

Lecture 4

Fisher's exact test: R example and case study

Outline

- Computing Fisher's exact p-value and CI with R
- A case study using Fisher's sharp null and exact p-values

Case study: the California alphabet lottery

[Randomization inference with natural experiments: An analysis of ballot effects in the 2003 California recall election. *Journal of the American statistical association*, 2006]

Problem background

- In the 2000 U.S. national election, George W. Bush became President by winning 537 more votes than Al Gore in Florida.
- This unusually close election result served as a reminder that the manner in which elections are administered can change outcomes.
- This paper studied the causal effect of the page placement of candidates in the 2003 California recall election
- dataset was collected by *The New York Times* in 2003 (not publicly available)

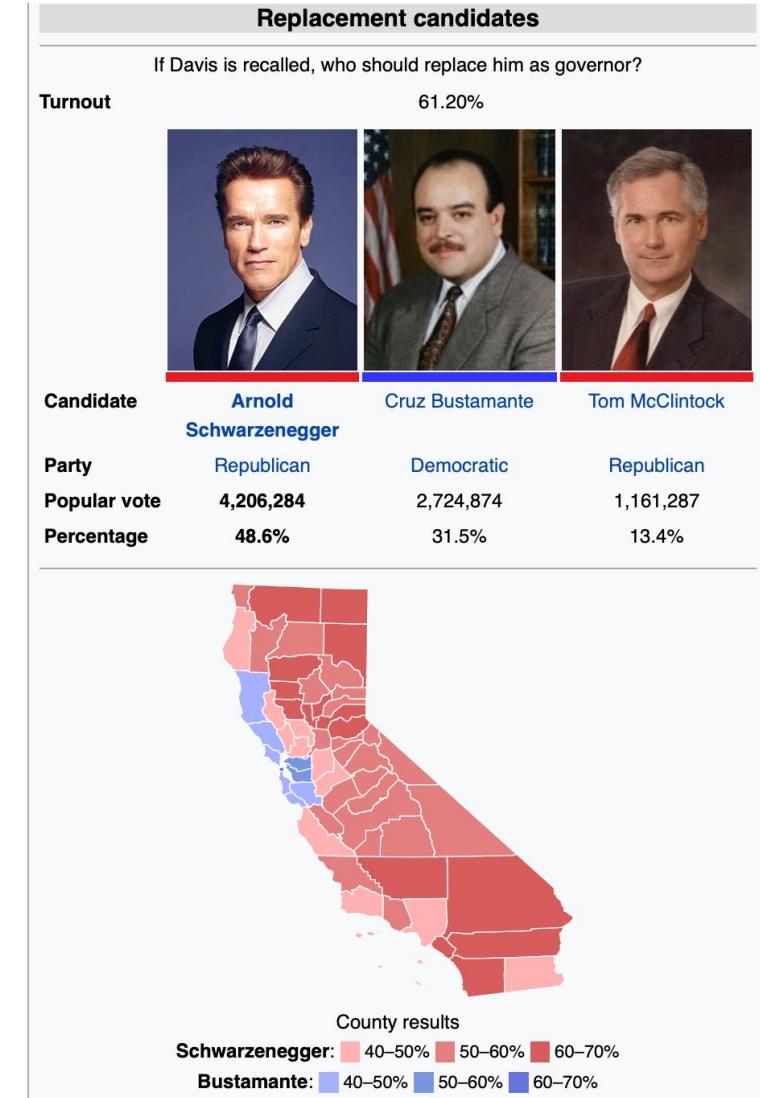
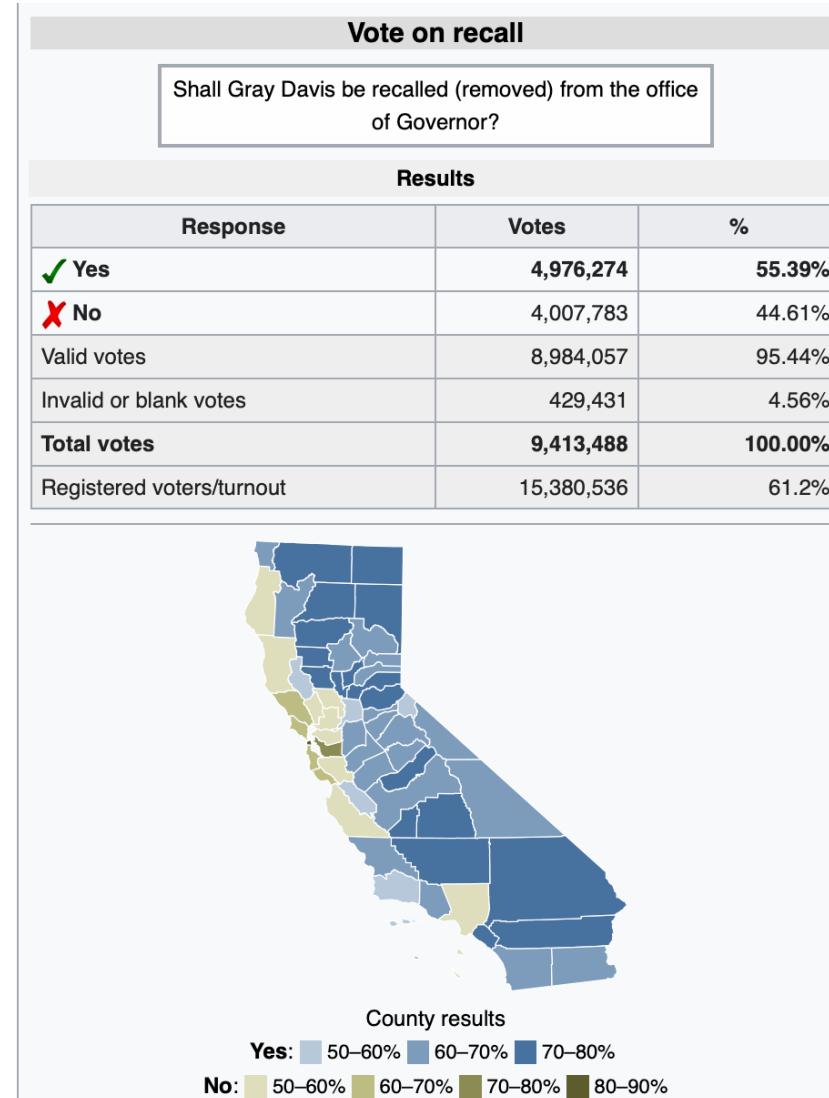
Case study: the California alphabet lottery

[Randomization inference with natural experiments: An analysis of ballot effects in the 2003 California recall election. *Journal of the American statistical association*, 2006]

Problem background

- Recall results

https://en.wikipedia.org/wiki/2003_California_gubernatorial_recall_election



The randomization-rotation procedure

- Since 1975, California law has mandated that the Secretary of State draw a random alphabet for each election to determine the order of candidates for the first assembly district [California Election Code § 13112 (2003)].
- California law further requires that the candidate order be systematically rotated throughout the remaining assembly districts.
- The procedure
 1. Randomize alphabet
 2. Sort candidates by randomized alphabet
 3. Rotate the candidate order from the first district

For the 2003 recall election, the actual randomized alphabet was

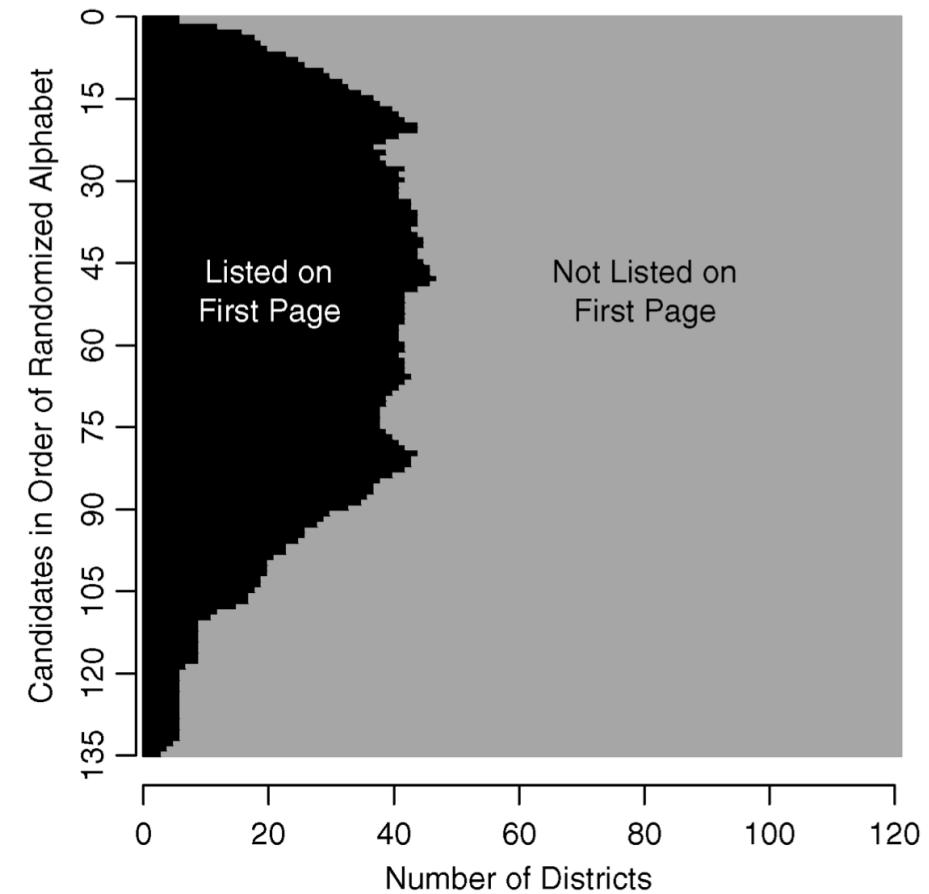
R W Q O J M V A H B S G Z X N T C I E K U P D Y F L

- The ballot order in the first assembly district was determined, starting from Robinson, Roscoe, Ramirez, and so on and proceeding to Lewis and Leonard.
- This candidate order was then rotated throughout the remaining assembly districts.

The randomization-rotation procedure

Challenges analyzing data with the randomization procedure

- Randomization is not done on each candidate
- The alphabets are randomized, but the 80 assembly districts order are not randomized
- an unprecedented total of 135 candidates, from Hollywood actor Arnold Schwarzenegger to child television star Gary Coleman
- Each of the 58 counties uses a different ballot format with varying numbers of pages, leading to 121 county-district combinations of ballot formats
- interactions across candidates



No complete randomization of page placement across candidates nor across districts

Set up the analysis framework

- Analyze the causal effect of page placement for each of the 135 candidates separately
- Each of 121 county-district combination is a **unit**: $Y_i(0)$ and $Y_i(1)$ for a district i and a particular candidate
- Treatment: $T_i = 1$ if candidate is placed on the first page, $T_i = 0$ otherwise
- Sharp null for a particular candidate: $H_0: Y_i(0) \equiv Y_i(1)$ for all $i = 1, \dots, 121$
- Test statistics:
 - Sample average treatment effect $W^D(\mathbf{T}) = \frac{\sum_{i=1}^{121} T_i y_i}{N_1} - \frac{\sum_{i=1}^{121} (1 - T_i) y_i}{N_0}$
 - Covariate-adjusted test statistics

$$W^L(\mathbf{T}) = (\mathbf{T}^\top \mathbf{M} \mathbf{T})^{-1} \mathbf{T}^\top \mathbf{M} \mathbf{y}, \quad (4)$$

where $\mathbf{y} = (y_1, y_2, \dots, y_{121})$, $\mathbf{M} = \mathbf{I} - \mathbf{X}(\mathbf{X}^\top \mathbf{X})^{-1} \mathbf{X}^\top$, and \mathbf{X} is the matrix of the observed pretreatment covariates.

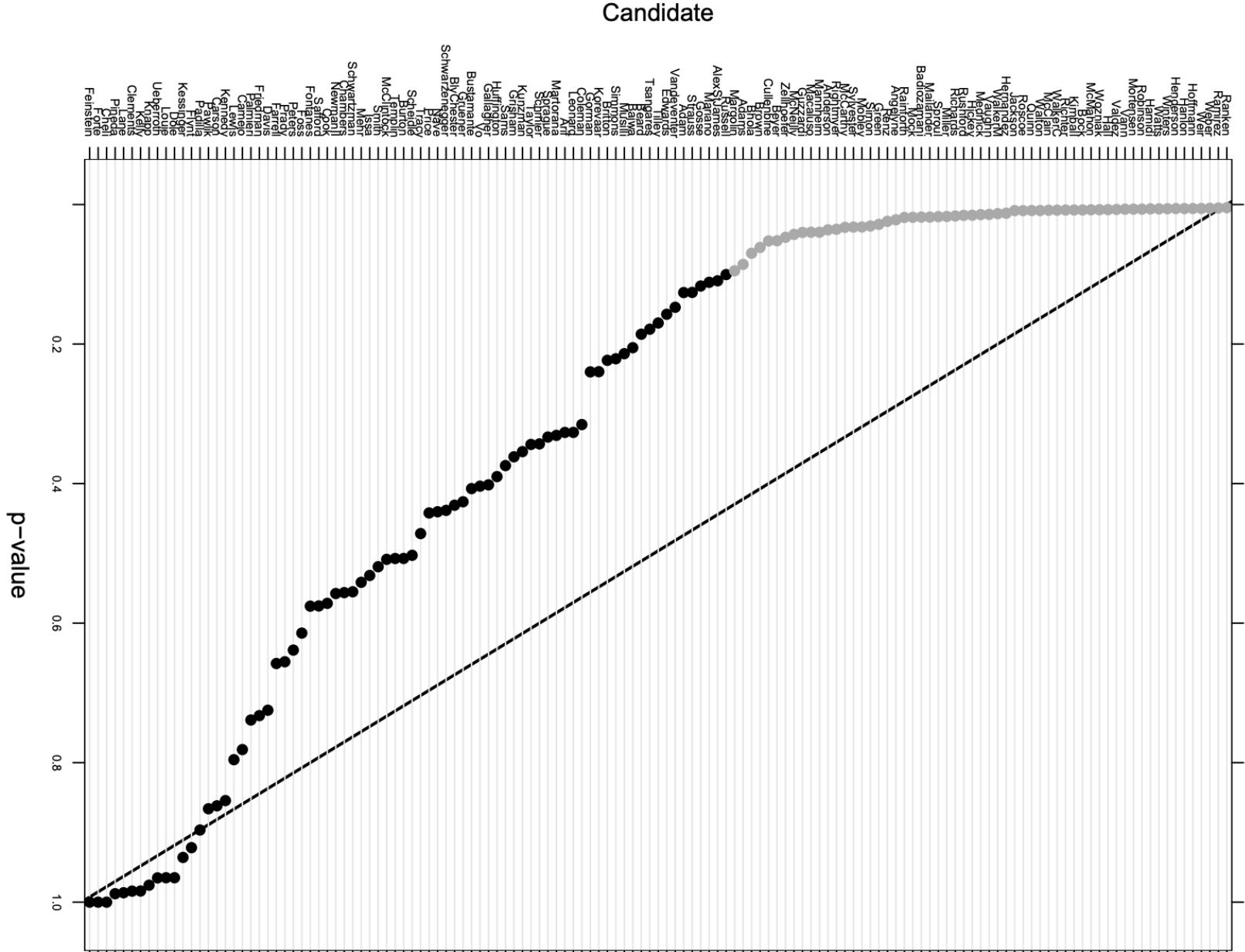
Set up the analysis framework

Implicit assumptions

- *Assumption 1* (No interference among units) The potential outcomes of one unit do not depend on the treatment of other units.
 - potential vote shares of a candidate in one district do not depend on the same candidate's ballot placement in another district.
 - Voters usually do not see ballots of other districts and hence are unlikely to be affected by such ballots.
 - focus on the estimation of a separate causal effect for each candidate
- *Assumption 2* (Known random assignment). Treatment is randomly assigned by a known mechanism. Formally, $p(T_i | Y_i(0), Y_i(1)) = p(T_i)$ is known for each i .
 - Assumes county page formats are independent of the randomized alphabet
 - Number of possible ballot pages is driven primarily by the type of voting technology, which is exogenous to the randomization

Distribution of Exact p-values across Candidates

- Authors computed the one-sided p-values
 - Reference distribution obtained via Monte Carlo
 - Candidates ranked based on their p-values
 - If the sharp null is true, these p-values should all be uniformly distributed



Confidence intervals under the constant additive effect model

- For each candidate, we assume $Y_i(0) - Y_i(1) \equiv \tau_0$ across all republican / democratic districts
- We construct confidence intervals by inverting the Fisher's randomization tests at a range of τ_0 values

