

STAT 35510

Lecture 3

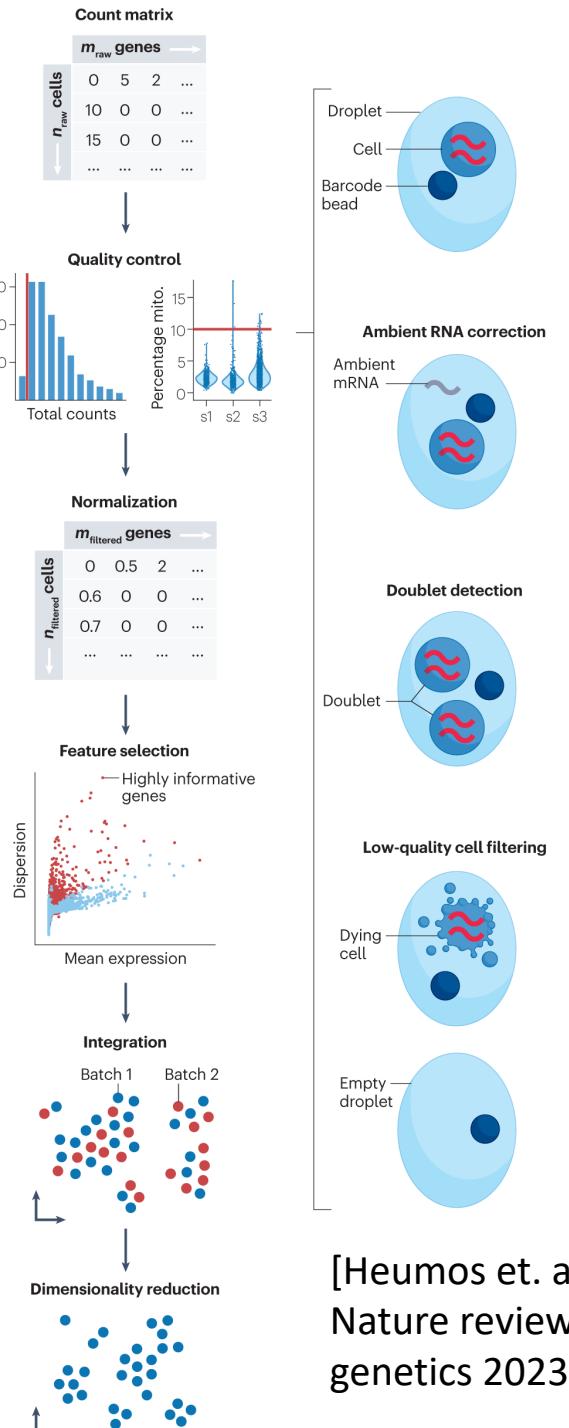
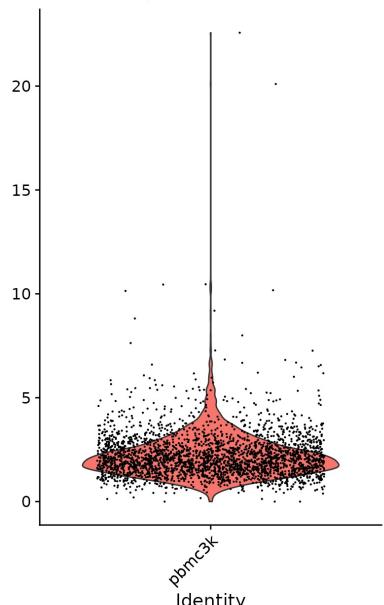
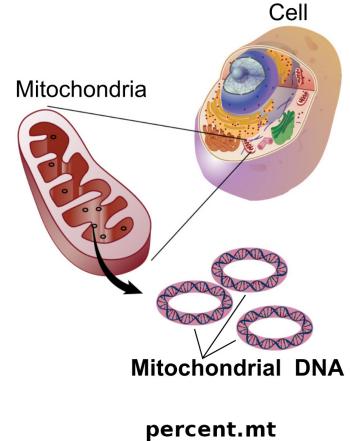
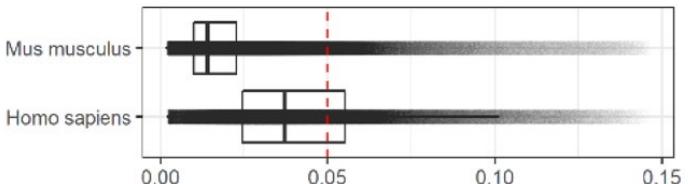
Spring, 2024
Jingshu Wang

Outline

- Standard scRNA-seq data analysis workflow: Seurat and Scanpy
- Dimensional reduction, highly variable gene selection, visualization

Standard pipeline for scRNA-seq preprocessing and visualization

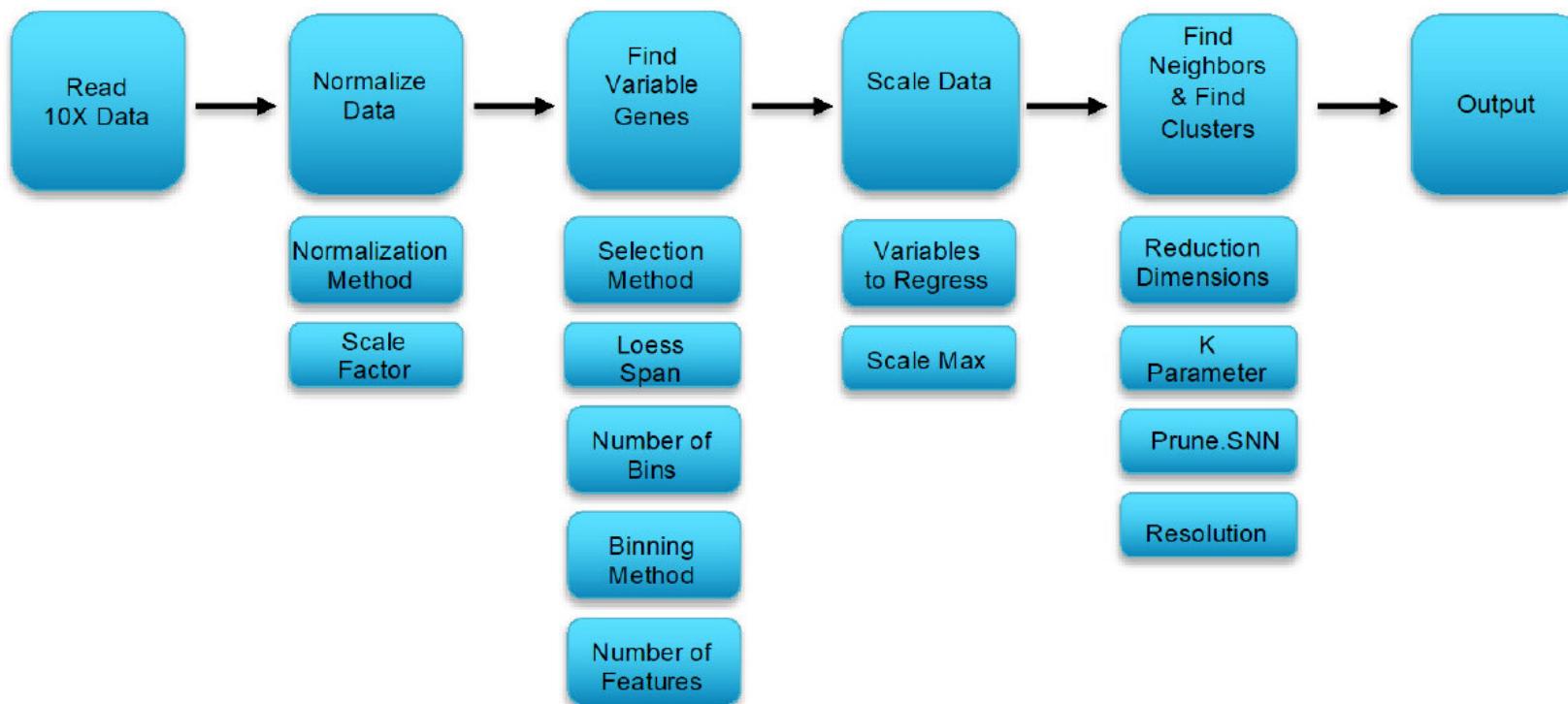
- Remove low-quality cells
 - Mitochondria also have DNA and can transcribe into RNA
 - Mitochondrial mRNA also have poly-A tail that are captured in scRNA-seq
 - High expression levels of mitochondrial genes can be an indicator of lysing cells
- Remove cells that have a high proportion of reads from mitochondrial genes (default 5%)
 - Maybe better to use 10% for human cells (Osorio and Cai, Bioinformatic 2021)



[Heumos et. al.,
Nature reviews
genetics 2023]

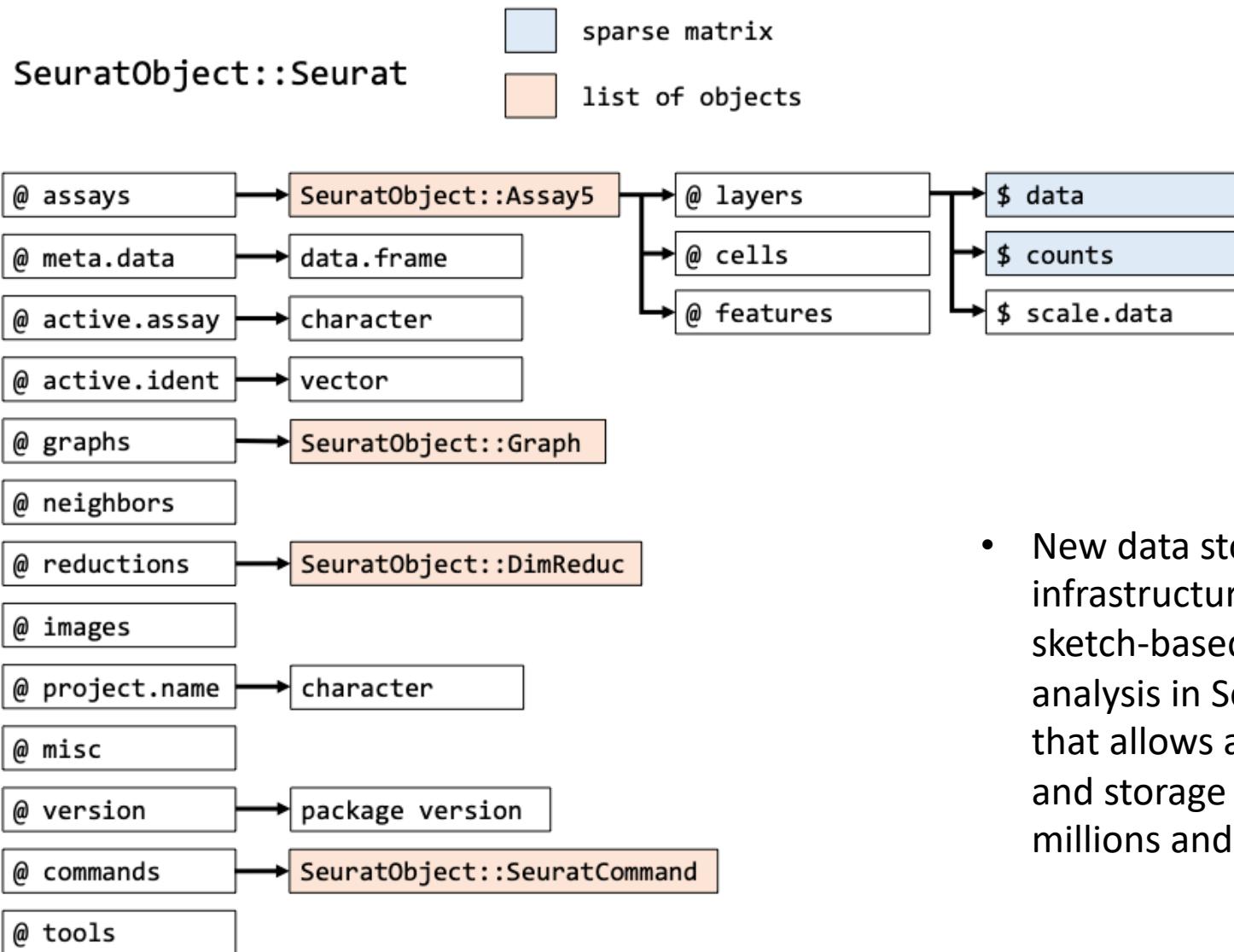
Seurat (Satija group)

- An R package that is widely used
- Current version v5 supports multi-modality and scalable analysis



Schneider, I., Cepela, J., Shetty, M., Wang, J., Nelson, A. C., Winterhoff, B., & Starr, T. K. (2021). Use of “default” parameter settings when analyzing single cell RNA sequencing data using Seurat: a biologist’s perspective. *J Transl Genet Genom*, 5, 37-49.

Seurat object



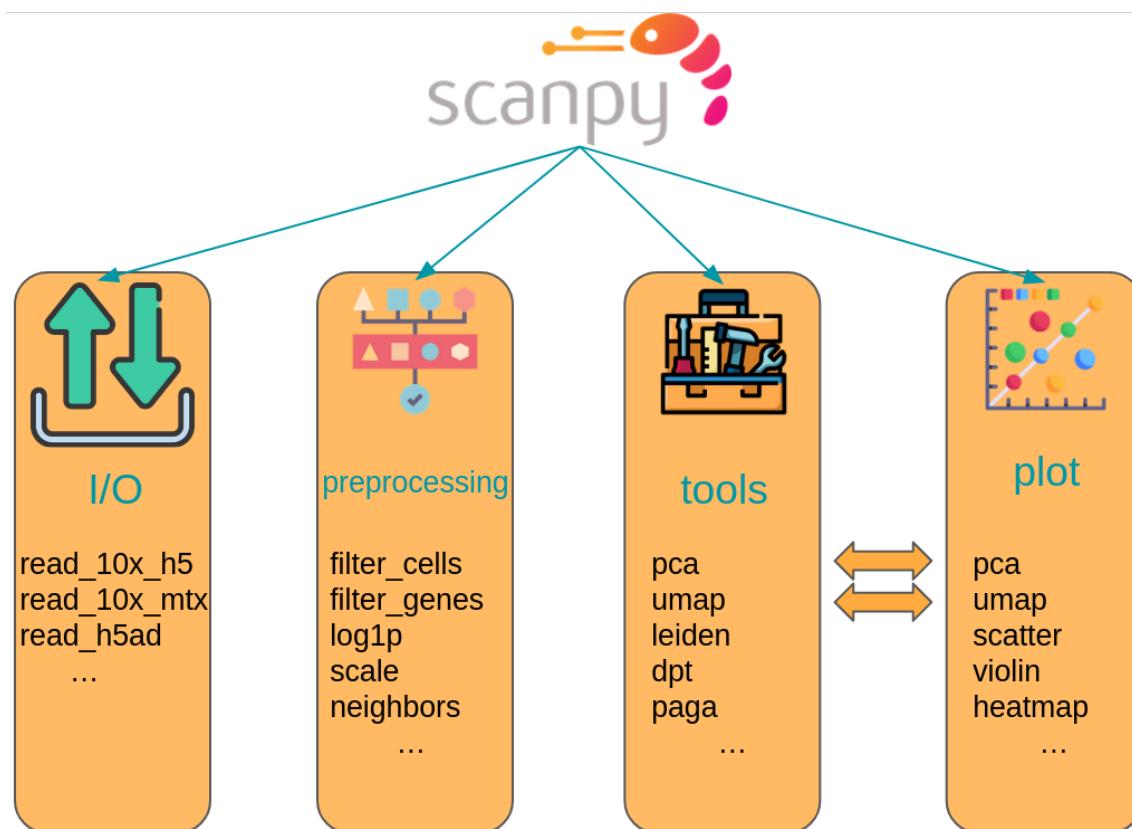
Reference tutorial:

https://sib-swiss.github.io/single-cell-training/day1/day1-2_analysis_tools_qc.html

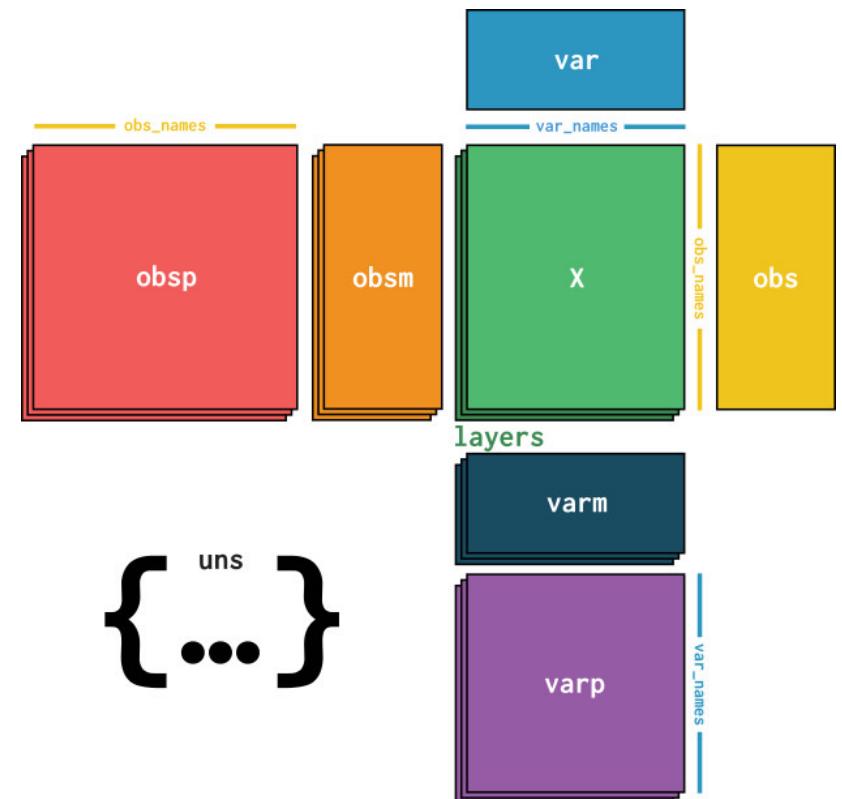
- New data storage infrastructure and sketch-based analysis in Seurat v5 that allows analysis and storage of millions and cells

Scanpy (Wolf et. al. Genome Biology 2018)

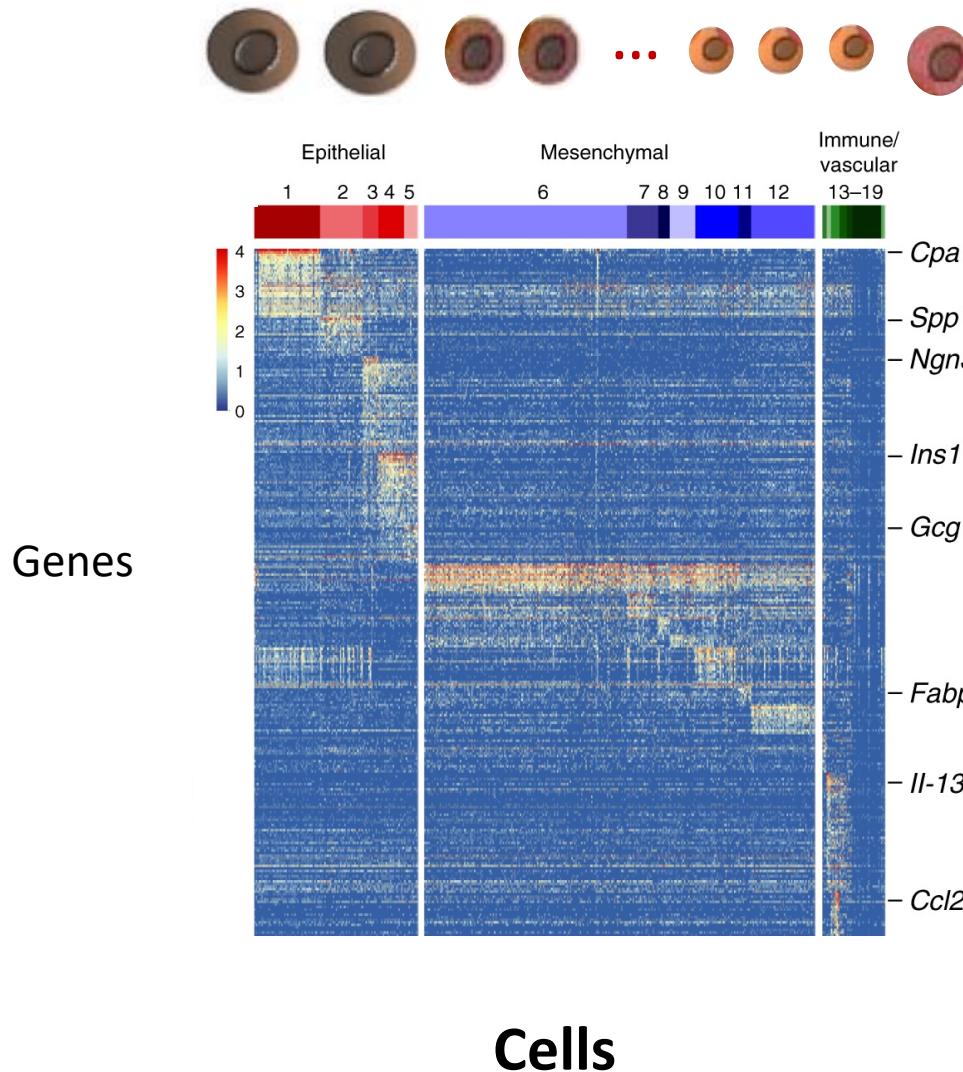
- A python package alternative to Seurat
- Handle large-scale data
- Easy to interface with deep-learning based methods



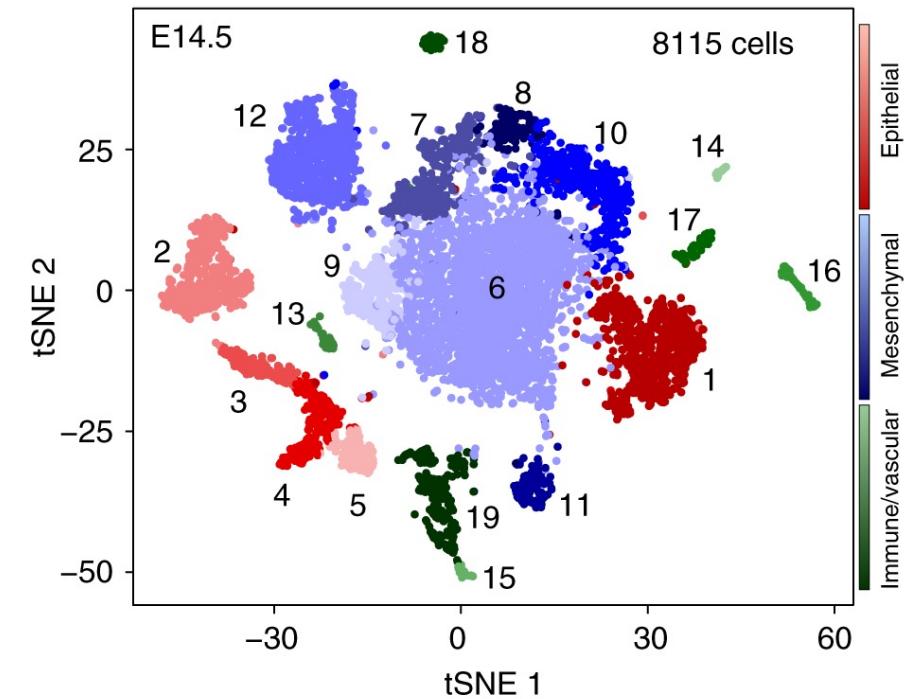
AnnData object



scRNA-seq dimension reduction and visualization

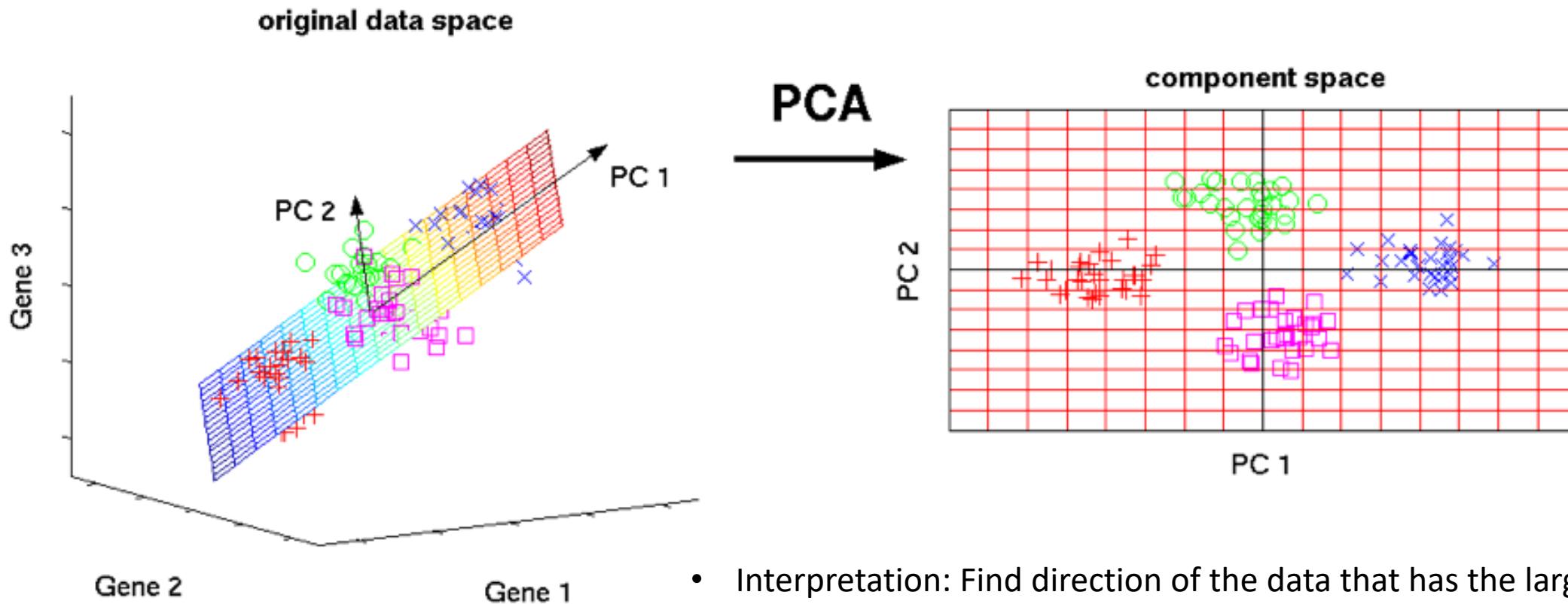


(Colors are determined by separate clustering methods!!)



Lineage dynamics of murine pancreatic development at single-cell resolution,
Byrnes et. al. *Nature Comm.* 2018

Linear dimension reduction: PCA



- Interpretation: Find direction of the data that has the largest variation
- Not ideal for visualization
- Requires proper normalization of the data for using Euclidean distance
- High-dimensional PCA is not accurate

scRNA-seq normalization

Why do we need normalization?

- Raw counts across cells are not comparable → adjust for library size
- Make the data more “Gaussian” before using linear methods like PCA

Shifted logarithm

- Library size normalization + taking logarithm

$$f(Y_{gc}) = \log\left(\frac{Y_{gc}}{s_c} + y_0\right)$$

- y_0 : pseudo-count to avoid $\log(0)$. Typically $y_0 = 1$ to make the normalized data sparse
- $s_c = l_c/L$ so that y_0 is not too influential. $L = 10^4$ (Seurat and Scanpy default)
- Shifted logarithm is approximately doing some variance stabilization

$$\text{var}\left(f(Y_{gc})\right) \approx \frac{s_c^2}{\mu_g^2} \text{var}(Y_{gc})$$

if $Y_{gc} \sim NB(\mu_g, \theta)$ then $\text{var}(Y_{gc}) = \mu_g + \theta\mu_g^2$, variance stabilized if θ or μ_g is large

- Scaling: standardize each gene across cells to have mean 0 and variance 1 after log-normalization

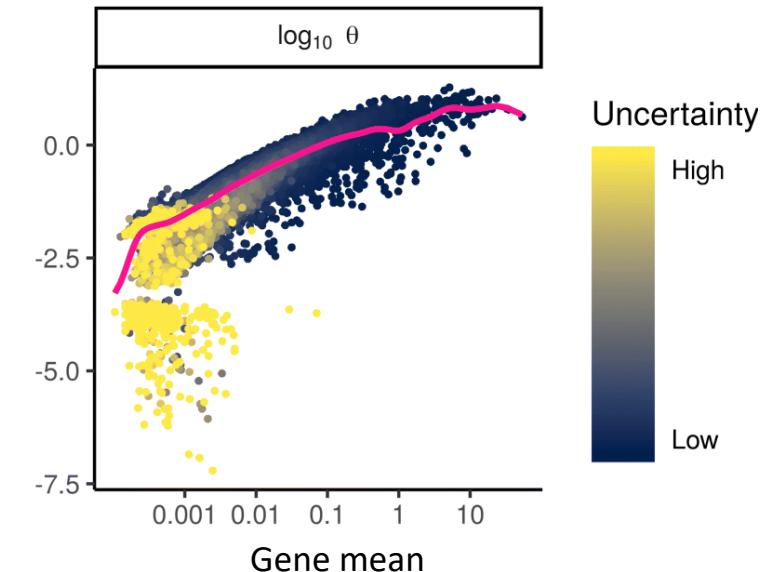
Pearson / deviance residuals

Sctransform (Hafemeister and Satija, Genome Biology 2019; Choudhary and Satija, Genome Biology 2022)

$$\begin{aligned}x_{gc} &\sim \text{NB}(\mu_{gc}, \theta_g) \\ \ln \mu_{gc} &= \beta_{g0} + \ln n_c,\end{aligned}$$

- n_c is the library size, x_{gc} is the observed count (Y_{gc})
- Assume $\mu_{gc} = n_c p_g$
- Pearson residual normalization

$$\begin{aligned}Z_{gc} &= \frac{x_{gc} - \mu_{gc}}{\sigma_{gc}} \\ \mu_{gc} &= \exp \beta_{g0} + \ln n_c \\ \sigma_{gc} &= \sqrt{\mu_{gc} + \frac{\mu_{gc}^2}{\theta_g}},\end{aligned}$$



- Estimate θ_g as a smoothed function of μ_g . $\theta_g = \infty$ for small μ_g
- If we are interested in heterogeneity across cells, then μ_{gc} contains non-interesting information
- Normalized data is not sparse any more

Pearson / deviance residuals

Deviance residuals (Townes et. al. Genome Biology 2019)

- For general definitions, check a GLM book
- The deviance residuals can look more normal than Pearson residuals
- Assume Poisson model on the observed counts

$$Z_{cg} = \text{sign}(X_{cg} - \hat{\mu}_{cg}) \sqrt{2 \left[X_{cg} \ln \frac{X_{cg}}{\hat{\mu}_{cg}} - (X_{cg} - \hat{\mu}_{cg}) \right]}$$

- Assume NB model on the observed counts

$$Z_{cg} = \text{sign}(X_{cg} - \hat{\mu}_{cg}) \sqrt{2 \left[X_{cg} \ln \frac{X_{cg}}{\hat{\mu}_{cg}} - (X_{cg} + \theta) \ln \frac{X_{cg} + \theta}{\hat{\mu}_{cg} + \theta} \right]}$$

(formula and notations copied from Lause et. al. Genome Biology 2021)

- Assume multinomial distribution (Townes et. al. Genome Biology 2019)

$$r_{ij}^{(d)} = \text{sign}(y_{ij} - \hat{\mu}_{ij}) \sqrt{2y_{ij} \log \frac{y_{ij}}{\hat{\mu}_{ij}} + 2(n_i - y_{ij}) \log \frac{n_i - y_{ij}}{n_i - \hat{\mu}_{ij}}}$$

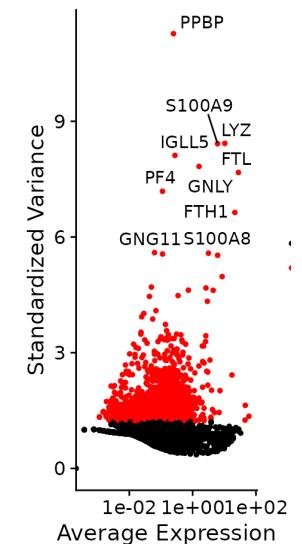
- Almost identical to the Poisson deviance

Selection of highly variable genes (HVG)

- High-dimensional PCA is not accurate when latent factors are not strong enough
- If a gene is expressed homogeneously across cells, it does not contain information about cell heterogeneity and only contribute noise to PCA
- Selection of HVG:
 - only use genes that have higher variability across cells than background when doing PCA
 - Identify a subset of 500-2000 genes
 - Using Sctransform Pearson residuals
 - Calculate variance of Z_{gc} for each g across c , select the top ones
 - Default method in Seurat: same idea, but a more straight-forward way to get Z_{gc}

$$Z_{gc} = \frac{Y_{gc} - \bar{Y}_g}{\sigma_g}$$

σ_g is calculated by fitting a smoothed mean-variance relationship



- Calculate residual deviance:
if Z_{gc} are deviance residuals, rank genes based on $\sum_c Z_{gc}^2$

Non-linear visualization: t-SNE & UMAP

- PCA for dimension reduction:
 - Only use HVG to perform PCA and get PC loadings
 - Selection top k ($k = 50$ in Seurat default) PCs to reduce data dimensions for further cell-level analyses
 - Systematic selection of k is possible but can be time consuming and may not worth it
- t-SNE: t-Distributed Stochastic Neighbor Embedding

Paper: https://lvdmaaten.github.io/publications/papers/JMLR_2008.pdf

Presentation: <https://www.youtube.com/watch?v=RJVL80Gg3IA&list=UUtXKDgv1AVoG88PLI8nGXmw>

- UMAP: Uniform Manifold Approximation and Projection

Paper: <https://arxiv.org/pdf/1802.03426.pdf>

Benchmark paper on scRNA-seq: <https://www.nature.com/articles/nbt.4314>

Presentation: <https://www.youtube.com/watch?v=nq6iPZVUxZU>

The idea of t-SNE

SNE (stochastic neighbor embedding)

- Preserve the similarity of high-dimensional points in low-dimensional points
- Measure similarity (conditional distributions) by Gaussian density

Original space:

$$p_{j|i} = \frac{\exp(-\|x_i - x_j\|^2 / 2\sigma_i^2)}{\sum_{k \neq i} \exp(-\|x_i - x_k\|^2 / 2\sigma_i^2)}$$

Find $\{y_i\}$ to minimize:

Low-dimensional space:

$$q_{j|i} = \frac{\exp(-\|y_i - y_j\|^2)}{\sum_{k \neq i} \exp(-\|y_i - y_k\|^2)}$$

$$C = \sum_i KL(P_i || Q_i) = \sum_i \sum_j p_{j|i} \log \frac{p_{j|i}}{q_{j|i}}$$

- Because of asymmetry in the KL divergence
 - large cost for using widely separated (y_i, y_j) to represent nearby (x_i, x_j)
 - Small cost for using nearby (y_i, y_j) to represent widely separated (x_i, x_j)
 - Only retain local structure of the data

The idea of t-SNE

SNE (stochastic neighbor embedding)

- Determination of the standard deviations σ_i
 - Smaller σ_i for denser regions and larger σ_i for sparser regions
 - For each i , find σ_i that reaches a pre-specified perplexity

$$Perp(P_i) = 2^{H(P_i)},$$

where $H(P_i)$ is the Shannon entropy of P_i measured in bits

$$H(P_i) = - \sum_i p_{j|i} \log_2 p_{j|i}.$$

- Decrease perplexity to preserve more global structures
- Solution obtained by gradient descent
 - Initialization: randomly sampled points from independent Gaussian
 - Large momentum to avoid poor local minima
 - Difficult to optimize and has “crowding problem”

The idea of t-SNE

t-SNE (t-distribution density [Cauchy])

Original space:

$$p_{j|i} = \frac{\exp(-\|x_i - x_j\|^2 / 2\sigma_i^2)}{\sum_{k \neq i} \exp(-\|x_i - x_k\|^2 / 2\sigma_i^2)} \quad p_{ij} = \frac{p_{j|i} + p_{i|j}}{2n}$$

Low-dimensional space:

~~$$q_{j|i} = \frac{\exp(-\|y_i - y_j\|^2)}{\sum_{k \neq i} \exp(-\|y_i - y_k\|^2)}$$~~

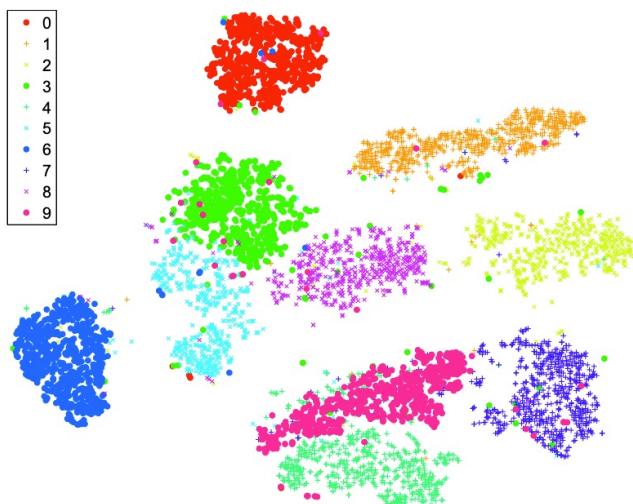
Find $\{y_i\}$ to minimize:

$$C = KL(P||Q) = \sum_i \sum_j p_{ij} \log \frac{p_{ij}}{q_{ij}}$$

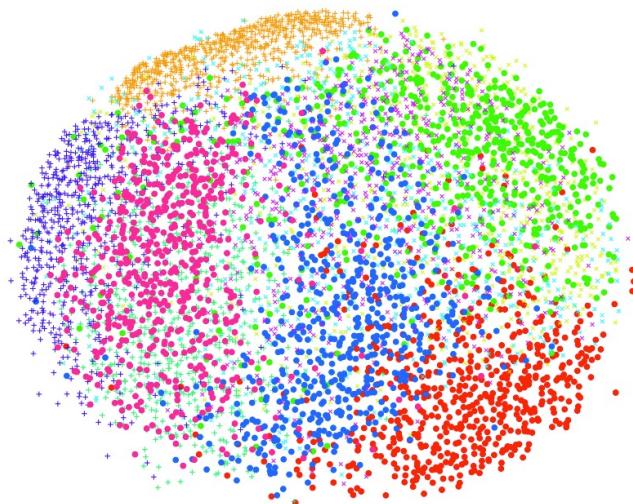
$$q_{ij} = \frac{(1 + \|y_i - y_j\|^2)^{-1}}{\sum_{k \neq l} (1 + \|y_k - y_l\|^2)^{-1}}$$

- Represent high-dimensional points better and keep moderately far-away points not too close
- Faster to optimize because calculation does not involve exponential
- Computational cost: $O(n^2)$

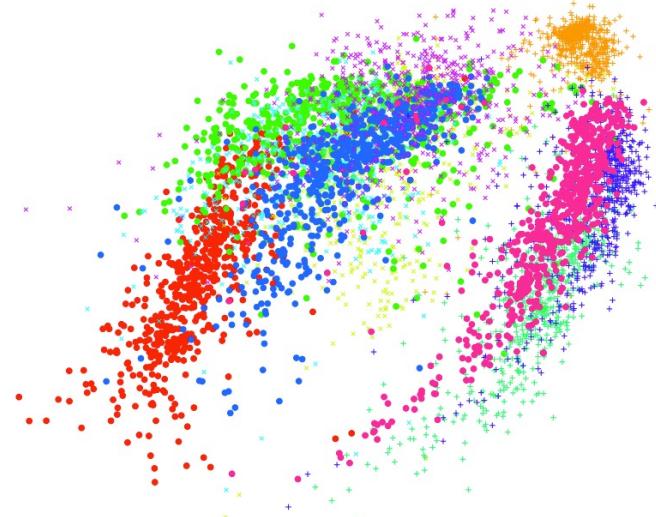
Visualization of MNEST data



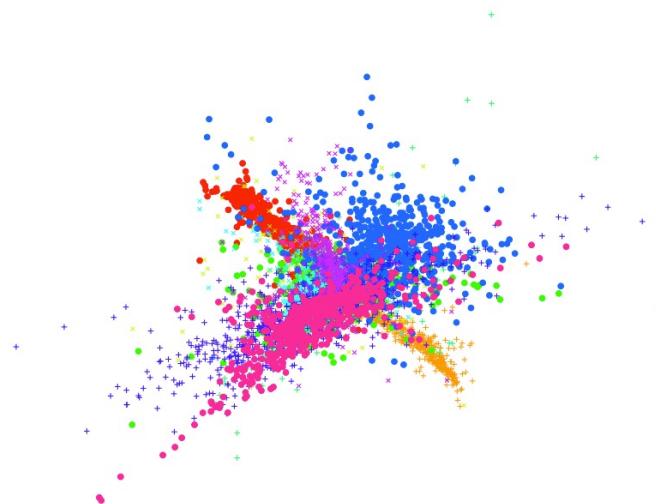
(a) Visualization by t-SNE.



(b) Visualization by Sammon mapping.



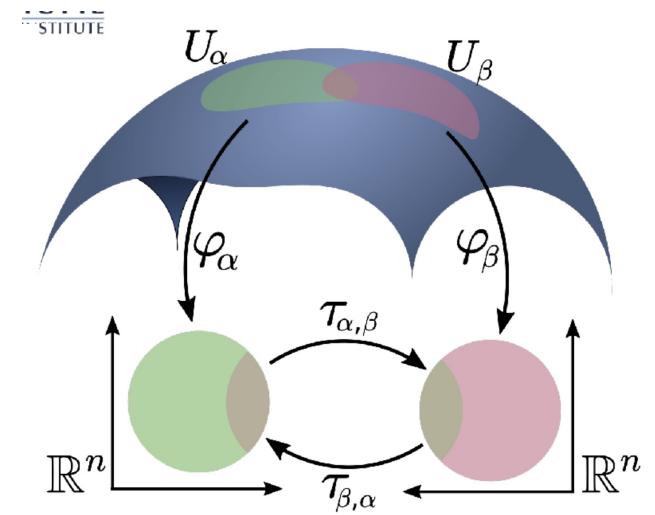
(a) Visualization by Isomap.



(b) Visualization by LLE.

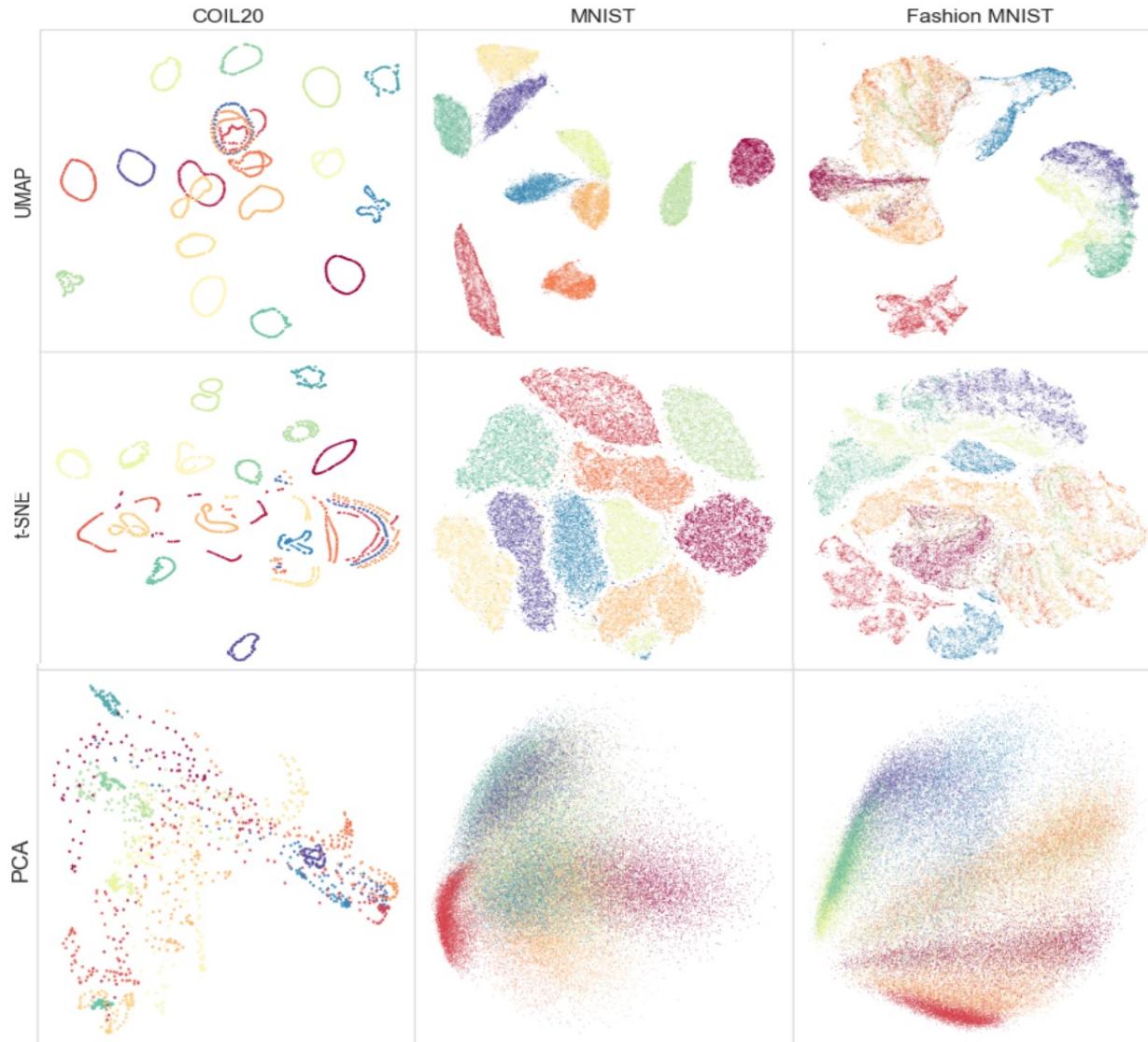
The (very high-level) idea of UMAP

- Construct topological representation of high-dimensional data
 - Assume that the data points uniformly lie on a low-dimensional manifold
 - Define local distance by k-nearest neighbors and construct a weighted k-neighbour graph
 - Based on the theory of local fuzzy simplicial set representations
- Represent the manifold by low-dimensional points
 - Minimize cross entropy of fuzzy simplicial set representation between the low and high-dimensional space
 - Use force-directed graph layout algorithm in low-dimensional space
- Computational cost: $O(n^{1.14})$



<https://www.youtube.com/watch?v=nq6iPZVUxZU>

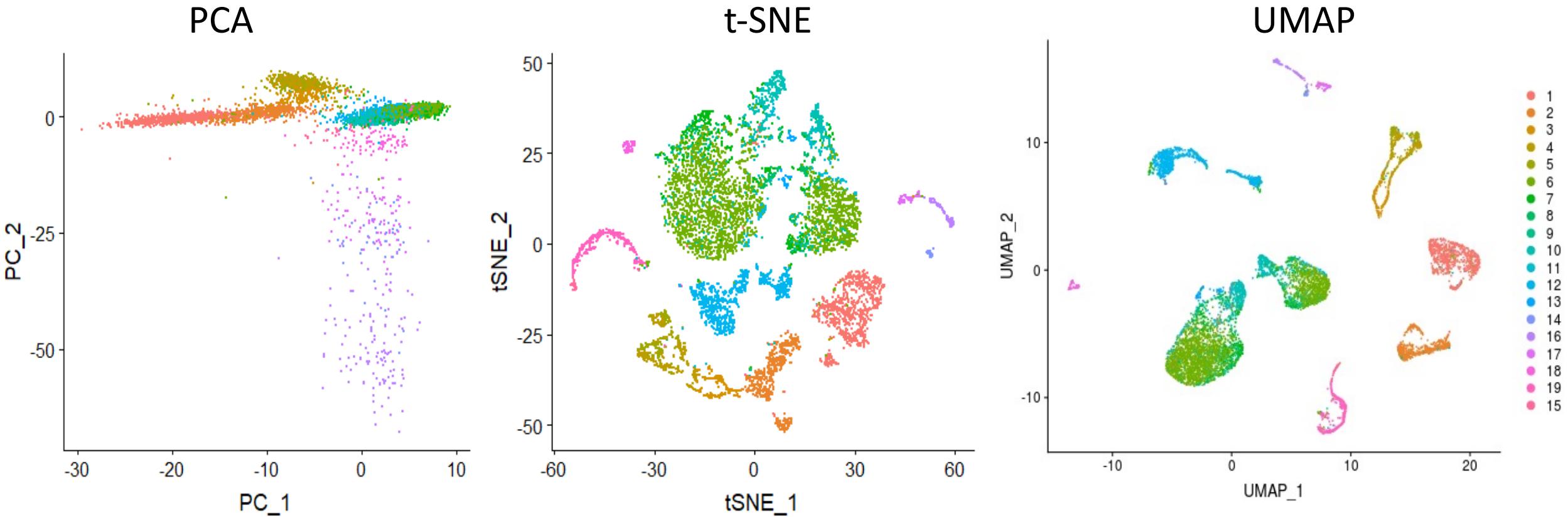
Compare PCA, t-SNE, UMAP



- PCA: keep global distance
- T-SNE: focus on local distance
- UMAP: focus on local distance, but may keep more global distance features

<https://arxiv.org/pdf/1802.03426.pdf>

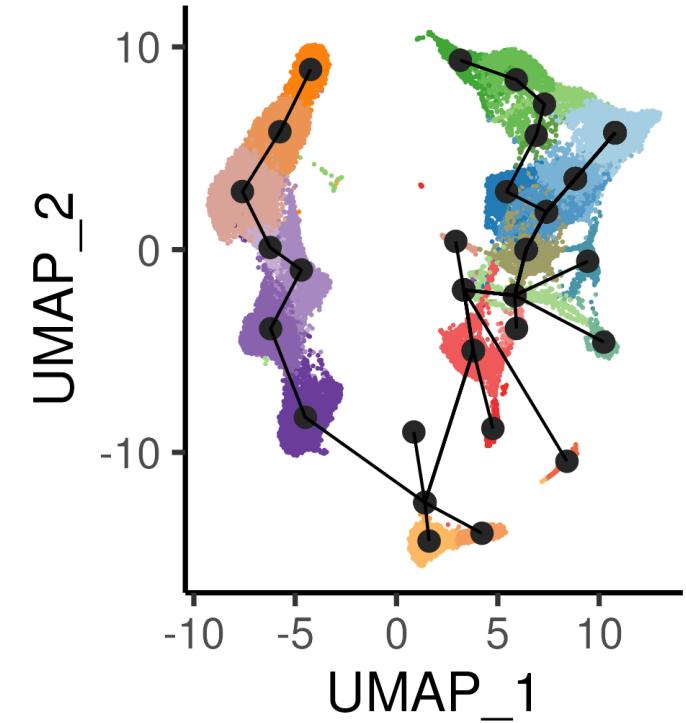
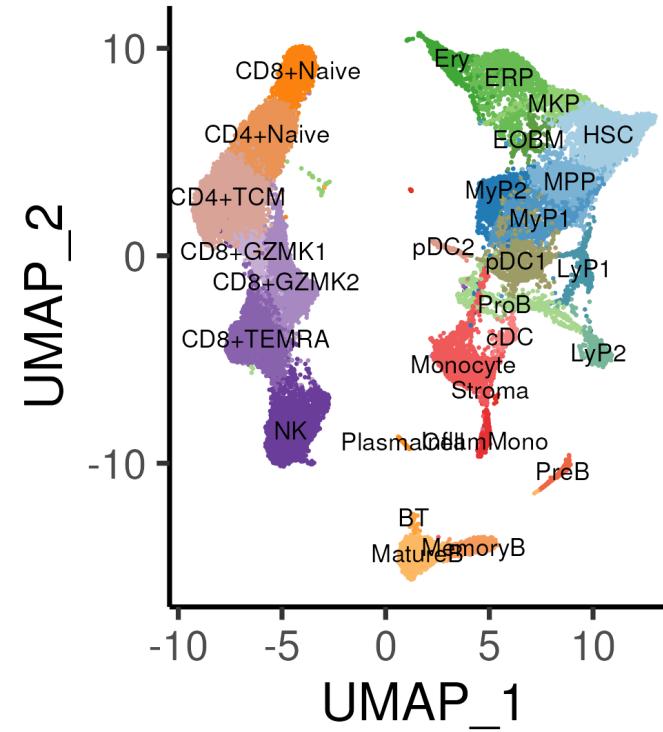
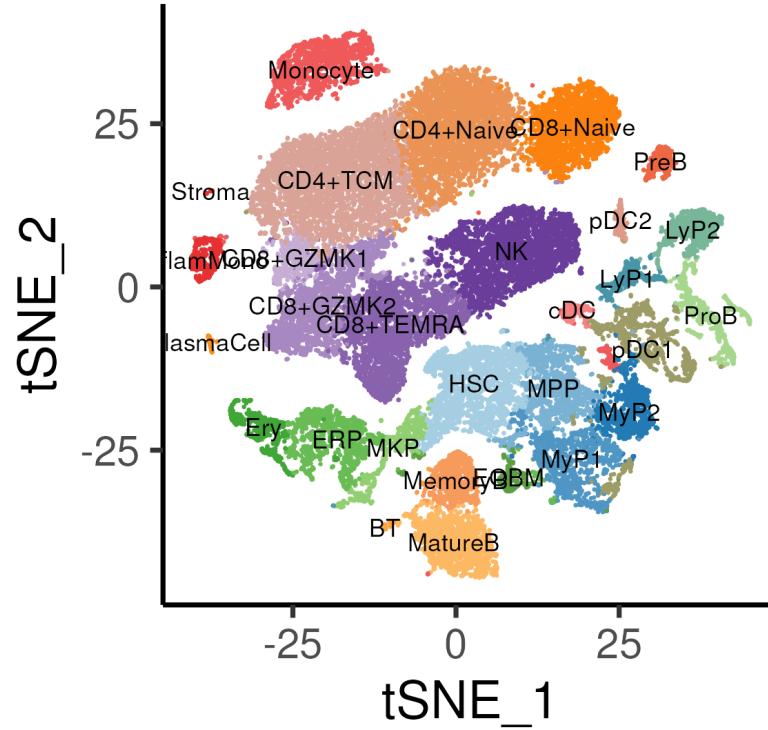
Visualize scRNA-seq using PCA, t-SNE, UMAP



Data from paper: Lineage dynamics of murine pancreatic development at single-cell resolution, Byrnes et. al. *Nature Comm.* 2018

Analysis pipeline see Seurat tutorial: https://satijalab.org/seurat/v3.0/pbmc3k_tutorial.html

UMAP is better at showing the cell lineages



Running time comparison



- Computation of UMAP is based on the construction of k-nearest-neighbor graph
- Nearest neighbors are obtained using the top PCs
- Computational cost: $O(n^{1.14})$

Related papers

- Osorio, D., & Cai, J. J. (2021). Systematic determination of the mitochondrial proportion in human and mice tissues for single-cell RNA-sequencing data quality control. *Bioinformatics*, 37(7), 963-967.
- Heumos, L., Schaar, A. C., Lance, C., Litinetskaya, A., Drost, F., Zappia, L., ... & Theis, F. J. (2023). Best practices for single-cell analysis across modalities. *Nature Reviews Genetics*, 24(8), 550-572.
- Wolf, F. A., Angerer, P., & Theis, F. J. (2018). SCANPY: large-scale single-cell gene expression data analysis. *Genome biology*, 19, 1-5.
- Hafemeister, C., & Satija, R. (2019). Normalization and variance stabilization of single-cell RNA-seq data using regularized negative binomial regression. *Genome biology*, 20(1), 296.
- Choudhary, S., & Satija, R. (2022). Comparison and evaluation of statistical error models for scRNA-seq. *Genome biology*, 23(1), 27.
- Townes, F. W., Hicks, S. C., Aryee, M. J., & Irizarry, R. A. (2019). Feature selection and dimension reduction for single-cell RNA-Seq based on a multinomial model. *Genome biology*, 20, 1-16.
- Lause, J., Berens, P., & Kobak, D. (2021). Analytic Pearson residuals for normalization of single-cell RNA-seq UMI data. *Genome biology*, 22, 1-20.