Example to understand $\mathbf{q}$

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March 29, 2024

$[q_0, q_1, q_2, q_3] = [0.9885, 0.0091, 0.0206, -0.1512]$:

1. Rotation about the x-axis (roll):

$$
\begin{aligned}
\text{roll} &= \arctan 2\left(2(q_0 q_1 + q_2 q_3), 1 - 2(q_1^2 + q_2^2)\right) \\
&= \arctan 2\left(2(0.9885 \times 0.0091 + 0.0206 \times -0.1512), 1 - 2(0.0091^2 + 0.0206^2)\right) \\
&= \arctan 2\left(2(0.0090 - 0.0031), 1 - 2(0.0001 + 0.0004)\right) \\
&= \arctan 2\left(2 \times 0.0059, 0.9990\right) \\
&= \arctan 2\left(0.0118, 0.9990\right) \\
&\approx 0.0118 \text{ rad} \\
&\approx 0.68°
\end{aligned}
$$

2. Rotation about the y-axis (pitch):

$$
\begin{aligned}
\text{pitch} &= \arcsin\left(2(q_0 q_2 - q_3 q_1)\right) \\
&= \arcsin\left(2(0.9885 \times 0.0206 - (-0.1512) \times 0.0091)\right) \\
&= \arcsin\left(2(0.0204 + 0.0014)\right) \\
&= \arcsin\left(2 \times 0.0218\right) \\
&= \arcsin\left(0.0436\right) \\
&\approx 0.0436 \text{ rad} \\
&\approx 2.50°
\end{aligned}
$$

3. Rotation about the z-axis (yaw):

$$
\begin{aligned}
\text{yaw} &= \arctan 2\left(2(q_0 q_3 + q_1 q_2), 1 - 2(q_2^2 + q_3^2)\right) \\
&= \arctan 2\left(2(0.9885 \times -0.1512 + 0.0091 \times 0.0206), 1 - 2(0.0206^2 + (-0.1512)^2)\right) \\
&= \arctan 2\left(2(-0.1495 + 0.0002), 1 - 2(0.0004 + 0.0228)\right) \\
&= \arctan 2\left(-0.2986, 0.9536\right) \\
&\approx -0.3046 \text{ rad} \\
&\approx -17.46°
\end{aligned}
$$

Therefore, the quaternion $[q_0, q_1, q_2, q_3] = [0.9885, 0.0091, 0.0206, -0.1512]$ represents the following rotations about the x, y, and z axes:

- Rotation about the x-axis (roll): approximately 0.68 degrees

- Rotation about the y-axis (pitch): approximately 2.50 degrees

- Rotation about the z-axis (yaw): approximately -17.46 degrees