

Feature Sensitive Mesh Subdivision

1st Given Name Surname
dept. name of organization (of Aff.)
name of organization (of Aff.)
City, Country
email address

2nd Given Name Surname
dept. name of organization (of Aff.)
name of organization (of Aff.)
City, Country
email address

3rd Given Name Surname
dept. name of organization (of Aff.)
name of organization (of Aff.)
City, Country
email address

Abstract—As biomedical visual data in 3D spatial spaces collected at an increasing speed, there is a growing demand for accurate and efficient application for comparative data analysis. For comparing purpose, mesh subdivision techniques are commonly used to adaptively generate and amplify tiny differences. Compared with current methods, the smoothing methods for each subdivision algorithm will moderate features in the original mesh. In this paper, we propose an edge features focused mesh subdivision method, which generates a visually sensitive and precise result compared with commonly used subdivision methods.

Index Terms—mesh, subdivision, keypoints, feature, biomedical

I. INTRODUCTION

Recent years, computer graphical technology works difusely as biomedical assistants, including the utilization of three dimensional CT (3D CT) reconstruction [1]–[4], presentation of skeletons and tissues [5], and comparison of genome [6]–[9], and etc, due to its visual obviousness. The technique is widely used for presenting facial bones and their connections, confirmation of borderline or size of diseased tissues and their relation to the adjacent tissues. With the aid of bio-images and bio-models, diagnosis and operations on clinical medicine accomplished a more accurate precision. Moreover, substantial effort has been made around world to determine spatial expression patterns of genes in mammalian genome using experimental techniques such as *in situ* hybridization(ISH) [10]. Performing ISH on multiple subjects yields expression images of various genes over the common anatomical structure and comparing these images reveals the spatial relations between genes, which are often key to understanding their functional relations [11] [12].

The pros of using Computer Graphics including 3D spatial and triangular meshes [13]–[15] to expose nicety of bioinformation has created an ever-titanic amount of spatial data (in the form of 2D medical images, and 3D bio-models) which requires efficient and accurate process and analysis consequently. The anatomical differentiations among these biomedical models aggravate the computational challenges involved in visual and medical comparison among data collected from different modes [16].

Restrained by the calculation performance of physical devices, medical detailed expression and accuracy of biomedical data, in which the rudimentary expression are 3D meshes (triangular form, quad form, combined form), is circumstanced.

Solutions emerge as medical processing require, one of which is using meshes without redundant vertices and faces. The contradiction of biomedical precision and data optimization generates as model shrinking. Hence, feature-based mesh subdivision present start-of-the-art results [16].

Using feature-based subdivision to suitably amplify medical precision and organize spatial data into multi-resolution versions, on which visual comparison and diagnosis will have a higher performance [16]. Meanwhile, the multi-resolution structure of a subdivision mesh further gives rise to fast algorithms for processing and accurate comparison for tiny biodivergence.

Commonly used mesh subdivision methods including Butterly method [27], Loop subdivision [25], and Catmull-Clark subdivision [26], whose experimental result will be illustrate in Fig.1. Moreover, Xie [17] provide a solid-shell element based triangular subdivision to avoid interpenetration. Liu [18] introduced a dense reconstruction algorithm for mesh subdivision. Amresh [19] developed a subdivision scheme derivate from Loop scheme and using watershed segmentation. Dirc Rose [20] proposed an adaptive process that stores the next splitting vertex and temporary triangle based on Modified Butterly scheme. Kobbelt [21] developed refinement for both his Kobbelt scheme and newly introduced $\sqrt{3}$ subdivision. Seeger [22] introduced a subdivision scheme based on Butterly Scheme using quark. In their method, subdivision is controlled by the faces of the original mesh and mesh features are not suitably preserved.

In this paper, we propose three mesh subdivision methods including Feature Sensitive Mesh subdivision, Advanced Feature Sensitive Mesh Subdivision, and Keypoints-based Feature Sensitive Mesh Subdivision to maintain original mesh features while give out a more adaptive derivative. Subdivision Result will be compared by using the same mesh after same time of iterations, whose visual difference is unambiguously discerned. Results show the promise of the proposed method on feature maintained mesh subdivisions.

The rest of the paper is organized as follows: In Section II, we present a imbalanced keypoints detection method to locate features on the mesh, and provide our basic idea of Feature Sensitive Mesh Subdivision with its improved version to efface model-related noise wave on the mesh. Finally we combined features detection method with our mesh subdivision algorithm. Experimental comparison and conclusion will be

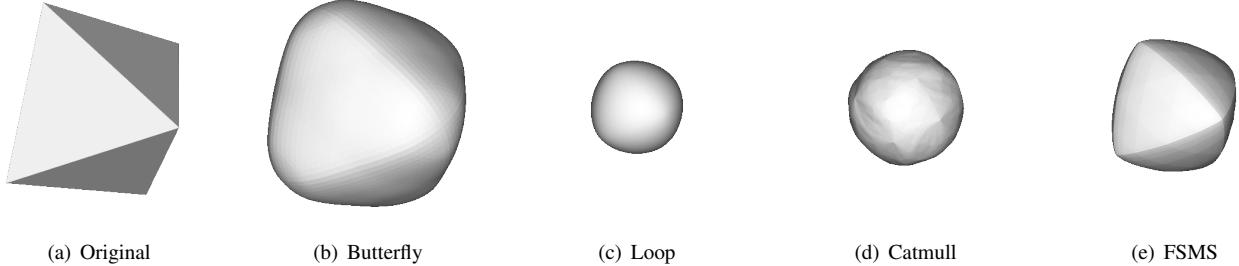


Fig. 1: a) Original Mesh. b) Butterfly Subdivision. c) Loop Subdivision. d) Catmull Subdivision. e) Feature Sensitive Mesh Subdivision. All subdivisions algorithm has been applied five times on original meshes

demonstrated in Section III and Section IV, respectively.

II. FEATURE SENSITIVE MESH SUBDIVISION

In this section, we first present our previous work on imbalanced vertices based feature detection [23] extended from Li's method [24] on imbalanced points detection. Moreover, we provide a series of mesh subdivision schemes including Feature Sensitive Mesh Subdivision, Advanced Feature Sensitive Mesh Subdivision, and Keypoints-based Feature Sensitive Mesh Subdivision. All three algorithms follow the same subdivision framework consisted of linear face subdivision and vertex smoothing.

A. Imbalanced Keypoints Detection

We first propose a geometric feature-based vertex operator, which is a two-dimensional implementation of keypoints detection, to pick up imbalanced keypoints. We extend Li's [24] previous work on imbalanced keypoints detection to triangular meshes.

The basic idea of vertex operator is using projections to transform 3D geometric features to two-dimensional space. Let $M(V, F, N_F)$ be a triangular mesh where V is the set of vertices, F is the set of faces, and N_F represents the set of face normals. Suppose a projection P_T will project any vector on the plane T , 3D geometric mesh features will be transformed to two-dimension when T is the tangent plane of given mesh vertex in vertex set V . Face normals N_F will then be transformed into its projection vector set N_F' . Finally, we settle all normalized vectors in N_F' to a polar coordinates and calculate the set of cross angles between each vector side by side, denote as A_F .

Imbalanced point detection in 2D images aims to minimize the occurrences of edge points [24]. Denote I a gray value image, p a local point, $\theta_i = \frac{(i-1)2\pi}{N}$, and $l_i = (\cos \theta_i, \sin \theta_i)$ for $i = 1, 2, \dots, N$. Denote $\frac{\partial I}{\partial l_i}(p)$ a directional derivative of p along l_i direction. We cluster $\{\frac{\partial I}{\partial l_i}(p)\}_{i=1}^N$ into two classes in terms of their magnitudes $|\frac{\partial I}{\partial l_i}(p)|$. If two clusters have the same size, the image point p is balanced.

The sorting method proposed in Li's method is to classify $\{\frac{\partial I}{\partial l_i}(p)\}_{i=1}^N$, which can be generalized to extract 3D imbalanced vertices with proposed operator. Let \maxDiff be the max difference and D be the index of maximum difference:

$$\maxDiff = \max_j(\alpha_{j+1} - \alpha_j)$$

$$D = \arg \min_j(\alpha_{j+1} - \alpha_j)$$

where α represents value in A_F , and $1 \leq j \leq N - 1$. Given a threshold on homogeneity ε , the imbalanced vertex can be defined under the condition that $\maxDiff < \varepsilon$:

$$IMB(v_i) = \begin{cases} 1 & D_i < \frac{N}{2} \\ 0 & \text{else} \end{cases}$$

B. Feature Sensitive Mesh Subdivision

Loops method can be expressed in terms of linear subdivision and an averaging scheme to approximate a spherical surface, which will efface edge and vertex features. The phenomenon is common when applying Catmull method, and Butterfly method. When the mesh is rigid, these iterations will fail as illustrated in Fig.1. To moderate the feature friction after subdivision and highlight original edge and vertex features, we provide a Feature Sensitive Mesh Subdivision.

Our proposed method implements linear one to four triangular mesh subdivision to increase the details in a mesh as illustrated in Fig.2, and smooths by proposed smoothing algorithm to accomplish surface approximation after subdivision while actualize shape retention.

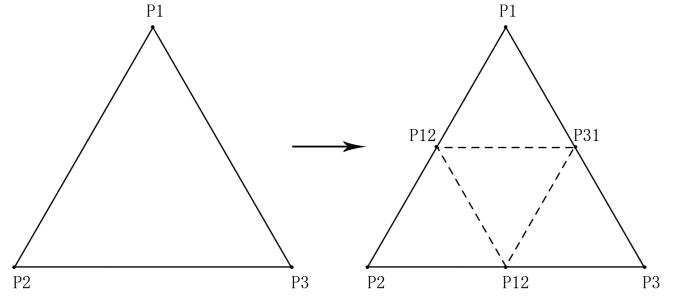


Fig. 2: Linear one to four subdivision for triangular subdivision schemes

Denote the original mesh as M_i , and edge point generator G_{FSMS} as illustrated in Fig.6. To perform linear triangular

Algorithm 1 Linear Triangular Mesh Subdivision

Input : Original Mesh M_i
Output: Subdivided Mesh M_{i+1}

```

for each triangular face  $f_i$  in  $M_i$  do
    get 3 middle point on edges of  $f_i$ ;
    generate 4 subsurfaces anticlockwisely;
end for

```

subdivision, we insert points created by generator G_{FSMS} on the edge of each triangle to the hash map $H_n(v_i, h_i)$ for storing vertices coordination v_i and their handles h_i as corresponding hash key. For each vertex on mesh $\{v_1, v_2, \dots, v_n\}$, check if the generated middle edge point v_k is already in the map. If so, get its handle h_k for the on-coming face generation, else, insert the point v_k into the mesh and create its handle h_k while update vertices hash map $H_{n+1}(v_i, h_i)$. Finally, form the new triangular surfaces using vertex handles geometrically anticlockwise, and eliminate elder redundant faces simultaneously. Each triangle will then be split into four sub-triangles and original mesh M_i is subdivided to M_{i+1} .

Smoothing for triangular meshes will be applied on not only the previous vertices on M_i but all vertices on the generated mesh M_{i+1} whose two-ring vertex set is derived from one-ring vertices of previous mesh M_i as illustrated in Fig.3. For each vertex on M_{i+1} , we use a one-ring neighbor weighted centroid method for averaging shown in Fig.4. The weight of each neighbor β is decided by the number of one-ring neighbors n .

$$\beta = \frac{5}{8} - \left(\frac{3}{8} + \frac{1}{4} \cos \frac{2\pi}{n} \right)^2$$

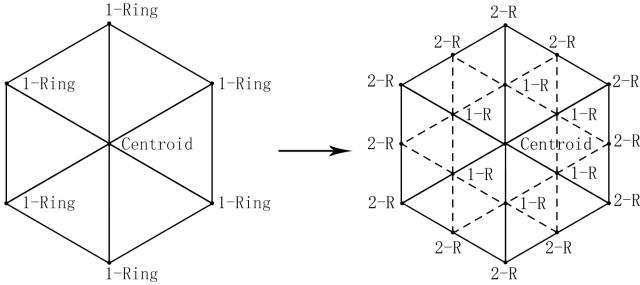


Fig. 3: One-ring vertices derivative

C. Advanced Feature Sensitive Mesh Subdivision

By using Feature Sensitive Mesh Subdivision, an edge and vertex sensitive result is generated. But the method will generate model-depend wavelike noise on the mesh, as illustrated in Fig.5.

We propose an Advanced Feature Sensitive Mesh Subdivision, a combination of Feature Sensitive Mesh Subdivision and Smoothness Focused Mesh Subdivision, to eliminate noise.

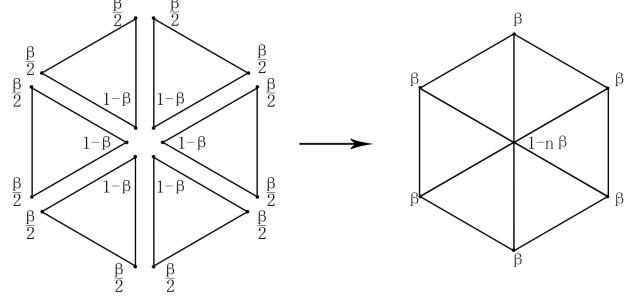


Fig. 4: One-ring neighbor weighted centroid method
for triangular vertices

Algorithm 2 Feature Sensitive Mesh Subdivision

Input : Original Mesh M_i
Output: Subdivided Mesh M_{i+1}

```

for each triangular face  $f_i$  in  $M_i$  do
    get 3 edge point on edges of  $f_i$  with  $G_{FSMS}$ ;
    generate 4 subsurfaces anticlockwisely;
end for
{NOTE:  $M_i$  is now subdivided into  $M_{i+1}$ }

for each vertex  $v_i$  in  $M_{i+1}$  do
     $N(v_i) = 1\text{-ring neighbors of } v_i \text{ on current mesh } M_{i+1};$ 
     $v_i = 1\text{-Ring Neighbor Weighted Centroid method on } v_i;$ 
end for
{NOTE:  $M_{i+1}$  is now smoothed}

```

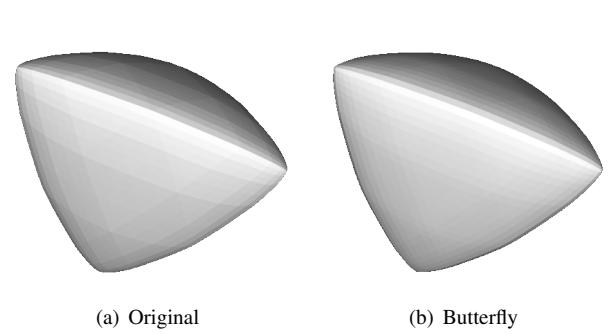


Fig. 5: a) Model-depend wavelike noise. b) Remove wavelike noise by Advanced Feature Sensitive Mesh Subdivision. All subdivisions algorithm has been applied five times on original meshes

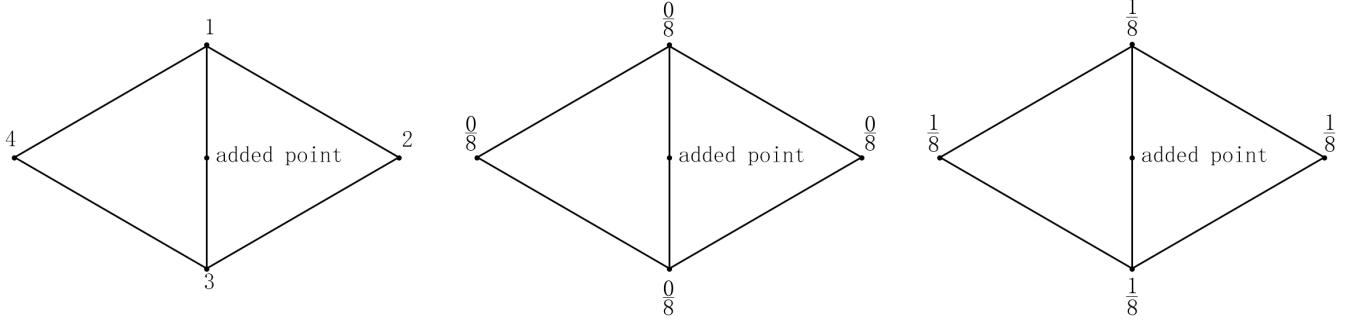


Fig. 6: Unit of edge point generator (left). Feature Sensitive Mesh Subdivision edge point generator (middle). Advanced Feature Sensitive edge point generator (right). Vertex weight of two generators are tags with its position

The Subdivision scheme consists of several iterations of Feature Sensitive Subdivision and an iteration of Smoothness-focused Mesh Subdivision at the end of algorithm. The implementation of Smoothness-focused Mesh Subdivision resembles the Feature Sensitive Mesh Subdivision, except the generation of edge points and smoothing operator.

Unlike edge points generator G_{FSMS} in Feature Sensitive Subdivision, edge points in the last iteration of Advanced Edge Sensitive Subdivision are generated based on weighted neighbors illustrated in Fig.6, where directly connected vertices take up a greater weight, say $3/8$ each, and indirect weight slightly less, say $1/8$ each.

Neighbor vertices for smoothing method depends on previous one-ring vertices on M_i , but not newly generated vertices on M_{i+1} , which is different from the implementation of Feature Sensitive Mesh Subdivision. The weight of each neighbor follows the same principle introduced in Feature Sensitive Mesh Subdivision.

Algorithm 3 Advanced Feature Sensitive Mesh Subdivision

Input : Original Mesh M_i
Output: Subdivided Mesh M_{i+1}

several iterations on Feature Sensitive Mesh Subdivision;
{NOTE: oncoming method is Smooth Focused Subdivision}

```

for each triangular face  $f_i$  in  $M_i$  do
    get 3 edge point on edges of  $f_i$  with  $G_{AFSMS}$ ;
    generate 4 subsurfaces anticlockwisely;
end for
{NOTE:  $M_i$  is now subdivided into  $M_{i+1}$ }

for each vertex  $v_i$  in  $M_{i+1}$  do
     $N(v_i)$  = 1-ring neighbors of  $v_i$  on previous mesh  $M_i$ ;
     $v_i$  = 1-Ring Neighbor Weighted Centroid method on  $v_i$ ;
end for
{NOTE:  $M_{i+1}$  is now smoothed}

```

D. Keypoints-based Feature Sensitive Mesh Subdivision

Based on imbalanced keypoints detection, all the vertices are classified into two categories. According to the properties of imbalanced vertices, our keypoints distributed mainly at the boundaries and detailed parts, which should be remained after subdivision but slightly adjusted with neighbors. We propose a subdivision scheme, which contains edge point generation and smooth operator, to balance the emphasis of original features and smoothing extent.

The unit of edge point generation in our provided method is a couple of triangles back to back as illustrated in Fig.6. Denote the edge point generator in Feature Sensitive Mesh Subdivision and Advanced Feature Sensitive Mesh Subdivision as $G_{FSMS}(N)$ and $G_{AFSMS}(N)$ respectively, where N is the set of neighbor points.

The balance can be guided by the amount of imbalanced keypoints in an isolated generation unit and implemented by a weighted combination of $G_{FSMS}(N)$ and $G_{AFSMS}(N)$. For a precise and accurate calculation, the weight of two generators should be geometrically related and symmetric. The weight of $G_{FSMS}(N)$ and $G_{AFSMS}(N)$ will be defined as $W_{G_{FSMS}}$ and $W_{G_{AFSMS}}$, which depend on the number of keypoints in a generation unit, and obey the following principles.

$$W_{G_{FSMS}} = \sum_{i=1}^{|N|} Key(N_i)$$

$$W_{G_{AFSMS}} = |N| - \sum_{i=1}^{|N|} Key(N_i)$$

$$Key(N_i) = \begin{cases} 1 & N_i \text{ is a keypoint} \\ 0 & \text{else} \end{cases}$$

A combination of two edge point generator $G(N)$ will be defined:

$$G(N) = W_{G_{FSMS}} G_{FSMS}(N) + W_{G_{AFSMS}} G_{AFSMS}(N)$$

Smooth operator will also be guided by keypoints, which extends the previous smoothing scheme. Neighbor vertices for

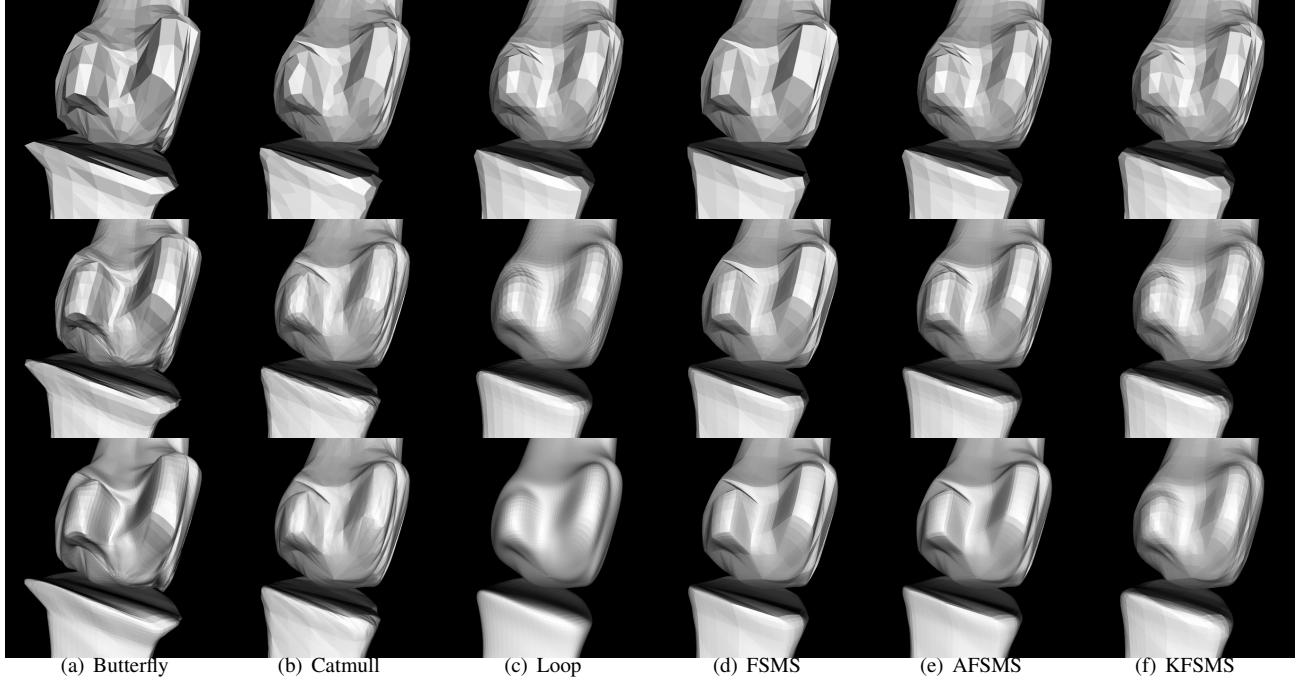


Fig. 7: Subdivision Comparison on biomedical data with iteration increasing by row. a) Butterfly Method. b) Catmull Method. c) Loop Method. d) Feature Sensitive Mesh Subdivision. e) Advanced Feature Sensitive Mesh Subdivision. f) Keypoints-based Feature Sensitive Mesh Subdivision.

Algorithm 4 Keypoints-based Sensitive Mesh Subdivision

Input : Original Mesh M_i

Output: Subdivided Mesh M_{i+1}

imbalanced keypoints detection;

for each triangular face f_i **in** M_i **do**

 get edge point e_{FSMS} on edges of f_i with G_{FSMS} ;
 get edge point e_{AFSMS} on edges of f_i with G_{AFSMS} ;
 average position of edge points by $G(N)$;
 generate 4 subsurfaces anticlockwisely;

end for

{NOTE: M_i is now subdivided into M_{i+1} }

for each vertex v_i **in** M_{i+1} **do**

if v_i is a keypoint **then**

$N(v_i)$ = 1-ring neighbors of v_i on previous mesh M_i ;
 v_i = 1-Ring Neighbor Weighted Centroid method on
 v_i ;

else

$N(v_i)$ = 1-ring neighbors of v_i on current mesh M_{i+1} ;
 v_i = 1-Ring Neighbor Weighted Centroid method on
 v_i ;

end if

end for

{NOTE: M_{i+1} is now smoothed}

smooth operator will be different which depends on whether the centroid vertex is a keypoint or not. If so, neighbor vertices should be elder one-ring vertices on M_i , else, one-ring vertices on M_{i+1} .

III. EXPERIMENT

We test different subdivision methods on several biomedical data, including phalanx (distal phalanx, middle phalanx, and proximal phalanx), cuneiform bone (intermediate cuneiform bones, entocuneiform, ectocuneiform), central anlrlle bone, cuboid bone, etc, provided by Florida State University.

As the time of subdivision methods iteration increased, the performance of different algorithm is becoming easier to differentiate, as illustrated in Fig.7. Except Catmull subdivision, all the subdivision methods implement one to four triangular faces subdivision, which means each mesh in a row has the same number of vertices and faces, except Catmull subdivision. The number of vertices and triangular faces in each iteration follows the following principles when apply one to four mesh subdivision schemes, where $V(\cdot)$ and $F(\cdot)$ represent total amount of vertices and faces on a mesh respectively.

$$V(M_{i+1}) = V(M_i) + \frac{3}{2}F(M_i)$$

$$F(M_{i+1}) = 4F(M_i)$$

Obviously, subdivision result after Loop method has a better spherical resembling appearance, while Feature Sensitive Subdivision provide a better property of edge feature remaining.

Butterfly subdivision leads mesh to a more boundary malleable shape, while Catmull present more vertex details. Advanced Feature Sensitive Subdivision accomplish an edge feature remaining and smooth result, but compared with Keypoint-based Feature Sensitive Subdivision, the later algorithm provides a better result.

Fig.8 and Fig.9 exhibit more experimental comparison of different mesh subdivisions on biomedical utilization.

IV. CONCLUSION

Proposed Feature Sensitive Subdivision method works well on edge features but it will generate troublesome model-shape noise waves. By using the idea in Advanced Feature Sensitive Subdivision, noise waves problem will be solved and generate a smoother result. Compared all the methods mentioned in the figures, keypoints-based Feature Sensitive Subdivision has a better feature based result.

REFERENCES

- [1] DENG J F, LI J X, LI L, et al. FPGA accelerate 3D CT reconstruction[J]. Application of Electronic Technique, 2010, 9: 030.
- [2] Koopman D, Dalen J, Arkies H, et al. A small voxel FDG-PET/CT reconstruction improves the visual evaluation of axillary lymph nodes in patients with breast cancer[J]. Journal of Nuclear Medicine, 2016, 57(supplement 2): 1502-1502.
- [3] Muthusami P, Shkumat N, Rea V, et al. CT reconstruction and MRI fusion of 3D rotational angiography in the evaluation of pediatric cerebrovascular lesions[J]. Neuroradiology, 2017: 1-9.
- [4] McCann M T, Nilchian M, Stampaoni M, et al. Fast 3D reconstruction method for differential phase contrast X-ray CT[J]. Optics express, 2016, 24(13): 14564-14581.
- [5] X. Tao. Application of 3D Reconstruction in Maxillofacial Surgery. Acta Universitatis Medicinae Tangi. 1999.
- [6] Lein E S, Hawrylycz M J, Ao N, et al. Genome-wide atlas of gene expression in the adult mouse brain[J]. Nature, 2007, 445(7124): 168.
- [7] Bonev B, Cavalli G. Organization and function of the 3D genome[J]. Nature Reviews Genetics, 2016, 17(11): 661-678.
- [8] Tjong H, Li W, Kalhor R, et al. Population-based 3D genome structure analysis reveals driving forces in spatial genome organization[J]. Proceedings of the National Academy of Sciences, 2016, 113(12): E1663-E1672.
- [9] Caspermeyer J. Principles of 3D Genome Folding and Gene Expression Studied across Species[J]. Molecular Biology and Evolution, 2017, 34(6): 1548-1548.
- [10] Carson J P, Thaller C, Eichele G. A transcriptome atlas of the mouse brain at cellular resolution[J]. Current opinion in neurobiology, 2002, 12(5): 562-565.
- [11] Ju T, Warren J, Eichele G, et al. A geometric database for gene expression data[C]//Symposium on geometry processing:[proceedings]/sponsored by the Eurographics Association in cooperation with ACM SIGGRAPH. Symposium on Geometry Processing. NIH Public Access, 2003, 2003: 166.
- [12] Schaefer S, Hakenberg J, Warren J. Smooth subdivision of tetrahedral meshes[C]//Proceedings of the 2004 Eurographics/ACM SIGGRAPH symposium on Geometry processing. ACM, 2004: 147-154.
- [13] Brentzen J A, Abd rashitov R, Singh K. Interactive shape modeling using a skeleton-mesh co-representation[J]. ACM Transactions on Graphics (TOG), 2014, 33(4): 132.
- [14] Guskov I V, Schrder P, Sweldens W. Non-uniform relaxation procedure for multiresolution mesh processing: U.S. Patent 8,830,235[P]. 2014-9-9.
- [15] Zhang Y, Ma L. A Scaling Method of Sensitive Objects Based on Loss Constraint Triangle Mesh Deformation[J]. Journal of Image and Graphics, 2015, 3(2).
- [16] Ju T, Carson J, Liu L, et al. Subdivision meshes for organizing spatial biomedical data[J]. Methods, 2010, 50(2): 70-76.
- [17] Xie Q, Sze K Y, Zhou Y X. Drape simulation using solid-shell elements and adaptive mesh subdivision[J]. Finite Elements in Analysis and Design, 2015, 106: 85-102.
- [18] Liu H H. An Improved refinement algorithm of triangular mesh subdivision based on minimum weight theory[C]//Applied Mechanics and Materials. Trans Tech Publications, 2014, 513: 2552-2555.
- [19] Amresh A, Farin G, Razdan A. Adaptive subdivision schemes for triangular meshes[J]. Hierarchical and Geometric Methods in Scientific Visualization, 2003, 13(4): 319-327.
- [20] Rose D, Kada M, Ertl T. On-the-Fly Adaptive Subdivision Terrain[C]//VMV. 2001: 87-92.
- [21] Kobelt L. $\sqrt{3}$ -subdivision[C]//Proceedings of the 27th annual conference on Computer graphics and interactive techniques. ACM Press/Addison-Wesley Publishing Co., 2000: 103-112.
- [22] Seeger S, Hormann K, Husler G, et al. A Sub-Atomic Subdivision Approach[C]//VMV. 2001: 77-86.
- [23] K. Feng, Q. Li, Y. Gong, J. Yang, S. Peng. Detection of Imbalanced Vertices in 3D meshes. ICDH 2016.
- [24] Li Q, Ye J, Kambhamettu C. Interest point detection using imbalance oriented selection[J]. Pattern Recognition, 2008, 41(2): 672-688.
- [25] Loop C. Smooth subdivision surfaces based on triangles[J]. 1987.
- [26] Catmull E, Clark J. Recursively generated B-spline surfaces on arbitrary topological meshes[J]. Computer-aided design, 1978, 10(6): 350-355.
- [27] Zorin D, Schrder P, Sweldens W. Interpolating Subdivision for Meshes with Arbitrary Topology[J]. ACM SIGGRAPH Computer Graphics, 1996, 30:pgs. 43-53.

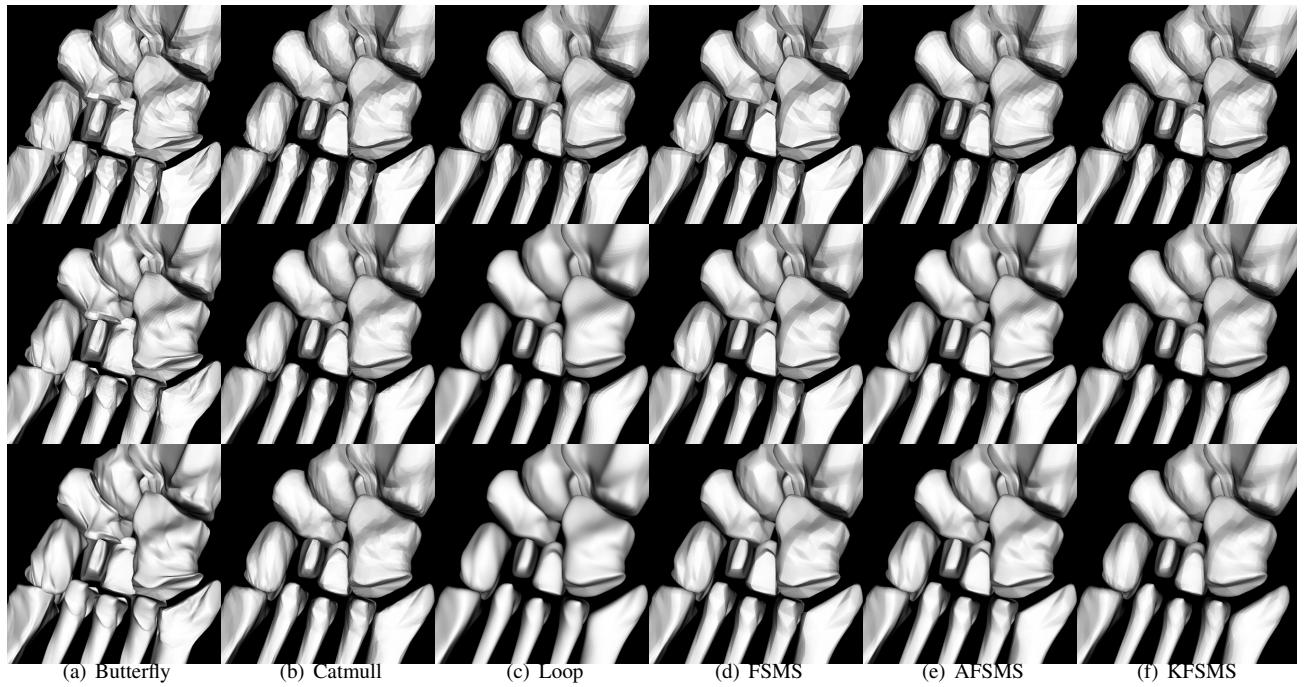


Fig. 8: Subdivision Comparison on biomedical data with iteration increasing by row. a) Butterfly Method. b) Catmull Method. c) Loop Method. d) Feature Sensitive Mesh Subdivision. e) Advanced Feature Sensitive Mesh Subdivision. f) Keypoints-based Feature Sensitive Mesh Subdivision.

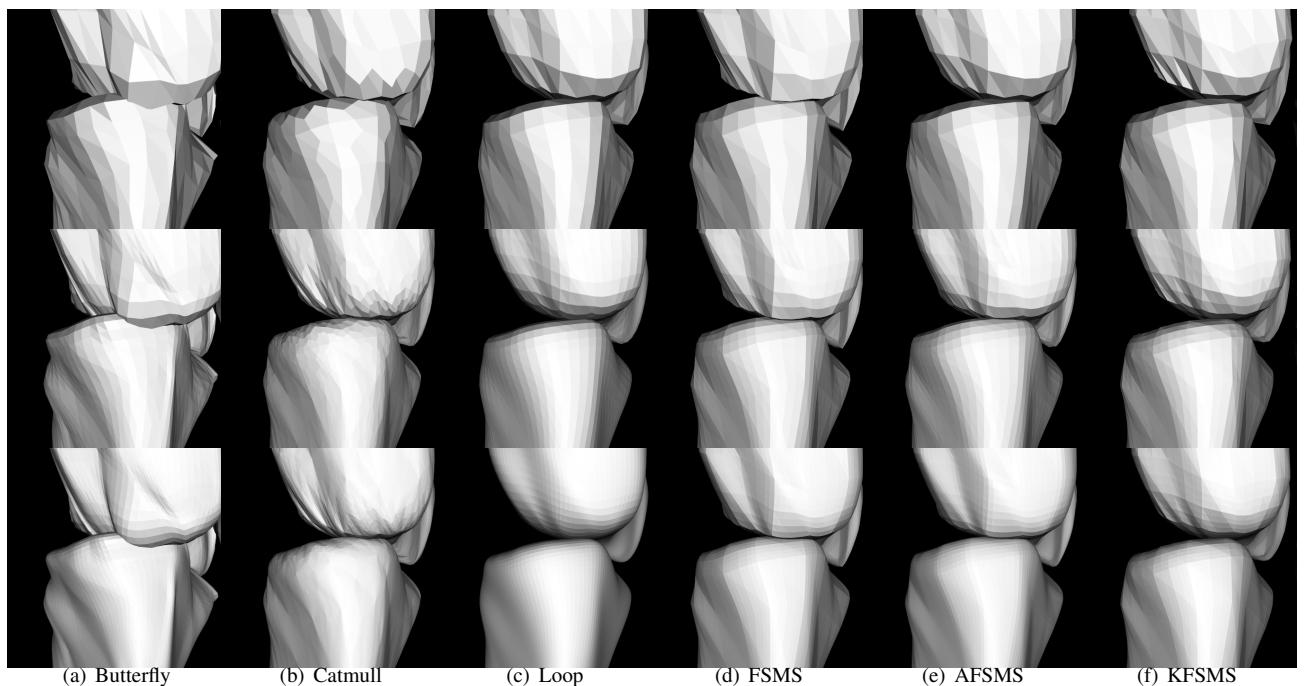


Fig. 9: Subdivision Comparison on biomedical data with iteration increasing by row. a) Butterfly Method. b) Catmull Method. c) Loop Method. d) Feature Sensitive Mesh Subdivision. e) Advanced Feature Sensitive Mesh Subdivision. f) Keypoints-based Feature Sensitive Mesh Subdivision.