CSci 5304	HW6	Due: Dec 10 2014

- Homework is due in class – no late homeworks accepted because it may be discussed in class.
- You can use Matlab for the numerical calculations here (especially in problem 1), but you do not need to turn in your Matlab code. Please include your intermediate results.

1. Let
$$A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ s & 0 & 2 \end{pmatrix}, \text{ with } s = -10^{-6}.$$
   (a) Compute all the eigenvalues λ_i and corresponding right eigenvectors $\mathbf{x}_i$ and left eigenvectors $\mathbf{y}_i$.
   (b) Compute the "overall condition number" for this eigenproblem based on the perturbation formula
$$|\lambda_{A+E} - \lambda_A| \le \|X\| \cdot \|X^{-1}\| \cdot \|E\|,$$
   where X is the matrix of right eigenvectors and E is some generic perturbation matrix.
   (c) Compute the condition numbers for every individual eigenvalue based on the perturbation approximation (ignoring higher order terms)
$$|\lambda_{A+E} - \lambda_A| \approx \frac{O(\|E\|)}{\cos\theta} = \frac{\|\mathbf{y}_i\|_2 \|\mathbf{x}_i\|_2}{\mathbf{y}_i^T \mathbf{x}_i} O(\|E\|),$$
   where $\mathbf{y}_i, \mathbf{x}_i$ are the left and right eigenvectors corresponding to the eigenvalue λ_A for the matrix A. Do this for each eigenvalue of A.
   (d) Compute the eigenvalues of the matrix $\widetilde{A}$ defined as the same as the matrix A above but with $s = 0$. How do the eigenvalues of $\widetilde{A}$ compare with what would be expected based on the condition number bounds?

2. Compute all the eigenvalues and eigenvectors of the complex symmetric matrix
$$A = \begin{pmatrix} 2i & 1 \\ 1 & 0 \end{pmatrix}$$

3. Consider the block upper triangular matrix
$$A = \begin{pmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{pmatrix}.$$
   (a) Suppose $A_{11}\mathbf{u} = \lambda\mathbf{u}$, but λ is not an eigenvalue of A_{22}. Find a vector $\mathbf{v}$ (in terms of $A_{ij}, \mathbf{u}$) such that the vector $\begin{pmatrix} \mathbf{u} \\ \mathbf{v} \end{pmatrix}$ is an eigenvector of A. What is the corresponding eigenvalue?
   (b) Suppose $A_{22}\mathbf{v} = \lambda\mathbf{v}$, but λ is not an eigenvalue of A_{11}. Find a vector $\mathbf{u}$ (in terms of $A_{ij}, \mathbf{v}$) such that the vector $\begin{pmatrix} \mathbf{u} \\ \mathbf{v} \end{pmatrix}$ is an eigenvector of A. What is the corresponding eigenvalue?
   (c) Repeat the above assuming λ is an eigenvalue for both A_{11} and A_{22}. Do any of the above cases fail?