CSci 5304, F'14	Homework # 1 $\boxed{\text{v2}}$	Due Date: 09-22-14

1. Given two vectors $u, v, \in \mathbb{R}^n$, and real scalars α, β, let $A = I + \alpha uv^T$, $B = I + \beta uv^T$.

   (a) If u, v, α are given, find β such that $B = A^{-1}$.

   (b) For which values of α is A singular, if any? For that particular value of α, give a non-zero vector x in the right nullspace of A. Write x in terms of u, v, α.

   (c) Prove of disprove: for any given pair of vector u, v, there always exists a value α such that A singular. To prove, show such an α always exists, giving a formula in terms of u, v. To disprove, give an example of a pair of non-zero vectors u, v for which no such α exists. In the latter case, what general property do u, v satisfy to prevent the existence of α? You can illustrate your answer with a 2×2 example.

   (d) Give a value of α (in terms of u, v) such that $A^2 = A$ (i.e., A is a projector).

   [Hint: Multiply out $(I + \alpha uv^T)(I + \beta uv^T)$ and find value for β to reduce the product to the desired result.]

2. Let $f_p(v) = \max_{\|u\|_p=1} |u^T v|$, where $\|v\|_p$ denotes the p-norm.

   (a) Prove or disprove: f_p is a vector norm. (check each property, or show one is violated).

   (b) Give a formula for f_p for $p = 1, 2$. Hint, the answers can be written in terms of $\|\cdot\|_2, \|\cdot\|_\infty$. For $p = 2$, use the Cauchy-Schwartz inequality.

3. Define the inner product among square matrices by $\langle A, B\rangle = \mathsf{trace}(A^T B)$, where A, B are $n \times n$ matrices.

   (a) What is the norm induced by this inner product: $\|A\|^2 = \langle A, A\rangle$? Answer this question for the general case for any A.

   - Now answer the remaining questions below using this specific matrix:

$$A = \begin{pmatrix} 6 & -2 & 1 \\ 7 & -7 & 3 \\ -4 & 5 & -2 \end{pmatrix}.$$

   (b) For this specific matrix A, what is the value of $\langle A, A\rangle$ and the corresponding induced norm $\|A\| = \sqrt{\langle A, A\rangle}$ from part (a)?

   (c) What is the p norm of A, for $p = 1$? Find a vector x s.t. $\boxed{\|x\|_p = 1 \text{ and}}$ $\|A\|_p = \|Ax\|_p$.

   (d) Repeat the above for $p = 2$. Use Matlab and write the result to 4 decimal places. Show your Matlab commands.

   (e) Use Matlab to help solve this problem: Find a vector x achieving the minimum in $\min_{\|x\|_p=1} \|Ax\|_p$. Do this for $p = 1, 2$.

Note: in this exercise, you can use Octave instead of Matlab.