Topics in Causal Inference

STAT41530

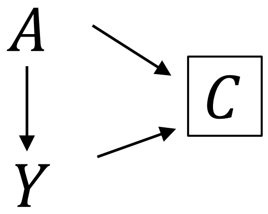
Jingshu Wang

Lecture 5

Topic:
causal directed acyclic graph (DAG)

- Selection bias
- IPW with do-operator

Collider bias



Conditional on a collider C creates non-causal association between A and Y

Example:

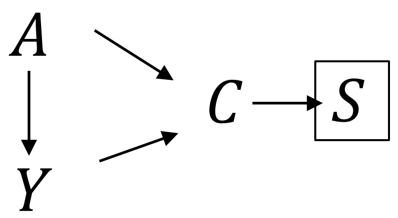
A: Give folic acid supplements to pregnant women shortly after conception

Y: fetus's risk of developing a cardiac malformation

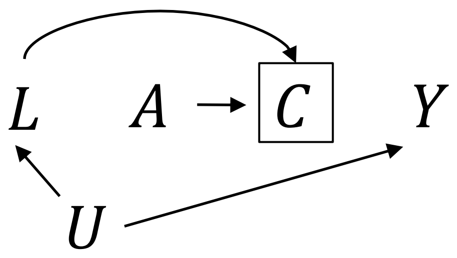
C: survival at birth

Selection bias exists even in randomized controlled experiments!

Other structures of selection bias

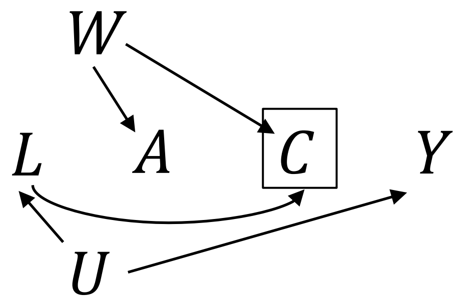


A: Give folic acid supplements to pregnant women shortly after conception
Y: fetus's risk of developing a cardiac malformation
C: survival at birth
S: parent grief



A: Antiretroviral treatment on HIV
Y: 3-year risk of death
U: high-level of immunosuppression
L: presence of symptoms
C: lost to follow-up

Healthy worker bias:
A: Occupational exposure to chemical
Y: Mortality
U/L: health status
C: being at work

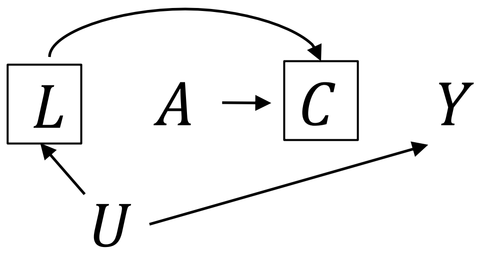


W: unmeasured lifestyle/personality/educational variables

Volunteer bias:
A: cigarette smoking
Y: coronary heart disease
U: family history of heart disease
L: disease awareness
C: agree to participate in the study
W: healthy lifestyle

Adjust for selection bias

- Avoid conditioning on post-treatment variables
- Selection bias is often unavoidable
- Adjustment: conditioning on both C and L



C is a collider that must be conditioned on. If there is L satisfying:

1. L satisfies the backdoor criterion
2. $(C \perp\!\!\!\perp Y \mid A, L)_G$

then

$$P(Y|do(A)) = \sum_l P(Y|A, C = 1, L = l)P(L = l)$$

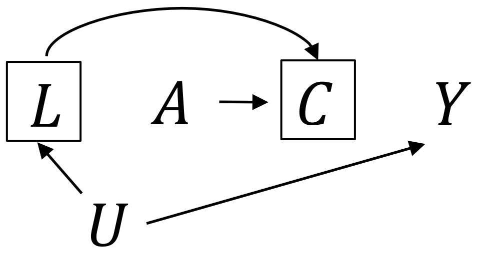
Need positivity assumption:

$P(A = a, C = 1, L = l) > 0$ for all l where $P[L = l] > 0$

- $P(A = a \mid L = l) > 0$
- $P(C = 1 \mid A = a, L = l) > 0$

$$\begin{aligned}\mathbb{P}[Y \mid do(A)] &= \sum_l \mathbb{P}[Y \mid L = l, A]\, \mathbb{P}[L = l] \\ &= \sum_l \mathbb{P}[Y \mid L = l, A, C = 1]\, \mathbb{P}[L = l]\end{aligned}$$

IPW in selection bias adjustment

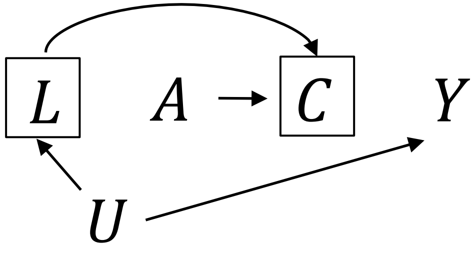


$$
\begin{aligned}
\mathbb{P}\left[Y \mid do(A)=a\right] &= \sum_{l} \mathbb{P}\left[Y \mid L=l, A=a, C=1\right] \\
&= \sum_{l} \frac{\mathbb{P}\left[Y, C=1 \mid L=l, A=a\right]}{\mathbb{P}\left[C=1 \mid A=a, L=l\right]} \mathbb{P}\left[L=l\right] \\
&= \sum_{l} \frac{\mathbb{P}\left[Y, C=1, A \mid L=l\right] \mathbb{P}\left[L=l\right]}{\mathbb{P}\left[A=a \mid L=l\right] \mathbb{P}\left[C=1 \mid A=a, L=l\right]} \\
&= \sum_{l} W_a(l) \mathbb{P}\left[Y, C=1, A=a, L=l\right]
\end{aligned}
$$

A completely randomized experiment example

$$W_a = \frac{1}{P[C=0 \mid A=a, L=l]}$$

$[P(A = a \mid L = l) \equiv 0.5]$



A: Wasabi intake
Y: one-year risk of death
L: heart disease
C: lost to follow-up ($C = 1$)

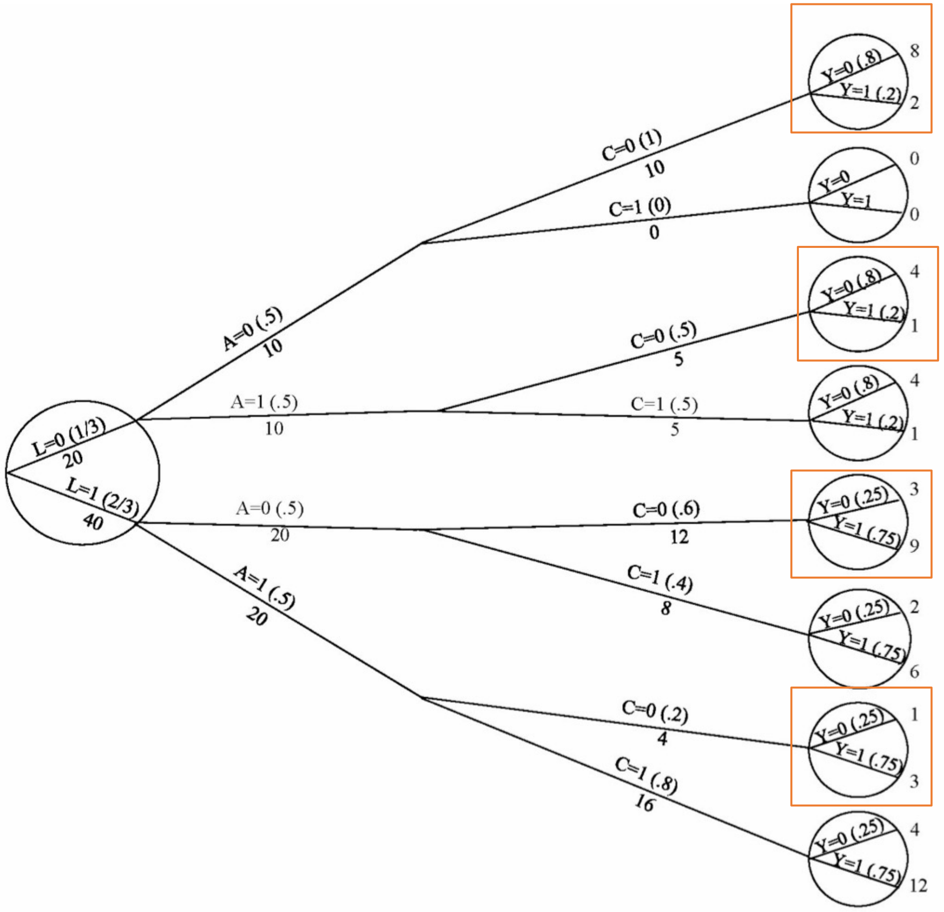


Figure 8.10

1

2

$\frac{5}{3}$

5

Risk ratio: $\frac{P(Y=1 \mid do(A)=1)}{P(Y=0 \mid do(A)=0)} = \frac{1*2+3*5}{2*1+9*5/3} = 1$

Collider bias

$$
\begin{tikzcd}
A \arrow[d] \arrow[r] & \boxed{C} \\
Y \arrow[ur] &
\end{tikzcd}
$$

Conditional on a collider C creates non-causal association between A and Y

Example:

A: Give folic acid supplements to pregnant women shortly after conception

Y: fetus's risk of developing a cardiac malformation

C: survival at birth

Selection bias exists even in randomized controlled experiments!