CSci 5304, F'14	Homework # 2	Due Date: 10/06/2014

1. Consider the two mathmatically equivalent formulas

   (i) $a = x^2 - y^2$

   (ii) $a = (x + y) * (x - y)$.

   (a) Compute both formulas in 2 digit decimal arithmetic using $x = 11$, $y = 10$. Which formula gives the best answer? Ignore any limits on the exponent.

   (b) Compute both formulas in 3-bit binary arithmetic using $x = 3/2$ (dec) and $y = 1$. Again, which formula gives the most accurate answer? Ignore any limits on the exponent.

   (c) In your favorite programming language and platform, find two numbers x, y such that the two formulas above give different answers. Which answer is more accurate. Report the precision used, the machine epsilon (unit round-off) and the general description of your machine. Can you find two numbers for which one of the answers has no accuracy whatsoever, but the other is almost OK? Note: in Matlab all arithmetic is in double precision, but you can force single precision by using the `single` function: `a = single(5/4)` forces all arithmeitc involving `a` to be on single precision.

2. Consider the system

$$Ax \equiv \begin{bmatrix} -0.001 & 1.001 \\ 0.001 & -0.001 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \equiv b$$

   whose solution is $x_1 = x_2 = 1$ and the system

$$(A + \Delta A)y = b + \Delta b$$

   where $\Delta A = \varepsilon|A|$, $\Delta b = \varepsilon|b|$. Here $|A|$ means take the absolute value of all the elements individually. In the following we let $\varepsilon = 10^{-4}$.

   (a) Compute $\kappa_\infty(A)$. Compute the actual value of $\|x - y\|_\infty / \|x\|_\infty$ and its estimate obtained from using the (standard) condition number κ_∞ (Theorem 2 in notes).

   Which is quite far from the actual error.

   (b) Now repeat the above, but this time use $\Delta b = \varepsilon \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

3. (a) Show that the following matrix is singular

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 1 & 1 \\ -1 & 1 & -2 \end{pmatrix}$$

   (b) What is the range or column space of A ? What is its null space? Give a basis for each subspace.

   (c) Consider the matrix B obtained from A by adding $\eta = 0.001$ to the entry (1,3) (So $B = A + \eta e_1 e_3^T$). Without computing the inverse of B, show that $\|B^{-1}\|_1 \geq 3,000$.

   (d) Find a lower bound for the condition number $\kappa_1(B)$.

Continued overleaf . . .

4. Consider the $n \times n$ matrix

$$A_n = \begin{pmatrix} 1 & & & & & \\ -2 & 1 & & & & \\ & -2 & 1 & & & \\ & & \ddots & \ddots & & \\ & & & \ddots & \ddots & \\ & & & & -2 & 1 \end{pmatrix}$$

What is A_n^{-1}? [Hint: Write $A_n = I - E_n$ and use expansion $I + E + E^2 + \cdots$.]

Calculate the condition numbers $\kappa_1(A_n)$ and $\kappa_\infty(A_n)$. Verify your results with matlab for the case $n = 10$.