Digital Signal Processing HW4 MATLAB Part

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Consider a causal discrete-time LTI system implemented using the difference equation,

$$y(n) = 0.1\,x(n) - 0.12\,x(n-1) + 0.1\,x(n-2) + 1.7\,y(n-1) - 0.8\,y(n-2)$$

The frequency response of the system is denoted $H^f(\omega)$.

1. What is the transfer function $H(z)$ of the system? (Not a Matlab question!)

Solution:

$$y(n) = 0.1x(n) - 0.12x(n-1) + 0.1x(n-2) + 1.7y(n-1) - 0.8y(n-2)$$

$$\therefore Y(z) = 0.1X(z) - 0.12X(z)z^{-1} + 0.1X(z)z^{-2} + 1.7Y(z)z^{-1} - 0.8Y(z)z^{-2}$$

$$Y(z)(1-1.7z^{-1}+0.8z^{-2}) = X(z)(0.1-0.12z^{-1}+0.1z^{-2})$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{0.1-0.12z^{-1}+0.1z^{-2}}{1-1.7z^{-1}+0.8z^{-2}}$$

2. Plot the magnitude of the frequency response $H^f(\omega)$ of the system using the Matlab function `freqz`:

```
>> [H,om] = freqz(b,a);
>> plot(om,abs(H));
```

where `b` and `a` are appropriately defined.

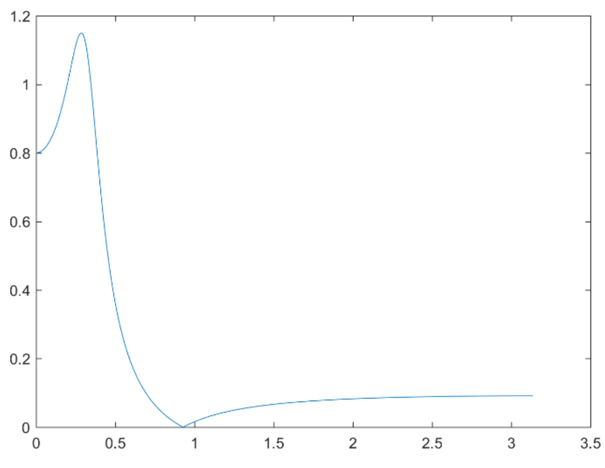
Solution:

.m file(s): jyz_HW4_2.m

Code:

```
a = [1 -1.7 0.8];
b = [0.1 -0.12 0.1];
[H, om] = freqz(b, a);
plot(om, abs(H));
```

Result(plot):



3. When the input signal is

$$x(n) = \cos(0.1\pi n)u(n) \tag{1}$$

find the output signal $y(n)$ for $-10 \le n \le 100$ using `filter`. Make stem plots of the input and output signals. (Use subplot in Matlab.) Comment on your observations.

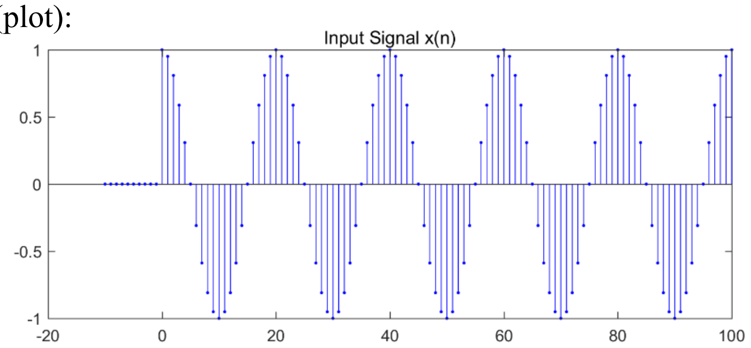
Solution:

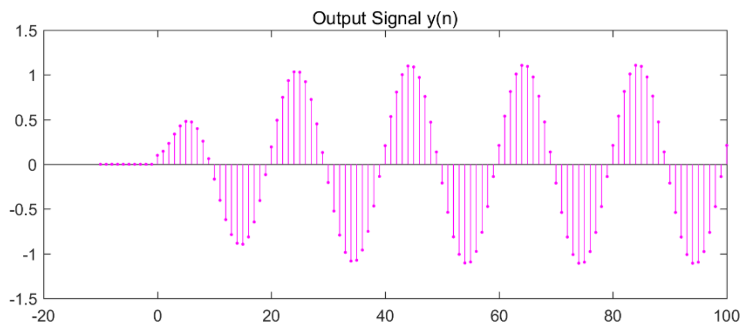
.m file(s): jyz_HW4_3.m

Code:

```
clear
close all

u = @(n) (n >= 0); %Function

a = [1 -1.7 0.8];
b = [0.1 -0.12 0.1];
n = -10 : 100;
x = cos(0.1 * pi * n).* u(n);
y = filter(b, a, x);
subplot(2,1,1)
stem(n, x, 'b.')
title('Input Signal x(n)')
subplot(2,1,2)
stem(n, y, 'm.')
title('Output Signal y(n)')
```

Result(plot):





Comment: At first, the output signal is less than the steady output signal in scale. After the transient response decays to zero, the steady output signal remains. And there is a phase change between the input and output signals.

4. Find the exact value of $H^f(\omega)$ at $\omega = 0.1\pi$. First, express $H^f(0.1\pi)$ as

$$H^f(0.1\pi) = \frac{B(\mathrm{e}^{\mathrm{j}0.1\pi})}{A(\mathrm{e}^{\mathrm{j}0.1\pi})}$$

where $B(z)$ and $A(z)$ are polynomials. Second, use `exp` to find the complex number $z = \mathrm{e}^{\mathrm{j}0.1\pi}$. Third, evaluate $B(z)$ and $A(z)$ at the complex value $z = \mathrm{e}^{\mathrm{j}0.1\pi}$ using `polyval` in Matlab.

Solution:

$$H^f(\omega) = H(\mathrm{z})\big|_{\mathrm{z}=\mathrm{e}^{\mathrm{j}\omega}}$$

$$H(\mathrm{z}) = \frac{0.1\mathrm{z}^2 - 0.12\mathrm{z} + 0.1}{\mathrm{z}^2 - 1.7\mathrm{z} + 0.8}$$

$$H^f(\omega) = \frac{0.1e^{j2\omega} - 0.12e^{j\omega} + 0.1}{1e^{j2\omega} - 1.7e^{j\omega} + 0.8}$$

$$H^f(0.1\pi) = \frac{0.1e^{j0.2\pi} - 0.12e^{j0.1\pi} + 0.1}{e^{j0.2\pi} - 1.7e^{j0.1\pi} + 0.8}$$

.m file(s): jyz_HW4_4.m

Code:

```
clear
close all

a = [1 -1.7 0.8];
b = [0.1 -0.12 0.1];

z = exp(i*0.1*pi);
Hf = polyval(b, z)/polyval(a, z);
disp(Hf)
```

Result:

$$H^f(0.1\pi) = 0.2109 - 1.0954\mathrm{j}$$

```
>> jyz_HW4_4
  0.2109 - 1.0954i
```

5. Recall that when the impulse response h is real,

$$\cos(\omega_o n) \longrightarrow \boxed{h(n)} \longrightarrow |H^f(\omega_o)| \cos\big(\omega_o n + \angle H^f(\omega_o)\big)$$

Note this input signal starts at $n = -\infty$. (There is no $u(n)$ term.) However, we are interested here in the case where the input signal starts at $n = 0$, namely the input given by Equation (1). In this case, we have

$$\cos(\omega_o n)u(n) \longrightarrow \boxed{h(n)} \longrightarrow |H^f(\omega_o)| \cos\big(\omega_o n + \angle H^f(\omega_o)\big)u(n) \ + \ \text{Transients}$$

The transients decay to zero, and the *steady-state* output signal is simply

$$s(n) = |H^f(\omega_o)| \cos\big(\omega_o n + \angle H^f(\omega_o)\big)u(n). \tag{2}$$

In Matlab, use `abs` and `angle` to compute $|H^f(0.1\pi)|$ and $\angle H^f(0.1\pi)$. Create a stem plot of the steady-state output signal

$$s(n) = |H^f(0.1\pi)| \cos(0.1\pi n + \angle H^f(0.1\pi)).$$

Compare your plot of $s(n)$ with your plot of $y(n)$ obtained using `filter`. You can plot both on the same graph using `plot(n,y,n,s)`. You should find that $s(n)$ and $y(n)$ agree after a while. After the transient response decays to zero, the steady-state output signal $s(n)$ remains.

Solution:

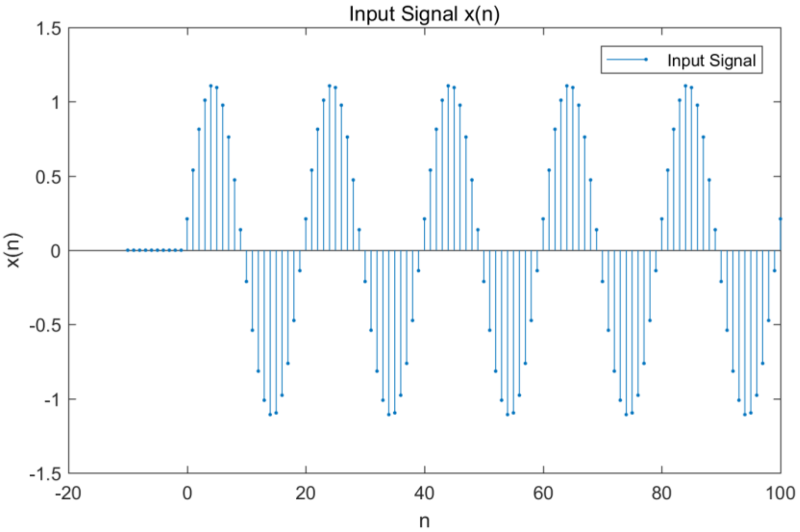
.m file(s): jyz_HW4_5.m

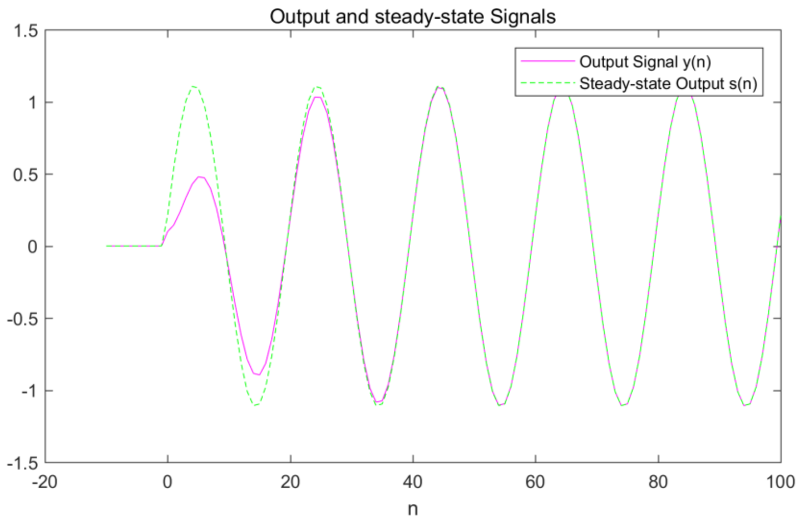
Code:

```
clear
close all

u = @(n) (n >= 0); %Function

a = [1 -1.7 0.8];
b = [0.1 -0.12 0.1];

n = -10:100;
x = cos(0.1 * pi * n).*u(n);
y = filter(b,a,x);
z = exp(i*0.1*pi);
Hf = polyval(b, z)/polyval(a, z);
Hfabs = abs(Hf);
Hfangle = angle(Hf);
s = Hfabs * cos(0.1*pi.*n+Hfangle).*u(n);
figure(1)
stem(n,s,'.')
xlabel('n')
ylabel('x(n)')
title('Input Signal x(n)')
```

```
legend('Input Signal')
figure(2)
plot(n,y,'m',n,s,'g--')
xlabel('n')
title('Output and steady-state Signals')
legend('Output Signal y(n)', 'Steady-state Output s(n)')
```

Result(Plots):





Comment: As we can see in the plots, approximately after n = 40, when the transient response decays to zero, y(n) remains the same as the steady-state output signal s(n).

6. When the input signal is

$$x(n) = \cos(0.3\pi n)u(n)$$

find the output signal $y(n)$ for $-10 \leq n \leq 100$ using `filter`. How could $y(n)$ be predicted from the frequency response of the system? What is the steady-state signal? Relate your explanation to the concept described in the previous question.

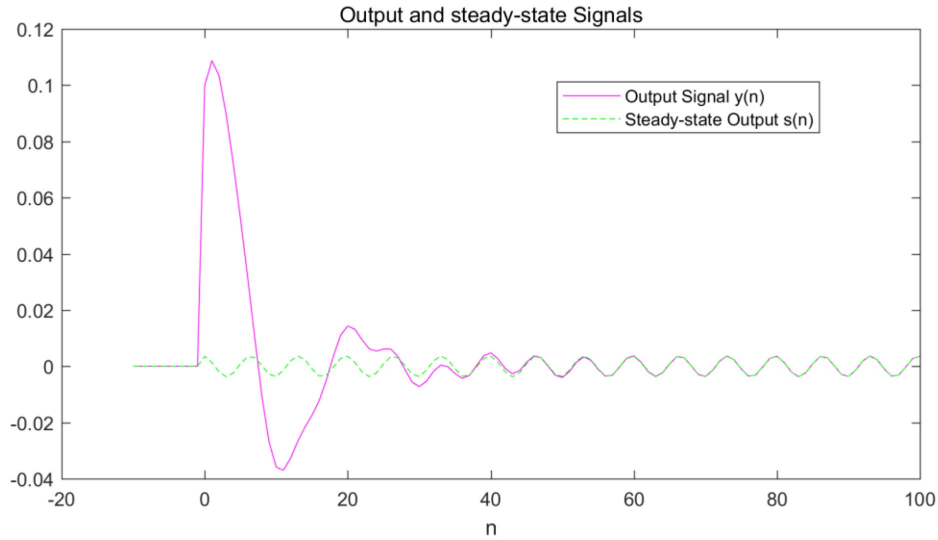
Solution(prediction): As we can see in the plot of problem 2, the magnitude of frequency response is nearly zero when $\omega = 0.3\pi$, which means the output signal would be considerably small in scale, when the transient response decays to zero, y(n) would remain the same as the steady-state output signal s(n). Besides, there will be a phase delay of $\angle H^f(0.3\pi)$ between the input and output signals.

.m file(s): jyz_HW4_6.m

Code:

```
clear
close all

u = @(n) (n >= 0); %Function

a = [1 -1.7 0.8];
b = [0.1 -0.12 0.1];
n = -10:100;
x = cos(0.3 * pi * n).*u(n);
y = filter(b,a,x);
z = exp(i*0.3*pi);
Hf = polyval(b, z)/polyval(a, z);
Hfabs = abs(Hf);
Hfangle = angle(Hf);
s = Hfabs * cos(0.3*pi.*n+Hfangle).*u(n);
plot(n,y,'m',n,s,'g--')
xlabel('n')
title('Output and steady-state Signals')
legend('Output Signal y(n)', 'Steady-state Output s(n)')
```

Result(Plots):



Comment: The output signal y(n) and the steady-state output signal s(n) generally consistent with the prediction.

7. Plot the poles and zeros of the transfer function using `zplane(b, a)` in Matlab. The shape of the frequency response magnitude $|H^f(\omega)|$ can be predicted from the pole-zero diagram.

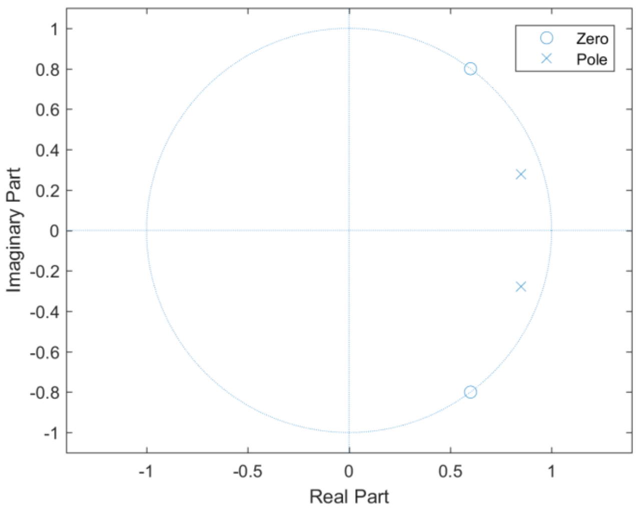
Solution:

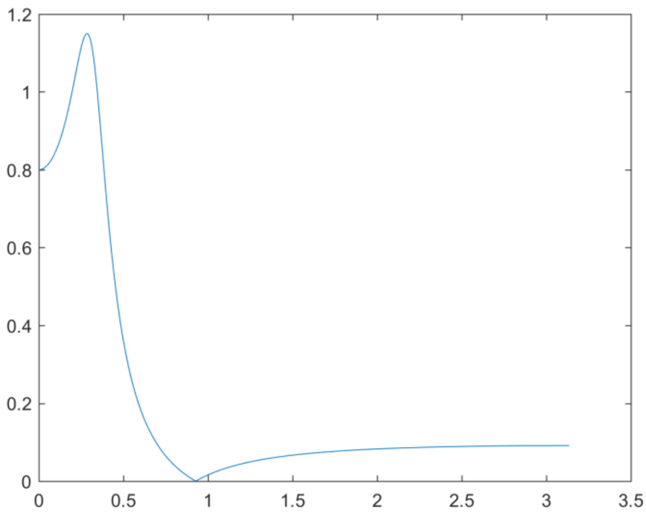
.m file(s): .m file(s): jyz_HW4_7.m

Code:

```
clear
close all

a = [1 -1.7 0.8];
b = [0.1 -0.12 0.1];

figure(1)
zplane(b,a,'g')
legend('Zero', 'Pole')
[H, om] = freqz(b, a);
figure(2)
plot(om, abs(H));
```

Result(plots):





Comment: In this problem, pole's angle is less than zero's angle, two zero points sits on the unit circle. From $\omega = 0$ to $\omega = \angle$pole, the frequency response magnitude $|H^f(\omega)|$ increases with ω, from $\omega = \angle$pole to $\omega = \angle$zero, $|H^f(\omega)|$ decreases with ω until it becomes 0, from $\omega = \angle$zero to $\omega = \pi$, $|H^f(\omega)|$ increase with ω.