

# Digital Signal Processing HW12 MATLAB Part

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Compare the effect of the following filters when applied horizontally on a sample image: IIR elliptic filter, FIR linear phase minimax filter, FIR minimum phase filter obtained from the linear phase filter by reflecting the zeros outside the unit circle. Design your filters to satisfy the following spec:

$$\begin{aligned} \left| |H(e^{j\omega})| - 1 \right| &\leq \Delta_p \quad 0 < \omega \leq \omega_p \\ |H(e^{j\omega})| &\leq \Delta_s \quad \omega_s < \omega \leq \pi \end{aligned}$$

where  $\Delta_p = 0.05$ ,  $\Delta_s = 0.05$ ,  $\omega_p = 0.15\pi$ ,  $\omega_s = 0.2\pi$ . Furthermore, apply `filtfilt` operation using your designed filters. You could choose to apply the filter on any image (including your own). You want to choose a relatively small image (size less than or equal to 512x512 to observe the delay effect) and has many vertical edges (if you use on an image with many horizontal edges then apply your filter vertically). A sample image "barbara.png" is available on NYU Classes "sample data" folder. Here is a link to some popular test images:

<https://homepages.cae.wisc.edu/~ece533/images/>

(Note that matlab does not handle all image formats well. You should just first use `imread()` to read the image in, and then use `imshow()` to see whether you could display it properly. If not, choose another image.)

Your MATLAB program should design the filter, plot their frequency response (magnitude and unwrapped phase, zero-pole pattern), perform filtering, and display the filtered images. You should also add your own comments / observations. You could follow the example code shown in `Wang_PhaseDistortionDemo2_Images.pdf`.

Optional: Instead of designing the minimum phase filter from the PM filter by merely reflecting zeros outside the unit circle, use the lifting procedure (the resulting filter should be of the same order as the previous minimum phase filter) described in `Selesnick_minphase_lifting.pdf`. And compare the two types of minimum phase filters in terms of their frequency response and filtered images.

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Solution:

.m file(s): jyz\_HW12.m

Code:

```
close all
clear

% load image and basic process ↓
img=imread('daytona_512.png');
[height, width] = size(img);
gimg = img;
dimg = double(gimg);

% set basic parameters↓
wp= 0.15 * pi; fp = wp / pi;
ws= 0.2 * pi; fs = ws / pi;
```

```

wc = (wp + ws) / 2; fc = wc / pi;
delp = 0.05; Rp = -20 * log10(1 - delp); % delp : pass-band ripple
dels = 0.05; Rs = -20 * log10(dels); % dels : stop-band ripple

[NIIR, nwp] = ellipord(fp, fs, Rp, Rs);
[bIIR, aIIR] = ellip(NIIR, Rp, Rs, nwp);
[HIIR, ~] = freqz(bIIR, aIIR);

[NFIR, f0, m0, w] = firpmord([wp ws] / pi, [1 0], [delp, dels]);
bFIR = firpm(NFIR, f0, m0, w);
[HFIR, ~] = freqz(bFIR, 1);

del = 0.0001; % a small number
rts = roots(bFIR);
inrts = rts(abs(rts) <= (1+del)); % INSIDE or ON the U.C.
outrts = rts(abs(rts) > (1+del)); % OUTSIDE the U.C.
reflectedrts = 1 ./ conj(outrts);
newrts = [inrts; reflectedrts];
newrts = leja(newrts); % Reorder roots using Leja to make poly work more stable
[minbFIR] = poly(newrts); % convert roots to polynomial coefficients
minbFIR = minbFIR * sum(bFIR) / sum(minbFIR);
[HminFIR, om] = freqz(minbFIR, 1);

disp(['The order of IIR filter: ', num2str(NIIR)])
disp(['The order of FIR filter: ', num2str(NFIR)])

% apply filters to each row of the image
IIRimg = zeros(height, width); % initialization
IIRffimg = zeros(height, width); % initialization
FIRimg = zeros(height, width); % initialization
FIRffimg = zeros(height, width); % initialization
minFIRimg = zeros(height, width); % initialization
minFIRffimg = zeros(height, width); % initialization
for i = 1 : height
    IIRimg(i, :) = filter(bIIR, aIIR, dimg(i, :));
    IIRffimg(i, :) = filtfilt(bIIR, aIIR, dimg(i, :));
    FIRimg(i, :) = filter(bFIR, 1, dimg(i, :));
    FIRffimg(i, :) = filtfilt(bFIR, 1, dimg(i, :));
    minFIRimg(i, :) = filter(minbFIR, 1, dimg(i, :));
    minFIRffimg(i, :) = filtfilt(minbFIR, 1, dimg(i, :));
end

figure (1)
subplot(2, 2, 1);
plot(om/pi, abs(HIIR), 'r', 'Linewidth', 1)
xlabel('\omega/\pi'); ylabel('|H(\omega)|')
ylim([0 1.1]); grid
title('Magnitude Response of IIR Elliptic Filter')
subplot(2, 2, 2);

```

```

plot(om/pi, abs(HFIR), 'g', 'Linewidth', 1)
xlabel('\omega/\pi'); ylabel('|H(\omega)|')
ylim([0 1.1]); grid
title('Magnitude Response of FIR Filter')
subplot(2, 2, 3);
plot(om/pi, abs(HminFIR), 'b', 'Linewidth', 1)
xlabel('\omega/\pi'); ylabel('|H(\omega)|')
ylim([0 1.1]); grid
title('Magnitude Response of Minimum Phase FIR Filter')
subplot(2, 2, 4);
plot(om/pi, abs(HIIR), ':r', om/pi, abs(HFIR), '-g', om/pi, abs(HminFIR), '--b',
'Linewidth', 1)
xlabel('\omega/\pi')
ylim([0 1.1]); grid
legend('IIR Ellipitic', 'FIR Filter', 'Minimum Phase FIR')
title('Magnitude Responses')

figure (2)
subplot(2, 2, 1);
plot(om / pi, unwrap(angle(HIIR) * 2) / 2, 'r', 'Linewidth', 1)
xlabel('\omega/\pi')
ylabel('\angle H(\omega)'); grid
title('Phase Response of IIR Ellipitic Filter ')
subplot(2, 2, 2);
plot(om / pi, unwrap(angle(HFIR) * 2) / 2, 'g', 'Linewidth', 1)
xlabel('\omega/\pi')
ylabel('\angle H(\omega)'); grid
title('Phase Response of FIR Filter ')
subplot(2, 2, 3);
plot(om / pi, unwrap(angle(HminFIR) * 2) / 2, 'b', 'Linewidth', 1)
xlabel('\omega/\pi')
ylabel('\angle H(\omega)'); grid
title('Phase Response of Minimum Phase FIR Filter ')
subplot(2, 2, 4);
plot(om/pi, unwrap(angle(HIIR)*2)/2, ':r', om/pi, unwrap(angle(HFIR)*2)/2, '-g', om/pi, unwrap(angle(HminFIR)*2)/2, '--b', 'Linewidth', 1);
xlabel('\omega/\pi'); grid
legend('IIR Ellipitic', 'FIR Filter', 'Minimum Phase FIR')
title('Phase Responses')

figure (3)
subplot(2, 2, 1)
zplane(bIIR, aIIR);
title('Zero-Pole Plot of IIR Elliptic Filter')
subplot(2, 2, 2)
zplane(bFIR, 1);
title('Zero-Pole Plot of FIR Filter')
subplot(2, 2, 3)
zplane(minbFIR, 1);

```

```

title('Zero-Pole Plot of Minimum Phase FIR Filter')
subplot(2, 2, 4)
zplane(minbFIR, 1);
title('Minimum Phase FIR Filter (Zoomed in)')
axis([-1.1 1.1 -0.5 0.5])

```

```

figure (4)
imshow(img, []);
title('Original Image');

```

```

figure (5)
subplot(1, 2, 1)
imshow(IIRimg, []);
title('IIR Filtered Image');
subplot(1, 2, 2)
imshow(IIRffimg, []);
title('IIR F-B Filtered Image');

```

```

figure (6)
subplot(1, 2, 1)
imshow(FIRimg, []);
title('FIR Filtered Image');
subplot(1, 2, 2)
imshow(FIRffimg, []);
title('FIR F-B Filtered Image');

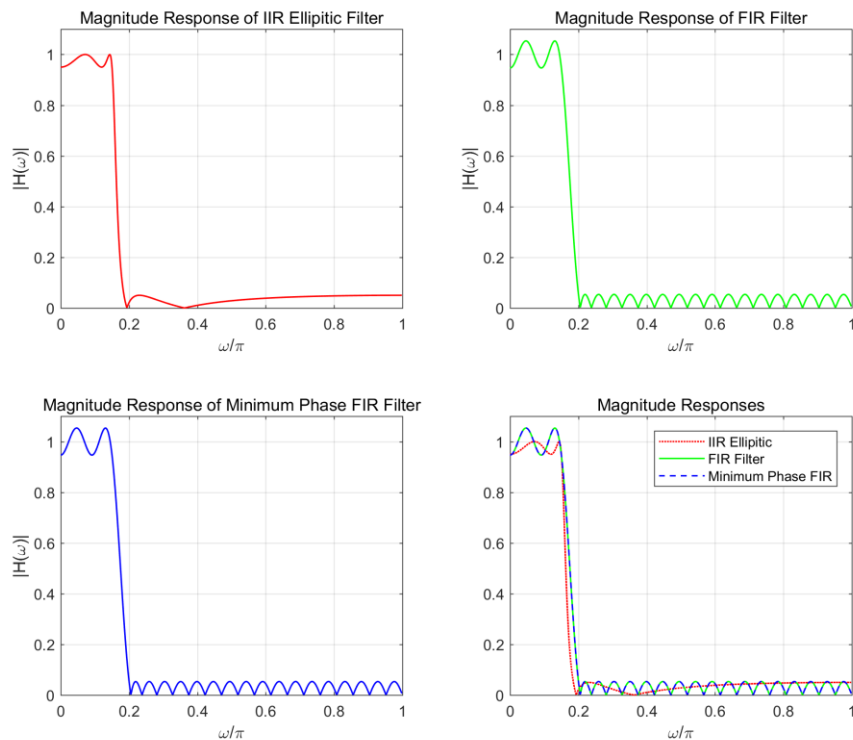
```

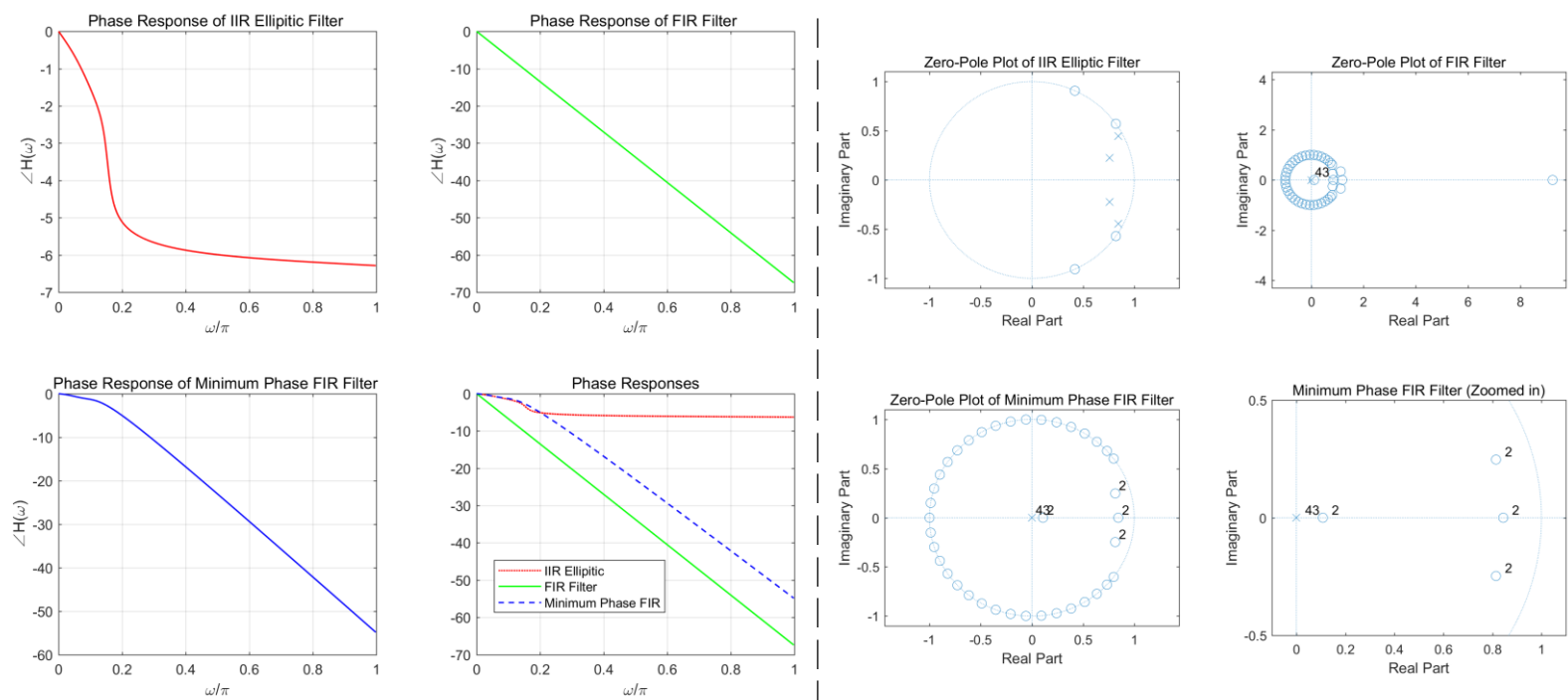
```

figure (7)
subplot(1, 2, 1)
imshow(minFIRimg, []);
title('Minimum Phase FIR Filtered Image')
subplot(1, 2, 2)
imshow(minFIRffimg, []);
title('Minimum Phase FIR F-B Filtered Image')

```

Result(plots):





>> jyz\_HW12

The order of IIR filter: 4

The order of FIR filter: 43

Original Image:

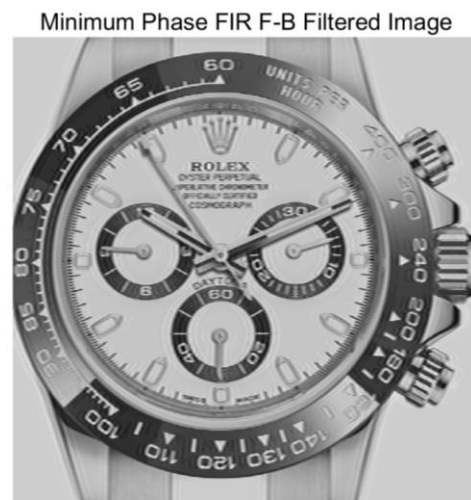
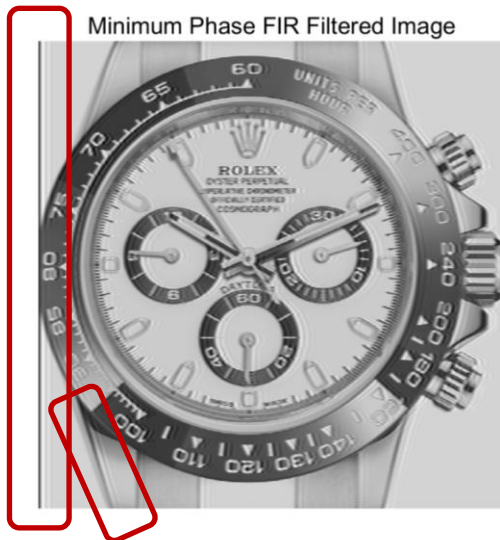


IIR Filtered Image



IIR F-B Filtered Image





Comments:

A low-pass filter would result in detail-loss, so I chose an image of a watch with many graduations on it to see the influence on the details.

After 6 filters are designed, it's easy to find that the IIR filter has considerably much smaller filter length than that of FIR filters (order: 4 against 43). The FIR filter and its minimum phase filter shares the same magnitude response, which is predictable from the definition of the '*minimum phase filter*'. When looking at the Pole-Zero diagrams, the minimum phase filter does not have any zeros outside U.C. after reflecting the zeros.

The IIR filter's phase response is non-linear according to the unwrapped phase plot while the FIR is a kind of linear phase filter (its minimum phase filter is nearly linear).

In all the 6 images filtered by different low-pass filters, their details are blurred, we now cannot distinct the graduations on the watch easily. As we can see in the plots, all three filtered images without Forwards-Backwards Filtering have ringing effect on the left side of the image and near the vertical edges to different extents. Besides, they all have delay effect to different extents, which leave blacks edges on the left side. Among them, the image filtered by FIR filter has the most obvious delay effect.

After Forwards-Backwards Filtering is implemented in three cases, both the ringing and delay effects are suppressed (but still, there are some influence remaining).