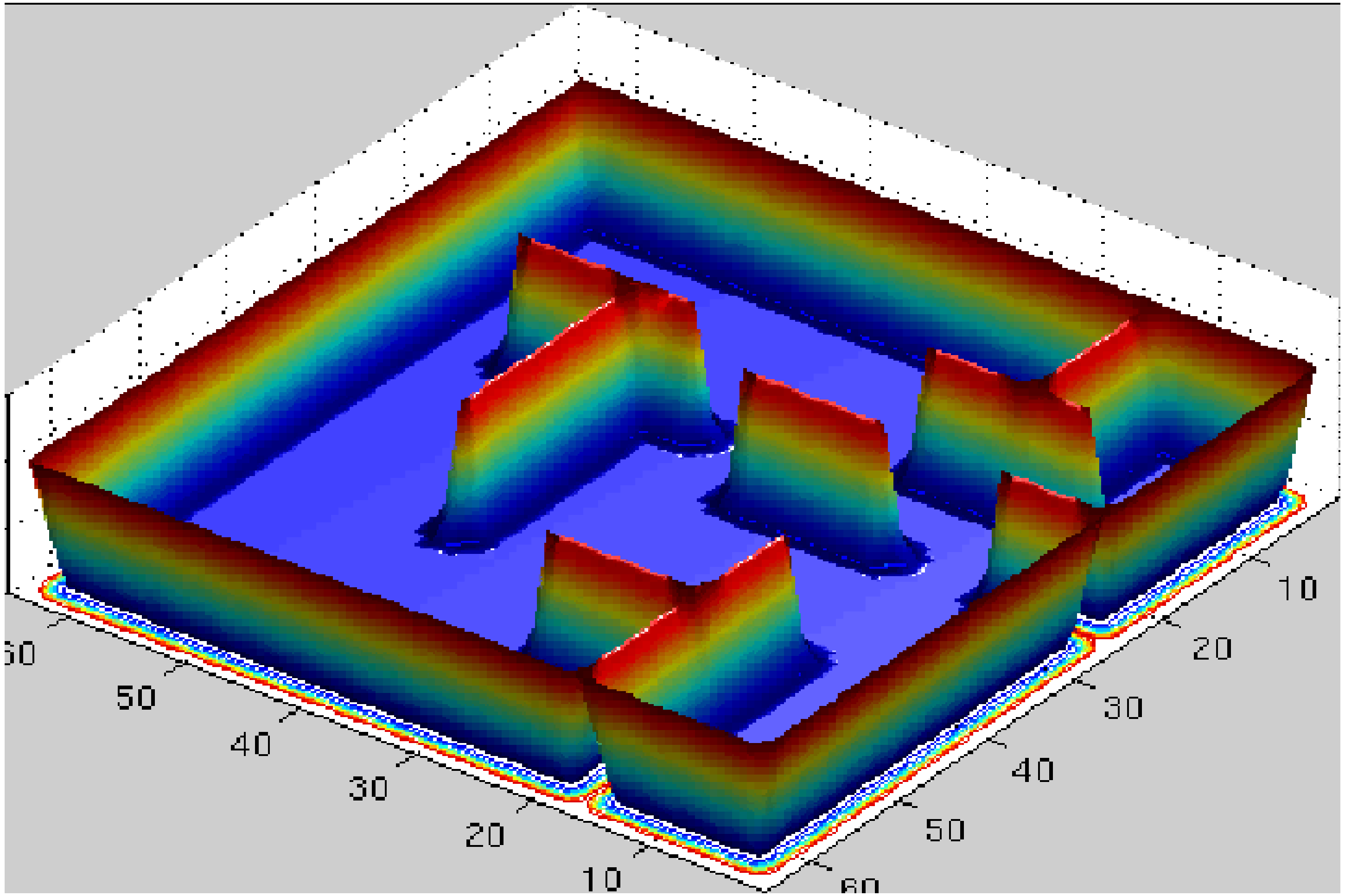
Compute wave front expansion along medial axis

NF2 – Step 3



Compute wave front expansion from medial axis

Harmonic Potentials

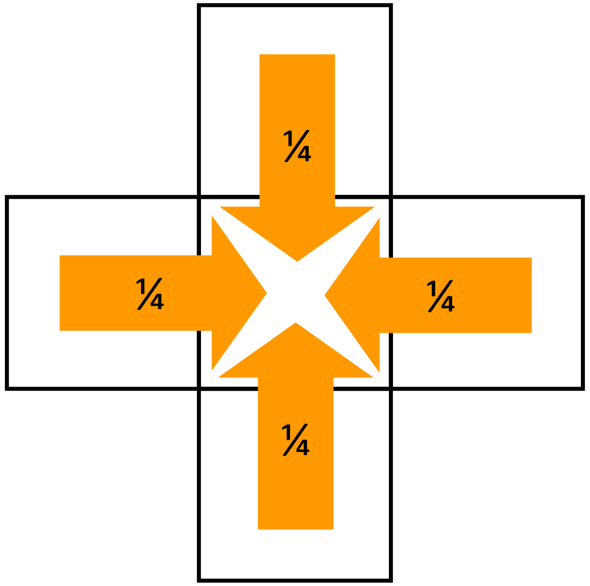


Courtesy of John Sweeney

Harmonic Potentials

- Harmonic functions
- Solutions to **Laplace's equation** (PDE)
- Intuition: heat transfer
- Numerical solutions: relaxation
- No local minima
- Require a lot of computation time
- Susceptible to numerical rounding error

Example: Jacobi Iteration



the new value of a cell for the next iteration
is ¼ of the sum of its 4-neighbors

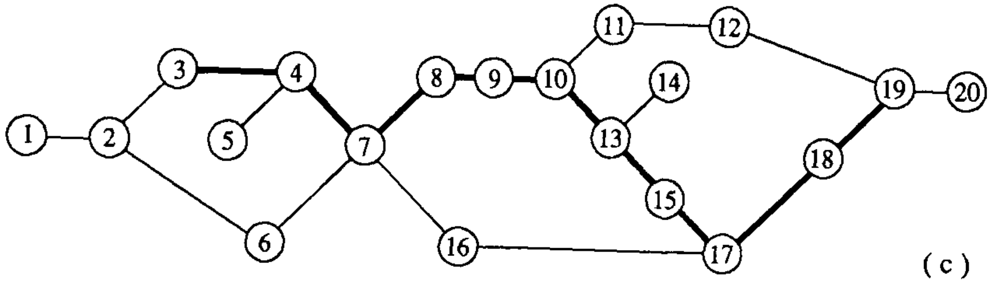
$$\text{cell}_{(x,y,t+1)} := \frac{\text{cell}_{(x-1,y,t)} + \text{cell}_{(x+1,y,t)} + \text{cell}_{(x,y-1,t)} + \text{cell}_{(x,y+1,t)}}{4}$$

Summary

- Local Methods
  - Potential Field Approach
  - Subject to local minima!
- Global Methods
  - Potential Field Approach with global navigation function
    - NF1, NF2, Harmonic Potential
  - Cell Decomposition
  - Visibility Method

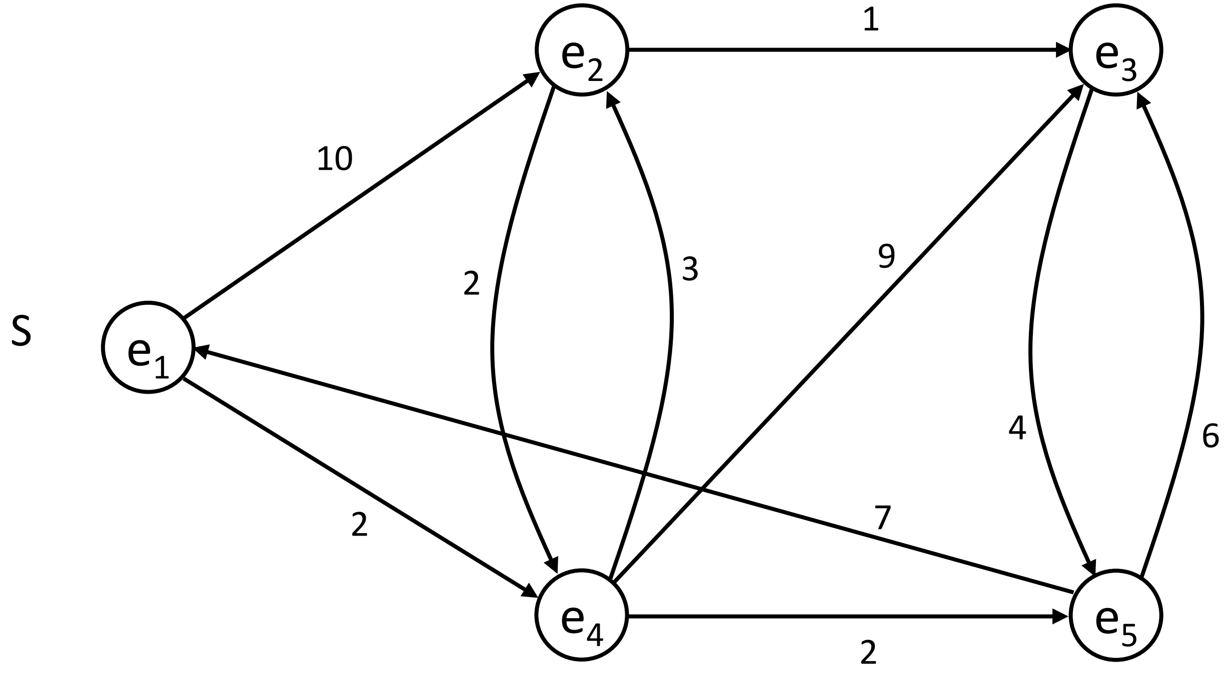
Sidebar: Dijkstra’s Algorithm

- Single-source shortest-path problem
- Weighted, directed graph G(V,E)
- Given a node e 2 E, what is the shortest path to all other reachable nodes?

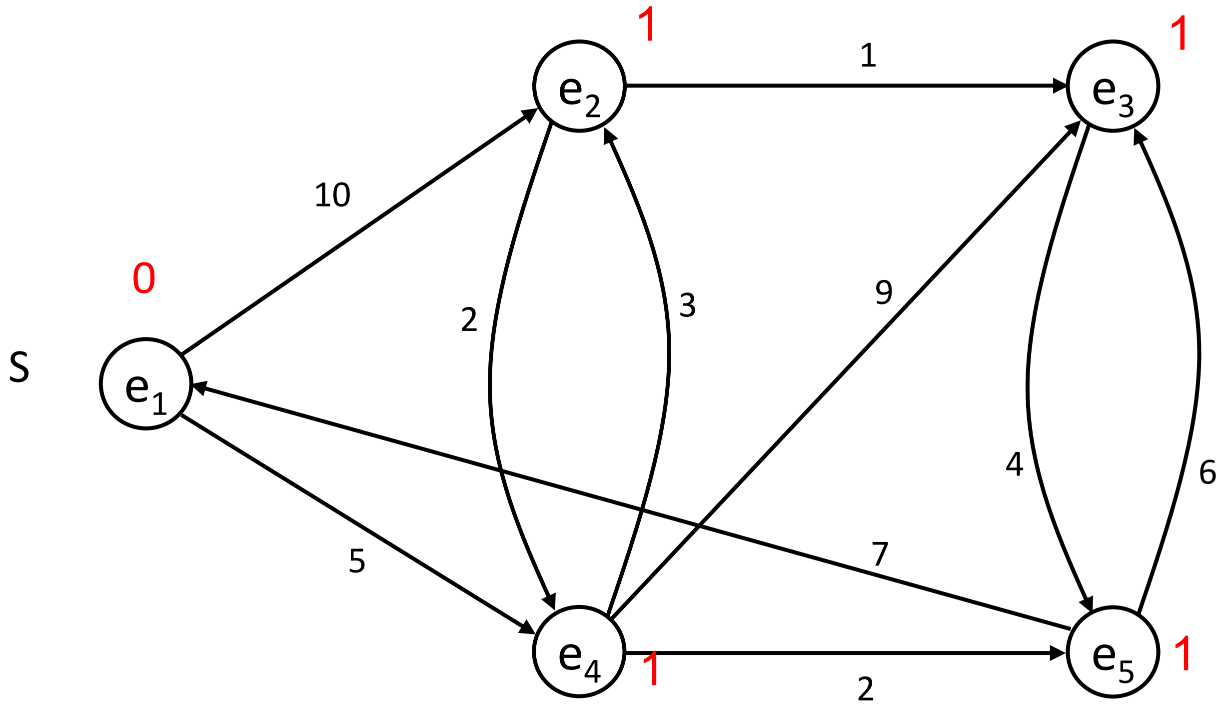


(c)

Sidebar: Dijkstra cont.



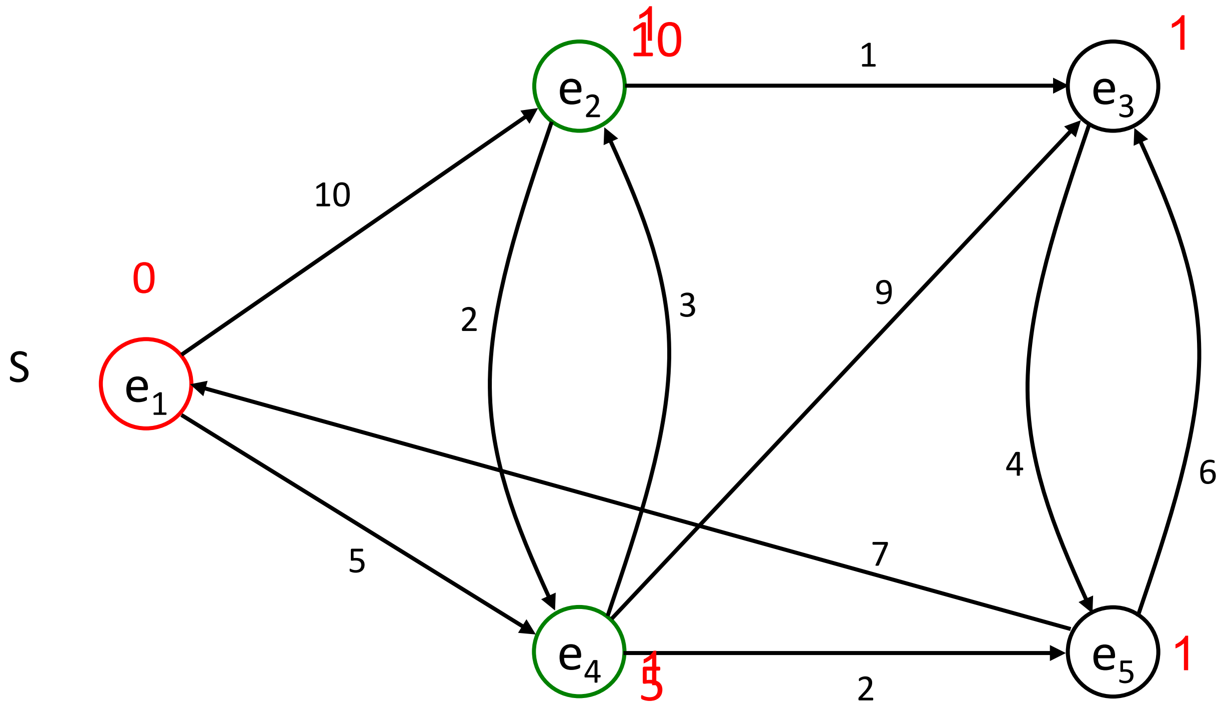
Sidebar: Dijkstra Initialization



Current nodes: $\{e_1=0, e_2=1, e_3=1, e_4=1, e_5=1\}$

Nodes completed: {;}

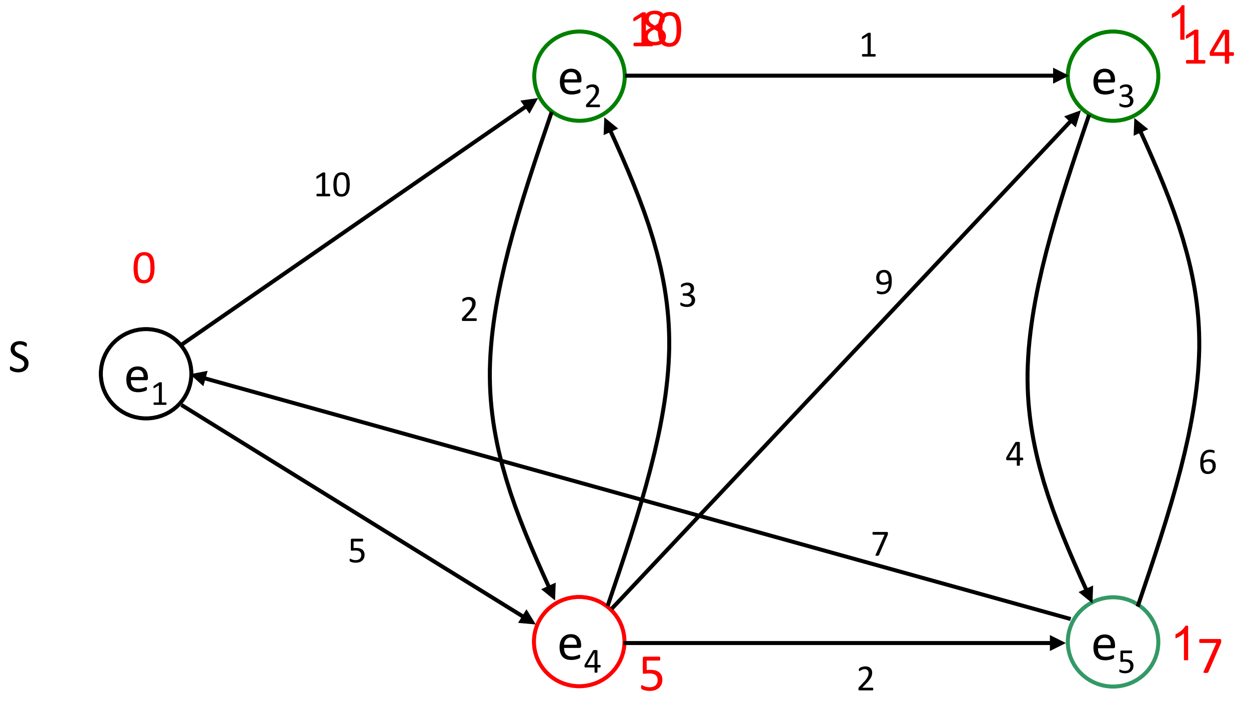
Sidebar: Dijkstra Relaxation 1



Current nodes: {e_1=0, e_2=10, e_3=1, e_4=1, e_5=1}

Nodes completed: {e_1}

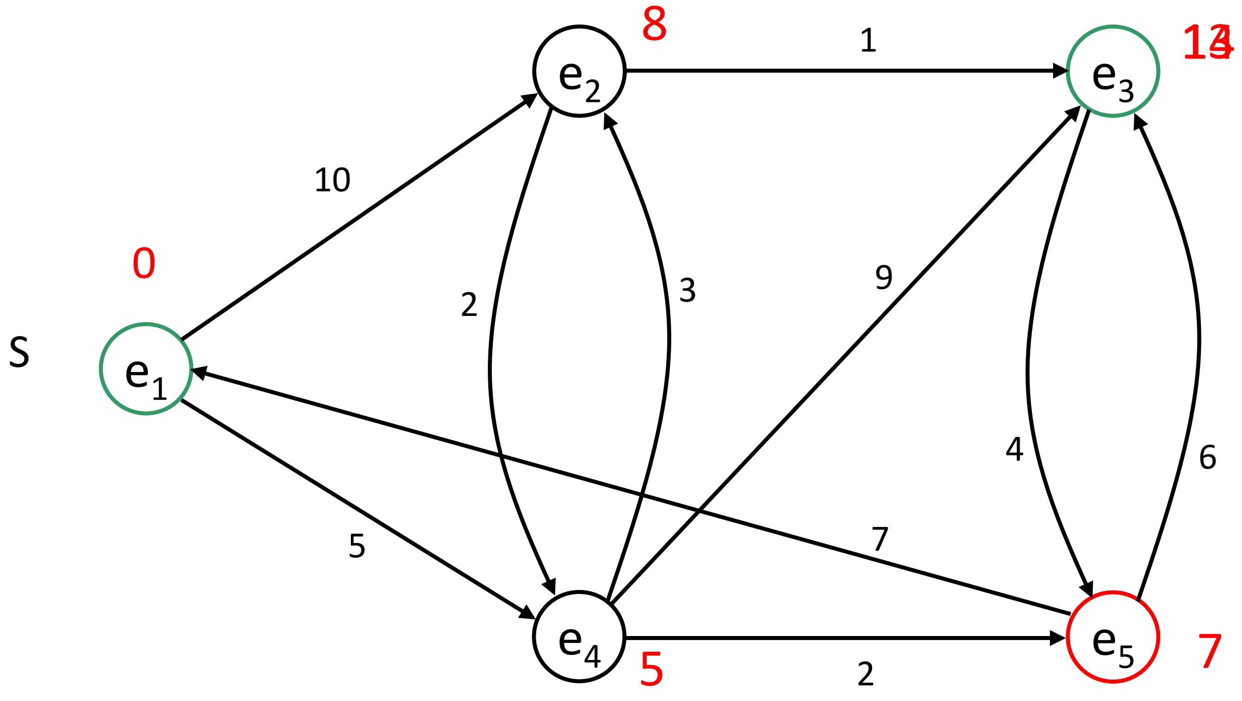
Sidebar: Dijkstra Relaxation 2



Current nodes: {e_5=7, e_2=8, e_3=14} {e_4=5, e_2=10, e_3=1} e_5=1}

Nodes completed: {e_1, e_4}

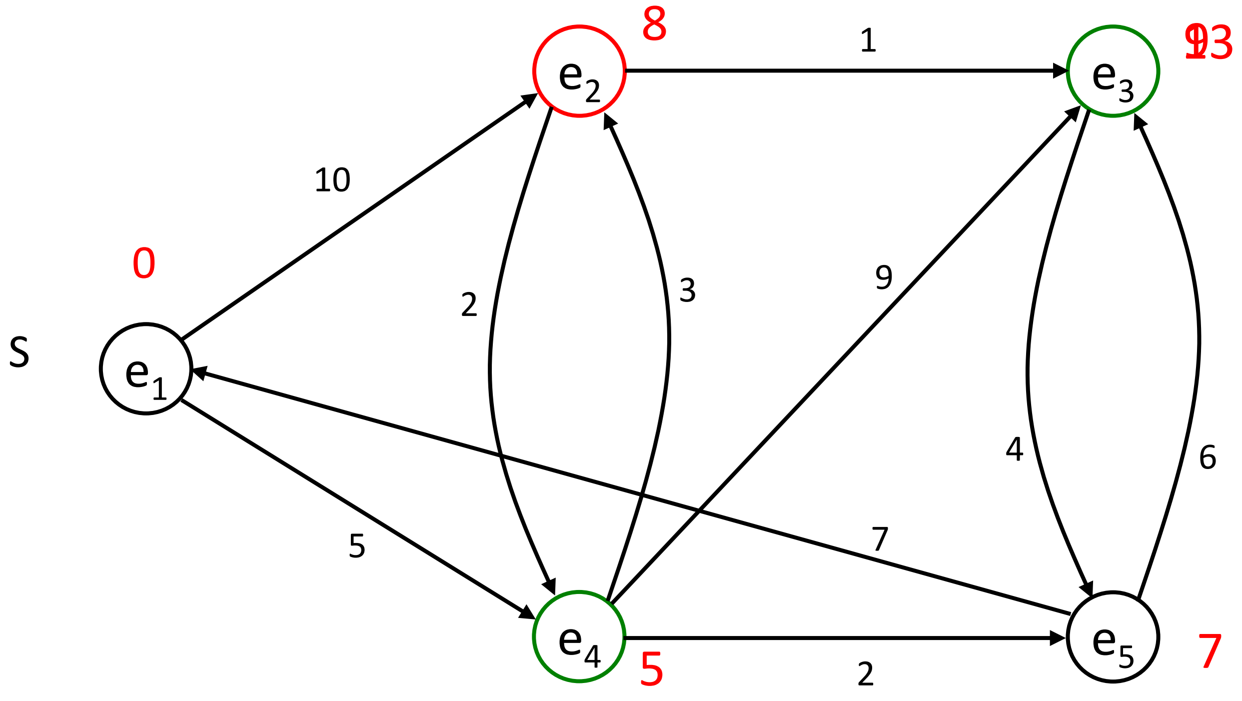
Sidebar: Dijkstra Relaxation 3



Current nodes: $\{e_2=8, e_3=13\}$

Nodes completed: $\{e_1, e_4, e_5\}$

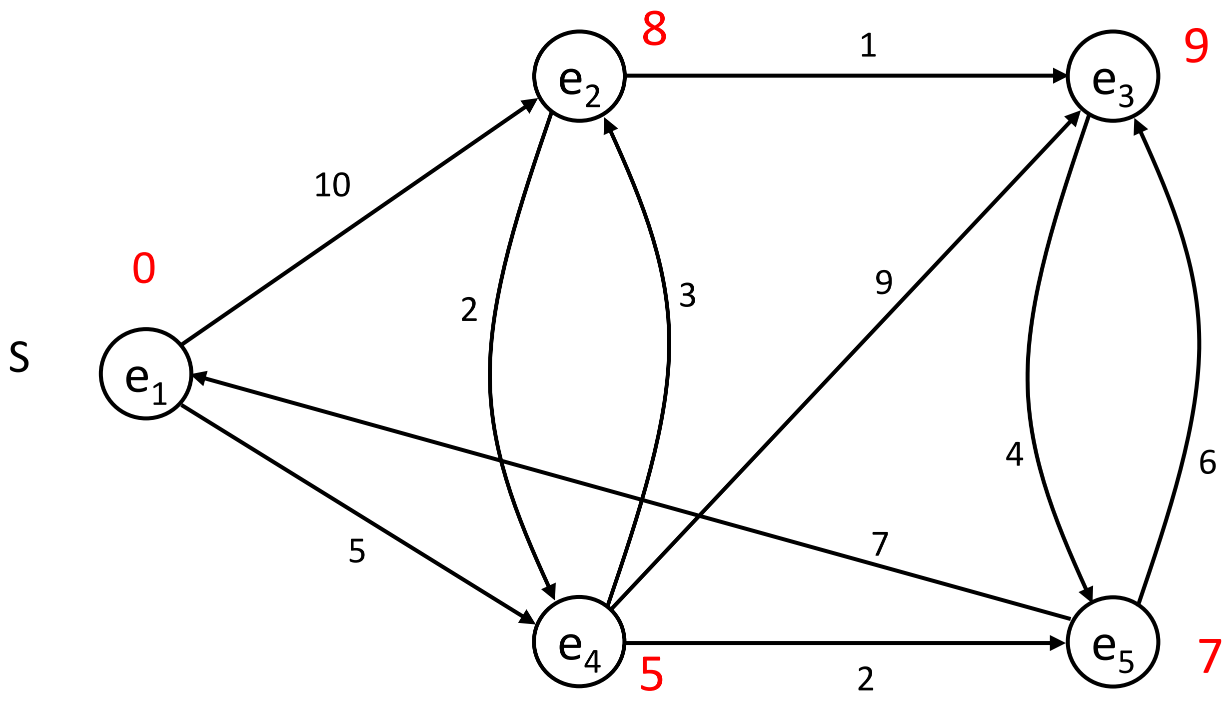
Sidebar: Dijkstra Relaxation 4



Current nodes: {e_3=9}

Nodes completed: {e_1,e_2,e_4,e_5}

Sidebar: Dijkstra Relaxation 5



Current nodes: $\{e_3=9\}$

Nodes completed: $\{e_1, e_2, e_4, e_5\}$

Sidebar: Dijkstra cont. III

```
Dijkstra(G,w,s)
Initialize-Single-Source(G,s)
S Ã ;
Q Ã V[G]
while Q ≠ 0
   do u Ã Extract-Min(Q)
        S Ã S [ {u}
        for each vertex v 2 Adj[u]
               do Relax (u,v,w)
```

```
Intialize-Single-Source(G,s)
   for each vertex v 2 V[G]
        do d[v]Ã 1
               π[v] Ã NIL
d[s] Ã 0
```

```
Relax(u,v,w)
//Is it shorter to reach v via u?
       if d[v] > d[u] + w(u,v)
       then d[v] Ã d[u] + w(u,v)
            π[v] Ã u
```