第 2 次作业题

第 2.1 题

阅读并总结第 2 周阅读材料.

第 2.2 题

For any subset X of $\mathbb{R}^n$, define

$$X^* := \{y \in \mathbb{R}^n \mid x^T y \leq 1 \text{ for each } x \in X\}.$$

(i) Show that for each convex cone C, C^* is a closed convex cone.

(ii) Show that for each closed convex cone C, $(C^*)^* = C$.

第 2.3 题

Let $C \subseteq \mathbb{R}^n$. Then C is a closed convex cone if and only if $C = \bigcap \mathcal{F}$ for some collection $\mathcal{F}$ of linear halfspaces.

(A subset H of $\mathbb{R}^n$ is called a *linear halfspace* if $H = \{x \in \mathbb{R}^n \mid c^T x \leq 0\}$ for some nonzero vector c.)

第 2.4 题

Let P be a polyhedron.

(i) Show that P^* is again a polyhedron.

(*Hint:* Use previous exercises.)

(ii) Show that P contains the origin if and only if $(P^*)^* = P$.

(iii) Show that the origin is an internal point of P if and only if P^* is bounded.

第 2.5 题

Prove that there exists a vector $x \geq 0$ such that $Ax \leq b$ if and only if for each $y \geq 0$ satisfying $y^T A \geq 0$ one has $y^T b \geq 0$.

第 2.6 题

Prove that there exists a vector $x > 0$ such that $Ax = 0$ if and only if for each y satisfying $y^T A \geq 0$ one has $y^T A = 0$. (Stiemke's theorem (Stiemke [1915]).)