The impact of cause-specific mortality forecasting on projections of life expectancy

Bachelor's Thesis and Thesis Seminar Actuarial Science

Projecting mortality rates

SUPERVISOR:

Frank van Berkum

JAY LEE (11651318)

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Statement of Originality

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1 Introduction

Since the medical systems and the people's acknowledgement of the health have been improved, average life expectancy has been increasing over time world-wide. The European Commission predicts that there will be a growth of 57.6% of life expectancy in 2061 while in 2013 there was a growth rate of 36% of life expectancy (Antonio et al., 2017). This is also backed up by the statistics of the World Bank. 73.4 the life expectancy in 1960 increased to 81.8 by 2018 in the Netherlands (The World Bank, 2018). Since actuarial products are strongly dependent on people's life or death, projecting future mortality rate is one of the most important topics among the actuaries. In the case of life insurances, for example, premiums, death benefits, and all the costs are calculated based on the mortality rates of the insured. Miscalculation of these values causes unexpected costs to the insurer. This is called the longevity risk and there are few steps to handle it. The first step is to obtain as precise predictions of future mortality rates as possible. This helps to quantify the longevity risk, which decreases the probability of making wrong calculations.

However, dealing with the uncertainties in the future is volatile while there is always room for un-expectancy. In this sense, small changes in factors such as the form of data set or the application of the model could bring totally different predictions. One of well-known factors is whether the model takes into account the causes of deaths to the projections (Alai et al., 2015). This is because future trends of the mortality rate rely on the trend of the causes of deaths. For instance, heart disease, cancers are the two main causes of total deaths. Although deaths caused by heart diseases are the largest, the mortality rate of cancer deaths would become a major factor in the future. Based on the data of the National Cancer Institute, the number of deaths by cancers was around 6 million in 1990 and became about 10 million in 2017, meaning cancer deaths have increased by almost 66% or more over about 30 years (Roser and Ritchie, 2015). Likewise, in the future, there are always the possibility that a new cause of deaths becomes the main cause of deaths. Also, on the other hand, one cause of deaths suddenly would become perfectly curable. This paper will answer the question how eliminating a single cause of deaths affects the projection of mortality rates. This paper particularly reviews all cancer deaths as mentioned previously. The estimation is made with the help of Lee and Carter model

and the projection will be following. The projection result will explain the impact of the one cause of deaths to the future mortality rates. Finally, the conclusion is based on the differences on the life expectancy, which is a general way to interpret the result of predicted mortality rates.

2 Literature Review

This section explains the key points of every literature that this paper refers to.

2.1 Lee and Carter's model

Lee and Carter (LC) model (1992) predicted the mortality rate with historical data over the periods between 1933 and 1987 in the U.S. This model starts with defining $D_{x,t}$ as the number of people dying age x at time t, and $E_{x,t}$ as the number of people age x exposure to the risk of dying at time t. Then, the central death rate is defined by

$$m_{x,t} = \frac{D_{x,t}}{E_{x,t}} \tag{1}$$

Then, a following relational equation is defined:

$$ln(m_{x,t}) = a_x + b_x k_t + \epsilon_{x,t} \tag{2}$$

a_x represents the average of $ln(m_{x,t})$, b_x specifies the effect of time, and k_t explains a time-dependent effect. In order to handle the identification problem for these two parameters b_x and k_t, below constraints are made .

$$\sum_t k_t = 0 \tag{3}$$

$$\sum_x b_x = 1 \tag{4}$$

Those three parameters (a_x, b_x, k_t) are estimated by the method of singular value decomposition (SVD) assuming homoscedasticity. After the estimation of parameters, $\hat{k_t}$ is re-estimated such that $D_{x,t} = E_{x,t} \times \mu_{x,t}$ with $\mu_{x,t} = e^{\hat{a_x} + \hat{b_x}\hat{k_t}}$, which is called the second stage estimation. However,

this estimation is found to be a non-critical property to apply LC model (Brouhns et al., 2002; Girosi and King, 2007). In this paper, the second-stage-estimation is skipped. For future time t, given the same age x, future mortality rate is dependent on the preiod effect variable k_t^{future}. In other words, the force of mortality rate of people age x at future time t is merely determined by k_t^{future}. Then, a following step is to figure out the way to estimate k_t^{future}. To do so, LC used random walk drift time series model. Since there was an influenza epidemic of 1918, they should include this factor to forecast the mortality rate. Therefore, the forecast model is defined by

$$\kappa_t = \kappa_{t-1} + \theta + \beta flu + e_t \tag{5}$$

θ is called the drift, β is the effect of flu to the κ, and e_t is the error terms. Substituting projected k_t^{future} to the equation (2) offers the projection of mortality rate. In conclusion, LC model successfully derived the future mortality rate of the U.S (Lee and Carter, 1992).

2.2 Applying LC model

Antonio and researchers (2017) used the method shown above for the multi-populations. Antonio predicted future mortality rate of the Netherlands and Belgium over the period between 1970 and 2009. The base of the model is the same as LC model but applying the Poisson regression approach. Given $\mu_{x,t}$ is the force of mortality rates for people age x at time t and $E_{x,t}$ is total number of people age x lived at time t, the total number of death $D_{x,t}$ is defined by

$$D_{x,t} \sim Poisson(E_{x,t}\, \mu_{x,t}) \tag{6}$$

This research made a successful prediction of future mortality rate, especially for the actuaries. For the uncertainties of the future, results of this research showed appropriate guidelines of projections of mortality rate (Antonio et al., 2017).

2.3 Causes of deaths

Lee and Cater extrapolated a forecast of mortality rate only using historical data, which means that this model neglects important assumptions affecting the projection of the future mortality

rate. In other words, the overall trend is a sum of trends over different ages. For example, Caselli and researchers (2019) stressed the importance of including the causes of death to predict future mortality rates. As time has passed, medical techniques and people's awareness of health have been improving. Caselli and researchers introduced 5 causes of death such as Cardiovascular diseases, Bronchial and lung cancers, Digestive cancers, Other tumors, and other diseases and violent death. In the research of LC, $D_{x,t}$ is set to the number of people age x dying at time t by all causes. On the other hand, $D_{x,t,i}$ is set to the number of people age x dying at time t by those 5 causes ($i = 1, \cdots, 5$) with England & Wales Data. This research concluded that the projection based on all-cause mortality showed more realistic results. However, this projection was not always the best method of projections, which means the cause-specific mortality rate could also produce desired results on the objective of the research. When determining which method to use, note that there is no predetermined rule that certain application is better over the other. Both models can extract desirable results based on the situation at the time and the purpose of the research. The point is it is important to choose the model that is most representative with current situation (Bengtsson and Keilman, 2019; Lee and Carter, 1992).

2.4 Elimination of Causes of death

A question is raised on some causes of deaths, which now become no longer valid. Alai and researchers (2015) explained the results of the elimination of causes of deaths and its consequences. Define $q_k(x,t)$ to be the mortality rate of people age x at time t with the cause k. Alai and researchers used a multi-nominal logistic model, which defines $D_i(x.t)$ as the number of deaths from cause i for age x at time t and $L(x,t)$ as the number of people survived. Then, $Y(x,t)$ is defined by the total number of people age x at time t. Define $Y(x,t)$ to follow multi-nominal distribution. Alai found that there was differences in the outcome of eliminating causes of deaths to the future mortality rate. (Alai et al., 2015).

3 Data

This section shows the notation used for the projection and the information of the data set.

3.1 Notation

Define $\mu_{x,t}$ to be equal to the force of mortality rates of someone age x at time t. A piece-wise constant assumption is imposed to $\mu_{x,t}$ saying that

$$\mu_{x+s,t} = \mu_{x,t}, \qquad 0 \leq s \leq 1 \tag{7}$$

This means that within a certain year, the force of mortality is the same. Once the force of mortality rate is known, the mortality rate and the survival rate for h period can be obtained by

$$\begin{aligned}
{}_hq_{x,t} &= 1 - e^{-\int_0^h \mu_{t+s,x+s}ds}, \qquad h \geq 0 && (8)\\
&= 1 - e^{-h\times\mu_{x,t}} && (9)\\
{}_hp_{x,t} &= 1 -{}_h\, q_{x,t} = e^{-h\times\mu_{x,t}} && (10)\\
q_{x,t} &= 1 - e^{-\mu_{x,t}}, \qquad for \quad h = 1 && (11)\\
p_{x,t} &= 1 - q_{x,t} = 1 - e^{-\mu_{x,t}} && (12)
\end{aligned}$$

Define $D_{x,t}$ is the number of deaths of people age x at time t before $t+1$, and $E_{x,t}$ is the total number of people age x exposures to the risks of death at time t, which is the total number of people age x living at time t until $t+1$. Then, $m_{x,t}$ which is the central death rate is defined by

$$m_{x,t} = \frac{D_{x,t}}{E_{x,t}} \tag{13}$$

Assuming that there are j people, and the available data is $\theta_{x,j,t}$,$\gamma_{x,j,t}$ such that proportions of individual j to the whole population,

$${}_{\theta_{x,j,t}}p_{x,t} \times \mu_{x+\theta_{x,j,t},t+\theta_{x,j,t}} = e^{-\theta_{x,j,t}\mu^j_{x,t}} \qquad h \geq 0 \tag{14}$$

Under the assumption of independence, the total likelihood function is defined as

$$L(\mu_{x,t}) = \prod_{i=1}^{j} e^{-\theta_{x,i,t} \times (\mu_{x,t})^{\gamma_{x,i,t}}} \tag{15}$$

$$= exp[-E_{x,t}\ \mu_{x,t}] \times (\mu_{x,t})^{D_{x,t}} \tag{16}$$

The maximum likelihood estimation (MLE) for $\mu_{x,t}$ is calculated such that $L(\mu_{x,t})$ has the maximum value with respect to $\mu_{x,t}$.

$$\hat{\mu}_{x,t} = m_{x,t} \tag{17}$$

3.2 Data set

Data set is downloaded from the World Health Organization (WHO). This contains the total number of deaths categorized by The International Classification of Diseases (ICD). Specifically, the definition of cause of deaths is described by ICD10 (Organization, 2004). By ICD10, the codes for all cancer deaths are C00 $\cdots$ C97 and D00 $\cdots$ D48. This data is collected from the year 1996 until 2017. The data set of the Netherlands has a format of age specification. For age 0 to 4, it has its own group (Death2, Death3, Death4, Death5, and Death6). After that, Deaths7, for example, shows the number of deaths categorized by the age group for age 5 to 9. Lastly, Death25 contains total number of deaths for age 95 and above. However, this format was not applicable for the year between 1996 and 1999. Hence the data is reduced to the year 2000 afterwards.

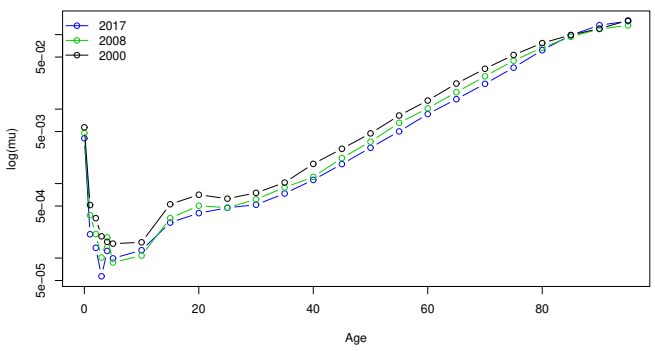

Figure 1: $\mu_{x,t}$ for the male

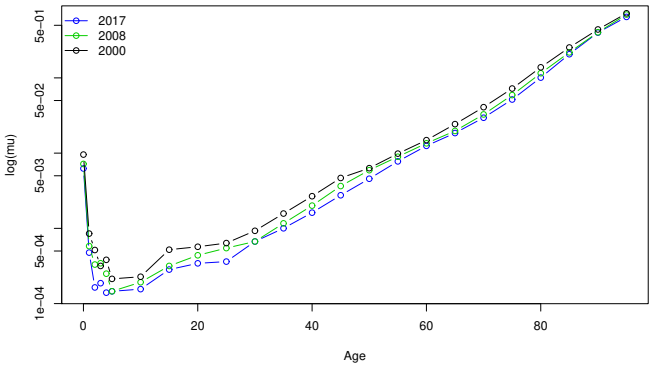

Figure 2: $\mu_{x,t}$ for the female

Figure (1), (2) show the force of mortality rate ($\mu_{x,t}$) by the gender. For these two figures, y axis shows the force of mortality rate in the log scale ($\log(\mu_{x,t})$) and x axis shows the age. Because of the infant deaths, $\mu_{x,t}$ is high in the beginning. After that, for all genders, $\mu_{x,t}$ is increasing over ages. In contrast, the force of mortality rate has decreased over time. In other words, $\mu_{x,t}$ depends on both the period and age.

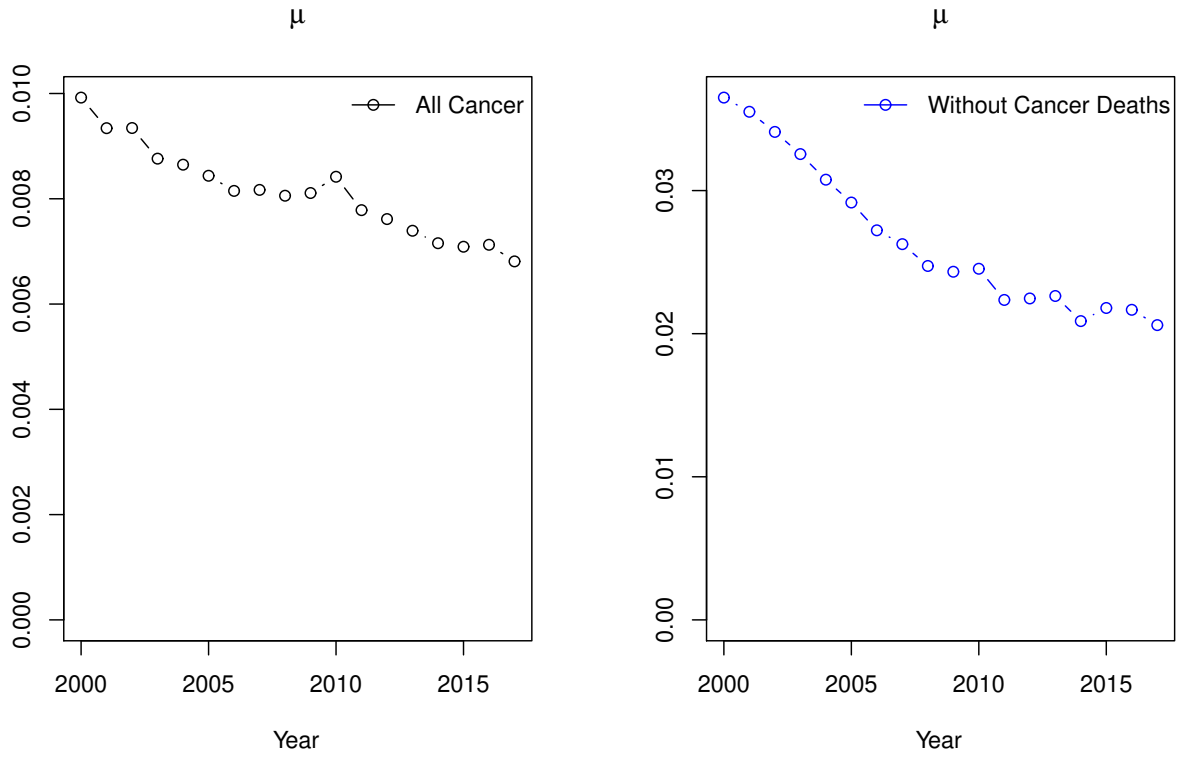

Figure 3: $\mu_{x,t}$ for age between 65 and 69 for all cancer deaths and without cancer deaths

To be more precise, figure (3) shows the force of mortality rates by the year. The black line shows the force of mortality rate of people age 65 considering all cancer deaths. The blue line shows the force of mortality rate of people age between 65 and 69 considering deaths without all cancer deaths. When it comes to the trends of $\mu_{x,t}$, it is clear to see that $\mu_{x,t}$ has decreased over time. Moreover, the force of mortality rate based on the all cancer deaths is more severe than the force of mortality based on deaths without all cancer deaths. This also means that the blue line goes faster to 0 than the black one.

4 Details of the model and methods

This section introduces the details of methods such as model structures, and the methods of estimation and projection.

4.1 Model Structure

The central death rate considering all the causes of deaths is defined by $m_{x,t}^{(a)}$, the central death rate of all cancer deaths by $m_{x,t}^{(c)}$, and the central death rate without all cancer deaths by $\tilde{m}_{x,t}^{(c)}$. The model is designed to estimate and forecast $m_{x,t}^{(a)}$, $m_{x,t}^{(c)}$, and $\tilde{m}_{x,t}^{(c)}$. In Section 3.1, the central death rate is the MLE for the force of mortality rate. Therefore the central death rate is not only dependent on the age also the period as explained in Section 3.2. Since the LC model handles both the period and age factor, this model is used to estimate the period effect and age effect to the mortality rate. As the LC model presented in their paper, the logarithm of the central death rate is following: (Lee and Carter, 1992):

$$\ln m_{x,t}^{(a)} = \alpha_x^{(a)} + \beta_x^{(a)}\kappa_t^{(a)} \tag{18}$$

$$\ln m_{x,t}^{(c)} = \alpha_x^{(c)} + \beta_x^{(c)}\kappa_t^{(c)} \tag{19}$$

$$\ln \tilde{m}_{x,t}^{(c)} = \tilde{\alpha}_x^{(c)} + \tilde{\beta}_x^{(c)}\tilde{\kappa}_t^{(c)} \tag{20}$$

In Section 2.1, $\kappa_t^{(a)}$, $\kappa_t^{(c)}$ and $\tilde{\kappa}_t^{(c)}$ are restricted to its sum to be equal to 0 for all t ($\sum_t K_t = 0$). $\beta_x^{(a)}, \beta_x^{(c)}$ and $\tilde{\beta}_x^{(c)}$ are restricted to its sum to be 1 for all age x such that $\sum_x \beta_x = 1$. Further details are explained in Section 4.2. Projecting the future force of mortality rate is done by fitting a time series model to the estimated period effects, $\kappa_t^{(a)}$, $\kappa_t^{(c)}$, and $\tilde{\kappa}_t^{(c)}$, and then forecasting using the estimated time series model. Details of the method is explained in Section 4.3. The future trends of the time dependent variable, K_t, $\kappa_t^{(c)}$, and $\tilde{\kappa}_t^{(c)}$ are represented by the random walk with drift time series model (RWD). This RWD is defined as following:

$$\kappa_{t+1}^{(a)} = \kappa_t^{(a)} + \theta^{(a)} + \delta_{t+1}^{(a)} \tag{21}$$

$$\kappa_{t+1}^{(c)} = \kappa_t^{(c)} + \theta^{(c)} + \delta_{t+1}^{(c)} \tag{22}$$

$$\tilde{\kappa}_{t+1}^{(c)} = \tilde{\kappa}_t^{(c)} + \tilde{\theta}^{(c)} + \tilde{\delta}_{t+1}^{(c)} \tag{23}$$

$\theta^{(a)}, \theta^{(c)}, \tilde{\theta}^{(c)}$ are called the drift, and $\delta^{(a)}_{t+1}, \delta^{(c)}_{t+1}, \tilde{\delta}^{(c)}_{t+1}$ are the error terms which have mean zero and constant variance. Dynamics of estimated time dependent variables ($\kappa^{(a)}_t$, $\kappa^{(c)}_t$, and $\tilde{\kappa}^{(c)}_t$.) are used to estimate future time $m^{(a)}_{x,t}$, $m^{(c)}_{x,t}$, $\tilde{m}^{(c)}_{x,t}$ by the equation (18), (19), (20).

4.2 Calibration

All parameters $(\alpha^{(a)}_x, \beta^{(a)}_x, \kappa^{(a)}_t, \alpha^{(c)}_x, \beta^{(c)}_x, \kappa^{(c)}_t, \tilde{\alpha}^{(c)}_x, \tilde{\beta}^{(c)}_x, \tilde{\kappa}^{(c)}_t)$ are calibrated by using Singular Value Decomposition (SVD). From SVD, a single matrix A $(n \times m)$ is redefined by $U \sum V^T$. U and V are eigen-vectors of $A^T A$. $\sum$ is a diagonal matrix which contains singular values of A $(diag(\sigma_1, \sigma_2 \dots))$. $E_{x,t}$, $D_{x,t}$ and $m^{(a)}_{x,t}$, $m^{(c)}_{x,t}$, $\tilde{m}^{(c)}_{x,t}$ are obtained from the aggregated WHO data over the time and age. Specifically, for all cancer deaths, the number of deaths for certain age group such as Deaths4 is 0. To avoid this, following methods are applying to the age of group over 35. And κ_t for future time t is projected after all the estimations.

4.2.1 Estimation

Rewriting the equation (18), (19), (20) in a general way is following:

$$\ln m_{x,t} = \alpha_x + \beta_x \kappa_t + \epsilon_{x,t} \tag{24}$$

$$\sum_{t=1}^{T} \ln m_{x,t} = T\alpha_x + \beta_x \sum_{t=1}^{T} \kappa_t \tag{25}$$

$$\sum_{t=1}^{T} \ln m_{x,t} = T\alpha_x \tag{26}$$

$$\alpha_x = \frac{1}{T} \sum_{t=1}^{T} \ln m_{x,t} \tag{27}$$

Estimation of α_x is obtained by the realizations of $\frac{1}{T}\sum_{t=1}^{T} \ln m_{x,t}$. Then, define a matrix $A_{x,t}$ to be equal to $A_{x,t} = \ln m_{x,t} - \alpha_x = \beta_x \kappa_t$.

$$SVD(A_{x,t}) = U \sum V^T \tag{28}$$

$$= \sigma_1 U_{x,1} V_{t,1} + \cdots + \sigma_r U_{x,r} V_{t,r} \tag{29}$$

$$= \sum_{r=1}^{rank(A_{x,t})} \sigma_r U_{x,r} V_{t,r} \tag{30}$$

By the LC specification, $rank(A_{x,t})$ to be equal to 1.

$$\hat{\alpha}_x = \hat{\beta}_x \hat{\kappa}_t = \sigma_1 U_{x,1} V_{t,1} \tag{31}$$

$$\hat{\beta}_x = (u_{1,1}, \ldots, u_{x,1})^T \tag{32}$$

$$\hat{\kappa}_t = \sigma_1 \times (v_{1,1}, \ldots, v_{t,1})^T \tag{33}$$

The sum of $\hat{\beta}_x$ for all age x should be 1. And, The sum of $\hat{\kappa}_t$ should be 0 for all time t. Therefore, rewriting the equation (32) and (33) is as following :

$$\hat{\beta}_x = \frac{1}{\sum_x u_{x,1}} (u_{1,1}, \ldots, u_{x,1})^T \tag{34}$$

$$\hat{\kappa}_t = \sum_x u_{x,1} \times \sigma_1 \times (v_{1,1}, \ldots, v_{t,1})^T \tag{35}$$

4.2.2 Forecasting

As represented in the equation (21), (22), and (23), θ is estimated using RWD model. Given the error terms have mean 0 and constant variances, the first step is to take the difference of time series data. Then, the mean of the difference data is the estimated value of θ. As a result, $\hat{\theta}^{(a)}$, $\hat{\theta}^{(c)}$, and $\hat{\tilde{\theta}}^{(c)}$ are following:

$$\hat{\theta}^{(a)} = \sum_{t=2}^{T} \frac{\kappa_t^{(a)} - \kappa_{t-1}^{(a)}}{T-1} \tag{36}$$

$$\hat{\theta}^{(c)} = \sum_{t=2}^{T} \frac{\kappa_t^{(c)} - \kappa_{t-1}^{(c)}}{T-1} \tag{37}$$

$$\hat{\tilde{\theta}}^{(c)} = \sum_{t=2}^{T} \frac{\tilde{\kappa}_t^{(c)} - \tilde{\kappa}_{t-1}^{(c)}}{T-1} \tag{38}$$

These fitted values are used to calculate future time mortality rates. Given that t_{max} is the latest time of the data, $\kappa_{t_{max}+1}^{(a)}$, $\tilde{\kappa}_{t_{max}+1}^{(c)}$, and $\tilde{\kappa}_{t_{max}+1}^{(c)}$ are following:

$$\kappa_{t_{max}+1}^{(a)} = \kappa_{t_{max}}^{(a)} + \hat{\theta}^{(a)} + \delta_{t_{max}+1}^{(a)} \tag{39}$$

$$\kappa_{t_{max}+1}^{(c)} = \kappa_{t_{max}}^{(c)} + \hat{\theta}^{(c)} + \delta_{t_{max}+1}^{(c)} \tag{40}$$

$$\tilde{\kappa}_{t_{max}+1}^{(c)} = \tilde{\kappa}_{t_{max}}^{(c)} + \hat{\tilde{\theta}}^{(c)} + \tilde{\delta}_{t_{max}+1}^{(c)} \tag{41}$$

$\kappa^{(a)}_{t_{max}+1}$, $\kappa^{(c)}_{t_{max}+1}$, and $\tilde{\kappa}^{(c)}_{t_{max}+1}$ are defined beyond t_{max}. With these predicted values, the future mortality rates are obtained for future time t as following:

$$m^{(a)}_{x,t} = e^{\hat{\alpha}^{(a)}_x + \hat{\beta}^{(a)}_x \hat{\kappa}^{(a)}_t} = \mu^{(a)}_{x,t} \tag{42}$$

$$m^{(c)}_{x,t} = e^{\hat{\alpha}^{(c)}_x + \hat{\beta}^{(c)}_x \hat{\kappa}^{(c)}_t} = \mu^{(c)}_{x,t} \tag{43}$$

$$\tilde{m}^{(c)}_{x,t} = e^{\hat{\tilde{\alpha}}^{(c)}_x + \hat{\tilde{\beta}}^{(c)}_t \hat{\tilde{\kappa}}^{(c)}_x} = \tilde{\mu}^{(c)}_{x,t} \tag{44}$$

In short, fit the period effect variable in time $t \in \{2000, \cdots, 2017\}$. Then, forecast the period effect variable for the future time $t \in \{2018, \cdots, 2061\}$.

4.2.3 Comparison

The mortality rates trends are obtained for all causes of deaths, all cancer deaths, and all deaths without cancer deaths for historical time $t \in \{2000, \cdots, 2017\}$, and future time $t \in \{2018, \cdots, 2060\}$. If there is no effect of the one cause of deaths at all, this equation should hold.

$$\mu^{(a)}_{x,t} = \mu^{(c)}_{x,t} + \tilde{\mu}^{(c)}_{x,t} \tag{45}$$

However, in case of existence of an effect of the one cause of deaths, a proper comparison tool is needed. One of the most common ways is to compare the life expectancy. By means of the life expectancy, the consequence of the one cause of deaths is able to be seen clearly. Define $\tilde{\mu^{(a)}_{x,t}}$ by

$$\tilde{\mu}^{(a)}_{x,t} = \mu^{(c)}_{x,t} + \tilde{\mu}^{(c)}_{x,t} \tag{46}$$

Then, the calculation of the mortality rate based on $\mu^{(a)}_{x,t}$ and $\tilde{\mu}^{(a)}_{x,t}$ is following:

$$q^{(a)}_{x,t} = 1 - e^{-\mu^{(a)}_{x,t}} \tag{47}$$

$$\tilde{q}^{(a)}_{x,t} = 1 - e^{-\tilde{\mu}^{(a)}_{x,t}} \tag{48}$$

The extensions to the life expectancy for someone age x is obtained by

$$e_x^{(a)} = \sum_t {}_tp_x^{(a)} \tag{49}$$

$$\tilde{e}_x^{(a)} = \sum_t {}_t\tilde{p}_x^{(a)} \tag{50}$$

where ${}_tp_x^{(a)}$ and ${}_t\tilde{p}_x^{(a)}$ is following:

$${}_tp_x^{(a)} = \prod_{i=0}^{t} p_{x+i}^{(a)} = \prod_{i=0}^{t} 1 - q_{x+i}^{(a)} \tag{51}$$

$${}_t\tilde{p}_x^{(a)} = \prod_{i=0}^{t} \tilde{p}_{x+i}^{(a)} = \prod_{i=0}^{t} 1 - \tilde{q}_{x+i}^{(a)} \tag{52}$$

The estimation is implemented over age 35 as mentioned in Section 4.2.1. The future force of mortality rates of people under age 35 are assumed to be equal to the one of the last year (2017).

5 Results

This section represents the output of the estimation and the projections. Based on these values, the projected life expectancy is shown.

5.1 Estimation

Figure (4) indicates the results of the estimation, using the method in Section 4.2.1. Blue lines are representing estimated values of the male. Red lines are calibrated values of the female. First column graphs are representing each $\alpha_x^{(a)}$, $\alpha_x^{(c)}$, and $\tilde{\alpha}_x^{(c)}$. Second column graphs are showing $\beta_x^{(a)}$, $\beta_x^{(c)}$, and $\tilde{\beta}_x^{(c)}$. And, the last column graphs are $\kappa_t^{(a)}$, $\kappa_t^{(c)}$, and $\tilde{\kappa}_t^{(c)}$. For estimated α, it shows generally increasing trend over ages for both the male and female. $\alpha_x^{(a)}$ which is the average mortality rate of people age x, is higher for the male than the female. However, $\tilde{\alpha}_x^{(c)}$ is the other way around. $\alpha_x^{(c)}$ tells that the average mortality rate caused by cancers are higher for the old male. $\beta_x^{(a)}$, and $\tilde{\beta}_x^{(c)}$ have almost the same trends. $\beta_x^{(c)}$ has shown the improvement in cancer deaths for the men around age 40 and 50. Furthermore, $\beta_x^{(c)}$ has shown 0 values for the male

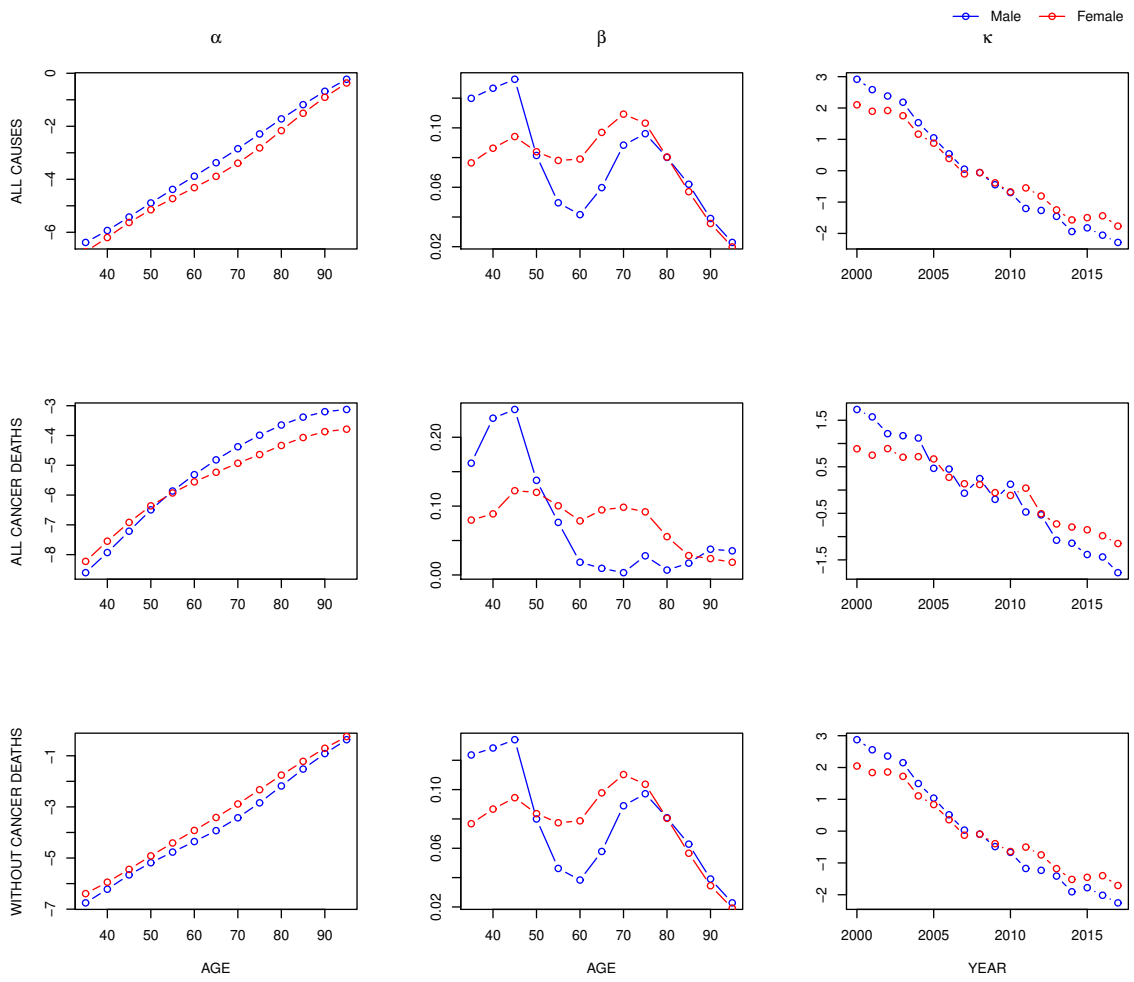

Figure 4: Estimation of parameters by causes of deaths

around age 70. Finally, $\kappa_t^{(a)}$, $\kappa_t^{(c)}$, and $\tilde{\kappa}_t^{(c)}$ show a decreasing trend for both the male and the female. Moreover, the female are affected by cancers more than the female as the time pasts.

5.2 Projection

Figure (5) shows the overall trends of the time dependent variable $\kappa_t^{(a)}$, $\kappa_t^{(c)}$, and $\tilde{\kappa}_t^{(c)}$. Both graphs are consists of two parts, which are a line part and a dot part. The line parts show the estimated $\kappa_t^{(a)}$, $\kappa_t^{(c)}$, and $\tilde{\kappa}_t^{(c)}$. The dot parts indicate the projected values of $\kappa_t^{(a)}$, $\kappa_t^{(c)}$, and $\tilde{\kappa}_t^{(c)}$. As the projection method indicates in Section 4.2.2, the dotted graphs are linearly decreasing. The decreasing speed of the force of mortality rate is higher for the male than the female. For both the male and the female, the decreasing speed of $\tilde{\kappa}_t^{(c)}$ is the largest and $\kappa_t^{(a)}$, $\kappa_t^{(c)}$ are following. It is obvious that $\kappa_t^{(c)}$ shows the totally different trend from the others, which means that all cancer deaths are more critical to the people than the other causes of deaths.

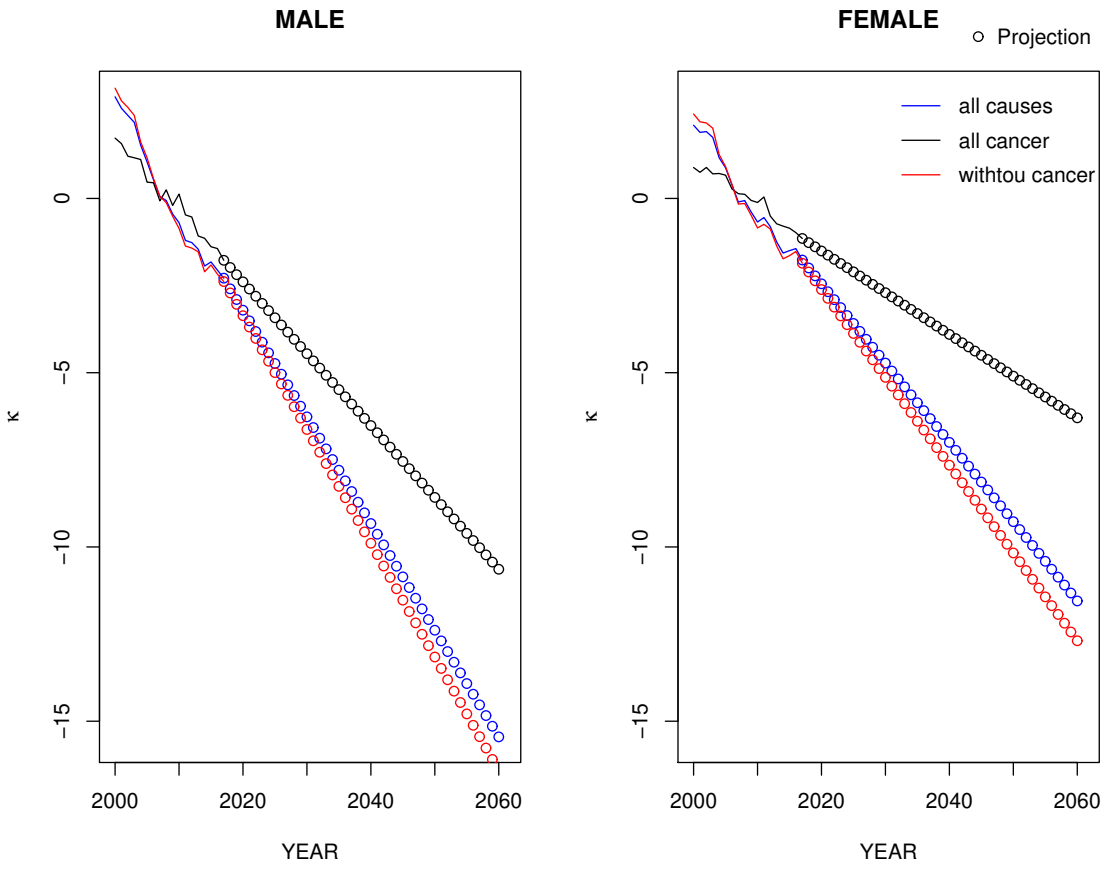

Figure 5: Projections of κ by causes of deaths

5.3 Life Expectancy

Figure (6) represents the life expectancy for $t \in \{2000, \ldots, 2060\}$, where the left graph is for the male and the right graph is for the female. Blue dotted-lines show the projection of the life expectancy based on the cause-of-death, black dotted-lines show the projected life expectancy based on the all-cause mortality, and the green dotted-lines represent the future life expectancy based on the all deaths without cancer deaths. The blue and black lines show almost the same increasing trend for the male. However, the future life expectancy of the female based on the cause-of-death is slightly less than the one based on the all-cause mortality. When it comes to the green lines, the elimination of all cancer deaths increases the life expectancy about 2 years for the male and the female. Furthermore, figure (6) shows the female will be affected by cancer deaths more than the male in the future.

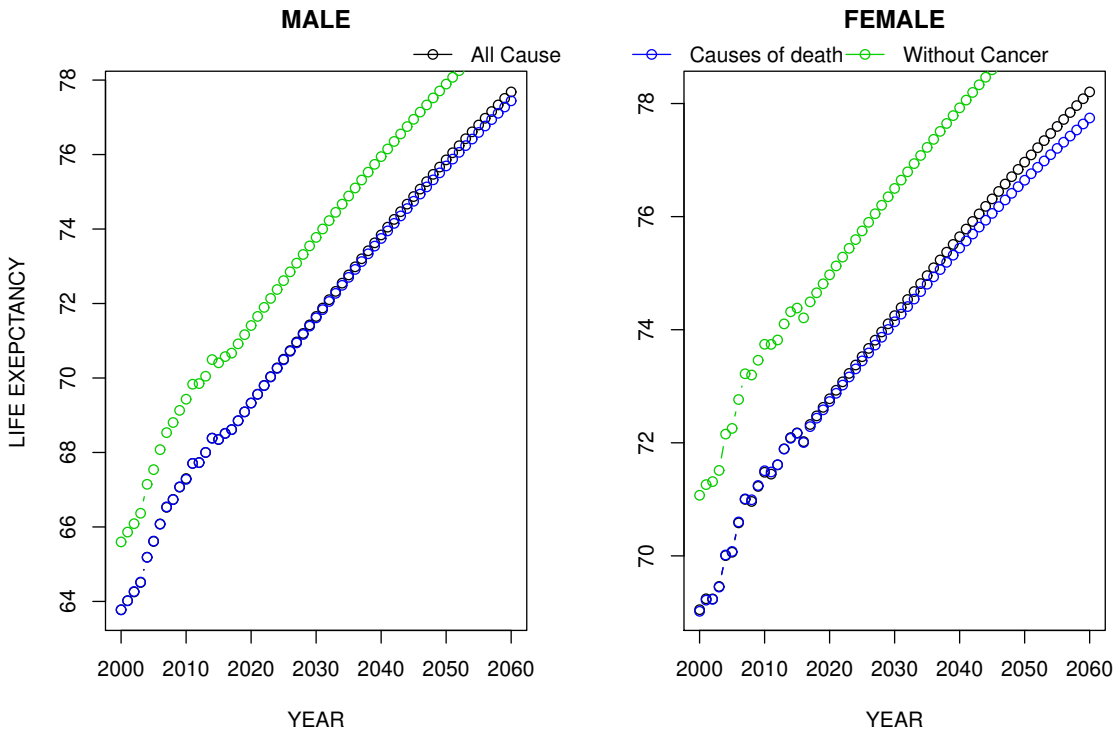

Figure 6: Projections of Life expectancy

6 Conclusion

This paper investigated the consequence of applying different models by eliminating one cause of deaths. As explained in Section 5.3, the all-cause mortality rate was almost the same as the mortality rate based on the causes-of-deaths (COD). Between year 2000 and 2017, they are perfectly equal as assumed in Section 4.2.3. For the future years, the constant force of mortality rate within the age group makes more possibilities to be the same. In 2050 afterwards, the total mortality rate based on COD is a little less than the all-cause mortality rate. This difference might be because for the mortality rate based on the cause-of-deaths was calculated by adding two projected $\mu_{x,t}^{(c)}$, $\tilde{\mu}_{x,t}^{(c)}$. Each $\mu_{x,t}^{(c)}$ and $\tilde{\mu}_{x,t}^{(c)}$ was estimated separately, which would affect the projections, or a different trend, which may be a better indication of future developments would be estimated. Moreover, the life expectancy based on the total death number without all cancer deaths is higher than the other. When the total number of deaths caused by cancer deaths were deducted, it was for sure that the projected life expectancy were higher. Therefore, eliminating one COD has an enormous effect on the projection of the mortality rate. As mentioned in Section 1, the total number of deaths by all cancers was the second largest number among the all COD.

If one COD was chosen to be the least number of deaths, there would be totally different outcomes. In this sense, the projection of mortality rate is strongly affected by eliminating causes of deaths. Hence, when the projection of mortality rate is based on COD, the selection of the COD might be one of most important factors to be consider.

The force of mortality rate is assumed to be the same within one age group. As mentioned in Section 2.1, the force of mortality rate is the age-dependent variable. Therefore, this assumption could be replaced by other assumptions such as linear interpolation between years. In Section 4.2.3, the future force of mortality rate is assumed to be the same as the one of the latest year (2017) in Section 4.2.3. Since the force of mortality rate depends on the time as well, the future mortality rate of under age 35 could be extrapolated.

Furthermore, future study could apply LC model using Poisson distribution. This paper only calibrated over age 35. However, future study can project the future mortality rate for all ages. Also, it would be a better approach if a more realistic assumption is made.

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