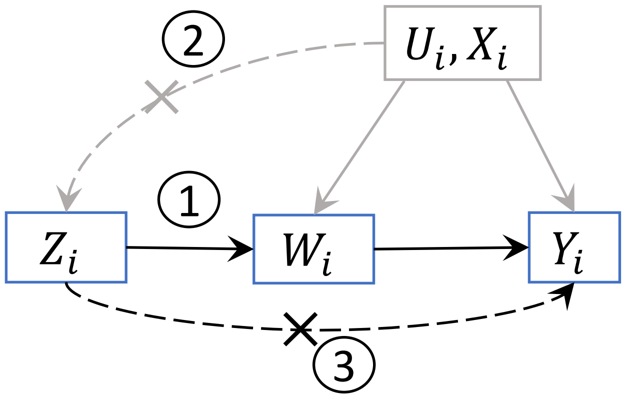
Lecture 10
Non-compliance in randomized experiments, instrumental variables
Part II

Outline

- Non-compliance in randomized experiment
  - Covariate adjustment
  - Connection with two-stage least square estimator
  - Weak instrument
- Textbook Chapters: Imbens and Rubin Chapters 23 & 24, Peng Chapter 21

Causal diagram for IV



- Treatment received may be affected by measured (X_i) and unmeasured (U_i) covariates
- Treatment assigned is randomized

Assumptions:

- Relevance: Z_i has an effect on W_i
- Randomization: Z_i are randomized
- Exclusion restriction: instrument affects the outcome only through treatment

- Monotonicity (only for binary W_i): no defiers

IV estimator with covariates adjustment

- We can generalize to incorporate covariates when estimating ITT_W and ITT_Y
  - Step 1: regress W_i^{obs} on Z_i and pre-assignment covariates X_i to get an estimate of $\widehat{\mathrm{ITT}}_W$
  - Step 2: regress Y_i^{obs} on Z_i and pre-assignment covariates X_i to get an estimate of $\widehat{\mathrm{ITT}}_Y$
  - Step 3: Take the ratio of estimated coefficients
    - If no covariates to adjust, the ratio estimator is exactly $\hat{\tau}^{iv}$

- How to estimate the variance of the ratio estimate?
  - Bootstrap:
    - Repeat M rounds, for each round:
      - Step 1: randomly sample N units from the triple $(Z_i, W_i^{\mathrm{obs}}, Y_i^{\mathrm{obs}})$
        - Sample with replacement
      - Step 2: for each bootstrap round, calculate the ratio estimator
    - Calculate the sample variance of ratio estimator across M rounds

Two-stage-least-square (2SLS) estimator

- Conventionally in econometrics, researchers use a two-stage least square approach for CATE
- The two-stage least square estimator is equivalent to $\hat{\tau}^{iv}$
- Two-stage least square
  - Stage 1: regress W_i^{obs} on Z_i :
    - the fitted coefficient on Z_i is $\widehat{\text{ITT}}_W$
    - Predict W_i^{obs} as $\widehat{W}_i^{\text{obs}} = \widehat{\text{ITT}}_W Z_i$
  - Stage 2: regress Y_i^{obs} on $\widehat{W}_i^{\text{obs}}$
  - The estimated coefficient of $\widehat{W}_i^{\text{obs}}$ on stage 2 is exactly $\hat{\tau}^{iv}$

$$\frac{\sum_{i=1}^{N} \widehat{\text{ITT}}_W (Z_i - \bar{Z})(Y_i^{\text{obs}} - \bar{Y}^{\text{obs}})}{\sum_{i=1}^{N} \widehat{\text{ITT}}_W^2 (Z_i - \bar{Z})^2} = \frac{\sum_{i=1}^{N} (Z_i - \bar{Z})(Y_i^{\text{obs}} - \bar{Y}^{\text{obs}})}{\widehat{\text{ITT}}_W \sum_{i=1}^{N} (Z_i - \bar{Z})^2} = \frac{\widehat{\text{ITT}}_Y}{\widehat{\text{ITT}}_W}$$

- We can generalize 2SLS to incorporate covariates in both stages

The Angrist draft lottery data

Background

- Policy makers are interested in whether veterans are adequately compensated for their service.
- Angrist (1991) aims to measure the long-term labor market consequences of military service during the Vietnam era
- Question: estimate the causal effect of serving in the military during the Vietnam War on earnings
- We can not directly compare veterans and non-veterans, as they can be systematically different in unobserved ways, even after adjusting for differences in observed covariates

- Serving in the military or not during the Vietnam War could not randomized directly, but the military draft lottery of the Vietnam War was randomized
- This is called a natural experiment

The Angrist draft lottery data

Randomization

- For each birth year of birth cohort 1950-1952, a random ordering of the 365 days was constructed, a cutoff number was pre-determined, young men of that birth year who had a birth date with order before the cutoff “won” the lottery
- Randomization of birth date, instead of the individuals

- Theoretically, each date should be a unit, but in the book example, we treat each individual as a unit and consider the experiment as a completely randomized experiment (it’s actually a stratified cluster randomized experiment).
  Consequence is that we will tend to under-estimate the uncertainty of the causal estimator.

Relevance and two-sided non-compliance:

- Drafted individuals were required to prepare to serve in the military if fit for the service
- To serve the military, drafted individuals need to pass medical tests and have achieved minimum education level
- Individuals who were not draft eligible also can volunteer to serve in the military

The Angrist draft lottery data

	Non-Veterans ($N_c = 6{,}675$)				Veterans ($N_t = 2{,}030$)			
	Min	Max	Mean	(S.D.)	Min	Max	Mean	(S.D.)
Draft eligible	0	1	0.24	(0.43)	0	1	0.40	(0.49)
Yearly earnings (in $1,000's)	0	62.8	11.8	(11.5)	0	50.7	11.7	(11.8)
Earnings positive	0	1	0.88	(0.32)	0	1	0.91	(0.29)
Year of birth	50	52	51.1	(0.8)	50	52	50.9	(0.8)

Check assumptions

- **Monotonicity**: appears to be a reasonable assumption
  - The lottery numbers impose restrictions on individuals' behaviors.
  - Monotonicity means that no one responds to these restrictions by serving only if they are not required to do so
  - It is possible that there are some individuals who would be willing to volunteer if they are not drafted but would resist the draft if required, but it must be a very small fraction and are likely ignorable

The Angrist draft lottery data

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Check assumptions

- **Exclusion restriction**: may be questionable
  - Consider the never-takers
  - Some never-takers are due to medical exemptions or exemptions due to their education or career choices. For them, the lottery numbers would likely not affect their future behaviors and the outcome
  - Some never-takers did have exemptions but changed their plan (enter graduate school or move to Canada) if they had a low draft number to avoid serving in the military. For them, exclusion restriction can be violated.

Analysis results

ITT Estimates:

- $\widehat{\mathrm{ITT}}_W = 0.1460, \widehat{\mathbb{V}}\left(\widehat{\mathrm{ITT}}_W\right) = 0.0108^2$
- $\widehat{\mathrm{ITT}}_Y = -0.2129, \widehat{\mathbb{V}}\left(\widehat{\mathrm{ITT}}_W\right) = \sum_{z=0}^{1} \frac{\bar{Y}_z^{\mathrm{obs}}(1-\bar{Y}_z^{\mathrm{obs}})}{N_z(N_z-1)} = 0.1980^2$
- 95% CI of ITT_Y: $(-0.6010, 0.1752)$

If we are willing to assume monotonicity and exclusion restriction

CATE estimate:

- $\hat{\tau}^{iv} = \frac{-0.2129}{0.1460} = -1.46$
- $\widehat{\mathbb{V}}\left(\hat{\tau}^{iv}\right) = 1.36^2$
- 95% CI of CATE: $(-4.13, 1.2)$

Weak instrument

- The instrumental variable is a weak instrument if the compliance probability (π_c or ITT_W) is small

- Problems using weak instrument
  - $\hat{\tau}^{iv} = \frac{\widehat{\mathrm{ITT}}_Y}{\widehat{\mathrm{ITT}}_W}$: the ratio is very unstable. If ITT_W is close to 0, then a small error (perturbation) in $\widehat{\mathrm{ITT}}_W$ can lead to a large error in $\hat{\tau}^{iv}$
  - If the exclusion restriction assumption is violated, the bias in our estimator assuming exclusion restriction is inversely proportional to π_c

- How to identify weak instrument?
  - In the first stage linear regression model $W_i^{\mathrm{obs}} = \alpha + \pi_c W_i + \varepsilon_i$, calculate the F-statistics to test whether $\pi_c = 0$
  - A rule of thumb is to check whether the F-statistics is larger to 10 or not.
  - F-statistics smaller than 10 indicates a weak instrument