Systematic review of SysID - model representations

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%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Jingxiao Liu
% Inferlab, CEE, CMU
% June, 2019
% This is my first Matlab live script for introducing system
% identification;
% Some contents are based on Arun K. Tangirala's SysID lecture in IIT
% and the textbook "System identification - an introduction" by Keesman
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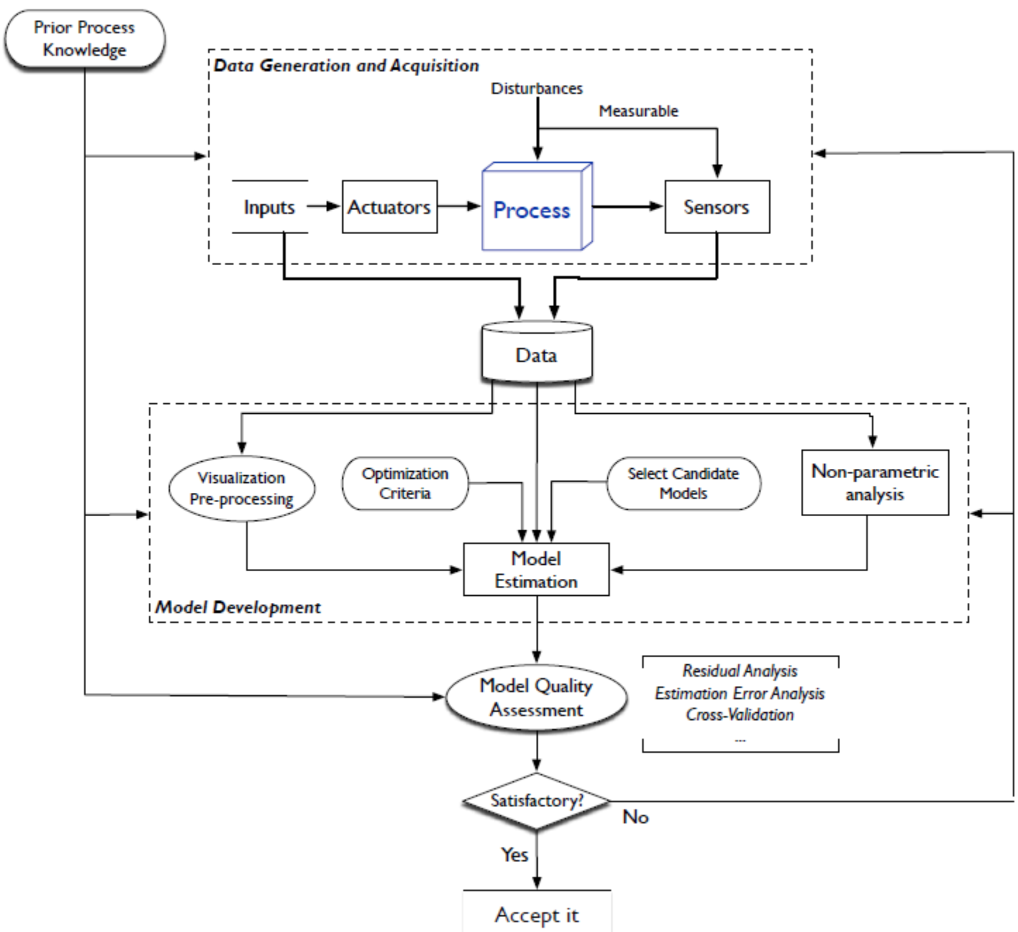
Workflow:



First-principles or Empirical

Empirical model: Non-parametric or Parametric; Black-box or Grey-box

LInear or Non-linear

Time-invariant or Time-varying

Single-scale or Multi-scale

Continuous or Discrete

Static or Dynamic

Lumped or Distributed

Deterministic or Stochastic

Deterministic model: the model that explains the effect of physical / measured input(s) on the output(s).

Stochastic model: the model which accounts for the unmeasured disturbances and sensor noise

The overall model developed through identification is a composite model. The deterministic model is driven by a physical input while the stochastic portion of the model is driven by a shock wave (fictitious and unpredictable)

A good identification exercise separates the deterministic and stochastic parts with reasonable accuracy.

Identifiability

Identifiability is guaranteed if the data set is sufficiently informatve. It provides the ability to estimate the parameters of a selected model using a given dataset, and to discriminate between two models of different structures.

Signal-to-Noise Ratio (SNR)

The ratio of variance of signal to the variance of noise in a measurement:

$$\mathrm{SNR} = \frac{{\sigma_{\mathrm{signal}}}^2}{{\sigma_{\mathrm{noise}}}^2}$$

Or a measure of the degree of certainty to uncertainty.

Linear time-invariant system

skip...but one interesting defination is:

A system is linear and time-invariant if and only if a sine wave of frequency ω input produces a resposne (at steady-state) of the same frequency ω (does not produce a new frequency reponse).

Since FRF is a function of the input frequency, every LTI system acts as a filter.

Part 0. Different representations of a dynamic system -- 2-DoF mass-spring-damper system

Define stiffness and mass matrices

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k1 c1 m1 k2 c2 m2

$M\ddot{X}+C\dot{X}+KX=F$

```
clc;
clear;
close all;
addpath ../functions/
m1=5; %mass 1 [kg]
m2=2; %mass 2 [kg]
k1=200; %spring 1 [N/m]
k2=100; %spring 2 [N/m]
c1=10; % damping coeff 1
c2=10; % damping coeff 2
M=[m1 0;0 m2]; %mass matrix
K=[k1+k2 -k2; -k2 k2]; %stiffness matrix
Damp=[c1+c2 -c2; -c2 c2]; % damping matrix
```

0.1 Differential equation model

Create time-domain signal by differential equation

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix}+\begin{bmatrix} c_1+c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}+\begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$

$X(t)=x_C(t)+X_P(t)$

Solution of the differential equations is sum of the complementary solution and the particular solution.

To obtain the complementary solution, we should solve:

$M\ddot{X}+C\dot{X}+KX=0$

The characteristic equation is

$Mr^2+Cr+K=0$

The roots of the characteristic equation are

$$r_{1,2}=\frac{-C\pm\sqrt{C^2-4MK}}{2M}$$

If $C^2-4MK=0$, the system is critical damping.

If $C^2 - 4MK < 0$, the system is under damping.

If $C^2 - 4MK > 0$, the system is over damping.

For civil structures, we consider the under-damped case.

Then, $r_{1,2} = -\frac{C}{2M} \pm \frac{\sqrt{C^2 - 4MK}}{2M} = -\xi \pm \omega_d i$,

$x_C(t) = \mathrm{const}_1 e^{-\xi t}\cos(\omega_d t) + \mathrm{const}_2 e^{-\xi t}\sin(\omega_d t)$

while $\mathrm{const}_1 = X(0), \mathrm{const}_2 = \frac{\dot{X}(0) + \xi X(0)}{\omega}$

To obtain the particular solution, we just need to assume a form of the particular solution and plug into the differential equations. For example, suppose

$X_P(t) = a\cos\omega_P t + b\sin\omega_P t$

Then

$\dot{X_P}(t) = -a\omega_P\sin\omega_P t + b\,\omega_P\cos\omega_P t$

$\ddot{X_P}(t) = -a\omega_P^2\cos\omega_P t - b\omega_P^2\sin\omega_P t$

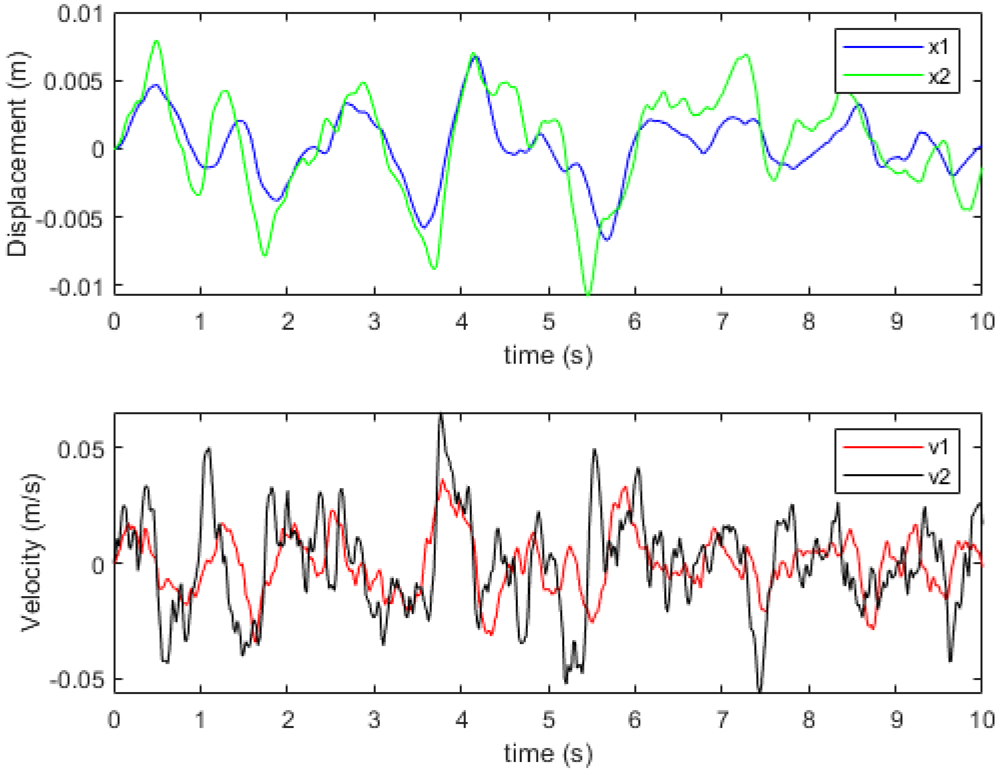
Thus,

M($-a\omega_P^2\cos\omega_P t - b\omega_P^2\sin\omega_P t$)+C$(-a\omega_P\sin\omega_P t + b\,\omega_P\cos\omega_P t) + K(a\cos\omega_P t + b\sin\omega_P t) = F$

Solve the above equation, we will obtain *a* and *b*, then the particular solution.

```
%% Solving system as ODE
T = 10; %length of time duration
nt = 500; %number of time stamps
Fs = nt/T;
tspan=linspace(0,T,nt);
x0=[0;0;0;0]; %initialization of states
rng(49)
F = randn(2,length(tspan)); % input ambient force
% F = zeros(2,length(tspan)); %input force
% hammer = zeros(length(tspan),1);
% hammer(51:80) = 1;
% hammer(251:280) = 1;
% hammer(451:480) = 1;
% F(1,:) = hammer;
% hammer2 = zeros(length(tspan),1);
% hammer2(101:150) = 1;
% hammer2(251:300) = 1;
% F(2,:) = hammer2;
% F(1,:) = 20*sin(10*tspan);
% F(2,:) = 10*sin(5*tspan);
odefun=@(t,y) solve_2dof(t,y,M,Damp,K,tspan,F); %ODE function
[tsol,ysol]=ode45(odefun,tspan,x0); %solve by ODE solver
```

```
figure(1)
plot_dv(tsol,ysol) %plot function
```

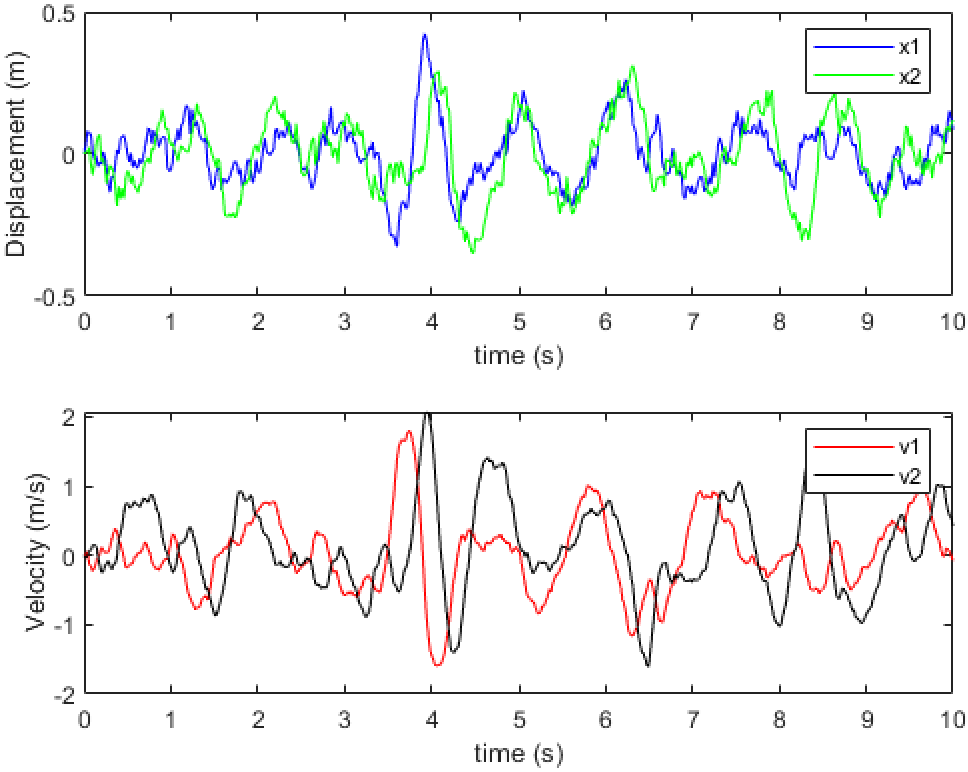


0.2 State-space model

Create time-domain signal by state-space model

```
C = [1,0,0,0;
     0,1,0,0;
     0,0,1,0;
     0,0,0,1]; %observation matrix
rs = 94; %random seed with my lucky number
pro_n_l = [0.,0.,1e-6,1e-5,1e-4,1e-3]; %process noise levels
obs_n_l = [0.,1e-4,1e-4,1e-4,1e-4,1e-4]; %observation noise levels
filenames = {'../data/2dof.csv','../data/2dof_noP.csv','../data/2dof_P6.csv','../data/2dof_P5.c
promu = [0;0;0;0]; %mean of process noise
obsmu = [0;0;0;0]; %mean of observation noise
% save input
csvwrite('../data/ambient.csv',F)
for i = 1:6
    prosi = pro_n_l(i)*eye(4); %covariance matrix of process noise
    obssi = obs_n_l(i)*eye(4); %covariance matrix of observation noise
    ysol1 = glti_2dofO_generator(m1,m2,k1,k2,c1,c2,T,nt,x0,F,C,...
                                    promu,prosi,obsmu,obssi,rs); %generate data by using ga
    % save output
    csvwrite(filenames{i},ysol1)
end
```

```
figure(2)
plot_dv(tspan,ysol1')
```



0.3 Impulse function model / frequency response function model / transfer function model

If the input function is a Dirac delta function, the unit-impulse response function is

$$h(t) = \begin{cases} \frac{1}{M\omega_d} e^{-\xi\omega_n t} \sin\omega_d t & t > 0 \\ 0 & t < 0 \end{cases}$$

where $\omega_n = \sqrt{\frac{K}{M}}$,

Then apply the Dhamel integral, the impulse response is

$$h(t-\tau) = \frac{1}{M\omega_d} e^{-\xi\omega_n(t-\tau)} \sin(\omega_d(t-\tau))$$

Thus,

$$X(t) = \int_0^t F(\tau)h(t-\tau)d\tau$$

```
winlen = size(F,2)/5;
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```
figure(4)
modalfrf(F',ysol(:,3:4),Fs,hann(winlen),0.5*winlen,'Sensor','dis');
```