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(a)

The instantaneous forward rate of the 1st year = 0.028438000000, approximately 0.02844

(b)

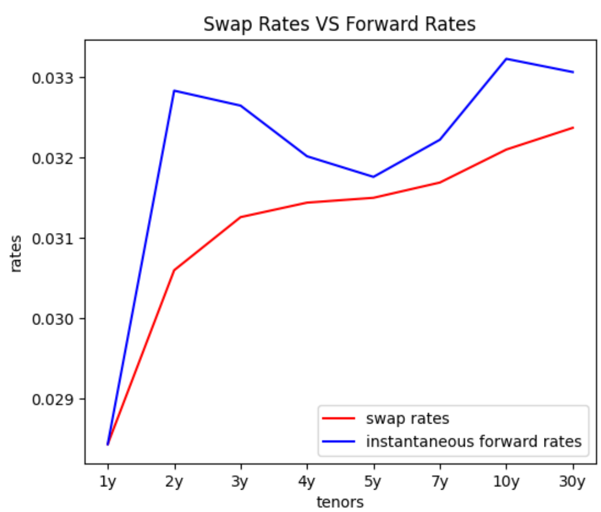
The instantaneous forward rate of the 2nd year = 0.032831688722, approximately 0.03283

(c)

The piecewise constant instantaneous forward rates:

0.02843800000009651
0.032831688722217214
0.03264584859443859
0.032016202894010345
0.03176021862818744
0.03222198917746936
0.033227493680078636
0.03306303548497454

The instantaneous forward rates V.S. swap rates:



My comments on the forward rates vs the swap rates:

For one swap contract, there is one fixed swap rate, the fixed leg equals the value of the floating leg which is the discounted value of a series of forward rates. The swap rates can thus be regarded as a weighted average of the forward rates. This naturally explains why the swap rate is always beneath the forward rates, but when the forward rates decline,
the swap rates still go up on a smaller slope.

(d)

The 15y swap rate = 0.03236999999999889

(e)

Discount factors: (from 0.5y till 30y with 0.5y between every pair of consecutive tenors)

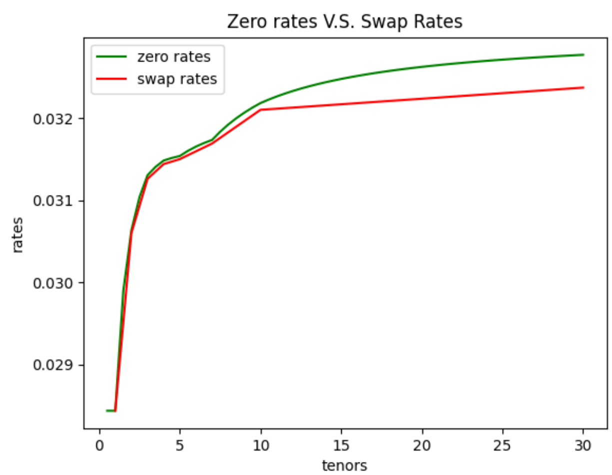
[0.9858816125460068, 0.9719625539563146, 0.9561372164669971, 0.9405695445694572, 0.9253413217742006, 0.9103596504124248, 0.8959025449104558, 0.8816750276809903, 0.8677845151956868, 0.8541128433614698, 0.840624915961653, 0.827030298479059, 0.8138127774189919, 0.8008064975471795, 0.7876120095606803, 0.7746349205510364, 0.7618716485440792, 0.749318670584036, 0.7369725217611137, 0.7248297942551045, 0.7129457588708349, 0.7012565696395984, 0.6897590319375051, 0.6784500035185105, 0.6673263936556489, 0.6563851622963462, 0.6456233192315811, 0.6350379232786695, 0.6246260814774468, 0.6143849482996304, 0.6043117248711439, 0.5944036582071925, 0.5846580404598797, 0.5750722081781596, 0.5656435415799236, 0.5563694638360202, 0.5472474403660144, 0.538274978145495, 0.5294496250247364, 0.5207689690585333, 0.5122306378470225, 0.5038322978873123, 0.4955716539357429, 0.48744644838060164, 0.47945446062512465, 0.4715935064806133, 0.4638614375695015, 0.4562561407382098, 0.4487755374796261, 0.4414175833650557, 0.434180267485484, 0.42706161190200065, 0.4200596711052341, 0.4131725314836496, 0.40639831080056343, 0.3997351576797336, 0.39318125109938307, 0.3867347998945199, 0.38039404226741635, 0.3741572453061144]

Zero rates: (from 0.5y till 30y with 0.5y between every pair of consecutive tenors)

0.02843800000009651, 0.02843800000009651, 0.02990256290747008, 0.030634844361156863, 0.031037045207813207, 0.031305179105584106, 0.03140675393250214, 0.03148293505269067, 0.03151374433885698, 0.031538391767790025, 0.031600536986851786, 0.03165232466940325, 0.03169614501617757, 0.0317337053134127, 0.03183329120452376, 0.03192042885924594, 0.031997315025177275, 0.032065658283782904, 0.03212680751516689, 0.03218184182341247, 0.032223803426344, 0.032261950338099935, 0.032296780127094485, 0.032328707433672825, 0.032358080555724894, 0.032385194206849884, 0.03241029943937302, 0.032433611441001646, 0.03245531571838002, 0.03247557304393318, 0.03249452344525709, 0.03251228944649826, 0.03252897872039148, 0.032544686272290985,

0.03255949624979623, 0.032573483450773405, 0.0325867145868329,
0.03259924934731031, 0.03261114129955811, 0.03262243865419352,
0.03263318491835892, 0.03264341945565929, 0.03265317796796895,
0.03266249291153726, 0.03267139385761364, 0.03267990780603453,
0.03268805945877794, 0.0326958714593237, 0.03270336460270433,
0.032710558020349736, 0.032717469343185514, 0.03272411484591223,
0.03273050957495114, 0.032736667462173796, 0.03274260142622472,
0.03274832346298811, 0.032753844726531735, 0.0327591756016773,
0.03276432576919081, 0.032769304264453875]

Zero rates V.S. swap rates:



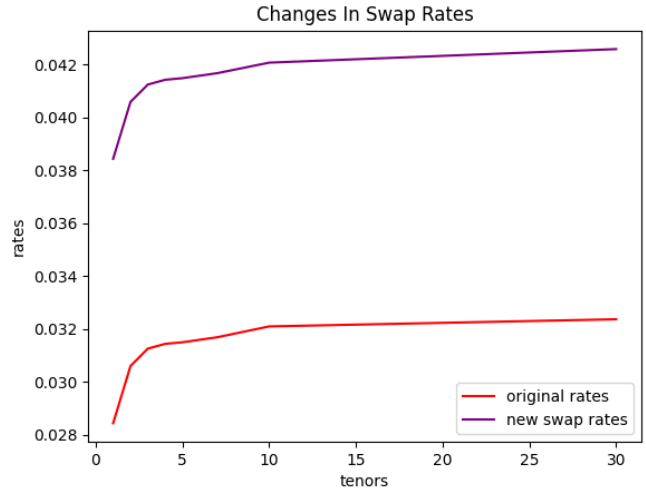
My comments on the differences between zero rates and swap rates:
The zero rates keep getting bigger because time has a positive value, and zero rates are forward rates with their expiry time = inception and maturity = T, naturally the rate gets bigger when the T gets bigger. The swap rates, as pointed out before, is a weighted average of forward rates, which are multipliers that connect one zero rate to another consecutive zero rate. We can see that as swap rates remain almost unchanged since the 3rd year, the zero rates go up at a seemingly ’constant speed’.

(f)

New swap rates: (of 1y, 2y, 3y, 4y, 5y, 7y, 10y, 30y)

0.03843800000009651, 0.040589022518679754,
0.04124579925673613, 0.04142636532208151,
0.04148763588278599, 0.04167585630840207,
0.04207427066979912, 0.04258647619645323

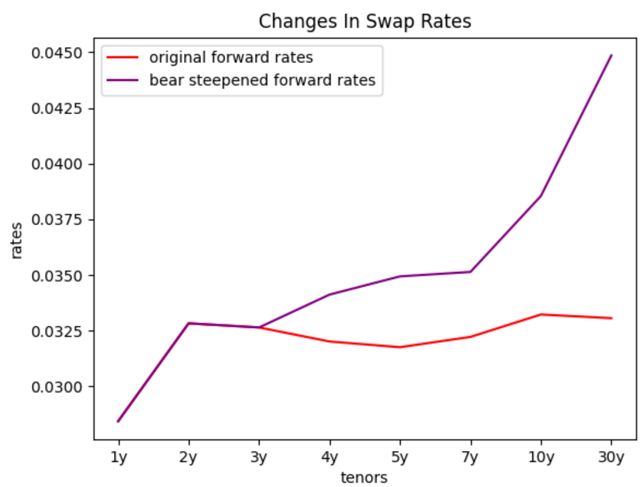
Plotting new swap rates against original swap rates:



(g)

New swap rates: (of 1y, 2y, 3y, 4y, 5y, 7y, 10y, 30y)
[0.028438, 0.0306, 0.03126, 0.03194, 0.0325, 0.03319, 0.0346, 0.03737]

(h)

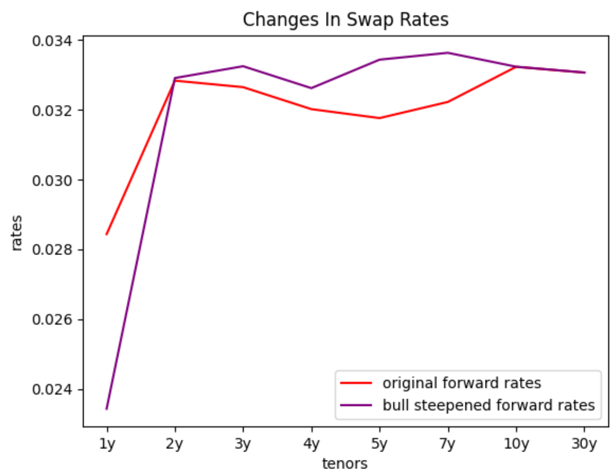


The bootstrapping results show that the steepening of the swap rates must correspond to a steepening of the forward rates because the swap rates are the weighted average of forward rates that correspond to non-overlapping time intervals. As we can see, the upward shift in the swap rates in long-term maturities corresponds to even more steep upward changes in long-term forward rates. Yet overall the swap rates stay beneath the forward rates because that is exactly a result of swap rates being an ’average’.

(i)

New swap rates: (of 1y, 2y, 3y, 4y, 5y, 7y, 10y, 30y)
[0.023438, 0.0281, 0.02976, 0.03044, 0.031, 0.03169, 0.0321, 0.03237]

(j)
New swap rates V.S. original swap rates:



Analogous to the bearish scenario, the downward shift in the swap rates in short-term maturities corresponds to even more steep downward changes in short-term forward rates. Yet overall the swap rates stay beneath the forward rates because that is also a result of swap rates being an 'average'.